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Solving the Type 2 Trapezoidal Intuitionistic Fuzzy Fractional Transportation Problem Using the Diagonal Optimal Approach

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Abstract:

This research develops a diagonal optimal technique for solving the Type 2 Trapezoidal Intuitionistic Fuzzy Fractional Transportation Problem (T2TIFFTP), aiming to minimize transportation costs within an optimal solution framework. By incorporating the intuitionistic fuzzy fractional approach, the method effectively addresses the uncertainty and imprecision commonly encountered in real-world transportation scenarios. Numerical results demonstrate the technique's efficiency and effectiveness. This highlighting its potential to significantly enhance the accuracy and quality of solutions in complex transportation situations. The diagonal optimal technique provides a robust framework for tackling fuzzy transportation problems, paving the way for more reliable and cost-efficient decision-making in uncertain environments.

Keywords: Fuzzy fractional transportation problem, trapezoidal intuitionistic fuzzy number, diagonal approach, optimal solution.

1. Introduction

The field of operations research (OR) uses advanced analytical methods to improve efficiency in decision-making. It utilizes tools from computer science, statistics, and mathematics to resolve complicated problems across a range of sectors, including manufacturing, banking, logistics, and healthcare. Determining the most economical way to get goods from a group of suppliers to a group of customers is the aim of the transportation problem, a type of optimization problem in OR. The goal is to meet supply and demand constraints while minimizing the overall cost of transportation. A generalization of classical fuzzy sets, intuitionistic fuzzy sets (IFS) are designed to address scenarios in which there is a great deal of uncertainty and hesitation about establishing membership and non-membership degrees. An expansion of the basic transportation problem, the trapezoidal intuitionistic fuzzy fractional transportation problem (TIFFTP) involves representing the parameters (supply and demand) as trapezoidal intuitionistic fuzzy numbers (TIFNs). By addressing ambiguity and uncertainty more skillfully, this method offers a more adaptable and accurate model for transportation-related problems. By adding Type 2 intuitionistic fuzzy sets and fractional programming to the classical transportation problem, an advanced mathematical model known as the Type 2 intuitionistic fuzzy fractional transportation problem (T2IFFTP) is created. When there is a lot of ambiguity and uncertainty in the decision-making process, this approach is especially efficient. The structure of this paper is as follows: Section 2 addresses the literature review of the presented problem. The paper's preliminary context is provided in Section 3. The problem with formulation and the several stages used in the proposed approach are explained in Section 4. The optimal solution for the T2TIFFTP can be obtained by solving the illustrative example in Section 5. Results are presented in Section 6, and Section 7 concludes.

2. Literature review

A survey of the literature on intuitionistic fuzzy sets, fractional programming, transportation problems, and the interactions of these topics is necessary to understand the T2IFFTP. The following is a formal framework for this kind of literature review. Hitchcock [8] was the one who initially came up with the fundamental transportation problem. Zadeh [19] introduced the fuzzy set, which has been described by membership degree. Atanassov [3] introduced the IFS theory in 1986 to help with uncertainty and hesitation. [13] formulated a transportation problem where the real, fuzzy, and intuitionistic fuzzy numbers represent the costs, supply, and demands, respectively. In 2012, Kumar and Hussain [9] introduced a method for resolving the intuitionistic fuzzy transportation problem (IFTP). The triangular intuitionistic fuzzy numbers have been ordered using a suggested accuracy function by Singh and Yadav [17], and this work has been extended to an IFTP of type 2. Researchers Shukla [16], Taghikhani [18], and Fu [6] have all examined various decision-making problems in the wider picture of T2IFS. The idea of the TIFFTP was introduced by SK Bharati [4], along with an approach for solving it that utilizes the expectation of trapezoidal intuitionistic fuzzy numbers. Gupta (Aggarwal and Gupta [1]; Anupum and Gupta [7]) demonstrated how to solve the intuitionistic fuzzy solid transportation problem using a novel ranking technique based on signed distance, as well as an effective way for addressing the intuitionistic fuzzy transportation problem of type 2. The diagonal optimum technique was established by Khalid [12] for the assignment problem (AP). Fuzzy AP was resolved by Dhanasekar [5] through the diagonal optimum algorithm and Vogel's approximation technique (VAM). It is suggested to use the diagonal optimum approach in [[14], [15]] to tackle fully fuzzy transportation problems. [10] created and presented a type 2 fuzzy set extension known as the generalized symmetric type 2 intuitionistic fuzzy set. This study provides a practical method for handling the T2IFFTP, which may be compared to the work of [2]. Thus, an algorithm known as the diagonal optimal method is investigated in this work while keeping in mind everything that was previously described and motivated by [11].

3. Preliminary

This section provides essential background and definitions pertaining to fuzzy set theory.

Definition 3.1 Consider T to be a nonempty set. An IFS $\hat{\Theta}$ in T is defined as an object of the form $\hat{\Theta} = \{(t, \mu_{\hat{\Theta}}(t), \nu_{\hat{\Theta}}(t)) \mid t \in T\}$, where the functions, $\mu_{\hat{\Theta}}(t) : T \to [0,1]$ and $\nu_{\hat{\Theta}}(t) : T \to [0,1]$ define the degree of membership and non-membership of the element $t \in T$ respectively, and $0 \leq \mu_{\hat{\Theta}}(t), \nu_{\hat{\Theta}}(t) \leq 1$ for every $t \in T$.

Definition 3.2 If a subset of the real line $\hat{\Theta} = \{(t, \mu_{\hat{\Theta}}(t), \nu_{\hat{\Theta}}(t)) \mid t \in T\}$ is held to be an intuitionistic fuzzy number, then it is referred to as an intuitionistic fuzzy number that satisfies the criteria listed below.

i) There exist $r_1 \in R$, $\mu_{\hat{\Theta}}(r_1) = 1$ and $\nu_{\hat{\Theta}}(r_1) = 0$.

ii) Assuming that $\mu_{\hat{\Theta}}(t) : R \to [0,1]$ is continuous, for every $t \in R$, $0 \le \mu_{\hat{\Theta}}(t), \nu_{\hat{\Theta}}(t) \le 1$ holds.

The membership and non-membership functions of $\hat{\Theta}$ are as follows:

$$\mu_{\hat{\Theta}}(t) = \begin{cases} k_1(t); t \in [r_1 - \tau_1, r_1] \\ 1; \quad t = r_1 \\ l_1(t); t \in (r_1, r_1 + \sigma_1] \\ 0; \ Otherwise \end{cases}$$

and

$$\nu_{\hat{\Theta}}(t) = \begin{cases} 1; \quad t \in (-\infty, r_1 - \tau_2) \\ k_2(t); \quad t \in [r_1 - \tau_2, r_1] \\ 0; \ t = r_1, t \in [r_1 + \sigma_2, \infty) \\ l_2(t); \ t \in (r_1, r_1 + \sigma_2] \end{cases}$$

where $k_i(t)$ and $l_i(t)$ (for i = 1, 2) are strictly increasing and decreasing functions in $[r_1 - \tau_i, r_1)$ and $(r_1, r_1 + \sigma_i]$, respectively. The left and right spreads of $\mu_{\hat{\Theta}}(t)$ and $\nu_{\hat{\Theta}}(t)$.

Definition 3.3 Let $\hat{C} = ((m_1, m_2, m_3, m_4), (m_1', m_2', m_3', m_4'))$ represents a trapezoidal intuitionistic fuzzy number, where $(m_1' \le m_1 \le m_2 \le m_3 \le m_4 \le m_4')$ has membership and non-membership functions described as follows:

$$\mu_{\hat{C}}(t) = \begin{cases} \frac{t - m_1}{m_2 - m_1}; \ m_1 \le t \le m_2 \\ 1; \quad m_2 \le t \le m_3 \\ \frac{m_4 - t}{m_4 - m_3}; \ m_3 \le t \le m_4 \\ 0; \ otherwise. \end{cases}$$

and

$$\nu_{\hat{C}}(t) = \begin{cases} \frac{m_2 - t}{m_2 - m_1'}; \ m_1' \le t \le m_2 \\ 0; \quad m_2 \le t \le m_3 \\ \frac{t - m_3}{m_4' - m_3}; \ m_3 \le t \le m_4' \\ 1; \ otherwise. \end{cases}$$

Definition 3.4 Let the two generalized trapezoidal intuitionistic fuzzy numbers be and $\hat{C} = \{(m_1', m_2', m_3', m_4',), (n_1', n_2', n_3', n_4'); \ \xi_A, \psi_A\}$ and $\hat{D} = \{(p_1', p_2', p_3', p_4'), (q_1', q_2', q_3', q_4'); \ \xi_B, \psi_B\}$.

The following arithmetic operations appear:

i) Addition: $\hat{C} + \widehat{D} = \{(m_1' + p_1', m_2' + p_2', m_3' + p_3', m_4' + p_4')\ (n_1' + q_1', n_2' + q_2', n_3' + q_3', n_4' + q_4'); \xi, \psi\}$ where $\xi = \min(\xi_A, \xi_B)$ and $\psi = \max(\psi_A, \psi_B)$.

ii) Subtraction: $\hat{C} - \widehat{D} = \{(m_1' - p_4', m_2' - p_3', m_3' - p_2', m_4' - p_1')\ (n_1' - q_4', n_2' - q_3', n_3' - q_2', n_4' - q_1'); \xi, \psi\}$ where $\xi = \min(\xi_A, \xi_B)$ and $\psi = \max(\psi_A, \psi_B)$.

iii) Multiplication: $\hat{C} * \widehat{D} = \{(m_1' * p_1', m_2' * p_2', m_3' * p_3', m_4' * p_4')\ (n_1' * q_1', n_2' * q_2', n_3' * q_3', n_4' * q_4'); \xi, \psi\}$ where $\xi = \min(\xi_A, \xi_B)$ and $\psi = \max(\psi_A, \psi_B)$.

iv) Scalar Multiplication: $\pi\hat{C} = \{(\pi m_1', \pi m_2', \pi m_3', \pi m_4',), (\pi n_1', \pi n_2', \pi n_3', \pi n_4'); \xi_A, \psi_A\}$ if $\pi > 0$= $\{(\pi m_4', \pi m_3', \pi m_2', \pi m_1',), (\pi n_4', \pi n_3', \pi n_2', \pi n_1'); \xi_A, \psi_A\}$ if $\pi < 0$.

v) Division: $\hat{C} \div \widehat{D} = \{(m_1' \div p_4', m_2' \div p_3', m_3' \div p_2', m_4' \div p_1')\ (n_1' \div q_4', n_2' \div q_3', n_3' \div q_2', n_4' \div q_1'); \xi, \psi\}$ where $\xi = \min(\xi_A, \xi_B)$ and $\psi = \max(\psi_A, \psi_B)$.

Let $\hat{C} = \{(m_1', m_2', m_3', m_4',), (n_1', n_2', n_3', n_4'); \xi_A, \psi_A\}$. In such C = $(m_2' = m_3' = n_2' = n_3')$:

At this point, the membership function is described as follows:

$$F(\chi_C) = \widehat{x_0}, \widehat{y_0} = \left(\frac{2m_1{\prime} + 7m_2{\prime} + 7m_3{\prime} + 2m_4{\prime}}{18}\right) * \left(\frac{7\xi_A}{18}\right).$$

Likewise, the non-membership function is described as follows:

$$F(\varphi_C) = \widehat{x_0}, \widehat{y_0} = \left(\frac{2n_1{\prime} + 7n_2{\prime} + 7n_3{\prime} + 2n_4{\prime}}{18}\right) * \left(\frac{11 + 7\psi_A}{18}\right).$$

Then, utilizing them, we define the rank as follows:

$$\mathscr{R}(C) = \frac{\xi_A F(\chi_C) + \psi_A F(\varphi_C)}{\xi_A + \psi_A}.$$

For the purpose of comparing two ranking generalized intuitionistic trapezoidal fuzzy numbers, $\hat{C}$ and $\widehat{D}$ in the ranking method.

i) $\hat{C} > \widehat{D}$ if $\mathscr{R}(\hat{C}) > \mathscr{R}(\widehat{D})$.

ii) $\hat{C} \leq \widehat{D}$ if $\mathscr{R}(\hat{C}) \leq \mathscr{R}(\widehat{D})$.

iii) $\hat{C} = \widehat{D}$ if $\mathscr{R}(\hat{C}) = \mathscr{R}(\widehat{D})$.

4. Problem-making

a) Fractional Transportation Problem (FTP)

The FTP involves minimizing interval-valued objective functions with interval costs. Here, the objective function coefficients $\frac{E_{ij}^{v}}{F_{ij}^{v}}$, represent source parameters C_i and destination parameters D_j, while E_v is the conveyance parameter. These coefficients are in interval form, where $C_i = [g_{L_i'}, g_{R_i'}]$ for (i = 1, 2, ,.., p) and $D_j = [h_{L_j'}, h_{R_j'}]$, for (j = 1, 2, ,.., q). The formulation is as follows:

$$\text{Min } Z^{v}(t_{ij}^{*}) = \frac{\sum_{i=1}^{p}\sum_{j=1}^{q}[E_{L_{ij}}^{v}, E_{R_{ij}}^{v}]t_{ij}^{*} + \delta}{\sum_{i=1}^{p}\sum_{j=1}^{q}[F_{L_{ij}}^{v}, F_{R_{ij}}^{v}]t_{ij}^{*} + \epsilon}, v = 1, 2, , . . , V$$

subject to the constraints

$$\sum_{j=1}^{q} t_{ij}^{*} = C_i = [g_{L_i'}, g_{R_i'}], (i = 1,2, , . . , p)$$

$$\sum_{i=1}^{p} t_{ij}^{*} = D_j = [h_{L_j'}, h_{R_j'}], (j = 1,2, , . . , q)$$

$$t_{ij} \geq 0, \forall \text{ i, j.}$$

A necessary and sufficient condition for the existence of a feasible solution is the balanced criterion.

$$[E_{L'_{ij}}^{v}, E_{R'_{ij}}^{v}]\ [F_{L'_{ij}}^{v}, F_{R'_{ij}}^{v}], (v = 1, 2, , . . , V),$$

$$W_{ij}^{v} = \left[W_{L'_{ij}}^{v}, W_{R'_{ij}}^{v}\right] = \frac{E_{ij}^{v}}{F_{ij}^{v}} = \frac{\left[E_{L'_{ij}}^{v}, E_{R'_{ij}}^{v}\right]}{\left[F_{L'_{ij}}^{v}, F_{R'_{ij}}^{v}\right]}$$

represents the uncertain cost interval for the transportation problem.

b. Type 2 intuitionistic fuzzy fractional transportation problem

The proposed strategy effectively determines the optimal solution for an intuitionistic fuzzy fractional transportation problem with real demand, supply, and transportation costs $\frac{e_{ij}^{l}}{f_{ij}^{l}}$ where i = 1, 2, ... p and j = 1, 2, ... q from the ith source to the jth destination, as outlined in Table 1.

We calculate the optimum solutions for the numerator and denominator separately, then divide the results to obtain the optimal solution for the T2IFFTP.

Table 1: T2IFFTP

	D_1'	D_2'	D_3'	D_3'	s_i'
S_1'	$\frac{e_{11}^l}{f_{11}^l}$	$\frac{e_{12}^l}{f_{12}^l}$	…	$\frac{e_{1q}^l}{f_{1q}^l}$	s_1'
S_2'	$\frac{e_{21}^l}{f_{21}^l}$	$\frac{e_{22}^l}{f_{22}^l}$	…	$\frac{e_{2q}^l}{f_{2q}^l}$	s_2'
…	…	…	…		
S_p'	$\frac{e_{p1}^l}{f_{p1}^l}$	$\frac{e_{p2}^l}{f_{p2}^l}$	…	$\frac{e_{pq}^l}{f_{pq}^l}$	s_p'
d_j'	d_1'	d_2'	…	…	d_q'

5. Algorithm

This section discusses our proposed algorithm. Influenced by the research of [11]. The following is a list of the stages that have to be taken to determine the optimal solution.

Step 1: Identify the two cells with the lowest cost and the next lowest cost in each row for the T2IFFTP. Calculate their difference (penalty) along the side of the table against the corresponding row. Repeat the same process for the columns.

Step 2: Select the highest penalty from each column and row. In the case of a tie, use the tie-breaking option. Choose a row or column. Continue this process until each row or column has a value assigned to it.

Step 3: Let A_{ij}^* be the assignment cost for column j. Calculating each cost c_j^* in the relevant cost matrix column by subtracting A_{ij}^*.

Step 4: Identify a rectangle for every unassigned cell, with two corners assigned and the other two corners unassigned. Calculate the value of $d_{ij}^{'}$ by summing the diagonals of the non-assigned cells. If all $d_{ij}^{'}$ are positive, proceed to stage 6.

Otherwise, move to stage 5.

Step 5: Select the most negative d_{ij} and exchange the assigned cell of the diagonals.

Repeat the process until all $d_{ij}^{'} > 0$.

Step 6: Allocate the maximum feasible amount from the given supplies and demands to the assigned cells. For non-assigned cells, allocate the remaining supply and demand to the cell with the lowest cost. Continue this process until there is no more supply or demand.

Step 7: Once the problem has (m+n-1) basic variables, the initial basic feasible solution is found.

Step 8: We determined the optimal value using stage 7, and the optimal value of a fuzzy fractional intuitionistic is = $\sum_{i=1}^{p}\sum_{j=1}^{q}(\frac{e_{ij}^{l}}{f_{ij}^{l}}) * (t_{ij}^{*})$.

6. **Numerical illustration**

According to [2], the T2TIFFTP is applied to confirm the results of the proposed computational approach. First, we ascertained the numerator's values.

These are the steps that are provided. Determine the penalties for each row and column, then apply the corresponding penalty to each row and column as shown in the table.

Table 2: T2TIFFTP

	D_1'	D_2'	D_3'	D_4'	s_i'
S_1'	(4,5,5,7)(2,5,5,9);0.6,0.3) / ((2,3,3,7)(1,3,3,8);0.5,0.2)	(3,4,4,7)(1,4,4,8);0.5,0.1) / ((2,4,4,5)(1,4,4,8);0.4,0.1)	((5,7,7,10)(3,7,7,12);0.9,0.5) / ((5,6,6,9)(3,6,6,11);0.3,0.1)	(7,9,9,11)(6,9,9,14);0.4,0.2) / ((5,7,7,10)(4,7,7,12);0.4,0.2)	14; 12
S_2'	((4,6,6,10)(2,6,6,12);0.5,0.3) / ((5,6,6,9)(4,6,6,12);0.6,0.3)	((7,8,8,13)(3,8,8,14);0.6,0.4) / ((2,5,5,6)(1,5,5,8);0.5,0.2)	((8,10,10,17)(4,10,10,18);0.7,0.6) / ((4,7,7,10)(2,7,7,13);0.4,0.1)	((9,11,11,13)(6,11,11,14);0.8,0.6) / ((9,12,12,14)(7,12,12,16);0.3,0.1)	15; 14
S_3'	((7,10,10,13)(6,10,10,15);0.7,0.2) / ((7,8,8,10)(5,8,8,11);0.4,0.2)	((10,13,13,15)(8,13,13,18);0.8,0.6) / ((8,9,9,12)(6,9,9,14);0.6,0.4)	((7,10,10,14)(6,10,10,15);0.8,0.5) / ((3,4,4,6)(2,4,4,7);0.5,0.3)	((12,14,14,16)(10,14,14,17);0.9,0.4) / ((5,9,9,10)(4,9,9,13);0.2,0.1)	13; 11
S_4'	((6,10,10,12)(5,10,10,13);0.6,0.1) / ((2,4,4,5)(1,4,4,9);0.7,0.5)	((9,13,13,14)(8,13,13,15);0.8,0.5) / ((6,8,8,10)(5,8,8,12);0.6,0.5)	((15,17,17,18)(13,17,17,20);0.9,0.3) / ((9,11,11,13)(7,11,11,15);0.4,0.1)	((8,10,10,12)(7,10,10,14);0.7,0.5) / ((8,10,10,12)(6,10,10,13);0.5,0.3)	10; 12
d_j'	20; 18	12; 11	12; 7	8; 13	

Table 3: T2TIFFTP (Numerator)

	D_1'	D_2'	D_3'	D_4'	s_i'
S_1'	((4,5,5,7)(2,5,5,9);0.6,0.3)	((3,4,4,7)(1,4,4,8);0.5,0.1)	((5,7,7,10)(3,7,7,12);0.9,0.5)	((7,9,9,11)(6,9,9,14);0.4,0.2)	14
S_2	((4,6,6,10)(2,6,6,12);0.5,0.3)	((7,8,8,13)(3,8,8,14);0.6,0.4)	((8,10,10,17)(4,10,10,18);0.7,0.6)	((9,11,11,13)(6,11,11,14);0.8,0.6)	15
S_3'	((7,10,10,13)(6,10,10,15);0.7,0.2)	((10,13,13,15)(8,13,13,18);0.8,0.6)	((7,10,10,14)(6,10,10,15);0.8,0.5)	((12,14,14,16)(10,14,14,17);0.9,0.4)	13
S_4'	((6,10,10,12)(5,10,10,13);0.6,0.1)	((9,13,13,14)(8,13,13,15);0.8,0.5)	((15,17,17,18)(13,17,17,20);0.9,0.3)	((8,10,10,12)(7,10,10,14);0.7,0.5)	10
d_j'	20	12	12	8	

Table 4: T2TIFFTP Penalty Table (Numerator)

	D_1'	D_2'	D_3'	D_4'	Fuzzy Penalty
S_1'	((4,5,5,7)(2,5,5,9);0.6,0.3)	((3,4,4,7)(1,4,4,8);0.5,0.1)	((5,7,7,10)(3,7,7,12);0.9,0.5)	((7,9,9,11)(6,9,9,14);0.4,0.2)	((-3,1,1,4)(-6,1,1,8);0.5,0.3)
S_2'	((4,6,6,10)(2,6,6,12);0.5,0.3)	((7,8,8,13)(3,8,8,14);0.6,0.4)	((8,10,10,17)(4,10,10,18);0.7,0.6)	((9,11,11,13)(6,11,11,14);0.8,0.6)	((-3,2,2,9)(-9,2,2,12);0.5,0.4)
S_3'	((7,10,10,13)(6,10,10,15);0.7,0.2)	((10,13,13,15)(8,13,13,18);0.8,0.6)	((7,10,10,14)(6,10,10,15);0.8,0.5)	((12,14,14,16)(10,14,14,17);0.9,0.4)	((-6,0,0,7)(-9,0,0,9);0.7,0.5)
S_4'	((6,10,10,12)(5,10,10,13);0.6,0.1)	((9,13,13,14)(8,13,13,15);0.8,0.5)	((15,17,17,18)(13,17,17,20);0.9,0.3)	((8,10,10,12)(7,10,10,14);0.7,0.5)	((-4,0,0,6)(-6,0,0,9);0.6,0.5)
Fuzzy penalty	((-3,1,1,6)(-7,1,1,10);.5,.3)	((0,4,4,10)(-5,4,4,13);0.5,0.4)	((-3,3,3,9)(-6,3,3,12);0.8,0.5)	((-3,1,1,5)(-7,1,1,8);0.4,0.5)	

Table 5: T2TIFFTP Penalty Table (Numerator)

	D_1'	D_3'	D_4'	fuzzy penalty
S_2'	((4,6,6,10)(2,6,6,12);0.5,0.3)	((8,10,10,17)(4,10,10,18);0.7,0.6)	((9,11,11,13)(6,11,11,14);0.8,0.6)	((-2,4,4,13)(-8,4,4,16);0.5,0.6)
S_3'	((7,10,10,13)(6,10,10,15);0.7,0.2)	((7,10,10,14)(6,10,10,15);0.8,0.5)	((12,14,14,16)(10,14,14,17);0.9,0.4)	((-6,0,0,7)(-9,0,0,9);0.7,0.5)
S_4'	((6,10,10,12)(5,10,10,13);0.6,0.1)	((15,17,17,18)(13,17,17,20);0.9,0.3)	((8,10,10,12)(7,10,10,14);0.7,0.5)	((-4,0,0,6)(-6,0,0,9);0.6,0.5)
fuzzy penalty	((-4,4,4,8)(-7,4,4,11);0.5,0.3)	((-6,0,0,10)(-11,0,0,12);0.7,0.6)	((-3,1,1,5)(-8,1,1,7);0.7,0.6)	

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Table 6: T2TIFFTP Penalty Table (Numerator)

	D_3'	D_4'	fuzzy penalty
S_3'	((7,10,10,14)(6,10,10,15);0.8,0.5)	((12,14,14,16)(10,14,14,17);0.9,0.4)	((-2,4,4,9)(-5,4,411);0.8,0.5)
S_4'	((15,17,17,18)(13,17,17,20);0.9,0.3)	((8,10,10,12)(7,10,10,14);0.7,0.5)	((3,7,7,10)(-1,7,7,13);0.7,0.5)
fuzzy penalty	((1,7,7,11)(-2,7,7,14);0.8,0.5)	((0,4,4,8)(-4,4,4,10);0.7,0.5)	

Table 7: T2TIFFTP Allocation Table (Numerator)

	D_1'	D_2'	D_3'	D_4'	s_i'
S_1'	((4, 5, 5, 7)(2, 5, 5, 9); 0.6, 0.3)**(2)**	((3, 4, 4, 7)(1, 4, 4, 8); 0.5, 0.1)**(12)**	((5, 7, 7, 10)(3, 7, 7, 12); 0.9, 0.5)	((7, 9, 9, 11)(6, 9, 9, 14); 0.4, 0.2)	14
S_2'	((4, 6, 6, 10)(2, 6, 6, 12); 0.5, 0.3)**(15)**	((7, 8, 8, 13)(3, 8, 8, 14); 0.6, 0.4)	((8, 10, 10, 17)(4, 10, 10, 18); 0.7, 0.6)	((9, 11, 11, 13)(6, 11, 11, 14); 0.8, 0.6)	15
S_3'	((7, 10, 10, 13)(6, 10, 10, 15); 0.7, 0.2)**(1)**	((10, 13, 13, 15)(8, 13, 13, 18); 0.8, 0.6)	((7, 10, 10, 14)(6, 10, 10, 15); 0.8, 0.5)**(12)**	((12, 14, 14, 16)(10, 14, 14, 17); 0.9, 0.4)	13
S_4'	((6, 10, 10, 12)(5, 10, 10, 13); 0.6, 0.1)**(2)**	((9, 13, 13, 14)(8, 13, 13, 15); 0.8, 0.5)	((15, 17, 17, 18)(13, 17, 17, 20); 0.9, 0.3)	((8, 10, 10, 12)(7, 10, 10, 14); 0.7, 0.5)**(8)**	10
d_j'	20	12	12	8	

The maximum penalty in this case is seen in the second column. Choose the column to assign the smallest intuitionistic fuzzy number (shown in bold) to that column. After assigning, eliminate the matching row and column.

$$\begin{bmatrix} ((4,5,5,7)(2,5,5,9);0.6,0.3) & \mathbf{((3,4,4,7)(1,4,4,8);0.5,0.1)} & ((5,7,7,10)(3,7,7,12);0.9,0.5) & ((7,9,9,11)(6,9,9,14);0.4,0.2) \\ ((4,6,6,10)(2,6,6,12);0.5,0.3) & ((7,8,8,13)(3,8,8,14);0.6,0.4) & ((8,10,10,17)(4,10,10,18);0.7,0.6) & ((9,11,11,13)(6,11,11,14);0.8,0.6) \\ ((7,10,10,13)(6,10,10,15);0.7,0.2) & ((10,13,13,15)(8,13,13,18);0.8,0.6) & ((7,10,10,14)(6,10,10,15);0.8,0.5) & ((12,14,14,16)(10,14,14,17);0.9,0.4) \\ ((6,10,10,12)(5,10,10,13);0.6,0.1) & ((9,13,13,14)(8,13,13,15);0.8,0.5) & ((15,17,17,18)(13,17,17,20);0.9,0.3) & ((8,10,10,12)(7,10,10,14);0.7,0.5) \end{bmatrix}$$

The remaining matrix is obtained by eliminating the first row and second column.

$$\begin{bmatrix} ((4,6,6,10)(2,6,6,12);0.5,0.3) & ((8,10,10,17)(4,10,10,18);0.7,0.6) & ((9,11,11,13)(6,11,11,14);0.8,0.6) \\ ((7,10,10,13)(6,10,10,15);0.7,0.2) & ((7,10,10,14)(6,10,10,15);0.8,0.5) & ((12,14,14,16)(10,14,14,17);0.9,0.4) \\ ((6,10,10,12)(5,10,10,13);0.6,0.1) & (15,17,17,18)(13,17,17,20);0.9,0.3) & ((8,10,10,12)(7,10,10,14);0.7,0.5) \end{bmatrix}$$

To obtain Table 4, proceed with steps 1 and 2. Repeat the process until each row or column has a value assigned to it. Subtracting the allocated cost from each element in the relevant column.

$$\begin{bmatrix} ((-6,-1,-1,3)(-10,-1,-1,7);0.5,0.3) & ((-4,0,0,4)(-7,0,0,7);0.5,0.1) & ((-9,-3,-3,3)(-12,-3,-3,6);0.8,0.5) & ((-5,-1,-1,3)(-8,-1,-1,7);0.4,0.5) \\ ((-6,0,0,6)(-10,0,0,10);0.5,0.3) & ((0,4,4,10)(-5,4,4,13);0.5,0.4) & ((-6,0,0,10)(-9,0,0,12);0.7,0.6) & ((-3,1,1,5)(-8,1,1,7);0.7,0.6) \\ ((-3,4,4,9)(-6,4,4,13);0.5,0.3) & ((3,9,9,12)(0,9,9,17);0.5,0.6) & ((-7,0,0,7)(-9,0,0,9);0.8,0.5) & ((0,4,4,8)(-4,4,4,10);0.7,0.5) \\ ((-4,4,4,8)(-7,4,4,11);0.5,0.3) & ((2,9,9,11)(0,9,9,14);0.5,0.5) & ((1,7,7,11)(-2,7,7,14);0.8,0.5) & ((-4,0,0,4)(-7,0,0,7);0.7,0.5) \end{bmatrix}$$

For every non-assigned cell,

$$d_{11}' = \begin{bmatrix} ((-6,-1,-1,3)(-10,-1,-1,7);0.5,0.3) & ((-4,0,0,4)(-7,0,0,7);0.5,0.1) \\ ((-6,0,0,6)(-10,0,0,10);0.5,0.3) & ((0,4,4,10)(-5,4,4,13);0.5,0.4) \end{bmatrix}$$

$$d_{13}' = \begin{bmatrix} ((-4,0,0,4)(-7,0,0,7);0.5,0.1) & ((-9,-3,-3,3)(-12,-3,-3,6);0.8,0.5) \\ ((3,9,9,12)(0,9,9,17);0.5,0.6) & ((-7,0,0,7)(-9,0,0,9);0.8,0.5) \end{bmatrix}$$

$$d_{14}' = \begin{bmatrix} ((-4,0,0,4)(-7,0,0,7);0.5,0.1) & ((-5,-1,-1,3)(-8,-1,-1,7);0.4,0.5) \\ ((2,9,9,11)(0,9,9,14);0.5,0.5) & ((-4,0,0,4)(-7,0,0,7);0.7,0.5) \end{bmatrix}$$

$$d'_{22} = \begin{bmatrix} ((-6,-1,-1,3)(-10,-1,-1,7);0.5,0.3) & ((-4,0,0,4)(-7,0,0,7);0.5,0.1) \\ ((-6,0,0,6)(-10,0,0,10);0.5,0.3) & ((0,4,4,10)(-5,4,4,13);0.5,0.4) \end{bmatrix}$$

$$d'_{23} = \begin{bmatrix} ((-6,0,0,6)(-10,0,0,10);0.5,0.3) & ((-6,0,0,10)(-9,0,0,12);0.7,0.6) \\ ((-3,4,4,9)(-6,4,4,13);0.5,0.3) & ((-7,0,0,7)(-9,0,0,9);0.8,0.5) \end{bmatrix}$$

$$d'_{24} = \begin{bmatrix} ((-6,0,0,6)(-10,0,0,10);0.5,0.3) & ((-3,1,1,5)(-8,1,1,7);0.7,0.6) \\ ((-4,4,4,8)(-7,4,4,11);0.5,0.3) & ((-4,0,0,4)(-7,0,0,7);0.7,0.5) \end{bmatrix}$$

$$d'_{31} = \begin{bmatrix} ((-6,0,0,6)(-10,0,0,10);0.5,0.3) & ((-6,0,0,10)(-9,0,0,12);0.7,0.6) \\ ((-3,4,4,9)(-6,4,4,13);0.5,0.3) & ((-7,0,0,7)(-9,0,0,9);0.8,0.5) \end{bmatrix}$$

$$d'_{32} = \begin{bmatrix} ((-4,0,0,4)(-7,0,0,7);0.5,0.1) & ((-9,-3,-3,3)(-12,-3,-3,6);0.8,0.5) \\ ((3,9,9,12)(0,9,9,17);0.5,0.6) & ((-7,0,0,7)(-9,0,0,9);0.8,0.5) \end{bmatrix}$$

$$d'_{34} = \begin{bmatrix} ((-7,0,0,7)(-9,0,0,9);0.8,0.5) & ((0,4,4,8)(-4,4,4,10);0.7,0.5) \\ ((1,7,7,11)(-2,7,7,14);0.8,0.5) & ((-4,0,0,4)(-7,0,0,7);0.7,0.5) \end{bmatrix}$$

$$d'_{41} = \begin{bmatrix} ((-6,0,0,6)(-10,0,0,10);0.5,0.3) & ((-3,1,1,5)(-8,1,1,7);0.7,0.6) \\ ((-4,4,4,8)(-7,4,4,11);0.5,0.3) & ((-4,0,0,4)(-7,0,0,7);0.7,0.5) \end{bmatrix}$$

$$d'_{42} = \begin{bmatrix} ((-4,0,0,4)(-7,0,0,7);0.5,0.1) & ((-5,-1,-1,3)(-8,-1,-1,7);0.4,0.5) \\ ((2,9,9,11)(0,9,9,14);0.5,0.5) & ((-4,0,0,4)(-7,0,0,7);0.7,0.5) \end{bmatrix}$$

$$d'_{43} = \begin{bmatrix} ((-7,0,0,7)(-9,0,0,9);0.8,0.5) & ((0,4,4,8)(-4,4,4,10);0.7,0.5) \\ ((1,7,7,11)(-2,7,7,14);0.8,0.5) & ((-4,0,0,4)(-7,0,0,7);0.7,0.5) \end{bmatrix}$$

Given that all of the $d_{ij}^{'}$ > 0, the assignments are optimal.

The optimal solution for the numerator is $t_{11}^{*} = 2,\ t_{12}^{*} = 12,\ t_{21}^{*} = 15,\ t_{31}^{*} = 1,\ t_{33}^{*} = 12,\ t_{41}^{*} = 2,\ t_{44}^{*} = 8.$

The optimal transportation cost for the numerator is ((4,5,5,7)(2,5,5,9);0.6,0.3) × 2 + ((3,4,4,7)(1,4,4,8);0.5,0.1) × 12 + ((4,6,6,10)(2,6,6,12);0.5,0.3) × 15 + ((7,10,10,13)(6,10,10,15);0.7,0.2) × 1 + ((7,10,10,14)(6,10,10,15);0.8,0.5) × 12+ ((6,10,10,12) (5,10,10,13); 0.6,0.1) × 2 + ((8,10,10,12)(7,10,10,14);0.7,0.5) × 8

= ((271,378,378,549)(190,378,378,627);0.5,0.5).

Subsequently, we ascertained the denominator's value. These are the tables that are provided. For the denominator, the following steps are listed:

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Table 8: T2TIFFTP (Denominator)

	$D_1^{'}$	$D_2^{'}$	$D_3^{'}$	$D_4^{'}$	$s_i^{'}$
$S_1^{'}$	((2,3,3,7)(1,3,3,8);0.5,0.2)	((2,4,4,5)(1,4,4,8);0.4,0.1)	((5,6,6,9)(3,6,6,11);0.3,0.1)	((5,7,7,10)(4,7,7,12);0.4,0.2)	12
$S_2^{'}$	((5,6,6,9)(4,6,6,12);0.6,0.3)	((2,5,5,6)(1,5,5,8);0.5,0.2)	((4,7,7,10)(2,7,7,13);0.4,0.1)	((9,12,12,14)(7,12,12,16);0.3,0.1)	14
$S_3^{'}$	((7,8,8,10)(5,8,8,11);0.4,0.2)	((8,9,9,12)(6,9,9,14);0.6,0.4)	((3,4,4,6)(2,4,4,7);0.5,0.3)	((5,9,9,10)(4,9,9,13);0.2,0.1)	11
$S_4^{'}$	((2,4,4,5)(1,4,4,9);0.7,0.5)	((6,8,8,10)(5,8,8,12);0.6,0.5)	((9,11,11,13)(7,11,11,15);0.4,0.1)	((8,10,10,12)(6,10,10,13);0.5,0.3)	12
$d_j^{'}$	18	11	7	13	

Table 9: T2TIFFTP Penalty Table (Denominator)

	$D_1^{'}$	$D_2^{'}$	$D_3^{'}$	$D_4^{'}$	Fuzzy Penalty
$S_1^{'}$	((2,3,3,7)(1,3,3,8);0.5,0.2)	((2,4,4,5)(1,4,4,8);0.4,0.1)	((5,6,6,9)(3,6,6,11);0.3,0.1)	((5,7,7,10)(4,7,7,12);0.4,0.2)	((-3,-1,-1,5)(-7,-1,-1,7);0.4,0.2)
$S_2^{'}$	((5,6,6,9)(4,6,6,12);0.6,0.3)	((2,5,5,6)(1,5,5,8);0.5,0.2)	((4,7,7,10)(2,7,7,13);0.4,0.1)	((9,12,12,14)(7,12,12,16);0.3,0.1)	((-2,2,2,8)(-6,2,2,12)0.4,0.2)
$S_3^{'}$	((7,8,8,10)(5,8,8,11);0.4,0.2)	((8,9,9,12)(6,9,9,14);0.6,0.4)	((3,4,4,6)(2,4,4,7);0.5,0.3)	((5,9,9,10)(4,9,9,13);0.2,0.1)	((-1,5,5,7)(-3,5,5,11)0.2,0.3)
$S_4^{'}$	((2,4,4,5)(1,4,4,9);0.7,0.5)	((6,8,8,10)(5,8,8,12);0.6,0.5)	((9,11,11,13)(7,11,11,15);0.4,0.1)	((8,10,10,12)(6,10,10,13);0.5,0.3)	((4,7,7,11)(-2,7,7,14)0.4,0.5)
Fuzzy Penalty	((-5,1,1,3)(-7,1,1,8);0.5,0.5)	((-3,1,1,4)(-7,1,1,7);0.4,0.2)	((-6,-2,-2,1)((-9,-2,-2,4);0.3,0.3))	((-5,-2,-2,5)(-9,-2,-2,8);0.2,0.2)	

Table 10: T2TIFFTP Penalty Table (Denominator)

	$D_2^{'}$	$D_3^{'}$	$D_4^{'}$	Fuzzy Penalty
$S_1^{'}$	$((2,4,4,5)(1,4,4,8);0.4,0.1)$	$((5,6,6,9)(3,6,6,11);0.3,0.1)$	$((5,7,7,10)(4,7,7,12);0.4,0.2)$	$((0,2,2,7)(-5,2,2,10);0.3,0.1)$
$S_2^{'}$	$((2,5,5,6)(1,5,5,8);0.5,0.2)$	$((4,7,7,10)(2,7,7,13);0.4,0.1)$	$((9,12,12,14)(7,12,12,16);0.3,0.1)$	$((-2,2,2,8)(-6,2,2,12);0.4,0.2)$
$S_3^{'}$	$((8,9,9,12)(6,9,9,14);0.6,0.4)$	$((3,4,4,6)(2,4,4,7);0.5,0.3)$	$((5,9,9,10)(4,9,9,13);0.2,0.1)$	$((-1,5,5,7)(-3,5,5,11)0.2,0.3)$
Fuzzy Penalty	$((-3,1,1,4)(-7,1,1,7);0.4,0.2)$	$((-6,-2,-2,1)(-9,-2,-2,4);0.3,0.3)$	$((-5,-2,-2,5)(-9,-2,-2,8);0.2,0.2)$	

Table 11: T2TIFFTP Penalty Table (Denominator)

	$D_2^{'}$	$D_4^{'}$	Fuzzy Penalty
$S_1^{'}$	$((2,4,4,5)(1,4,4,8);0.4,0.1)$	$((5,7,7,10)(4,7,7,12);0.4,0.2)$	$((0,3,3,8)(-4,3,3,11);0.4,0.2)$
$S_2^{'}$	$((2,5,5,6)(1,5,5,8);0.5,0.2)$	$((9,12,12,14)(7,12,12,16);0.3,0.1)$	$((3,7,7,12)(-1,7,7,15);0.3,0.2)$
Fuzzy Penalty	$((-3,1,1,4)(-7,1,1,7);0.4,0.2)$	$((-1,5,5,9)(-5,5,5,12);0.3,0.2)$	

Table 12: T2TIFFTP Allocation Table (Denominator)

$D_1^{'}$	$D_2^{'}$	$D_3^{'}$	$D_4^{'}$	$s_i^{'}$	
$S_1^{'}$	((2,3,3,7)(1,3,3,8);0.5,0.2)	((2,4,4,5)(1,4,4,8);0.4,0.1)	((5,6,6,9)(3,6,6,11);0.3,0.1)	((5,7,7,10)(4,7,7,12);0.4,0.2) **(12)**	12
$S_2^{'}$	((5,6,6,9)(4,6,6,12);0.6,0.3)**(2)**	((2,5,5,6)(1,5,5,8);0.5,0.2)**(11)**	((4,7,7,10)(2,7,7,13);0.4,0.1)	((9,12,12,14)(7,12,12,16);0.3,0.1)**(1)**	14
$S_3^{'}$	((7,8,8,10)(5,8,8,11);0.4,0.2)**(4)**	((8,9,9,12)(6,9,9,14);0.6,0.4)	((3,4,4,6)(2,4,4,7);0.5,0.3)**(7)**	((5,9,9,10)(4,9,9,13);0.2,0.1)	11
$S_4^{'}$	((2,4,4,5)(1,4,4,9);0.7,0.5)**(12)**	((6,8,8,10)(5,8,8,12);0.6,0.5)	((9,11,11,13)(7,11,11,15);0.4,0.1)	((8,10,10,12)(6,10,10,13);0.5,0.3)	12
$d_j^{'}$	18	11	7	13	

Determine the penalties for each row and column, then apply the corresponding penalty to each row and column as shown in the table.

The maximum penalty in this case is seen in the fourth row. Choose the row to assign the smallest intuitionistic fuzzy number (shown in bold) to that row. After assigning, eliminate the matching row and column.

$$\begin{bmatrix} ((2,3,3,7)(1,3,3,8);0.5,0.2) & ((2,4,4,5)(1,4,4,8);0.4,0.1) & ((5,6,6,9)(3,6,6,11);0.3,0.1) & ((5,7,7,10)(4,7,7,12);0.4,0.2) \\ ((5,6,6,9)(4,6,6,12);0.6,0.3) & ((2,5,5,6)(1,5,5,8);0.5,0.2) & ((4,7,7,10)(2,7,7,13);0.4,0.1) & ((9,12,12,14)(7,12,12,16);0.3,0.1) \\ ((7,8,8,10)(5,8,8,11);0.4,0.2) & ((8,9,9,12)(6,9,9,14);0.6,0.4) & ((3,4,4,6)(2,4,4,7);0.5,0.3) & ((5,9,9,10)(4,9,9,13);0.2,0.1) \\ \mathbf{(2,4,4,5)(1,4,4,9);0.7,0.5)} & ((6,8,8,10)(5,8,8,12);0.6,0.5) & ((9,11,11,13)(7,11,11,15);0.4,0.1) & ((8,10,10,12)(6,10,10,13);0.5,0.3) \end{bmatrix}$$

The remaining matrix is obtained by eliminating the fourth row and first column.

$$\begin{bmatrix} ((2,4,4,5)(1,4,4,8);0.4,0.1) & ((5,6,6,9)(3,6,6,11);0.3,0.1) & ((5,7,7,10)(4,7,7,12);0.4,0.2) \\ ((2,5,5,6)(1,5,5,8);0.5,0.2) & ((4,7,7,10)(2,7,7,13);0.4,0.1) & ((9,12,12,14)(7,12,12,16);0.3,0.1) \\ ((8,9,9,12)(6,9,9,14);0.6,0.4) & ((3,4,4,6)(2,4,4,7);0.5,0.3) & ((5,9,9,10)(4,9,9,13);0.2,0.1) \end{bmatrix}$$

To obtain Table 9, proceed with steps 1 and 2. Repeat the process until each row or column has a value assigned to it. Subtracting the allocated cost from each element in the relevant column.

$$\begin{bmatrix} ((-3,-1,-1,5)(-8,-1,-1,7);0.5,0.5) & ((-4,-1,-1,3)(-7,-1,-1,7);0.4,0.2) & ((-1,2,2,6)(-4,2,2,9);0.3,0.3) & ((-5,0,0,5)(-8,0,0,8);0.4,0.2) \\ ((0,2,2,7)(-5,2,2,11);0.6,0.5) & ((-4,0,0,4)(-7,0,0,7);0.5,0.2) & ((-2,3,3,7)(-5,3,3,11);0.4,0.3) & ((-1,5,5,9)(-5,5,5,12);0.3,0.2)) \\ ((2,4,4,8)(-4,4,4,10);0.4,0.5) & ((2,4,4,10)(-2,4,4,13);0.5,0.4) & ((-3,0,0,3)(-5,0,0,5);0.5,0.3) & ((-5,2,2,5)(-8,2,2,9);0.2,0.2) \\ ((-3,0,0,3)(-8,0,0,8);0.7,0.5) & ((0,3,3,8)(-3,3,3,11);0.5,0.5) & ((3,7,7,10)(0,7,7,13);0.4,0.3) & ((-2,3,3,7)(-6,3,3,9);0.4,0.3) \end{bmatrix}$$

For every non assigned cell,

$$d'_{11} = \begin{bmatrix} ((-3,-1,-1,5)(-8,-1,-1,7);0.5,0.5) & ((-5,0,0,5)(-8,0,0,8);0.4,0.2) \\ ((-3,0,0,3)(-8,0,0,8);0.7,0.5) & ((-2,3,3,7)(-6,3,3,9);0.4,0.3) \end{bmatrix}$$

$$d'_{12} = \begin{bmatrix} ((-4,-1,-1,3)(-7,-1,-1,7);0.4,0.2) & ((-5,0,0,5)(-8,0,0,8);0.4,0.2) \\ ((-4,0,0,4)(-7,0,0,7);0.5,0.2) & ((-1,5,5,9)(-5,5,5,12);0.3,0.2) \end{bmatrix}$$

$$d'_{13} = \begin{bmatrix} ((-1,2,2,6)(-4,2,2,9);0.3,0.3) & ((-5,0,0,5)(-8,0,0,8);0.4,0.2) \\ ((-3,0,0,3)(-5,0,0,5);0.5,0.3) & ((-5,2,2,5)(-8,2,2,9);0.2,0.2) \end{bmatrix}$$

$$d'_{21} = \begin{bmatrix} ((0,2,2,7)(-5,2,2,11);0.6,0.5) & ((-4,0,0,4)(-7,0,0,7);0.5,0.2) \\ ((-3,0,0,3)(-8,0,0,8);0.7,0.5) & ((0,3,3,8)(-3,3,3,11);0.5,0.5) \end{bmatrix}$$

$$d'_{23} = \begin{bmatrix} ((-4,0,0,4)(-7,0,0,7);0.5,0.2) & ((-2,3,3,7)(-5,3,3,11);0.4,0.3) \\ ((2,4,4,10)(-2,4,4,13);0.5,0.4) & ((-3,0,0,3)(-5,0,0,5);0.5,0.3) \end{bmatrix}$$

$$d'_{24} = \begin{bmatrix} ((-4,-1,-1,3)(-7,-1,-1,7);0.4,0.2) & ((-5,0,0,5)(-8,0,0,8);0.4,0.2) \\ ((-4,0,0,4)(-7,0,0,7);0.5,0.2) & ((-1,5,5,9)(-5,5,5,12);0.3,0.2)) \end{bmatrix}$$

$$d'_{31} = \begin{bmatrix} ((2,4,4,8)(-4,4,4,10);0.4,0.5) & ((-3,0,0,3)(-5,0,0,5);0.5,0.3) \\ ((-3,0,0,3)(-8,0,0,8);0.7,0.5) & ((3,7,7,10)(0,7,7,13);0.4,0.3) \end{bmatrix}$$

$$d'_{32} = \begin{bmatrix} ((-4,0,0,4)(-7,0,0,7);0.5,0.2) & ((-2,3,3,7)(-5,3,3,11);0.4,0.3) \\ ((2,4,4,10)(-2,4,4,13);0.5,0.4) & ((-3,0,0,3)(-5,0,0,5);0.5,0.3) \end{bmatrix}$$

$$d'_{34} = \begin{bmatrix} ((-1,2,2,6)(-4,2,2,9);0.3,0.3) & ((-5,0,0,5)(-8,0,0,8);0.4,0.2) \\ ((-3,0,0,3)(-5,0,0,5);0.5,0.3) & ((-5,2,2,5)(-8,2,2,9);0.2,0.2) \end{bmatrix}$$

$$d'_{42} = \begin{bmatrix} ((0,2,2,7)(-5,2,2,11);0.6,0.5) & ((-4,0,0,4)(-7,0,0,7);0.5,0.2) \\ ((-3,0,0,3)(-8,0,0,8);0.7,0.5) & ((0,3,3,8)(-3,3,3,11);0.5,0.5) \end{bmatrix}$$

$$d'_{43} = \begin{bmatrix} ((2,4,4,8)(-4,4,4,10);0.4,0.5) & ((-3,0,0,3)(-5,0,0,5);0.5,0.3) \\ ((-3,0,0,3)(-8,0,0,8);0.7,0.5) & ((3,7,7,10)(0,7,7,13);0.4,0.3) \end{bmatrix}$$

$$d'_{44} = \begin{bmatrix} ((-3,-1,-1,5)(-8,-1,-1,7);0.5,0.5) & ((-5,0,0,5)(-8,0,0,8);0.4,0.2) \\ ((-3,0,0,3)(-8,0,0,8);0.7,0.5) & ((-2,3,3,7)(-6,3,3,9);0.4,0.3) \end{bmatrix}$$

Given that all of the assignments are optimal. The optimal solution for the denominator is $t^*_{14}=12,\ t^*_{21}=2,\ t^*_{22}=11,\ t^*_{24}=1,\ t^*_{31}=4,\ t^*_{33}=7,\ t^*_{41}=12.$

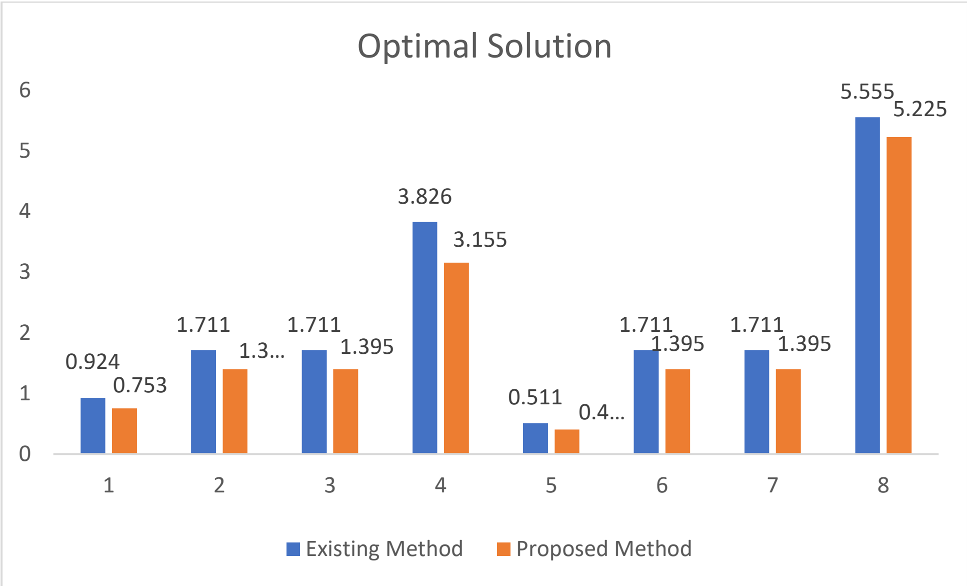
The optimal transportation cost for the denominator is ((5,7,7,10)(4,7,7,12);0.4,0.2) × 12 + ((5,6,6,9)(4,6,6,12);0.6,0.3) × 2 + ((2,5,5,6)(1,5,5,8);0.5,0.2) × 11 + ((9,12,12,14)(7,12,12,16);0.3,0.1) × 1 + ((7,8,8,10)(5,8,8,11);0.4,0.2) × 4 + ((3,4,4,6)(2,4,4,7); 0.5,0.3) × 7 + ((2,4,4,5)(1,4,4,9);0.7,0.5) × 1
= ((174,271,271,360)(120,271,271,473);0.3,0.5).

After determining the optimal solution for the numerator and denominator, we can divide the previously computed numerator and denominator to compute the T2TIFFTP. In other words, the optimal solution to the T2IFFTP is represented by the following, based on calculations:

((0.753,1.395,1.395,3.155)(0.402,1.395,1.395,5.255);0.3,0.5).

7. Results and Discussion

This investigation draws inspiration from [2] and will play a significant role in tackling transportation problems that involve uncertainty. This work does not require identification of the first basic viable solution, as the suggested technique immediately yields the optimal solution. It reduces computational effort and time. Our suggested method demonstrates an effective approach to identifying the best solution for T2IFFTP. To achieve this, we independently computed the numerator and denominator to determine the optimal solution. The approach employed herein is straightforward and accomplishes its objective swiftly and precisely for T2TIFFTP, in contrast to the methodologies documented in the literature. In this case, determining the first basic viable solution for the numerator and denominator individually is unnecessary, as it directly yields the optimal solution. The diagonal optimum approach improves the existing zero-centered value method for addressing the T2IFFTP. The proposed approach provides a more cost-effective option. By accounting for the diagonal parts, this method gives more accurate results when managing type 2 intuitionistic fuzzy sets, which include membership, non-membership, and hesitation degrees. This allows for a more accurate calculation of transportation expenses, resulting in a more optimal solution. The proposed method enhances the current strategy for tackling the T2TIFFTP. The diagonal optimal method offers a more economical alternative. By examining the diagonal components, such as the degrees of membership, non-membership, and hesitation, this method provides more precise results for managing type 2 intuitionistic fuzzy sets. This allows for a more accurate calculation of transportation costs, resulting in a more optimal solution. The proposed approach regularly yields reduced transportation expenses compared to the current strategy. Although the diagonal optimum technique produces positive outcomes, its applicability to more intricate transportation issues require further investigation in future studies. The figure and table below compare the suggested approach with the current method.



S.no.	Researchers	Optimal Solution
1.	Existing method [2]	((0.924,1.711,1.711,3.826)(0.511,1.711,1.711,5.555);0.2,0.6)
2.	Proposed method	((0.753,1.395,1.395,3.155)(0.402,1.395,1.395,5.225);0.3,0.5)

8. Conclusion

This study demonstrates a novel method for solving the T2TIFFTP by checking non-assigned cells in a planned manner using given diagonals. The approach ensures an optimal solution while greatly reducing computing complexity, providing a major advance over existing strategies. A significant decrease in transportation costs further supports the approach's practical usefulness, establishing it as a vital tool for resource allocation. The strategy improves resource management efficiency and has potential applications in a wide range of industries. Future studies should concentrate on improving the model and investigating its applicability to a variety of logistical difficulties, ensuring its continuous relevance and influence in optimization efforts.

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