Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 9s (2025)

Dual Hesitant Fuzzy Set based Knowledge and Accuracy Measure with its Application to Power Crisis and Pattern Detection

[1]Alisha Aggarwal, [2]Gurdas Ram, [3]Anirudh, [4]Ravinder

[1,2,3,4]Department of Mathematics, Maharishi Markandeshwar (Deemed to be University), Mullana-Ambala, 133207, Haryana (INDIA)

[1]aggarwalalisha95@gmail.com,[2]gurdasdadwal@mmumullana.org,
[3]anirudhkundu95@gmail.com,[3]ravindersheoran103@gmail.com

Article History:

Received: 12-01-2025

Revised: 15-02-2025

Accepted: 01-03-2025

Abstract:
Fuzzy sets have proven useful in the investigation of unclear phenomena. A number of researchers suggested intuitionistic fuzzy sets (IFSs) and hesitant fuzzy sets (HFSs) as an extension of fuzzy sets (FSs), and these sets have been applied in various contexts. The study of Dual hesitant fuzzy sets contains two type of hesitancy function one is membership and other is non-membership, they carry out the hesitation scenario and provides an adequate way to provide values corresponding to each element present in domain. The FSs, IFSs and HFSs as special cases are all included in the DHFSs. Compared with IFS and HFS, DHFS is more advantageous in dealing with multiple attribute decision making problems. In this paper, a new knowledge and accuracy measure for DHFS has been proposed. The main motive of this paper is to investigate knowledge and accuracy measures for DHFSs and to compare the performance of a proposed knowledge and accuracy measure in the DHF environment with other current measures. We also demonstrate the application of knowledge measure and accuracy measure that we have developed to tackle the problem of power crisis in a developing country. We demonstrate how our suggested accuracy measure of DHFSs outperforms certain similarity and distance measurements.

Keywords: Dual hesitant fuzzy set, Knowledge measure, Accuracy measure, Multi attribute decision making.

1. Introduction

Since uncertainty exists in practically every aspect of our everyday lives, numerous technologies have been created to identify and address it. Researchers have created a number of useful instruments and approaches to deal with ambiguity and imprecision in practical settings, but there are still a number of circumstances in which people are apprehensive or suspicious for various reasons. There is a great deal of vagueness and uncertainty in these issues. Sometimes hesitation and uncertainty can enforce even a simple situation to become too complicated. There are several ways to express uncertainty, including unpredictability, fuzziness, incompleteness, etc. The first fuzzy theory to address uncertainty issues in a variety of domains was proposed by Zadeh [1]. However, it is limited in how it can handle unclear data. Accordingly, traditional FS has minimal limitations. Several fuzzy set extensions have been introduced to address this scenario, such as the type-2 fuzzy set by Dubois et al.[2], the intuitionistic fuzzy set (IFS) by Atanassov [3], the IVIFS by Bustince et al. and Turksen [4, 5], and so on. Boekee and Lubbe [6] provided an explanation of R-Norm IM in probability distributions. Additionally, Bhandari and Pal [7] investigates some measures like information of FSs and offers multiple methods for the entropy. However, in many complex situations, people find it difficult to approach the ultimate agreement and hesitate to make a decision. In order to address these situations, Xia and Xu [8, 9] and Torra [10] defined new extensions called hesitant fuzzy sets (HFSs). Many scholars were formerly fascinated by HFSs, leading them to publish a variety of creative solutions in the literature, including as decision-making techniques, entropy measures, distance measures, and similarity measures. Types of HFSs were examined along with closeness, distance measure and HFEs entropy which was provided by Xu [11] and Zhang [12]. Furthermore, Torra [10] demonstrated the connection between HFS and IFS, upon which some operational laws of HFSs was provided by Xia and Xu [8] . The set of alternative values provides the membership and non-membership degrees for a dual hesitant fuzzy set ($DHFS$), which was proposed by Zhu and Xu [13] by combining the ideas of IFS and HFS. The correlation coefficient of DHFSs was defined by Jun Ye[14]. Compared to previous fuzzy set extensions like IFS and HFS, DHFS is able to more accurately capture human fuzziness and reluctance. It also treats IFS and HFS as exceptional circumstances. Additionally, Zhu et al. examined the fundamental features and operations of DHFSs, that include FSs, IFSs, and HFSs as specific examples. Decision-makers can reach judgements more softly and effectively with the help of DHFSs, that are better suited to handle decision-making challenges. A set that encompasses multiple existing sets, Two sets of possible values indicate the membership degrees of DHFS, which is a complete set. Scholars have given entropy a lot of attention, and many positive outcomes have been attained. The concept of entropy for IFSs was initially introduced by Burillo and Bustince [15]. Mao [16] improved the entropy constructive principle for IFSs. Xu and Xia [9] defined entropy and created a number of entropy measurements specifically for HFS. Zhao and Xu [17] defined entropy measures for DHFSs. Several basic distance measures for DHFSs were developed by Su et al. [18] and applied to pattern recognition. Ye [19] introduced cross entropy of DHFSs for multi attribute decision making. A new distance measure for DHFss is introduced by Wang et.al. [20] and applied on MADM problem. Later on, Zhang [21] constructed distance and entropy measures for DHFSs. A novel knowledge measure was presented by Szmidt et al. [22] for IFSs, these were examined by Attanassov. Recently, a knowledge measure for IFSs was proposed by Nguyen [23]. Joshi [24] talked about using innovative fuzzy knowledge measures to support MCDM. [25] provide new information measure for HFSs. Singh and Kumar [26] explained Intuitionistic fuzzy entropy based on knowledge and accuracy measure. Dual-hesitant fuzzy sets are the basis for Singh’s [27] knowledge and accuracy measure. For fuzzy sets, the knowledge measure can be viewed as a counterpart measure of entropy.

However, to the best of our knowledge, not much study has been done on the DHFSs knowledge measure. In this manuscript, a new knowledge and accuracy measure is proposed for DHFSs. With the help of entropy we can quantify the randomness related to any DHFSs, whereas the knowledge passed by any DHFSs is quantified by knowledge measure. For fuzzy sets, the knowledge measure can be viewed as a counterpart measure of entropy. The hesitant fuzzy information relating to each alternative was aggregated using the hesitant fuzzy weighted averaging operator. Then, the alternatives were ranked, and the most desirable one or ones were chosen based on the scoring function for hesitant fuzzy sets. The HFSs hypothesis and methodology are widely applied in MADM. In this manuscript, we solve MADM problem in DHF environment with the help of new knowledge measure. The MADM problem is described as choosing the right alternative from the given alternative(s) in order to accomplish a goal. Every option has a set of requirements. Making decisions in a scenario with several homogeneous options and determining the best ones are part of the MADM process. While we're making decisions, attribute weighting is quite important. The MADM theory and mechanism have been applied recently to modern decision science and management science. MADM issues have been discussed by Bajaj and Kumar [28] using the Pythagorean fuzzy intuitive distance measure. As a result, numerous decision-making strategies based on real-world issues were suggested. Popular methods for approaching MADM are TOPSIS, VIKOR and ELECTRE. These techniques typically combine near-optimal alternatives with DHFSs to get a better option. While selecting the best alternative, different decision makers have different priorities based on how much risk they are willing to take. A new mode that combines TOPSIS method of MADM has been used in this knowledge and accuracy based paper, that takes into account people's risk priorities. Using an exact measure of "closeness" to the positive ideal solution, this method determines which alternative is optimal. The proposed knowledge measure is effectively applicable to solve MADM problems where the attribute weights are completely unknown.The intended effort is motivated by the following factors: Researchers have recently created a variety of distance metrics. The concepts and importance of it demonstrate how divergent these measures are. Furthermore, most applications have not produced consistent and dependable findings from several measures. In order to address these issues, we present a novel solution to DHF entropy, which we refer to as DHF knowledge measure. Moreover, a situation is deemed paradoxical if a comparison measure—such as distance, divergence, similarity, etc. It is unable to distinguish between two DHFSs that appear to be distinct from one another. False classification results can occur under unreasonable settings, such as when two or more separate datasets cannot be distinguished from one another. Therefore, we require an ideal set of metrics that minimizes the drawbacks of the current metrics and produces more precise and categorical answers in order to address pattern recognition concerns. This led us to develop a different type of comparison measure for DHFSs, which we call the dual hesitant accuracy measure.The paper's primary contribution can be summed up as follows. We suggest utilizing dual-hesitant fuzzy sets as the foundation for a knowledge and accuracy metric. We show that the proposed DHF knowledge measure is beneficial in the calculation problem for attribute weight. Additionally, we compare how well the suggested measures work in a MADM situation. Using the suggested dual-hesitant accuracy measure, we examine the numerical analysis of a few pattern recognition issues and compare the results with certain traditional comparison measures within the dual-hesitant structure. The paper is organized as follows, Firstly it provides an overview of the HFSs, Secondly, contains the definitions and fundamental ideas of DHFSs. After that it presents a proposed DHF knowledge measure. The validity of the proposed measure is verified by comparing its outcome with many existing measures. In addition, a DHF accuracy measure has generalization of the DHF knowledge measure is presented in this section. Next, TOPSIS method of MADM is used in decision-making process based on the suggested knowledge measure. It presents a real-world example to demonstrate the suitability of this innovative approach to decision-making. The use of pattern detection is provided in to evaluate the suggested accuracy measure's performance with that of some other measures. Finally settles the conclusion.

2 Preliminaries

In this section, we present numerous examples of entropy and distance measurements for DHFSs along with standard definitions.

2.1 HFSs, DHFSs and their related properties.

Definition 2.1 (Torra and Narukawa)[10] Consider a finite collection set ζ, and define HFSs $\mathfrak{J}$ in terms of function on ζ.

$he_{\mathfrak{J}}: \zeta \rightarrow R(J)$, and $he_{\mathfrak{J}} \neq \phi$ and definite set for any $u \in \zeta$.

A hesitant fuzzy set usually gets expressed by $\mathfrak{J} = \{\langle u, he_{\mathfrak{J}}(u)\rangle | u \in \zeta\}$

Here $R(J)$ represents power set of [0,1] , $he_{\mathfrak{J}}(u)$ represents hesitant fuzzy element (HFE).

Definition 2.2 (Zhu et al.)[11] Let ζ be a fixed set, a dual hesitant fuzzy set (DHFS) M on ζ is defined as $M = \{(u, he(u), t(u)) | u \in \zeta\}$,

In which $he(u)$ and $t(u)$ are two sets of some values in [0,1], denoting the possible membership degrees and non-membership degrees of the element $u \in \zeta$ to the set M, respectively.For convenience, DHFE are represented by the pair $m_F(u) = (he_F(u), t_F(u))$, with the condition:

$0 \leq \theta, \vartheta \leq 1, 0 \leq \theta + \vartheta \leq 1$,

where $\theta \in he_F(u), \vartheta \;\; int_F(u), \theta^+ \in he_F^+(u) = \cup_{\theta \in he_F(u)} max\{\theta\}$,

And $\vartheta^+ \in t_F^+(u) = \cup_{\vartheta \in t_F(u)} max\{\vartheta\}$

Initially, we define a few unique DHFEs. Assuming a DHFE, d_F, we have

(1) Complete uncertainty: $d_F = \{\{0\}\}, \{\{1\}\}$;

(2) Complete certainty: $d_F = \{\{1\}\}, \{\{0\}\}$;

(3) Complete ill-known (all is possible): $d_F = [0,1]$;

(4) Nonsense element: $d_F = \Phi, i.e., he_F = \phi, t_F = \phi$.

Basic operations and properties of DHFSs

Definition 2.3 (Zhu et al.)[13] Given a DHFE d_E, and $d_E \neq \Phi$, its complement is as follow:

Complement: $d_E^c = (\cup_{\theta \in h_E} \{\theta\}, \cup_{\vartheta \in k_E} \{\vartheta\})$, if $h_E \neq \phi$ and $k_E \neq \phi$

$(\cup_{\theta \in h_E} \{1-\theta\}, \{\phi\})$, if $h_E \neq \phi$ and $k_E = \phi$ $(\{\phi\}, \cup_{\vartheta \in k_E} \{1-\vartheta\})$, if $h_E = \phi$ and $k_E \neq \phi$.

Definition 2.4 (Zhu et al.)[13] Let $d_{E_1}, d_{E_2} \in DHFFs$. Then

(a) **Union:** $d_{E_1} \cup d_{E_2} = \{\theta \in (h_{E1} \cup h_{E_2}) | \theta \geq max(h_{E_1}^-, h_{E_2}^-), \vartheta \in (k_{E_1} \cap k_{E_2}) | \vartheta \leq min(k_{E_1}^+, k_{E_2}^+)\}$

(b)**Intersection** : $d_{E_1} \cap d_{E_2} = \{\theta \in (h_{E1} \cap h_{E_2}) | \theta \leq min(h_{E_1}^+, h_{E_2}^+), \vartheta \in (k_{E_1} \cup k_{E_2}) | \vartheta \geq max(k_{E_1}^-, k_{E_2}^-)\}$

Definition 2.5 (Zhu et al.)[13] Let $d_E = \{h_E, k_E\}$ be any two DHFEs. Then score function of d_E is given as follow:

$s(d_E) = \frac{1}{t_{h_E}} \sum_{\theta \in h_E} \theta - \frac{1}{tk_E} \sum_{\vartheta \in k_E} \vartheta$,

Where $t(h_E)$ and $t(k_E)$ are length of h_E and k_E, respectively.

The accuracy function of d_E is given below:

$$A_c(d_E) = \frac{1}{t_{h_E}} \sum_{\theta \in h_E} \theta + \frac{1}{tk_E} \sum_{\vartheta \in k_E} \vartheta$$

Let $d_{E_1}, d_{E_2} \in DHFEs$, the comparison rule between $DHFEs$ are given below:

(a) if $s(d_{E_1}) < s(d_{E_2})$, then d_{E_1} is superior to d_{E_2}, denoted by $d_{E_1} > d_{E_2}$;

(b) if $s(d_{E_1}) = s(d_{E_2})$, then

(i) if $A_c(d_{E_1}) = A_c(d_{E_2})$, then d_{E_1} is equivalent to d_{E_2}, denoted by $d_{E_1} \sim d_{E_2}$.

(ii) if $A_c(d_{E_1}) > A_c(d_{E_2})$, then d_{E_1} is superior than d_{E_2}, denoted by $d_{E_1} > d_{E_2}$.

Definition 2.6 (Zhang) [21] Let $d_E = (\{h_1^1, h_1^2, \ldots, h_1^m\}, \{k_1^1, k_1^2, \ldots, k_1^n\})$ and $f_E(\{h_1^1, h_1^2, \ldots, h_1^p\}, \{k_1^1, k_1^2, \ldots, k_1^q\})$ be two DHFEs. If an entropy e fulfills the following axiomatic conditions, it is a real-valued function $e: DHFE \rightarrow [0,1]$.

(E1) $e(d_E) = 0$ iff $d_E = (\{1\}, \{0\})$ or $d_E = (\{0\}, \{1\})$.

(E2) $e(d_E) = 1$ iff $h_1^{\sigma(j)} = k_1^{\sigma(j)}$ where $h_1^{\sigma(j)}$ and $k_1^{\sigma(j)}$ are the smallest value of h_1 and k_1.

(E3) $e(d_E) \geq e(f_E)$, if $max_i h_1^i \leq min_s h_2^s, max_j k_1^j \leq min_t k_2^t$ for $max_s h_2^s \leq min_t k_2^t$ or $min_i h_1^i \leq max_s h_2^s$, $max_j g_1^j \leq min_t g_2^t$ for $min_s h_2^s \geq max_t g_2^t, (i = 1,2,\ldots,m; j = 1,2,\ldots,n; s,t = 1,2,\ldots,q)$.

(E4) $e(d_E) = e(d_E^c)$, where d_E^c represents compliment of d_E.

Definition 2.7 (Wang)[29] For $d_E, f_E \in DHFEs(u)$, Let d be a mapping $d: DHFE(u) \times DHFE(u) \rightarrow [0,1], d(d_E, f_E)$ is a distance measure between $DHFEs$ d_E and f_E, if $d(d_E, f_E)$ satisfies the following properties:

(i) $0 \leq d(d_E, f_E) \leq 0$;

(ii) $d(d_E, f_E) = 0$ if and only if $d_E = f_E$;

(iii) $d(d_E, f_E) = d(f_E, d_E)$.

2.2 Some existing entropies for DHFSs

Here, we discussed some distance and entropy measures which are already available in literature. Entropies of DHFSs are extensively studied, for instance Zhao and Xu [17]:

$$E_1(d_E) = \frac{1}{t} \sum_{j=1}^{t} \left[1 - \frac{|h_1^{\sigma(j)} + k_1^{\sigma(j)}|^\alpha + |h_1^{\sigma(j)} - k_1^{\sigma(j)}|^\alpha}{2}\right]; \alpha \geq 0. \quad (1)$$

$$E_2(d_E) = \frac{1}{t} \sum_{j=1}^{t} \left[\frac{1 - (|h^{\sigma(j)} - k^{\sigma(j)}|)(2 - h^{\sigma(j)} - k^{\sigma(j)})}{2}\right]; \alpha \geq 0. \quad (2)$$

$$E_3(d_E) = \frac{1}{t} \sum_{j=1}^{t} \left[1 - \frac{(|h^{\sigma(j)} - k^{\sigma(j)}|)^2 (1 - h^{\sigma(j)} + k^{\sigma(j)}|)}{2}\right]; \alpha \geq 0. \quad (3)$$

2.2.1 Existing distance based entropy for DHFSs

Zhang [21] defined distance based entropy for DHFEs as follow:

$$E_4(d_E,f_E) = 1 - [\frac{0.5}{m+p}(\sum_{i=1}^{m} \min_j |h_i^1 - h_j^2|^{\frac{\alpha}{2-\beta}} + \sum_{j=1}^{p} \min_i |h_j^1 - h_i^2|^{\frac{\alpha}{2-\beta}}) + \frac{0.5}{n+p}(\sum_{i=1}^{n} \min_j |k_i^1 - k_j^2|^{\frac{\alpha}{2-\beta}} + \sum_{j=1}^{q} \min_i |k_j^1 - k_i^2|^{\frac{\alpha}{2-\beta}})]^{\frac{2-\beta}{\alpha}}$$

(4)

2.3 Knowledge measure of DHFS

We present the notion of the DHF set knowledge measure in this section.

Definition 2.8 Let $d_E = (\{h_1^1, h_1^2, \dots, h_1^m\}, \{k_1^1, k_1^2, \dots, k_1^n\})$ and $f_E = (\{h_2^1, h_2^2, \dots, h_2^p\}, \{k_2^1, k_2^2, \dots, k_2^q\})$ be two DHFEs. If knowledge measure N_M fulfills the following axiomatic conditions, it is a real-valued function $N_M: DHFE \to [0,1]$

$(N1)$ $N_M(d_E) = 1$ iff $d_E = (\{1\},\{0\})$ or $d_E = (\{0\},\{1\})$.

$(N2)$ $N_M(d_E) = 0$ iff $d_E = (\{0\},\{0\})$

$(N3)$ $N_M(d_E) \le N_M(f_E)$, if $max_i h_1^i \le min_s h_2^s, max_j k_1^j \le min_t k_2^t$ for $max_s h_2^s \le min_t k_2^t$ or $min_i h_1^i \le max_s h_2^s$, $max_j g_1^j \le min_t g_2^t$ for $min_s h_2^s \ge max_t g_2^t$, $(i = 1,2,\dots,m; j = 1,2,\dots,n; s,t = 1,2,\dots,q)$.

$(N4)$ $N_M(d_E) = N_M(d_E^c)$, where d_E^c represents compliment of d_E.

Definition 2.9 Attribute weights computation

In modeling, a multi-attribute decision-making issue, attributes weights plays a vital role. Chen and Li [29] provided a method to find out the weight.

$$w_j = \frac{N_M(r_{ij})}{\sum_{i=1}^{m} N_M(r_{ij})}$$

Set $w = \{w_1, w_2, \dots, w_T\}$ is said to be weight of attributes if $w_j \ge 0$ and $\sum_{j=1}^{T} w_j = 1$

3 Proposed knowledge measure for DHFSs

We propose a DHF- knowledge measure that is described as:

$$N_M(d_E) = [\sum_{i=1}^{n} (\frac{1}{t}\sum_{j=1}^{t} |h^{\sigma(j)}(u_i) - k^{\sigma(j)}(u_i)|^{\frac{\alpha}{2-\beta}})]^{\frac{2-\beta}{\alpha}}, \alpha > 0, 0 < \beta < 2. \quad (5)$$

We now assess the validity of the proposed measure $N_M(d_E)$.

Theorem 1 The DHF knowledge measure $N_M(d_E)$, as stated in Eq. (5) is valid.

Proof (N1) Suppose that $d_E = (\{1\},\{0\})$ or $d_E = (\{0\},\{1\})$, we have, $N_M(d_E) = 1$.

Now, suppose $N_M(d_E) = 1$, we have

$$(\frac{1}{t}\sum_{j=1}^{t} |h^{\sigma(j)}(u_i) - k^{\sigma(j)}(u_i)|^{\frac{\alpha}{2-\beta}})^{\frac{2-\beta}{\alpha}} = 1$$

This is possible only if $d_E = (\{1\},\{0\})$ or $d_E = (\{0\},\{1\})$.

(N2) Suppose $h_1^{\sigma(j)} = k_1^{\sigma(j)}$, we have $N_M(d_E) = 0$.

Conversely, Suppose $N_M(d_E) = 0$, we have

$(\frac{1}{t}\sum_{j=1}^{t} |h^{\sigma(j)}(u_i) - k^{\sigma(j)}(u_i)|^{\frac{\alpha}{2-\beta}})^{\frac{2-\beta}{\alpha}} = 0,$

to $h^{\sigma(j)}$=$k^{\sigma(j)} = 0$.

(N3) Let $d_E = (h_1^{\sigma(j)}, k_1^{\sigma(j)})$=$(\{h_1^1, h_1^2, \dots, h_1^m\}, \{k_1^1, k_1^2, \dots, k_1^n\})$

$f = (h_2^{\sigma(j)}, k_2^{\sigma(j)})$=$(\{h_1^1, h_1^2, \dots, h_1^p\}, \{k_1^1, k_1^2, \dots, k_1^q\})$ be two DHFSs. we have

$N_M(d_E) = (\frac{1}{t}\sum_{j=1}^{t} |h_1^{\sigma(j)} - k_1^{\sigma(j)}|^{\frac{\alpha}{2-\beta}})^{\frac{2-\beta}{\alpha}}, \alpha > 0, 0 < \beta < 2.$

$N_M(f_E) = (\frac{1}{t}\sum_{j=1}^{t} |h_2^{\sigma(j)} - k_2^{\sigma(j)}|^{\frac{\alpha}{2-\beta}})^{\frac{2-\beta}{\alpha}}, \alpha > 0, 0 < \beta < 2.$

If $max_i h_1^{\sigma(j)} \leq min_i h_2^{\sigma(j)}, min_i k_1^{\sigma(j)} \geq max_i k_2^{\sigma(j)}$ for $max_i h_2^{\sigma(j)} \leq min_i g_2^{\sigma(j)}$, we have

$h_1^{\sigma(j)} - k_1^{\sigma(j)} \leq h_2^{\sigma(j)} - k_2^{\sigma(j)} \leq 0$ and

$k_1^{\sigma(j)} - h_1^{\sigma(j)} \leq k_2^{\sigma(j)} - h_2^{\sigma(j)} \leq 0,$

Therefore,

$|h_1^{\sigma(j)} - k_1^{\sigma(j)}| \geq |h_2^{\sigma(j)} - k_2^{\sigma(j)}|$

$(\frac{1}{t}\sum_{j=1}^{t} |h_1^{\sigma(j)} - k_1^{\sigma(j)}|^{\frac{\alpha}{2-\beta}})^{\frac{2-\beta}{\alpha}} \geq (\frac{1}{t}\sum_{j=1}^{t} |h_2^{\sigma(j)} - k_2^{\sigma(j)}|^{\frac{\alpha}{2-\beta}})^{\frac{2-\beta}{\alpha}}$

$N_M(d) \geq N_M(f)$

Similarly, if $min_i h_1^{\sigma(j)} \geq max_i h_2^{\sigma(j)}, max_i k_1^{\sigma(j)} \leq min_i k_2^{\sigma(j)}$ for $min_i h_2^{\sigma(j)} \geq max_i k_2^{\sigma(j)}$, We have

$N_M(d_E) \geq N_M(f_E)$

(N4) We have

$N_M(d_E) = (\frac{1}{t}\sum_{j=1}^{t} |h_1^{\sigma(j)} - k_1^{\sigma(j)}|^{\frac{\alpha}{2-\beta}})^{\frac{2-\beta}{\alpha}}, \alpha > 0, 0 < \beta < 2.$

So, $N_M(d_E^c) = (\frac{1}{t}\sum_{j=1}^{t} |k_1^{\sigma(j)} - h_1^{\sigma(j)}|^{\frac{\alpha}{2-\beta}})^{\frac{2-\beta}{\alpha}} = (\frac{1}{t}\sum_{j=1}^{t} |h_1^{\sigma(j)} - k_1^{\sigma(j)}|^{\frac{\alpha}{2-\beta}})^{\frac{2-\beta}{\alpha}} = N_M(d_E)$

Hence, $N_M(d_E^c) = N_M(d_E)$

As a result, $N_M(d_E)$ is an acceptable DHF knowledge measure.

The comparison of the DHF knowledge measure with a few DHF entropy measures that are currently in use is presented in the next section.

3.1 Comparative Study

Applying the weight calculation mechanism, we evaluate the existing DHF entropy measurements given in Eq.(1)-(3) and compare these with our suggested DHF knowledge measure given in Eq.(5). Assume that we have a MADM issue with two attributes $\{Q_1, Q_2\}$, and three alternatives $\{A_1, A_2, A_3\}$. The weight calculating process is displayed in Singh et al.'s 2019 publication. In Examples 1-3, we use DHF entropy and DHF knowledge measure to compute weights. Examples 1–3 given below show how our suggested DHF knowledge measure performs in comparison to some of the existing DHF entropies.

Example 1 Consider D_1 is a decision matrix corresponding to set of alternatives $\{A_1, A_2, A_3\}$ and a set of attributes $\{Q_1, Q_2\}$ established in an hesitant fuzzy environment.

$D_1 =$ Q_1, Q_2

A_1 = ({.3, .4, .6,}, {.1, .2, .4}), ({.3, .5, .6}, {.5, .6, .7})

A_2 = ({.5, .6, .7,}, {.3, .5, .6}), ({.2, .3, .4}, {.4, .5, .6})

A_3 = ({.4, .5, .6,}, {.2, .3, .4}), ({.1, .2, .4}, {.4, .5, .6})

3.2 Comparative Study

Applying the weight calculation mechanism, we evaluate the existing DHF entropy measurements given in Eq.(1)-(3) and compare these with our suggested DHF knowledge measure given in Eq.(5). Assume that we have a MADM issue with two attributes $\{Q_1, Q_2\}$, and three alternatives $\{A_1, A_2, A_3\}$. The weight calculating process is displayed in Singh et al.'s 2019 publication. In Examples 1-3, we use DHF entropy and DHF knowledge measure to compute weights. Examples 1–3 given below show how our suggested DHF knowledge measure performs in comparison to some of the existing DHF entropies.

Example 1 Consider D_1 is a decision matrix corresponding to set of alternatives $\{A_1, A_2, A_3\}$ and a set of attributes $\{Q_1, Q_2\}$ established in an hesitant fuzzy environment.

$D_1 =$ Q_1, Q_2

A_1 = ({.3, .4, .6,}, {.1, .2, .4}), ({.3, .5, .6}, {.5, .6, .7})

A_2 = ({.5, .6, .7,}, {.3, .5, .6}), ({.2, .3, .4}, {.4, .5, .6})

A_3 = ({.4, .5, .6,}, {.2, .3, .4}), ({.1, .2, .4}, {.4, .5, .6})

3.3 Comparative Study

Applying the weight calculation mechanism, we evaluate the existing DHF entropy measurements given in Eq.(1)-(3) and compare these with our suggested DHF knowledge measure given in Eq.(5). Assume that we have a MADM issue with two attributes $\{Q_1, Q_2\}$, and three alternatives $\{A_1, A_2, A_3\}$. The weight calculating process is displayed in Singh et al.'s 2019 publication. In Examples 1-3, we use DHF entropy and DHF knowledge measure to compute weights. Examples 1–3 given below show how our suggested DHF knowledge measure performs in comparison to some of the existing DHF entropies.

Example 1 Consider D_1 is a decision matrix corresponding to set of alternatives $\{A_1, A_2, A_3\}$ and a set of attributes $\{Q_1, Q_2\}$ established in an hesitant fuzzy environment.

$D_1 =$ Q_1, Q_2

A_1 = ({.3, .4, .6,}, {.1, .2, .4}), ({.3, .5, .6}, {.5, .6, .7})

A_2 = ({.5, .6, .7,}, {.3, .5, .6}), ({.2, .3, .4}, {.4, .5, .6})

A_3 = ({.4, .5, .6,}, {.2, .3, .4}), ({.1, .2, .4}, {.4, .5, .6})

Table 1: Comparison of N_M with E_1

	$N_M(\alpha = 2, \beta = 1)$	$E_1(\alpha = 2)$
w_{Q_1}	0.5279	0.5000
w_{Q_2}	0.4721	0.5000

Table 1 shows that both attributes have the same weights assigned to them by the ambiguity content as assessed by the current entropy measure. However, it is evident that the suggested N_M gives various weights.

Example 2 Let us consider the D_2 decision matrix.

$D_2 =$ Q_1, Q_2

A_1 = ({.2, .3, .5}, {.1, .4, .6}), ({.2, .3, .6}, {.3, .4, .6})

A_2 = ({.3, .4, .6}, {.2, .3, .6}), ({.3, .4, .6}, {.2, .5, .5})

A_3 = ({.2, .4, .6}, {.3, .5, .5}), ({.1, .3, .6}, {.2, .4, .5})

Table 2: Comparison of N_M with E_2

	$N_M(\alpha = 2, \beta = 1)$	$E_2(\alpha = 2)$
w_{Q_1}	0.5279	0.5000
w_{Q_1}	0.4721	0.5000

Table 2 shows that both attributes have the same weights assigned to them by the ambiguity content as assessed by the current entropy measure. However, it is evident that the suggested N_M gives various weights.

Example 3 Consider the decision matrix D_3. $D_3 =$ Matrix Example

Q_1, Q_2

$A_1 = (\{0.2,0.4,0.8\},\{0.3,0.6,0.7\})(\{0.3,0.4,0.401\},\{0.1,0.2,0.37\})$

$A_2 = (\{0.21,0.3,0.9\},\{0.6,0.7,0.8\})(\{0.2,0.6,0.7\},\{0.3,0.4,0.8\})$

$A_3 = (\{0.2,0.6,0.9\},\{0.3,0.4,0.4\})(\{0.2,0.3,0.9\},\{0.5,0.6,0.7\})$

Table 3: Comparison of N_M with E_3

	$N_M(\alpha = 2, \beta = 1)$	$E_3(\alpha = 2)$
w_{Q_1}	0.4613	0.5000
w_{Q_2}	0.5387	0.5000

We note that the existing entropy measure, which determines the ambiguity content, gives equal weights to the two attributes. Nonetheless, it is evident that the suggested knowledge measure gives various weights. We note from example 1-3 that we obtain the same weights for all the qualities when we use the current entropy measures. However, we obtain different weights if we apply our proposed DHF knowledge measure. As such, a new strategy is always required.

Furthermore, we compare our proposed knowledge measure given in Eq.(5) with distance based entropy given in Eq.(4).

Example 4 Let d_E and f_E be two DHFEs, and $d_E = \{\{.1,.3,.6\},\{.2,.5,.7\}\}$, $f_E = \{\{.2,.4,.6\},\{.3,.7,.7\}\}$.

Table 4: Comparison of N_M with $E(d_E, f_E)$

DHFEs	$E(d_E, f_E)(\alpha = 2, \beta = 1)$	$N_M(\alpha = 2)$
d_E	0.1000	0.1414
f_E	0.1000	0.1915

The data in Table 4 show that the entropy $E(d_E, f_E)$ have no difference between DHFEs d_E and f_E. However, the proposed knowledge measure of N_M can clearly distinguish the entropy of DHFEs d_E and f_E. Our knowledge measure is better than the entropy measure proposed by Zhang [21] for this example.

3.4 Proposed DHF accuracy measure

There is a correspondence between the amount of reluctant fuzzy knowledge and the quantity of hesitant fuzzy accuracy. As a generalization of the DHF knowledge measure, we propose DHF accuracy measure.

$$I_{acc}(d_E, f_E) = \left(\frac{1}{l}\sum_{i=1}^{n}\sum_{j=1}^{t} \left|(h_1^{\sigma(j)}(u_i))^{\frac{1}{2}}(h_2^{\sigma(j)}(u_i))^{\frac{1}{2}} - (k_1^{\sigma(j)}(u_i))^{\frac{1}{2}}(k_2^{\sigma(j)}(u_i))^{\frac{1}{2}}\right|^{\frac{\alpha}{2-\beta}}\right)^{\frac{2-\beta}{\alpha}}, \alpha > 0, \quad (6)$$

Where $d_E = ((h_1^{\sigma(j)}(u_i), k_1^{\sigma(j)}(u_i))$ and $f_E = ((h_2^{\sigma(j)}(u_i), k_2^{\sigma(j)}(u_i))$ are two DHFSs.

$d_E = f_E$ in Eq.(6) yields $I_{acc}(d_E, f_E) = N_M(d_E)$.

By multiplying with w_i, we obtain the weighted version $I_{acc}^{w}(d_E, f_E)$ of the DHF accuracy from Eq.(6). It is evident from Eq.(6) that the DHF accuracy $I_{acc}(d_E, f_E)$ for $d_E = f_E$ is obtained as a particular instance of the DHF knowledge measure $N_M(d_E)$. We suggest using $I_{acc}(d_E, f_E)$ as an accuracy measure to identify the asymmetric comparison of two sets. We demonstrate the use of the TOPSIS approach to apply our proposed DHF knowledge measure in MADM in the next section.

4 Evaluation method of MADM

MADM difficulties are related to discrete choice spaces where there are several predefined alternatives. It is employed to select the best option from a range of options. Making decisions in a scenario with several homogeneous options and determining the best ones are part of the MADM process. While we're making decisions, attribute weighting is quite important. The MADM mechanism and theory have been employed recently in modern decision science and management science. As an objective weight computation tool, the TOPSIS technique of MADM in a DHF environment with a DHF knowledge measure is introduced in this part. Suppose $S = \{A_1, A_2, \ldots, A_n\}$, $Q = \{Q_1, Q_2, \ldots, Q_T\}$, where A_i, s represent alternatives and Q_j, s represent attributes.

Let $w = (w_1, w_2, \ldots, w_T)$, where w_i, s are weight of attributes and $w_j \geq 0$; $\sum_{j=1}^{T} w_j = 1$

If dual-hesitant fuzzy elements are used by the maker of decisions to assess the attributes of the available alternative. We therefore have a decision matrix of dual-hesitant fuzzy elements, $D = (a_{ij})_{nm}$. The optimum alternative can be obtained by following a few MADM procedures. In light of our proposed knowledge measure, we now take into consideration the TOPSIS approach with minor modifications.

Algorithm

Step1: Build the DHF decision matrix $D = [a_{ij}]_{nm}$ with the decision-maker's ratings in it..

Step2: Applying the optimistic principle—that is, repeating the maximum value in DHFEs-make all DHFEs the same length.

Step3: Convert the $D = [a_{ij}]_{nm}$ decision matrix into a normalized decision matrix $R = [r_{ij}]_{nm}$ as

$r_{ij} = \{h^{\sigma(j)}(u_i), k^{\sigma(j)}(u_i)\}, v_j$ is benefit criteria;

$\{k^{\sigma(j)}(u_i), h^{\sigma(j)}(u_i)\}, v_j$ is cost criteria.

Step4: Find the DHF ideal solutions, S^+ and S^-, that are fuzzy positive and negative, respectively.

$S^+ = \{max h^{\sigma(j)}(u_i), min k^{\sigma(j)}(u_i), v_j$ is benefit criteria;

$\{min h^{\sigma(j)}(u_i), max k^{\sigma(j)}(u_i), v_j$ is cost criteria.

$S^- = \{min h^{\sigma(j)}(u_i), max k^{\sigma(j)}(u_i), v_j$ is benefit criteria;

$\{max h^{\sigma(j)}(u_i), min k^{\sigma(j)}(u_i), v_j$ is cost criteria.

Step5: Calculate the attribute weights using the DHF knowledge measure.

$$w_j = \frac{N_M(r_{ij})}{\sum_{i=1}^{m} N_M(r_{ij})}$$
(7)

Step6: Determine the distance between the dual-hesitant fuzzy positive ideal solutions or the negative ideal solutions and the alternative A_i as:

$$U^+ = I_w^{acc}(A_i, S^+)$$
(8)

$$U^- = I_w^{acc}(A_i, S^-)$$
(9)

Step7: Find the relative closeness coefficients from U^+ and U^- for each of the alternatives.

$$C_i = \frac{U^-}{U^+ + U^-}$$
(10)

Step8: Rank each alternative by closeness coefficient in descending order.

5 Application of proposed knowledge measure in MADM problem

Many developing nations are currently experiencing a serious power crisis as a result of rapidly growing needs and a significant discrepancy between supply and demand. The planning commission must choose the business that is thought to offer the finest service based on consumer satisfaction in order to resolve this problem. There are four alternatives in this situation: A_1 Power Company 1, A_2 Power Company 2, A_3 Power Company 3, and A_4 Power Company 4. While assessing these four potential power producing companies, the following four key attributes have been identified:

(a) Cost and Tariff (Q_1): The rate, fee, and terms and conditions for the production of electricity, as well as the transmission and distribution of services to consumers.

(b) Reliability and performance (Q_2): The extent to which consumers receive electric power within predetermined parameters.

(c) Installing and being accountable (Q_3): The assurance that all work, wiring, and equipment are installed and maintained safely by the business for the benefit of its clients.

(d) Safety and protection (Q_4): Enclosures that prevent employees from unintentionally coming into contact with electrical equipment and prevent unauthorised use of the electric service.

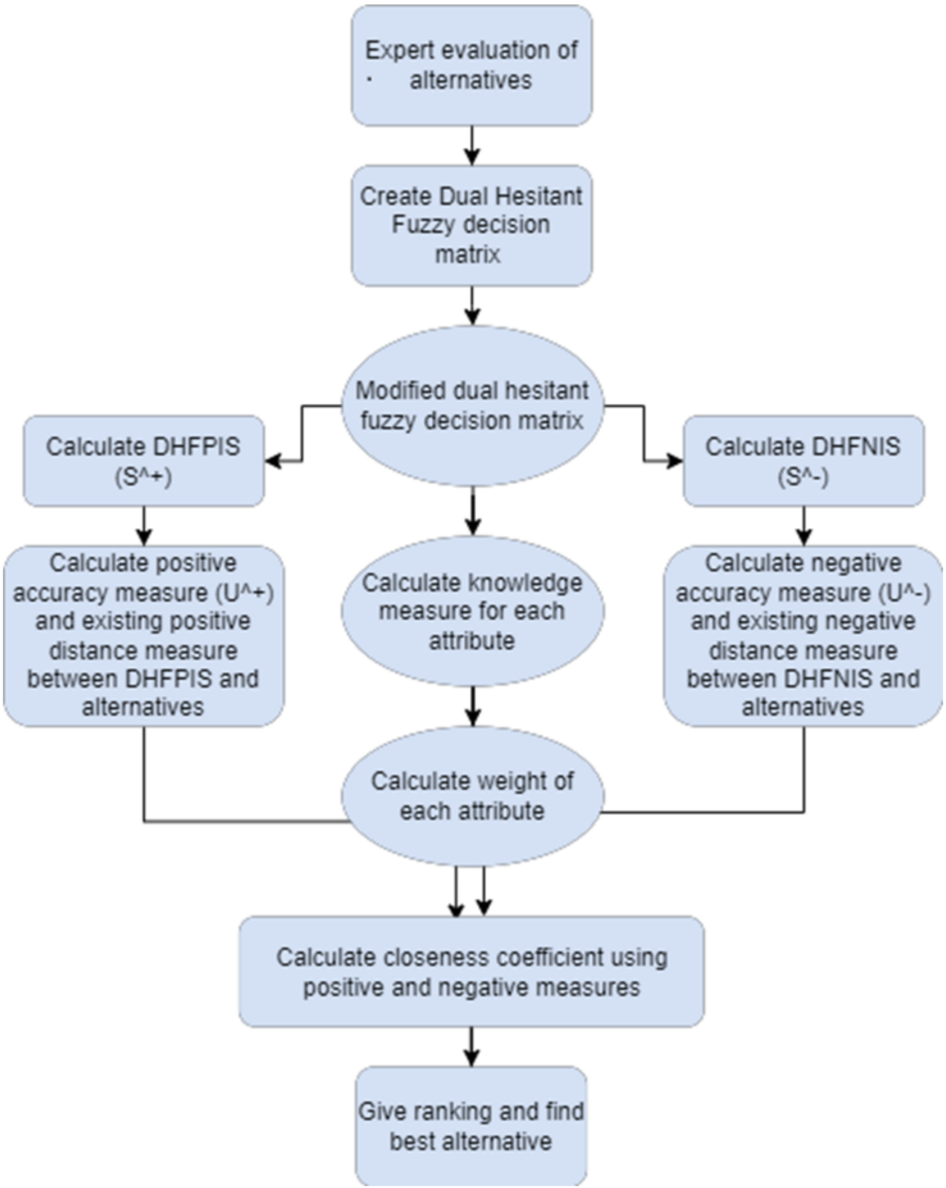

Figure 1: Step-by-step flowchart of the suggested methodology.

Assume that a group of decision-makers consisting of professionals from four related fields has been given permission to assess each alternative's degree of satisfaction and determine which one has the finest qualities. This issue can be seen as a MADM issue and solve it with the algorithm described in Sect. 4. Figure 1. stands for the proposed approach's operational steps.

Table 5: DHF decision matrix

	Q_1	Q_2	Q_3	Q_4
A_1	({.2, .3, .4},{.1, .2})	({.1, .3, .5},{.3})	({.1, .3, .4},{.2, .3})	({.2, .3, .4},{.1, .2})
A_2	({.1, .3, .3},{.2})	({.2, .4, .4},{.1, .2})	({.2, .3, .4},{.1, .2})	({.3, .4, .5},{.3, .4})
A_3	({.4, .5, .5},{.3, .4, .5})	({.1, .2, .3},{.2, .3})	({.3, .4, .5},{.3, .4})	({.1, .2, .3},{.3})
A_4	({.2, .4, .4},{.2})	({.1, .4, .5},{.2, .3})	({.1, .2, .3},{.3})	({.2, .4, .4},{.2})

Table 6: Modified DHF decision matrix

	Q_1	Q_2	Q_3	Q_4
A_1	({.2, .3, .4},{.1, .2, .2})	({.1, .3, .5},{.3, .3, .3})	({.1, .3, .4},{.2, .3, .3})	({.2, .3, .4},{.1, .2, .2})
A_2	({.1, .3, .3},{.2, .2, .2})	({.2, .4, .4},{.1, .2, .2})	({.2, .3, .4},{.1, .2, .2})	({.3, .4, .5},{.3, .4, .4})
A_3	({.4, .5, .5},{.3, .4, .5})	({.1, .2, .3},{.2, .3, .3})	({.3, .4, .5},{.3, .4, .4})	({.1, .2, .3},{.3, .3, .3})
A_4	({.2, .4, .4},{.2, .2, .2})	({.1, .4, .5},{.2, .3, .3})	({.1, .2, .3},{.3, .3, .3})	({.2, .4, .4},{.2, .2, .2})

Table 7: DHFPIS and DHFNIS

	Q_1	Q_2	Q_3	Q_4
S^+	({.4, .5, .5},{.1, .2, .2})	({.2, .4, .5},{.1, .2, .2})	({.3, .4, .5},{.1, .2, .2})	({.3, .4, .5},{.1, .2, .2})
S^-	({.1, .3, .3},{.3, .4, .5})	({.1, .2, .3},{.3, .3, .3})	({.1, .2, .3},{.3, .4, .4})	({.1, .2, .3},{.3, .4, .4})

Table 8: Objective weights

	Q_1	Q_2	Q_3	Q_4
N_M	.2517	.2887	.3266	.2582
w_j	.2237	.2566	.2903	.2294

Table 9: Positive (U^+) and negative (U^-) accuracy measure and distance measure ($\alpha = 2, \beta = 1$)

	I_{acc}^w		D_a	
	U^+	U^-	U^+	U^-
A_1	.3693	.1720	.1200	.1347
A_2	.3489	.1532	.1148	.1490
A_3	.2906	.2213	.1493	.1155
A_4	.3487	.2022	.1191	.1300

Table 10: Positive (U^+) and negative (U^-) accuracy measure and distance measure ($\alpha = 1.5, \beta = 1.5$)

	I_{acc}^w		D_a	
	U^+	U^-	U^+	U^-
A_1	.3141	.1626	.1229	.1503
A_2	.2888	.1384	.1432	.1613
A_3	.2512	.1885	.1587	.1404

A_4	.2991	.1800	.1216	.1419

Table 11: Closeness coefficient using positive and negative measures ($\alpha = 2, \beta = 1$)

	$C_i(I_{acc}^w)$	Rank	$C_i(D_a)$	Rank
A_1	.3178	3	.5289	4
A_2	.3051	4	.4128	1
A_3	.4323	1	.4362	2
A_4	.3670	2	.5219	3

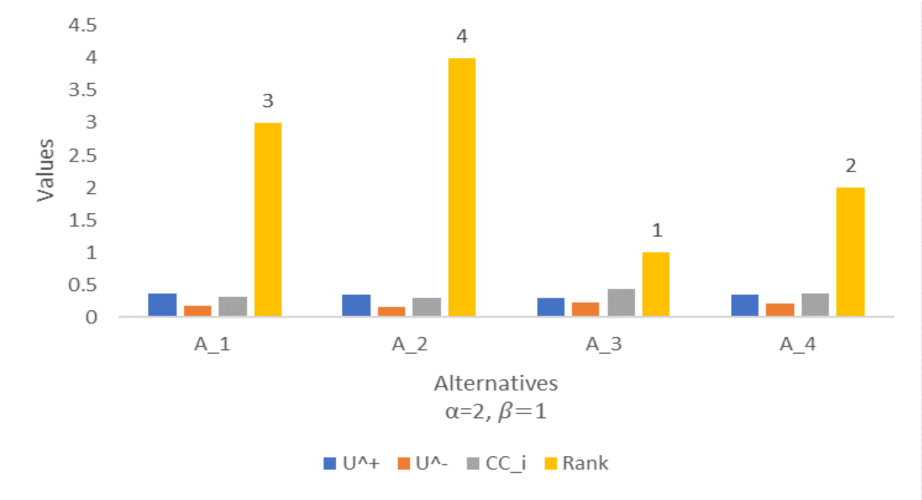

Figure 2: U^+ and U^-, Closeness coefficients and Rank of proposed accuracy measure

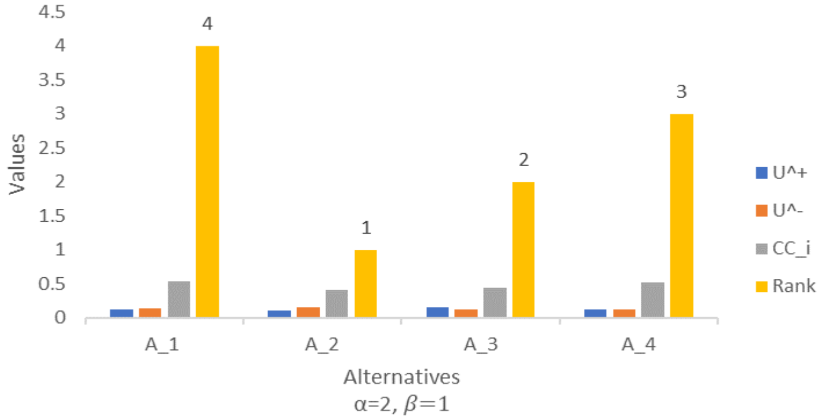

Figure 3: U^+ and U^-, Closeness coefficients and Rank of distance measure

Table 12: Closeness coefficient using positive and negative measures ($\alpha = 1.5, \beta = 1.5$)

	$C_i(I_{acc}^w)$	Rank	$C_i(D_a)$	Rank
A_1	.3410	3	.5501	4
A_2	.3240	4	.5297	2

A_3	.4287	1	.4694	1
A_4	.3757	2	.5385	3

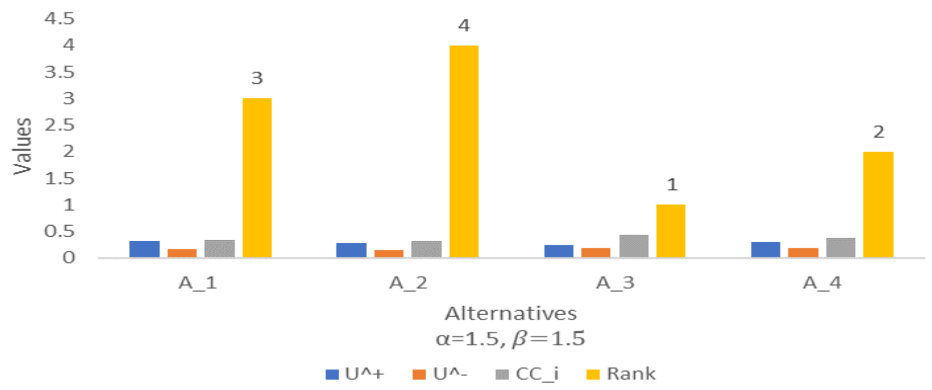

Figure 4: U^+ and U^-, Closeness coefficients and Rank of proposed accuracy measure

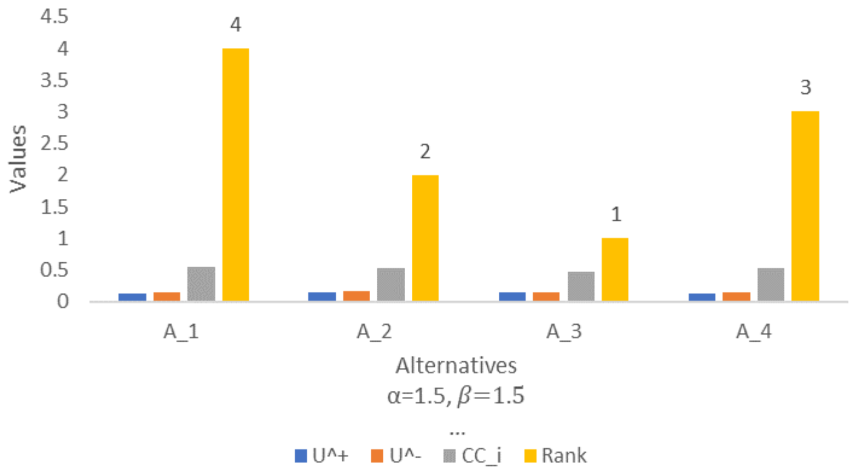

Figure 5: U^+ and U^-, Closeness coefficients and Rank of distance measure

Table 13: Ranking of alternatives

$I_{acc}^{w}(\alpha = 2, \beta = 1)$	$A_3 > A_4 > A_1 > A_2$
$I_{acc}^{w}(\alpha = 1.5, \beta = 1.5)$	$A_3 > A_4 > A_1 > A_2$
$D_1(\alpha = 2, \beta = 1)$	$A_1 > A_4 > A_3 > A_2$
$D_1(\alpha = 1.5, \beta = 1.5)$	$A_1 > A_4 > A_2 > A_3$

The algorithm is implemented step-by-step as follows:

Step1: Build the DHF decision matrix $D = [a_{ij}]_{5\times4}$ with the decision-maker's ratings in it. In Table 5 , the fuzzy decision matrix is displayed.

Step2: Apply the optimistic principle to ensure that every DHFE has the same length by repeating the maximum value in each DHFE. The outcomes are displayed in Table 6.

Step3: Since every attribute belongs to the benefit type, Table 6's normalised decision matrix corresponds to it.

Step4: Using Eqs.(8) and (9) determine the DHFPIS and DHFNIS; the findings are displayed in Table 7.

Step5: Using Eq. (7) and the DHF knowledge measure Eq.(5), determine the attribute weights. The findings are displayed in Table 8.

Step6: Utilizing equations (8) and (9) compute DHFPIS and DHFNIS; the results are displayed in Tables 9 and 10. We conducted a comparison between the outcomes produced with our proposed DHF accuracy measure and the results produced by the currently used DHF distance measurement. We use the following comparison measure as a point of reference.

$D_\alpha(A,B) = (\sum_{j=1}^{m} w_j\{\frac{1}{2l_{x_i}}\sum_{i=1}^{l_{x_i}} |h_A^{\sigma(j)} - h_B^{\sigma(j)}|^\alpha + \frac{1}{2l_{x_i}}\sum_{i=1}^{l_{x_i}} |k_A^{\sigma(j)} - k_B^{\sigma(j)}|^\alpha\}); \alpha > 0$ Su et al. [?]

Step7: Using Equation (10), get the closeness coefficient. The findings are displayed in Tables 11 and 12.

Step8: Table 13 and Fig. 2, 3, 4 and 5 presents the ranking of all the alternatives based on the closeness coefficient in descending order. Table 13 and Fig.2, 4 shows that, with our proposed DHF weighted accuracy measure, the optimum alternative stays the same for a range of α and β values; the parameter changes, though. However, even when the value of parameter α and β are changed, the overall ranking of the alternatives change for the current distance measure as shown in Fig.3 and 5.

6 Detecting patterns using the suggested DHF accuracy measure

Next, the accuracy measure is applied to address the pattern detection problem with DHF-set.

Problem: Examining n patterns, which are represented by a DHF-set

$R_j = \{\langle u_i, h_{R_i}^{\sigma(j)}(u_i), k_{R_i}^{\sigma(j)}(u_i) : u_i \in U\rangle\}, (i = 1,2,\ldots,n)$ defined on non empty set $U = \{u_1, u_2, \ldots, u_n\}$. Consider any unknown pattern $T = \{\langle u_i, h_T^{\sigma(j)}(u_i), k_T^{\sigma(j)}(u_i) : u_i \in U\rangle\}$. The objective is to assign pattern T to one of the identified patterns R_i the following methods to solve the above mentioned issue.

Similarity/ accuracy measure based detection: If $S(R_i, T)$ be the T's similarity/ accuracy pattern from R_i., at that point T is identified as pattern R_{i^*}, where

$S(T, R_{i^*}) = max_{i=1,2,\ldots,n}\{S(T, R_i)\}$.

Dissimilarity measure based detection: If $D(R_i, T)$ represents distance of pattern T from R_i. Then T is recognized as pattern R_{i^*},

Where $D(T, R_{i^*}) = min_{i=1,2,\ldots,n}\{D(T, R_i)\}$, We identify the pattern with our proposed DHF accuracy measure in the empirical investigations that follow, and we compare the recognition outcomes with well-known DHF distance and similarity measures.The distance measures $D_i(A,B)(i=1,2)$ and similarity measures $S_i(A,B)(i=1,2)$ that are currently in use are first listed as follows:

$D_1(A,B)=\frac{1}{2n}\sum_{i=1}^{n}\{\frac{1}{l_{x_i}}\sum_{j=1}^{l_{x_i}}|h_A^{\sigma(j)}-h_B^{\sigma(j)}|+\frac{1}{m_{x_i}}\sum_{j=1}^{m_{x_i}}|k_A^{\sigma(j)}-k_B^{\sigma(j)}|\}$, Singh[31]

$D_2(A,B)=\sum_{i=1}^{n}\{\frac{1}{n}(\frac{1}{l_{x_i}}\sum_{j=1}^{l_{x_i}}|h_A^{\sigma(j)}-h_B^{\sigma(j)}|^{\alpha}+\frac{1}{m_{x_i}}\sum_{j=1}^{m_{x_i}}|k_A^{\sigma(j)}-k_B^{\sigma(j)}|^{\alpha})\}^{\frac{1}{\alpha}}$, Wang et al.[29]

$S_1(A,B)=\frac{1}{2n}\sum_{i=1}^{n}\frac{\sum_{j=1}^{l_{x_i}}[min\{|h_A^{\sigma(j)},h_B^{\sigma(j)}|\}]+\sum_{j=1}^{m_{x_i}}[min\{|k_A^{\sigma(j)},k_B^{\sigma(j)}|\}]}{\sum_{j=1}^{l_{x_i}}[max\{|h_A^{\sigma(j)},h_B^{\sigma(j)}|\}]+\sum_{j=1}^{m_{x_i}}[max\{|k_A^{\sigma(j)},k_B^{\sigma(j)}|\}]}$, Singh [31]

$S_2(A,B)=\frac{1}{2n}\frac{\sum_{j=1}^{n}(h_A^{\sigma(j)},h_B^{\sigma(j)})+\sum_{j=1}^{n}(k_A^{\sigma(j)},k_B^{\sigma(j)})}{max\{\sum_{j=1}^{n}(h_A^{\sigma(j)})^2+(h_B^{\sigma(j)})^2,\sum_{j=1}^{n}(k_A^{\sigma(j)})^2+(k_B^{\sigma(j)})^2\}}$, Singh [31]

Example 5 Let $R_1=\{\langle\{.2,.3,.3\},\{.3,.5,.5\}\rangle,\langle\{.6,.7,.8\},\{.5,.5,.5\}\rangle\}$,

$R_2=\{\langle\{.1,.3,.3\},\{.9,.9,.9\}\rangle,\langle\{.7,.9,.9\},\{.7,.7,.7\}\rangle\}$,

$R_3=\{\langle\{.5,.5,.5\},\{.3,.4,.5\}\rangle,\langle\{.3,.7,.7\},\{.4,.5,.5\}\rangle\}$ be three known patterns.

Let $T=\{\langle\{.4,.6,.6\},\{.7,.8,.8\}\rangle,\langle\{.3,.3,.3\},\{.7,.7,.7\}\rangle\}$ be unknown pattern. We use a dissimilarity technique to classify T to R_i.

Table 14: Pattern detection using I_{acc} and D_1

	(R_1,T)	(R_2,T)	(R_3,T)
$I_{acc}(\alpha=1.5,\beta=1)$	.1832	.2666	.5674
$D_1(\alpha=1.5,\beta=1)$	.2417	.3000	.2417

Table 14 shows, $I_{acc}=(R_3,T)$ is maximum. Therefore T is classified to R_1. But dissimilarity measure D_1 is unable to classify T to R_i because $D_1(R_1,T)$ and $D_1(R_3,T)$ have same values.

Example 6 Let $R_1=\{\langle\{.3,.4,.4\},\{.1,.2,.3\}\rangle,\langle\{.3,.4,.4\},\{.1,.2,.3\}\rangle\}$, $R_2=\{\langle\{.4,.5,.5\},\{.2,.3,.5\}\rangle,\langle\{.7,.8,.9\},\{.2,.5,.6\}\rangle\}$, $R_3=\{\langle\{.5,.6,.6\},\{.3,.3,.3\}\rangle,\langle\{.35,.4,.4\},\{.5,.6,.8\}\rangle\}$ be three known patterns.

Let $T=\{\langle\{.5,.6,.7\},\{.8,.8,.8\}\rangle,\langle\{.4,.5,.6\},\{.2,.3,.4\}\rangle\}$ be unknown pattern. We use a dissimilarity technique to classify T to R_i.

Table 15: Pattern detection using I_{acc} and D_2

	(R_1,T)	(R_2,T)	(R_3,T)
$I_{acc}(\alpha=1.5,\beta=1)$	.1288	.3918	.2882

$D_2(\alpha = 1.5, \beta = 1)$	.4492	.4678	.4492

Table 15 shows, $I_{acc} = (R_3, T)$ is maximum. Therefore T is classified to R_1. But dissimilarity measure D_2 is unable to classify T to R_i because $D_2(R_1, T)$ and $D_2(R_3, T)$ have same values.

Example 7 Let $R_1 = \{\langle\{.7, .8, .9\}, \{.1, .2, .3\}\rangle, \langle\{.3, .4, .4\}, \{.1, .2, .3\}\rangle\}$,

$R_2 = \{\langle\{.4, .5, .5\}, \{.2, .3, .5\}\rangle, \langle\{.7, .8, .9\}, \{.2, .5, .5\}\rangle\}$,

$R_3 = \{\langle\{.1, .2, .2\}, \{.4, .5, .6\}\rangle, \langle\{.4, .5, .5\}, \{.5, .6, .8\}\rangle\}$ be three known patterns.

Let $T = \{\langle\{.2, .6, .7\}, \{.6, .7, .8\}\rangle, \langle\{.2, .5, .5\}, \{.1, .2, .3\}\rangle\}$ be unknown pattern. We use a similarity technique to classify T to R_i.

Table 16: Pattern detection using I_{acc} and S_1

	(R_1, T)	(R_2, T)	(R_3, T)
$I_{acc}(\alpha = 1.5, \beta = 1)$	.3354	.3257	.2997
$S_1(\alpha = 1.5, \beta = 1)$	.1885	.1573	.1885

Table 16 shows, $I_{acc} = (R_2, T)$ is maximum. Therefore T is classified to R_2. But dissimilarity measure S_1 is unable to classify T to R_i because $S_1(R_1, T)$ and $S_1(R_3, T)$ have same values.

Example 8 Let $R_1 = \{\langle\{.6, .7, .8\}, \{.1, .2, .2\}\rangle, \langle\{.4, .5, .5\}, \{.2, .2, .2\}\rangle\}$,

$R_2 = \{\langle\{.3, .4, .5\}, \{.5, .6, .7\}\rangle, \langle\{.6, .7, .8\}, \{.2, .2, .2\}\rangle\}$,

$R_3 = \{\langle\{.5, .6, .7\}, \{.3, .5, .7\}\rangle, \langle\{.5, .5, .5\}, \{.4, .6, .7\}\rangle\}$ be three known patterns.

Let $T = \{\langle\{.2, .5, .6\}, \{.6, .8, .8\}\rangle, \langle\{.6, .6, .6\}, \{.2, .3, .4\}\rangle\}$ be unknown pattern. We use a similarity technique to classify T to R_i.

Table 17: Pattern detection using I_{acc} and S_2

	(R_1, T)	(R_2, T)	(R_3, T)
$I_{acc}(\alpha = 1.5, \beta = 1)$	.1877	.3911	.5275
$S_2(\alpha = 1.5, \beta = 1)$	.2245	.1514	.2245

Table 17 shows, $I_{acc} = (R_3, T)$ is maximum. Therefore T is classified to R_3. But dissimilarity measure S_2 is unable to classify T to R_i because $S_2(R_1, T)$ and $S_2(R_3, T)$ have same values.

Comparative examinations of similarity and dissimilarity metrics show that no measure is appropriate for every pattern identification task in examples $4-7$. Consequently, an alternate approach is needed for problems involving pattern identification. The suggested accuracy measure might be more effective than the current similarity and dissimilarity measures in particular pattern recognition scenarios. Here, we offer a measure to clearly classify a pattern T into one of the existing patterns. Therefore, for this pattern detection problem, the suggested accuracy measure technique performs effectively.

7 Conclusion

The membership and non-membership degrees of the DHFS are represented by two sets of possible values. The DHFS is a comprehensive set that encompasses various existing sets. It appears to be a more versatile strategy and has its own set of desirable qualities and advantages. It is found that in the objective weight computation tasks, the DHF knowledge measure suggested in this research outperforms the traditional DHF entropy measures. The current study provides four instances to evaluate the efficacy of the proposed DHF - Knowledge measure. While dealing with MADM situations where the attribute weights are unknown, the suggested knowledge measure for DHFSs is more appropriate and very useful in a variety of scenarios. We also use the proposed DHF accuracy metric in pattern detection and compare its performance with that of many other measures. Compared to the traditional dual-hesitant measures of comparison, the suggested DHF accuracy measure is more effective in identifying the unknown pattern. Numerous fields, such as speech recognition, picture thresholding, and feature recognition, can get benefit from the application of the recommended knowledge and accuracy metrics.

Refrences

[1] L. Zadeh, "Fuzzy sets," Inform Control, vol. 8, pp. 338–353, 1965.
[2] D. Dubois, W. Ostasiewicz, and H. Prade, "Fuzzy sets: history and basic notions," Fundamentals of fuzzy sets, pp. 21–124, 2000.
[3] K. T. Atanassov, Intuitionistic fuzzy sets. Springer, 1999.
[4] H. Bustince, J. Fernandez, A. Kolesarova, and R. Mesiar, "Generation of linear orders for intervals by means of aggregation functions," Fuzzy Sets and Systems, vol. 220, pp. 69–77, 2013.
[5] I. B. Turksen, "Interval valued fuzzy sets based on normal forms," Fuzzy sets and systems, vol. 20, no. 2, pp. 191–210, 1986.
[6] D. E. Boekee and J. C. Van der Lubbe, "The r-norm information measure," Information and control, vol. 45, no. 2, pp. 136–155, 1980.
[7] D. Bhandari and N. R. Pal, "Some new information measures for fuzzy sets," Information Sciences, vol. 67, no. 3, pp. 209–228, 1993.
[8] M. Xia and Z. Xu, "Hesitant fuzzy information aggregation in decision making," International journal of approximate reasoning, vol. 52, no. 3, pp. 395–407, 2011.
[9] Z. Xu and M. Xia, "Distance and similarity measures for hesitant fuzzy sets," Information Sciences, vol. 181, no. 11, pp. 2128–2138, 2011.
[10] V. Torra, "Hesitant fuzzy sets," International journal of intelligent systems, vol. 25, no. 6, pp. 529–539, 2010.
[11] Z. Xu and M. Xia, "Hesitant fuzzy entropy and cross-entropy and their use in multiattribute decisionmaking," International Journal of Intelligent Systems, vol. 27, no. 9, pp. 799–822, 2012.
[12] Z. Xu and X. Zhang, "Hesitant fuzzy multi-attribute decision making based on topsis with incomplete weight information," Knowledge-Based Systems, vol. 52, pp. 53–64, 2013.
[13] B. Zhu, Z. Xu, and M. Xia, "Dual hesitant fuzzy sets," Journal of Applied mathematics, vol. 2012, pp. 1–13, 2012.
[14] J. Ye, "Correlation coefficient of dual hesitant fuzzy sets and its application to multiple attribute decision making," Applied Mathematical Modelling, vol. 38, no. 2, pp. 659–666, 2014.

[15] P. Burillo and H. Bustince, "Entropy on intuitionistic fuzzy sets and on interval-valued fuzzy sets," Fuzzy sets and systems, vol. 78, no. 3, pp. 305–316, 1996.
[16] J. Mao, D. Yao, and C. Wang, "A novel cross-entropy and entropy measures of ifss and their applications," Knowledge-Based Systems, vol. 48, pp. 37–45, 2013.
[17] N. Zhao and Z. Xu, "Entropy measures for dual hesitant fuzzy information," in 2015 Fifth international conference on communication systems and network technologies, pp. 1152–1156, IEEE, 2015.
[18] Z. Su, Z. Xu, H. Liu, and S. Liu, "Distance and similarity measures for dual hesitant fuzzy sets and their applications in pattern recognition," Journal of Intelligent & Fuzzy Systems, vol. 29, no. 2, pp. 731–745, 2015.
[19] J. Ye, "Cross-entropy of dual hesitant fuzzy sets for multiple attribute decision-making," International Journal of Decision Support System Technology (IJDSST), vol. 8, no. 3, pp. 20–30, 2016.
[20] R. Wang, W. Li, T. Zhang, and Q. Han, "New distance measures for dual hesitant fuzzy sets and their application to multiple attribute decision making," Symmetry, vol. 12, no. 2, p. 191, 2020.
[21] H. Zhang, "Distance and entropy measures for dual hesitant fuzzy sets," Computational and Applied Mathematics, vol. 39, pp. 1–16, 2020.
[22] E. Szmidt, J. Kacprzyk, and P. Bujnowski, "How to measure the amount of knowledge conveyed by atanassov's intuitionistic fuzzy sets," Information Sciences, vol. 257, pp. 276–285, 2014.
[23] H. Nguyen, "A new knowledge-based measure for intuitionistic fuzzy sets and its application in multiple attribute group decision making," Expert Systems with Applications, vol. 42, no. 22, pp. 8766–8774, 2015.
[24] R. Joshi, "Multi-criteria decision making based on novel fuzzy knowledge measures," Granular Computing, vol. 8, no. 2, pp. 253–270, 2023.
[25] G. Ram, S. Kumar, et al., "Information measure for hesitant fuzzy sets," in 2023 First International Conference on Advances in Electrical, Electronics and Computational Intelligence (ICAEECI), pp. 1–6, IEEE, 2023.
[26] A. Singh and S. Kumar, "Intuitionistic fuzzy entropy-based knowledge and accuracy measure with its applications in extended vikor approach for solving multi-criteria decision-making," Granular Computing, vol. 8, no. 6, pp. 1609–1643, 2023.
[27] S. Singh, "Knowledge and accuracy measure based on dual-hesitant fuzzy sets with application to pattern recognition and site selection for solar power plan t," Granular Computing, vol. 8, no. 1, pp. 157–170, 2023.
[28] J. Bajaj and S. Kumar, "Pythagorean fuzzy intuitive distance measure with its applications in madm issues," Evolutionary Intelligence, pp. 1–16, 2024.
[29] L. Wang, Q. Wang, S. Xu, and M. Ni, "Distance and similarity measures of dual hesitant fuzzy sets with their applications to multiple attribute decision making," in 2014 IEEE international conference on progress in informatics and computing, pp. 88–92, IEEE, 2014.
[30] T.-Y. Chen and C.-H. Li, "Determining objective weights with intuitionistic fuzzy entropy measures: a comparative analysis," Information Sciences, vol. 180, no. 21, pp. 4207–4222, 2010.
[31] P. Singh, "Distance and similarity measures for multiple-attribute decision making with dual hesitant fuzzy sets," Computational and Applied Mathematics, vol. 36, pp. 111–126, 2017.