Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 31 No. 2 (2024)

Coincidence Point Results for a New Class of Fuzzy Contractive Mappings

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Article History:

Received: *22-01-2024*

Revised: *28-03-2024*

Accepted: *20-04-2024*

Abstract:

This work is mainly aimed to enhance the concept of fuzzy $\mathcal{Z}$ -contractive mapping [11] by establish a different contraction called fuzzy $(\mathcal{Z}, g)$ - contraction. we obtain some sufficient conditions for the existence and uniqueness of point of coincidence of fuzzy $(\mathcal{Z}, g)$ - contraction mappings in the context of fuzzy metric spaces and prove some intriguing results.

Keywords: Fixed point, M-completeness, Fuzzy metric space, Fuzzy Z-contractive mapping. 2010 Mathematics Subject Classification: 54H25; 47H10

1. Introduction

In light of its numerous uses across a variety of fields, the analysis of fixed points of mappings following contraction conditions has become the focus of numerous research initiatives. Regarding this, the classical findings of Banach [1] and Edelstien [2] have served as the inspiration for several writers working in the field of metric fixed-point theory.

Grabiec [4], who pioneered the introduction of fixed-point theory in fuzzy metric spaces, extended the discoveries of Banach [1] and Edelstein [2] in the framework of fuzzy metric spaces in the design of

Kramosil and Michlek [8]. The fixed-point conclusion proposed by Grabiec in his work [4] was based on the notion of the -completeness of fuzzy metric spaces, which George and Veeramani [3] weakened by introducing. Many authors explored and examined various contractions in fuzzy metric spaces, and they came up with insightful results. (See [5], [6], [9], [12], [13]).
In order to integrate various types of fuzzy contractive mappings, Shukla et al. [11] recently introduced fuzzy $Z-$contractive mappings. They demonstrated that this new type is more thorough than the well-known earlier ones and properly incorporates the classes that were proposed by numerous researchers ([5], [9], [12], and [13]) in fuzzy metric spaces in the sense of George and Veeramani [3].
By expanding on the work of Shukla et al. [11], we introduce the concept of fuzzy $(\mathrm{Z}, g)-$contraction in this paper. In the context of fuzzy metric spaces, we also develop certain discoveries regarding the presence and uniqueness of coincidence points for such contractions.

2. Preliminaries

Let us recall some prefaces here:

Definition 2.1. [10] A continuous t-norm is a binary operation $*:[0,1]\times[0,1]\to[0,1]$ which satisfies the following conditions $\forall l,p,r,t\in[0,1]$

(1) $l*1=l$

(2) $l*p=p*l$

(3) $l*p\le r*t$ whenever $l\le r\ \&\ p\le t$

(4) $l*(p*r)=(l*p)*r$

Definition 2.2. [3] A fuzzy metric space is a 3-tuple $(X,\mathrm{M},*)$ where X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set in $X\times X\times(0,\infty)$, satisfying the following conditions:

(1) $\mathrm{M}(a,b,t)>0$,

(2) $\mathrm{M}(a,b,t)=1,\ \forall t>0\Leftrightarrow a=b$,

(3) $\mathrm{M}(a,b,t)=\mathrm{M}(b,a,t)$,

(4) $\mathrm{M}(a,c,t+r)\le\mathrm{M}(a,b,t)*\mathrm{M}(b,c,r)$,

(5) $\mathrm{M}(a,b,\cdot):(0,\infty)\to(0,1]$ is a continuous function, $\forall a,b,c\in X\ \&\ t,r>0$.

Lemma 2.3. [4] $\mathrm{M}(a,b,\cdot)$ is nondecreasing $\forall a,b\in X$.

Definition 2.4. [3] Let $(X,\mathrm{M},*)$ be a fuzzy metric space.

A sequence $\{s_q\}$ in X is a M-Cauchy sequence, if $\forall\varepsilon\in(0,1)\ \&\ t>0$, there exists $q_0\in N$ such that $\mathrm{M}(s_p,s_q,t)>1-\varepsilon,\ \forall\ p\ge q_0$.

A sequence $\{s_q\}$ in X is convergent to $x\in X$ if $\lim_{q\to\infty}\mathrm{M}(s_q,x,t)=1$ for each $t>0$.

If every $\mathrm{M}-$Cauchy sequence in X is convergent, then such fuzzy metric space X is said to be $\mathrm{M}-$complete.

Definition 2.5. [7] On a fuzzy metric space $(X, \mathrm{M}, *)$, two self-mappings ω & ξ are said to be compatible if $\lim_{q\to\infty} \mathrm{M}(\omega\xi x_q, \xi\omega x_q, t) = 1$ for each $t > 0$, whenever $\{x_q\}$ is a sequence in X ∍ $\lim_{q\to\infty} \omega x_q = \lim_{q\to\infty} \xi x_q = x \in X$.

Definition 2.6. [7] Two self-maps ω & ξ of a fuzzy metric space $(X, \mathrm{M}, *)$ are said to be weakly compatible if they commute at their coincidence points; $\omega x = \xi x$ for some $x \in X$ implies that $\omega\xi x = \xi\omega x$.

Remark 2.7. Let ω & ξ be weakly compatible self-maps of a set X. If ω & ξ have a unique point of coincidence $w = \omega x = \xi x$, then w is the unique common fixed point of ω & ξ.

Definition 2.8. [11] Let the family of all functions $\chi : (0,1] \times (0,1] \to \mathbb{R}$ satisfying,

$$\chi(t, r) > r, \ \forall t, r \in (0,1)$$

be denoted by Z.

Remark 2.9. [11] From the above definition, we can see that $\chi(t,t) > t, \ \forall t \in (0,1)$.

Example 2.10. [11] Consider the following functions $\chi : (0,1] \times (0,1] \to \mathbb{R}$ by:

(1) $\chi(p, q) = \psi(q)$, where $\psi : (0,1] \to (0,1]$ is a function such that $q < \psi(q), \ \forall q \in (0,1)$,

(2) $\chi(p, q) = \dfrac{1}{q + p} + p$,

(3) $\chi(p, q) = \dfrac{q}{p}$.

Then, in all the cases $\chi \in \mathrm{Z}$.

Definition 2.11. [11] Let $\kappa : X \to X$ be a self - mapping on a fuzzy metric space $(X, \mathrm{M}, *)$. Suppose, $\exists \chi \in \mathrm{Z}$ such that,

$$\mathrm{M}(\kappa x, \kappa y, t) \geq \chi(\mathrm{M}(\kappa x, \kappa y, t), \mathrm{M}(x, y, t)) \tag{2.1}$$

for all $x, y \in X$, $\kappa x = \kappa y$, $t > 0$. Then κ is called a fuzzy $\mathrm{Z}-$contractive mapping with respect to $\chi \in \mathrm{Z}$.

Remark 2.12. Even in the $\mathrm{M}-$complete fuzzy metric space, a fuzzy $\mathrm{Z}-$contractive mapping may not have a fixed point. (See example 3.10. of [11])

S. Shukla et al. [11] considered a space with the additional characteristic listed below to guarantee the existence of the fixed point of a fuzzy $\mathrm{Z}-$contractive mapping.

Definition 2.13. [11] Let $\kappa : X \to X$ be a mapping in a fuzzy metric space $(X, \mathrm{M}, *)$ and $\chi \in \mathrm{Z}$. Then, we say that the quadruple $(X, \mathrm{M}, \kappa, \chi)$ has the property(S), if for any Picard sequence $\{s_q\}$ with initial value $x \in X$, *i.e.*, $s_q = \kappa^q x, \ \forall q \in N$ such that $\inf_{p>q} \mathrm{M}(s_q, s_p, t) \leq \inf_{p>q} \mathrm{M}(s_{q+1}, s_{p+1}, t), \ \forall q \in N, \ t > 0$ implies that

$$\lim_{q\to\infty} \inf_{p>q} \chi(\mathrm{M}(s_{q+1}, s_{p+1}, t), \mathrm{M}(s_q, s_p, t)) = 1 \ \forall t > 0. \tag{2.2}$$

Theorem 2.14. [11] Let $\kappa : X \to X$ be a fuzzy $\mathrm{Z}-$contraction on an $\mathrm{M}-$complete fuzzy metric space $(X, \mathrm{M}, *)$. If the quadruple $(X, \mathrm{M}, \kappa, \chi)$ has the property (S), then f has a unique fixed point $z \in X$.

3. Main Results

Definition 3.1. Let $\kappa \& \upsilon$ be two self-mappings on a fuzzy metric space $(X, \mathrm{M}, *)$. Then κ is called a fuzzy $(\mathrm{Z}, \upsilon)-$contraction if $\exists \chi \in \mathrm{Z}$ such that

$$\mathrm{M}(\kappa x, \kappa y, t) \geq \chi\left(\mathrm{M}(\kappa x, \kappa y, t), \mathrm{M}(\upsilon x, \upsilon y, t)\right) \tag{3.1}$$

for all $x, y \in X, \upsilon x \neq \upsilon y, t > 0$.

Remark 3.2. If $\upsilon = I$ (identity mapping) in (3.1), then we get fuzzy $\mathrm{Z}-$contraction (Definition 2.11).

Now, we state our result for the notion of fuzzy $(\mathrm{Z}, \upsilon)-$contraction.

Theorem 3.3. Let $(X, \mathrm{M}, *)$ be a fuzzy metric space and $\kappa, \upsilon : X \to X$ be self-mappings. Let κ be a fuzzy $(\mathrm{Z}, \upsilon)-$contraction and the quadruple $(X, \mathrm{M}, \kappa, \chi)$ has the property (S). Also assume that, at least one of the following conditions hold:

(i) $\kappa(X)$ *or* $\upsilon(X)$ is complete.

(ii) X is complete, υ is continuous and κ, υ are commuting.

(iii) X is complete, υ is continuous and κ, υ are compatible. Then κ & υ have unique point of coincidence.

Proof. Firstly, we shall show that the point of coincidence of $\kappa \& \upsilon$, if exists is unique. Let us suppose that z_1 & z_2 are distinct points of coincidence of $\kappa \& \upsilon$ which follows that there exist two points w_1 & $w_2 (w_1 = w_2)$ such that $\kappa w_1 = \upsilon w_1 = z_1$ and $\kappa w_2 = \upsilon w_2 = z_2$.

In view of (3.1) and the property of χ, we obtain

$$\begin{aligned} \mathrm{M}(z_1, z_2, t) = \mathrm{M}(\kappa w_1, \kappa w_2, t) &\geq \varsigma\left(\mathrm{M}(\kappa w_1, \kappa w_2, t), \mathrm{M}(\upsilon w_1, \upsilon w_2, t)\right) \\ &> \mathrm{M}(\upsilon w_1, \upsilon w_2, t) = \mathrm{M}(z_1, z_2, t) \end{aligned} \tag{3.2}$$

which is a contradiction.

Let us consider a sequence $\{y_q\}$ such that $y_q = \kappa x_q = \upsilon x_{q+1}$ where $q \in N \bigcup \{0\}$.

If $y_{q_0} = y_{q_0+1}$ for any $q_0 \in N \cup \{0\}$, then $\upsilon x_{q_0+1} = y_{q_0} = y_{q_0} + 1 = \kappa x_{q_0+1}$ which shows that $\kappa \& \upsilon$ have a point of coincidence. Thus, we assume that $y_q = y_{q+1}$ for all $q \in N \cup \{0\}$.

Consider

$$\begin{aligned} \mathrm{M}\left(y_{q+1}, y_{q+2}, t\right) = \mathrm{M}\left(\kappa x_{q+1}, \kappa x_{q+2}, t\right) &\geq \chi\left(\mathrm{M}\left(\kappa x_{q+1}, \kappa x_{q+2}, t\right), \mathrm{M}\left(\upsilon x_{q+1}, \upsilon x_{q+2}, t\right)\right) \\ &> \mathrm{M}(\upsilon x_{q+1}, \upsilon x_{q+2}, t) \\ &= \mathrm{M}\left(y_q, y_{q+1}, t\right) \end{aligned} \tag{3.3}$$

which gives $\quad \mathrm{M}\left(y_q, y_{q+1}, t\right) < \mathrm{M}\left(y_{q+1}, y_{q+2}, t\right) \quad \forall t > 0.$

If $y_q = y_p$ for some $q < p$, then we have $y_{q+1} = \kappa x_{q+1} = \kappa x_{p+1} = y_{p+1}$.

Now using (3.1) and (3.3), one can get

$$\mathrm{M}\left(y_q, y_{q+1}, t\right) < \mathrm{M}(y_p, y_{p+1}, t) = \mathrm{M}\left(\mathrm{M}\left(y_q, y_{q+1}, t\right)\right), \quad \forall t > 0.$$

which is a contradiction. Thus, we assume that $y_q = y_p$ for all distinct $p, q \in N$.

Now we prove $\{y_q\}$ is a Cauchy sequence:

For $t > 0$, let us suppose that $a_q(t) = \inf_{p>q} \mathrm{M}(y_p, y_p, t)$.

Consider

$$\begin{aligned}\mathrm{M}\left(y_{q+1}, y_{p+1}, t\right) &= \mathrm{M}(\kappa x_{q+1}, \kappa x_{p+1}, t)\\ &\geq \chi\left(\mathrm{M}\left(\kappa x_{q+1}, \kappa x_{p+1}, t\right), \mathrm{M}\left(\upsilon x_{q+1}, \upsilon x_{p+1}, t\right)\right)\\ &> \mathrm{M}\left(\upsilon x_{q+1}, \upsilon x_{p+1}, t\right) = \mathrm{M}(y_q, y_p, t), \quad \forall t > 0. \end{aligned} \tag{3.4}$$

Thus, $\mathrm{M}\left(y_q, y_p, t\right) < \mathrm{M}\left(y_{q+1}, y_{p+1}, t\right), \ \forall q < p.$

Taking infimum over all $p(> q)$ in the above inequality, we get

$$\inf_{p>q} \mathrm{M}\left(y_q, y_p, t\right) \leq \inf_{p>q} \mathrm{M}(y_{q+1}, y_{p+1}, t)$$

i.e., $a_q(t) \leq a_{q+1}(t), \ \forall q \in N.$

Thus, $\{a_q(t)\}$ is monotonic and bounded for each $t > 0$.

We prove $\lim_{q\to\infty} a_q(t) = 1, \ \forall t > 0$. On the contrary, let us suppose that $\exists$ some t_0, where $0 < t_0 < t$ such that $\lim_{q\to\infty} a_q(t_0) = a(t_0) < 1$. For such t_0, using the fact that the quadruple $(X, \mathrm{M}, \kappa, \chi)$ have the property (S) $\left(\text{Since}\, y_q = q = \gamma^q(x)\right)$, we obtain

$$\lim_{q\to\infty} \inf_{p>q} \chi\left(\mathrm{M}\left(y_{q+1}, y_{p+1}, t_0\right), \mathrm{M}\left(y_q, y_p, t_0\right)\right) = 1 \tag{3.5}$$

$$\inf_{p>q} \mathrm{M}\left(y_{q+1}, y_{p+1}, t_0\right) \geq \inf_{p>q} \chi\left(\mathrm{M}\left(\kappa x_{q+1}, \kappa x_{p+1}, t_0\right), \mathrm{M}\left(\upsilon x_{q+1}, \upsilon x_{p+1}, t_0\right)\right) \geq \varepsilon_{p>q} \mathrm{M}\left(y_q, y_p, t_0\right)$$

$$\Rightarrow \inf_{p>q} \mathrm{M}\left(y_{q+1}, y_{p+1}, t_0\right) \geq \inf_{p>q} \chi\left(\mathrm{M}\left(y_{q+1}, y_{p+1}, t_0\right), \mathrm{M}\left(y_q, y_p, t_0\right)\right) \geq \inf_{p>q} \mathrm{M}\left(y_q, y_p, t_0\right)$$

$$i.e., \quad a_{q+1}(t_0) \geq \inf_{p>q} \chi\left(\mathrm{M}\left(y_{q+1}, y_{p+1}, t_0\right), \mathrm{M}\left(y_q, y_p, t_0\right)\right) \geq a_q(t_0)$$

Letting $q \to \infty$ in the above inequality and using (3.5), we get $a(t_0) = 1$ which is a contradiction to our assumption. Thus,

$$1\lim_{q\to\infty} \inf_{p>q} \mathrm{M}(y_q, y_p, t) = 1, \ \forall t > 0.$$

Hence from the definition of a_q, we get

$$1\lim_{p,q\to\infty} \mathrm{M}(y_q, y_p, t) = 1, \ \forall t > 0. \tag{3.6}$$

which shows $\{y_q\}$ is an $\mathrm{M}-$Cauchy sequence.

Suppose that (i) holds: *i.e.*, $\upsilon(X)$ is complete.

Then since $\{y_q\}$ is in $\upsilon(X)$, it has a limit in $\upsilon(X)$. Let $y_q \to \upsilon u$ *i.e.*, $\upsilon x_q \to \upsilon u$ as $q \to \infty$ for $u \in X$. Now, we shall prove that u is the coincidence point of κ and υ *i.e.*, $\kappa u = \upsilon u$.

It is clear that, we can suppose $y_q \neq \kappa u, \upsilon u$ for all $n \in N \bigcup \{0\}$.

In view of (3.1) and the property of χ, we have

$$\mathrm{M}\left(\upsilon x_q, \upsilon u, t\right) < \chi\left(\mathrm{M}\left(\kappa x_q, \kappa u, t\right), \mathrm{M}\left(\upsilon x_q, \upsilon u, t\right)\right) \leq \mathrm{M}(\kappa x_q, \kappa u, t)$$

Letting $q \to \infty$ in the above inequality and considering the extremities gives

$$\mathrm{M}\left(\upsilon u, \upsilon u, t\right) \leq \lim_{q\to\infty} \mathrm{M}\left(\kappa x_n, \kappa u, t\right) \Rightarrow 1 \leq \lim_{q\to\infty} \mathrm{M}\left(\kappa x_q, \kappa u, t\right).$$

Thus, $\kappa x_q \to \kappa u$ as $q \to \infty$ which implies $y_q \to ku$ as $q \to \infty$.

Since limit is unique, we obtain $\kappa u = \upsilon u$. Hence, $\kappa u = \upsilon u$ is a (unique) point of coincidence of κ & υ.

Similarly, we can show that $\kappa u = \upsilon u$ is a (unique) point of coincidence of κ & υ when $\kappa(X)$ is complete.

Suppose that (ii) holds:

Since X is complete, there exists $u \in X$ such that $y_q \to u \Rightarrow \upsilon x_q \to u$ as $q \to \infty$. Since υ is continuous, we have $\upsilon^2 x_q \to \upsilon$ as $q \to \infty$.

In view of (3.1) and the property of χ, we have

$$\mathrm{M}\left(\upsilon\left(\upsilon x_q\right), \upsilon u, t\right) < \chi\left(\mathrm{M}\left(\kappa\left(\upsilon x_q\right), \kappa u, t\right), \mathrm{M}\left(\upsilon\left(\upsilon x_q\right), \upsilon u, t\right)\right) \leq \mathrm{M}\left(\kappa\left(\upsilon x_q\right), \kappa u, t\right)$$

Letting $q \to \infty$ in the above inequality and considering the extremities gives

$$\mathrm{M}\left(\upsilon u, \upsilon u, t\right) \leq \lim_{q\to\infty} \mathrm{M}\left(\kappa\upsilon x_q, \kappa u,\right) t = \lim_{q\to\infty} \mathrm{M}\left(\upsilon\kappa x_q, \kappa u, t\right) \Rightarrow 1 \leq \lim_{q\to\infty} \mathrm{M}\left(\upsilon\kappa x_n, \kappa u, t\right).$$

Thus, $\upsilon\kappa x_q \to \kappa u$ as $q \to \infty$. But from the definition of y_q, we get $\upsilon\kappa x_q = \upsilon\upsilon x_{q+1} = \upsilon^2 x_{q+1}$ from which the result follows *i.e.*, $\kappa u = \upsilon u$ is a (unique) point of coincidence of κ & υ.

Suppose that (iii) holds:

Since X is complete, there exists $u \in X$ such that $y_q \to u \Rightarrow \kappa x_q \to u$ as $q \to \infty$. Since ν is continuous, we have $\upsilon\kappa x_q \to \upsilon u$ as $q \to \infty$.

In view of (3.1) and the property of χ and $\upsilon\kappa x_q = \upsilon\upsilon x_{q+1} = \upsilon^2 x_{q+1}$, we have

$$\mathrm{M}\left(\upsilon\kappa x_{q-1}, \upsilon u, t\right) = \mathrm{M}\left(\upsilon\left(\upsilon x_q\right), \upsilon u, t\right) < \chi\left(\mathrm{M}\left(\kappa\left(\upsilon x_q\right), \kappa u, t\right), \mathrm{M}\left(\upsilon\left(\upsilon x_q\right), \upsilon u, t\right)\right) \leq \mathrm{M}\left(\kappa\left(\upsilon x_q\right), \upsilon u, t\right)$$

Letting $q \to \infty$ in the above inequality and considering the extremities gives

$$\mathrm{M}\left(\upsilon u, \upsilon u, t\right) \leq \lim_{q\to\infty} \mathrm{M}\left(\kappa\upsilon x_q, \kappa u, t\right) \Rightarrow 1 \leq \lim_{q\to\infty} \mathrm{M}\left(\kappa\upsilon x_q, \kappa u, t\right).$$

Thus $\kappa\upsilon x_q \to \kappa u$ as $q \to \infty$.

Since κ, υ are compatible, we have $\lim_{q\to\infty} \mathrm{M}\left(\kappa\upsilon x_q, \upsilon\kappa x_q, t\right) = 1,\ t > 0$.

Consider $\mathrm{M}\left(\kappa u, \upsilon u, t\right) \leq \mathrm{M}\left(\kappa u, \kappa\upsilon x_q, \frac{t}{3}\right) * \mathrm{M}\left(\kappa\upsilon x_n, \upsilon\kappa x_q, \frac{t}{3}\right) * \mathrm{M}\left(\upsilon\kappa x_q, \upsilon u, \frac{t}{3}\right)$

Letting $q \to \infty$ in the above inequality gives $\kappa u = \upsilon u$ thus showing that $\kappa u = \upsilon u$ is a (unique) point of coincidence of $\kappa \& \upsilon$.

Hence, the result is proved for all three cases (i), (ii) and (iii), *i.e.*, the mappings $\kappa \& \upsilon$ have a unique point of coincidence.

Theorem 3.4. Besides the hypotheses of Theorem 3.3, if $\kappa \& \upsilon$ are weakly compatible, then they have a unique common fixed point in X.

Proof. Using Theorem 3.3, $\kappa \& \upsilon$ have unique point of coincidence. Further, if $\kappa \& \upsilon$ are weakly compatible, then according to Remark 2.7, they have a unique common fixed point in X.

Example 3.5. Let $X = [0,1]$ and M be the fuzzy set on $X \times X \times (0,\infty)$ defined by $\mathrm{M}(x,y,t) = \frac{t}{t+|x-y|}$ and $*$ is minimum norm. Then, $(X,\mathrm{M},*)$ is an complete fuzzy metric space. Let $\{x_q\}$ be a sequence given by $\forall q \in N$. Consider the function $\chi : (0,1] \times (0,1] \to \mathbb{R}$ by $\chi(s,t) = \frac{s}{t} \quad \forall t,s \in (0,1]$. Let κ, υ be given by

$$\kappa x = \begin{cases} 0 & if \;\; x \in \left[0,\frac{1}{4}\right] \\ \frac{7}{20} & if \;\; x \in \left(\frac{1}{4},\frac{1}{2}\right) \\ \frac{4}{25} & if \;\; x \in \left[\frac{1}{2},1\right] \end{cases} \quad \text{and} \quad \upsilon x = \begin{cases} \frac{x}{3} & if \;\; x \in \left[0,\frac{1}{4}\right] \\ \frac{2}{5} & if \;\; x \in \left(\frac{1}{4},\frac{1}{2}\right) \\ \frac{2}{3} & if \;\; x \in \left[\frac{1}{2},1\right] \end{cases}$$

Then $\chi \in \mathrm{Z}$ and the quadruple $(X,\mathrm{M},\kappa,\chi)$ has the property (S). Furthermore, the mapping κ is a fuzzy $(\mathrm{Z},\upsilon)-$contractive mapping with respect to the function χ. Thus, all the conditions of Theorem (3.4) are satisfied *i.e.*, the mappings κ and υ have a coincidence point $x = 0$. On the other word, they have a unique common fixed point *i.e.*, at $x = 0$.

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