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An operations and relations on the Cartesian Product of Interval Valued Intuitionistic Fuzzy Matrices

R. Amutha[1,2] and C. Ragavan[1]

[1]PG & Research Department of Mathematics, Sri Vidya Mandir Arts & Science College (Autonomous), Katteri – 636 902, Uthangarai, Tamilnadu, India

[2]Department of Mathematics, Periyar University, Salem – 636 011, Tamilnadu, India

[1]saraswathiamutha@gmail.com, [2]ragavanshana@gmail.com

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Abstract:

In this paper, we study an operations and relations on the cartesian product over interval valued intuitionistic fuzzy matrices are introduced and its some properties are explored. We prove some equality based on the operation and the relation over interval valued intuitionistic fuzzy matrices. Finally, we introducing some cartesian formulas $\times_1$, $\times_2$, $\times_3$, $\times_4$, $\times_5$ in cartesian product of interval valued intuitionistic fuzzy matrices.

Keywords: fuzzy sets, intuitionistic fuzzy sets, fuzzy matrix, cartesian product over intuitionistic fuzzy sets, operation, geometric interpretation, interval valued intuitionistic fuzzy set.

1. Introduction

In 1986, the theory of intuitionistic fuzzy sets was presented initially by Atanassov [1] and a series of operations and concepts is defined in [2]. Later, the interval valued intuitionistic fuzzy sets [3] was presented by Atanassov in 1989, and it has achieved a tremendous amount of research and development in various fields. For example, Zeng and Hu studied the necessity operator and the possibility operator of interval valued intuitionistic fuzzy sets [4]. T. Muthuraji, S. Sriram, P. Murugadas have proposed Decomposition of Intuitionistic Fuzzy Matrices [5]. H. Bustince, E. Barrenechea, M. Pagola, J. Fernandez proposed Interval-valued fuzzy sets constructed from matrices [17]. B. Chetia and P. K. Das. introduced Some Results of Intuitionistic Fuzzy Soft Matrix Theory [9]. Jian-qiang Wang, Rong-rong Nie, Hong-yu Zhang, Xiao-hong Chen initiated Intuitionistic fuzzy multi-criteria decision-making method based on evidential reasoning [12]. S. Senthilkumar, Eswari Prem and C. Ragavan have explored the concept of Intuitionistic fuzzy translation of anti-intuitionistic fuzzy T-ideals of subtraction BCK/BCI-algebras [20]. D. Pandey and Kamesh Kumar have achieved their results in the Interval Valued Intuitionistic Fuzzy Sets in Medical Diagnosis [22]. For distance measures, further research was done by Khalid and Abbas [23]. Atanassov defined five versions of Cartesian products of two IFSs. The sixth cartesian product over IFSs was defined by Velin Andonov in 2008 [24]. The seventh, eighth and ninth cartesian products over IFSs were defined by Annie Varghese and Sunny Kuriakose in 2012 [25], they also defined the tenth and eleventh Cartesian products over IFSs in the same year [26], that is to say $\times 7$, $\times 8$, $\times 9$, $\times 10$ and $\times 11$, respectively. And the corresponding equations for some operations and relations over IFSs were proved. According to the comparison of interval valued fuzzy sets and intuitionistic fuzzy set, Atanassov introduced and proposed five kinds of Cartesian products of two interval valued intuitionistic fuzzy sets. As we all know, there are eleven kinds of Cartesian products of IFSs. However, there are only five types of Cartesian products of interval valued intuitionistic fuzzy sets.

Finally, we prove that interval valued intuitionistic fuzzy matrices is a closed algebraic system for all these operations as fuzzy sets of intuitionistic fuzzy sets, and interval valued intuitionistic fuzzy sets. Therefore, this paper generalizes the interval valued intuitionistic fuzzy set theory and provides some valuable conclusions for the field of application research of interval valued intuitionistic fuzzy matrices and it is also useful to the generalization of interval valued intuitionistic fuzzy reasoning.

In this article, the intuitionistic fuzzy set approach using interval belief degrees is introduced to interval valued intuitionistic fuzzy matrices with interval belief structures. Then a fuzzy set analytical algorithm is developed to aggregate decision attributes of alternatives for multicriteria decision making problems with interval valued intuitionistic fuzzy matrices and incomplete decision information. A series of non-linear programming models are constructed based on criteria weights intervals, belief degrees intervals and fuzzy evidential reasoning analytical algorithm, then the genetic algorithm is employed to solve the non-linear models yielding the minimal and maximal fuzzy utilities of each alternative. With our proposed method, procedures involving arithmetic operations in aforementioned literature are not needed, thereby removing the limitations in those works.

2. Objectives

- We introduced an operation and relations on the Cartesian product of interval valued intuitionistic fuzzy matrices and some properties of the interval valued intuitionistic fuzzy matrices are discussed.
- An interval valued intuitionistic fuzzy matrices play an important role in the field of fuzzy system modelling. An interval valued intuitionistic fuzzy matrices are extension of the ordinary matrices.
- Five new operations are introduced over extended intuitionistic fuzzy matrices set and over their simpler cases, such as interval valued intuitionistic fuzzy matrices, extended interval valued intuitionistic fuzzy matrices.
- The concepts of an intuitionistic fuzzy set with elements being intuitionistic fuzzy matrix and of an extended interval valued intuitionistic fuzzy matrices with elements being predicates are introduced.
- Some operations, relations and operators over these new types of matrices are defined. Some properties of these ideas are discussed and statements are expressed.

3. Methods

In this section, several fundamental notions about interval valued intuitionistic fuzzy matrices are discussed. Fuzzy matrices play a vital role in scientific development, A possible application of the newly proposed interval valued intuitionistic fuzzy matrices of the student's training is discussed.

Definition 3.1. A Fuzzy matrix may be matrix that has its parts from [0, 1]. Consider a matrix $A = [aij]_{3\times 3}$ where $aij \in [0,1]$, $1 \leq j \leq n$. Then A is a Fuzzy Matrix [FM].

Definition 3.2. An intuitionistic fuzzy set (IFS) A in E is defined as an object of the following form A = $\{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in$ E} where the functions $\mu_A : E \to [0,1]$ define the membership and the degree of non-membership of the element $x \in$ E, respectively, and for every $x \in$ E, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. obviously, each ordinary fuzzy set may be written as $\{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in$ E$\}$.

Definition 3.3. The Cartesian products of two IFSs A and B are defined as follows, The Cartesian product "$\times_1$" then $A \times_1 B = \{\langle\langle x, y\rangle, \mu_A(x).\mu_B(y), \nu_A(x).\nu_B(y)\rangle \,|\, x \in E_1 \& y \in E_2\}$.

Definition 3.4. The Cartesian products of two IFSs A and B are defined as follows. The Cartesian product "$\times_2$" then $A \times_2 B = \{\langle\langle x, y\rangle, \mu_A(x) + \mu_B(y) - \mu_A(x).\mu_B(y), \nu_A(x).\nu_B(y)\rangle \,|\, x \in E_1 \& y \in E_2\}$.

Definition 3.5. The Cartesian products of two IFSs A and B are defined as follows. The Cartesian product "$\times_3$" then $A \times_3 B = \{\langle\langle x, y\rangle, \mu_A(x).\mu_B(y), \nu_A(x) + \nu_B(y) - \nu_A(x).\nu_B(y)\rangle \,|\, x \in E_1 \& y \in E_2\}$.

Definition 3.6. The Cartesian products of two IFSs A and B are defined as follows. The Cartesian product "$\times_4$" then $A \times_4 B = \{\langle\langle x, y\rangle, min\,(\mu_A(x).\mu_B(y)), max(\nu_A(x).\nu_B(y))\rangle \,|\, x \in E_1 \& y \in E_2\}$.

Definition 3.7. The Cartesian products of two IFSs A and B are defined as follows, The Cartesian product "$\times_5$" then $A \times_5 B = \{\langle\langle x, y\rangle, max\,(\mu_A(x).\mu_B(y)), min(\nu_A(x).\nu_B(y))\rangle \,|\, x \in E_1 \& y \in E_2\}$.

Definition 3.8. An interval valued fuzzy set (IVFS) A (over a basic set E) is specified by a function $M_A : E \to INT\ ([0,1])$, where $INT\ (X[0,1])$ is the set of all intervals within $[0,1]$, for all $x \in E$, $M_A(x)$ is an interval $[a, b]$, $0 \leq a \leq b \leq 1$.

Definition 3.9. An interval valued intuitionistic fuzzy set A is defined by the membership function $M_A : E \to INT\ ([0,1])$, The non-membership function $N_A : E \to INT\ ([0,1])$, Where $INT([0,1])$ is the set of all subsets of the unit interval.

4. Results

Main Theorem of Cartesian Product of Interval Valued Intuitionistic Fuzzy Matrices

Theorem 4.1. If Ạ $\times_1$ Ḅ are an interval valued intuitionistic fuzzy matrices, then $\Pi_{\alpha,\beta}$(Ạ $\times_1$ Ḅ) and $\Omega_{\alpha,\beta}$(Ạ $\times_1$ Ḅ) is also an interval valued intuitionistic fuzzy matrices.

Proof. Let us consider Ạ have 3×4 matrix and Ḅ have 4×3 matrix both are an interval valued intuitionistic fuzzy matrices. Then $\Pi_{\alpha,\beta}$(Ạ $\times_1$ Ḅ) = [(min $[\alpha, \mu_A^L.\,\mu_{\boldsymbol{B}}^L]$, min $[\alpha, \mu_A^U.\mu_B^U]$)], [(max $[\beta, \lambda_A^L.\,\lambda_{\boldsymbol{B}}^L]$, max $[\beta, \lambda_A^U.\lambda_B^U]$)] and $\Omega_{\alpha,\beta}$(Ạ $\times_1$ Ḅ) = [(max $[\alpha, \mu_A^L.\,\mu_{\boldsymbol{B}}^L]$, max $[\alpha, \mu_A^U.\mu_B^U]$)], [(min $[\beta, \lambda_A^L.\,\lambda_{\boldsymbol{B}}^L]$, min $[\beta, \lambda_A^U.\lambda_B^U]$)].

Ạ $\times_1$ Ḅ = $\begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{pmatrix}$ and Ạ $\times_1$ Ḅ = $\begin{pmatrix} \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31} & \omega_{32} & \omega_{33} \end{pmatrix}$

$\Pi_{\alpha,\beta}$(Ạ$\times_1$Ḅ)=

$$\begin{pmatrix} [\epsilon_{11} & \theta_{11} & \sigma_{11} & \pi_{11}] & [\epsilon_{12} & \theta_{12} & \sigma_{12} & \pi_{12}] & [\epsilon_{13} & \theta_{13} & \sigma_{13} & \pi_{13}] & [\epsilon_{14} & \theta_{14} & \sigma_{14} & \pi_{14}] \\ [\epsilon_{21} & \theta_{21} & \sigma_{21} & \pi_{21}] & [\epsilon_{22} & \theta_{22} & \sigma_{22} & \pi_{22}] & [\epsilon_{23} & \theta_{23} & \sigma_{23} & \pi_{23}] & [\epsilon_{24} & \theta_{24} & \sigma_{24} & \pi_{24}] \\ [\epsilon_{31} & \theta_{31} & \sigma_{31} & \pi_{31}] & [\epsilon_{32} & \theta_{32} & \sigma_{32} & \pi_{32}] & [\epsilon_{33} & \theta_{33} & \sigma_{33} & \pi_{33}] & [\epsilon_{34} & \theta_{34} & \sigma_{34} & \pi_{34}] \end{pmatrix} \times_1$$

$$\begin{pmatrix} [\psi_{11} & \rho_{11} & \phi_{11} & \chi_{11}] & [\psi_{12} & \rho_{12} & \phi_{12} & \chi_{12}] & [\psi_{13} & \rho_{13} & \phi_{13} & \chi_{13}] \\ [\psi_{21} & \rho_{21} & \phi_{21} & \chi_{21}] & [\psi_{22} & \rho_{22} & \phi_{22} & \chi_{22}] & [\psi_{23} & \rho_{23} & \phi_{23} & \chi_{23}] \\ [\psi_{31} & \rho_{31} & \phi_{31} & \chi_{31}] & [\psi_{32} & \rho_{32} & \phi_{32} & \chi_{32}] & [\psi_{33} & \rho_{33} & \phi_{33} & \chi_{33}] \\ [\psi_{41} & \rho_{41} & \phi_{41} & \chi_{41}] & [\psi_{42} & \rho_{42} & \phi_{42} & \chi_{42}] & [\psi_{43} & \rho_{43} & \phi_{43} & \chi_{43}] \end{pmatrix}$$

τ_{11} = [min (α, $\epsilon_{11} \cdot \psi_{11}$), min (α, $\theta_{11} \cdot \rho_{11}$)], [max (β, $\sigma_{11} \cdot \phi_{11}$), max (β, $\pi_{11} \cdot \chi_{11}$)] +[min (α, $\epsilon_{12} \cdot \psi_{21}$), min (α, $\theta_{12} \cdot \rho_{21}$)], [max (β, $\sigma_{12} \cdot \phi_{21}$), max (β, $\pi_{12} \cdot \chi_{21}$)] +[min (α, $\epsilon_{13} \cdot \psi_{31}$), min (α, $\theta_{13} \cdot \rho_{31}$)], [max (β, $\sigma_{13} \cdot \phi_{31}$), max (β, $\pi_{13} \cdot \chi_{31}$)] +[min (α, $\epsilon_{14} \cdot \psi_{41}$), min (α, $\theta_{14} \cdot \rho_{41}$)], [max (β, $\sigma_{14} \cdot \phi_{41}$), max (β, $\pi_{14} \cdot \chi_{41}$)]. Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}.

τ_{11} = max ([min (α, $\epsilon_{11} \cdot \psi_{11}$), min (α, $\theta_{11} \cdot \rho_{11}$)], [max (β, $\sigma_{11} \cdot \phi_{11}$), max (β, $\pi_{11} \cdot \chi_{11}$)], [min (α, $\epsilon_{12} \cdot \psi_{21}$), min (α, $\theta_{12} \cdot \rho_{21}$)], [max (β, $\sigma_{12} \cdot \phi_{21}$), max (β, $\pi_{12} \cdot \chi_{21}$)]) +[min (α, $\epsilon_{13} \cdot \psi_{31}$), min (α, $\theta_{13} \cdot \rho_{31}$)], [max (β, $\sigma_{13} \cdot \phi_{31}$), max (β, $\pi_{13} \cdot \chi_{31}$)] +[min (α, $\epsilon_{14} \cdot \psi_{41}$), min (α, $\theta_{14} \cdot \rho_{41}$)], [max (β, $\sigma_{14} \cdot \phi_{41}$), max (β, $\pi_{14} \cdot \chi_{41}$)]. Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}.

τ_{11} = max (max ([min (α, $\epsilon_{11} \cdot \psi_{11}$), min (α, $\theta_{11} \cdot \rho_{11}$)], [max (β, $\sigma_{11} \cdot \phi_{11}$), max (β, $\pi_{11} \cdot \chi_{11}$)],[min (α, $\epsilon_{12} \cdot \psi_{21}$), min (α, $\theta_{12} \cdot \rho_{21}$)], [max (β, $\sigma_{12} \cdot \phi_{21}$), max (β, $\pi_{12} \cdot \chi_{21}$)]), [min (α, $\epsilon_{13} \cdot \psi_{31}$), min (α, $\theta_{13} \cdot \rho_{31}$)], [max (β, $\sigma_{13} \cdot \phi_{31}$), max (β, $\pi_{13} \cdot \chi_{31}$)])+ [min (α, $\epsilon_{14} \cdot \psi_{41}$), min (α, $\theta_{14} \cdot \rho_{41}$)], [max (β, $\sigma_{14} \cdot \phi_{41}$), max (β, $\pi_{14} \cdot \chi_{41}$)]. Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}.

τ_{11} = max (max (max ([min (α, $\epsilon_{11} \cdot \psi_{11}$), min (α, $\theta_{11} \cdot \rho_{11}$)], [max (β, $\sigma_{11} \cdot \phi_{11}$), max (β, $\pi_{11} \cdot \chi_{11}$)], [min (α, $\epsilon_{12} \cdot \psi_{21}$), min (α, $\theta_{12} \cdot \rho_{21}$)], [max (β, $\sigma_{12} \cdot \phi_{21}$), max (β, $\pi_{12} \cdot \chi_{21}$)]),[min (α, $\epsilon_{13} \cdot \psi_{31}$), min (α, $\theta_{13} \cdot \rho_{31}$)], [max (β, $\sigma_{13} \cdot \phi_{31}$), max (β, $\pi_{13} \cdot \chi_{31}$)]), [min (α, $\epsilon_{14} \cdot \psi_{41}$), min (α, $\theta_{14} \cdot \rho_{41}$)], [max (β, $\sigma_{14} \cdot \phi_{41}$), max (β, $\pi_{14} \cdot \chi_{41}$)]). Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}. And

ω_{11} = [max (α, $\epsilon_{11} \cdot \psi_{11}$), max (α, $\theta_{11} \cdot \rho_{11}$)], [min (β, $\sigma_{11} \cdot \phi_{11}$), min (β, $\pi_{11} \cdot \chi_{11}$)] + [max (α, $\epsilon_{12} \cdot \psi_{21}$), max (α, $\theta_{12} \cdot \rho_{21}$)], [min (β, $\sigma_{12} \cdot \phi_{21}$), min (β, $\pi_{12} \cdot \chi_{21}$)] +[max (α, $\epsilon_{13} \cdot \psi_{31}$), max (α, $\theta_{13} \cdot \rho_{31}$)], [min (β, $\sigma_{13} \cdot \phi_{31}$), min (β, $\pi_{13} \cdot \chi_{31}$)] +[max (α, $\epsilon_{14} \cdot \psi_{41}$), max (α, $\theta_{14} \cdot \rho_{41}$)], [min (β, $\sigma_{14} \cdot \phi_{41}$), min (β, $\pi_{14} \cdot \chi_{41}$)]. Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

ω_{11} = max ([max (α, $\epsilon_{11} \cdot \psi_{11}$), max (α, $\theta_{11} \cdot \rho_{11}$)], [min (β, $\sigma_{11} \cdot \phi_{11}$), min (β, $\pi_{11} \cdot \chi_{11}$)],[max (α, $\epsilon_{12} \cdot \psi_{21}$), max (α, $\theta_{12} \cdot \rho_{21}$)], [min (β, $\sigma_{12} \cdot \phi_{21}$), min (β, $\pi_{12} \cdot \chi_{21}$)]) + [max (α, $\epsilon_{13} \cdot \psi_{31}$), max (α, $\theta_{13} \cdot \rho_{31}$)], [min (β, $\sigma_{13} \cdot \phi_{31}$), min (β, $\pi_{13} \cdot \chi_{31}$)] +[max (α, $\epsilon_{14} \cdot \psi_{41}$), max (α, $\theta_{14} \cdot \rho_{41}$)], [min (β, $\sigma_{14} \cdot \phi_{41}$), min (β, $\pi_{14} \cdot \chi_{41}$)]. Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

ω_{11} = max (max ([max (α, $\epsilon_{11} \cdot \psi_{11}$), max (α, $\theta_{11} \cdot \rho_{11}$)], [min (β, $\sigma_{11} \cdot \phi_{11}$), min (β, $\pi_{11} \cdot \chi_{11}$)], [max (α, $\epsilon_{12} \cdot \psi_{21}$), max (α, $\theta_{12} \cdot \rho_{21}$)], [min (β, $\sigma_{12} \cdot \phi_{21}$), min (β, $\pi_{12} \cdot \chi_{21}$)]),[max (α, $\epsilon_{13} \cdot \psi_{31}$), max (α, $\theta_{13} \cdot \rho_{31}$)], [min (β, $\sigma_{13} \cdot \phi_{31}$), min (β, $\pi_{13} \cdot \chi_{31}$)]) + [max (α, $\epsilon_{14} \cdot \psi_{41}$), max (α, $\theta_{14} \cdot \rho_{41}$)], [min (β, $\sigma_{14} \cdot \phi_{41}$), min (β, $\pi_{14} \cdot \chi_{41}$)]. Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

ω_{11} = max (max (max ([max (α, $\epsilon_{11} \cdot \psi_{11}$), max (α, $\theta_{11} \cdot \rho_{11}$)], [min (β, $\sigma_{11} \cdot \phi_{11}$), min (β, $\pi_{11} \cdot \chi_{11}$)],[max (α, $\epsilon_{12} \cdot \psi_{21}$), max (α, $\theta_{12} \cdot \rho_{21}$)], [min (β, $\sigma_{12} \cdot \phi_{21}$), min (β, $\pi_{12} \cdot \chi_{21}$)]), [max (α, $\epsilon_{13} \cdot \psi_{31}$), max (α, $\theta_{13} \cdot \rho_{31}$)], [min (β, $\sigma_{13} \cdot \phi_{31}$), min (β, $\pi_{13} \cdot \chi_{31}$)]), [max (α, $\epsilon_{14} \cdot \psi_{41}$), max (α, $\theta_{14} \cdot \rho_{41}$)], [min (β, $\sigma_{14} \cdot \phi_{41}$), min (β, $\pi_{14} \cdot \chi_{41}$)]). Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

Hence $\Pi_{\alpha,\beta}$(Ạ $\times_1$ Ḅ) and $\Omega_{\alpha,\beta}$(Ạ $\times_1$ Ḅ) is an interval valued intuitionistic fuzzy matrices.

Theorem 4.2. If Ạ $\times_2$ Ḅ are an interval valued intuitionistic fuzzy matrices, then $\Pi_{\alpha,\beta}$(Ạ $\times_2$ Ḅ) and $\Omega_{\alpha,\beta}$(Ạ $\times_2$ Ḅ) is also an interval valued intuitionistic fuzzy matrices.

Proof. Let us consider Ạ have 3×4 matrix and Ḅ have 4×3 matrix both are an interval valued intuitionistic fuzzy matrices. Then $\Pi_{\alpha,\beta}$(Ạ $\times_2$ Ḅ) = [(min [α, μ_A^L+ $\mu_{\boldsymbol{B}}^L - \mu_A^L \cdot \mu_{\boldsymbol{B}}^L$], min [$\alpha, \mu_A^U + \mu_B^U - \mu_A^U . \mu_B^U$])], [(max [β, $\lambda_A^L . \lambda_{\boldsymbol{B}}^L$], max [β, $\lambda_A^U . \lambda_B^U$])] and $\Omega_{\alpha,\beta}$(Ạ $\times_2$ Ḅ) = [(max [$\alpha, \mu_A^L + \mu_{\boldsymbol{B}}^L - \mu_A^L \cdot \mu_{\boldsymbol{B}}^L$], max [$\alpha, \mu_A^U + \mu_B^U - \mu_A^U . \mu_B^U$])], [(min [β, $\lambda_A^L . \lambda_{\boldsymbol{B}}^L$], min [β, $\lambda_A^U . \lambda_B^U$])]. Ạ $\times_2$ Ḅ = $\begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{pmatrix}$ and Ạ $\times_2$ Ḅ = $\begin{pmatrix} \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31} & \omega_{32} & \omega_{33} \end{pmatrix}$

$\Pi_{\alpha,\beta}$(Ạ$\times_2$Ḅ)=

$$\begin{pmatrix} [\epsilon_{11} & \theta_{11} & \sigma_{11} & \pi_{11}] & [\epsilon_{12} & \theta_{12} & \sigma_{12} & \pi_{12}] & [\epsilon_{13} & \theta_{13} & \sigma_{13} & \pi_{13}] & [\epsilon_{14} & \theta_{14} & \sigma_{14} & \pi_{14}] \\ [\epsilon_{21} & \theta_{21} & \sigma_{21} & \pi_{21}] & [\epsilon_{22} & \theta_{22} & \sigma_{22} & \pi_{22}] & [\epsilon_{23} & \theta_{23} & \sigma_{23} & \pi_{23}] & [\epsilon_{24} & \theta_{24} & \sigma_{24} & \pi_{24}] \\ [\epsilon_{31} & \theta_{31} & \sigma_{31} & \pi_{31}] & [\epsilon_{32} & \theta_{32} & \sigma_{32} & \pi_{32}] & [\epsilon_{33} & \theta_{33} & \sigma_{33} & \pi_{33}] & [\epsilon_{34} & \theta_{34} & \sigma_{34} & \pi_{34}] \end{pmatrix} \times_2$$

$$\begin{pmatrix} [\psi_{11} & \rho_{11} & \phi_{11} & \chi_{11}] & [\psi_{12} & \rho_{12} & \phi_{12} & \chi_{12}] & [\psi_{13} & \rho_{13} & \phi_{13} & \chi_{13}] \\ [\psi_{21} & \rho_{21} & \phi_{21} & \chi_{21}] & [\psi_{22} & \rho_{22} & \phi_{22} & \chi_{22}] & [\psi_{23} & \rho_{23} & \phi_{23} & \chi_{23}] \\ [\psi_{31} & \rho_{31} & \phi_{31} & \chi_{31}] & [\psi_{32} & \rho_{32} & \phi_{32} & \chi_{32}] & [\psi_{33} & \rho_{33} & \phi_{33} & \chi_{33}] \\ [\psi_{41} & \rho_{41} & \phi_{41} & \chi_{41}] & [\psi_{42} & \rho_{42} & \phi_{42} & \chi_{42}] & [\psi_{43} & \rho_{43} & \phi_{43} & \chi_{43}] \end{pmatrix}$$

τ_{11} = [min (α, $\epsilon_{11} + \psi_{11} - \epsilon_{11} \cdot \psi_{11}$), min (α, $\theta_{11} + \rho_{11} - \theta_{11} \cdot \rho_{11}$)], [max (β, $\sigma_{11} \cdot \phi_{11}$), max (β, $\pi_{11} \cdot \chi_{11}$)] +[min (α, $\epsilon_{12} + \psi_{21} - \epsilon_{12} \cdot \psi_{21}$), min (α, $\theta_{12} + \rho_{21} - \theta_{12} \cdot \rho_{21}$)], [max (β, $\sigma_{12} \cdot \phi_{21}$), max (β, $\pi_{12} \cdot \chi_{21}$)] +[min (α, $\epsilon_{13} + \psi_{31} - \epsilon_{13} \cdot \psi_{31}$), min (α, $\theta_{13} + \rho_{31} - \theta_{13} \cdot \rho_{31}$)], [max (β, $\sigma_{13} \cdot \phi_{31}$), max (β, $\pi_{13} \cdot \chi_{31}$)] +[min (α, $\epsilon_{14} + \psi_{41} - \epsilon_{14} \cdot \psi_{41}$), min (α, $\theta_{14} + \rho_{41} - \theta_{14} \cdot \rho_{41}$)], [max (β, $\sigma_{14} \cdot \phi_{41}$), max (β, $\pi_{14} \cdot \chi_{41}$)]. Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}.

τ_{11} = max ([min (α, $\epsilon_{11} + \psi_{11} - \epsilon_{11} \cdot \psi_{11}$), min (α, $\theta_{11} + \rho_{11} - \theta_{11} \cdot \rho_{11}$)], [max (β, $\sigma_{11} \cdot \phi_{11}$), max (β, $\pi_{11} \cdot \chi_{11}$)],[min (α, $\epsilon_{12} + \psi_{21} - \epsilon_{12} \cdot \psi_{21}$), min (α, $\theta_{12} + \rho_{21} - \theta_{12} \cdot \rho_{21}$)], [max (β, $\sigma_{12} \cdot \phi_{21}$), max (β, $\pi_{12} \cdot \chi_{21}$)]) +[min (α, $\epsilon_{13} + \psi_{31} - \epsilon_{13} \cdot \psi_{31}$), min (α, $\theta_{13} + \rho_{31} - \theta_{13} \cdot \rho_{31}$)], [max (β, $\sigma_{13} \cdot \phi_{31}$), max (β, $\pi_{13} \cdot \chi_{31}$)] +[min (α, $\epsilon_{14} + \psi_{41} - \epsilon_{14} \cdot \psi_{41}$), min (α, $\theta_{14} + \rho_{41} - \theta_{14} \cdot \rho_{41}$)], [max (β,

$\sigma_{14} \cdot \phi_{41}$), max (β, $\pi_{14} \cdot \chi_{41}$)]. Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}.

τ_{11} = max (max ([min (α, $\epsilon_{11} + \psi_{11} - \epsilon_{11} \cdot \psi_{11}$), min (α, $\theta_{11} + \rho_{11} - \theta_{11} \cdot \rho_{11}$)], [max (β, $\sigma_{11} \cdot \phi_{11}$), max (β, $\pi_{11} \cdot \chi_{11}$)],[min (α, $\epsilon_{12} + \psi_{21} - \epsilon_{12} \cdot \psi_{21}$), min (α, $\theta_{12} + \rho_{21} - \theta_{12} \cdot \rho_{21}$)], [max (β, $\sigma_{12} \cdot \phi_{21}$), max (β, $\pi_{12} \cdot \chi_{21}$)]),[min (α, $\epsilon_{13} + \psi_{31} - \epsilon_{13} \cdot \psi_{31}$), min (α, $\theta_{13} + \rho_{31} - \theta_{13} \cdot \rho_{31}$)], [max (β, $\sigma_{13} \cdot \phi_{31}$), max (β, $\pi_{13} \cdot \chi_{31}$)]) +[min (α, $\epsilon_{14} + \psi_{41} - \epsilon_{14} \cdot \psi_{41}$), min (α, $\theta_{14} + \rho_{41} - \theta_{14} \cdot \rho_{41}$)], [max (β, $\sigma_{14} \cdot \phi_{41}$), max (β, $\pi_{14} \cdot \chi_{41}$)]. Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}.

τ_{11} = max (max (max ([min (α, $\epsilon_{11} + \psi_{11} - \epsilon_{11} \cdot \psi_{11}$), min (α, $\theta_{11} + \rho_{11} - \theta_{11} \cdot \rho_{11}$)], [max (β, $\sigma_{11} \cdot \phi_{11}$), max (β, $\pi_{11} \cdot \chi_{11}$)], [min (α, $\epsilon_{12} + \psi_{21} - \epsilon_{12} \cdot \psi_{21}$), min (α, $\theta_{12} + \rho_{21} - \theta_{12} \cdot \rho_{21}$)], [max (β, $\sigma_{12} \cdot \phi_{21}$), max (β, $\pi_{12} \cdot \chi_{21}$)]),[min (α, $\epsilon_{13} + \psi_{31} - \epsilon_{13} \cdot \psi_{31}$), min (α, $\theta_{13} + \rho_{31} - \theta_{13} \cdot \rho_{31}$)], [max (β, $\sigma_{13} \cdot \phi_{31}$), max (β, $\pi_{13} \cdot \chi_{31}$)]),[min (α, $\epsilon_{14} + \psi_{41} - \epsilon_{14} \cdot \psi_{41}$), min (α, $\theta_{14} + \rho_{41} - \theta_{14} \cdot \rho_{41}$)], [max (β, $\sigma_{14} \cdot \phi_{41}$), max (β, $\pi_{14} \cdot \chi_{41}$)]). Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}. And

ω_{11} = [max (α, $\epsilon_{11} + \psi_{11} - \epsilon_{11} \cdot \psi_{11}$), max (α, $\theta_{11} + \rho_{11} - \theta_{11} \cdot \rho_{11}$)], [min (β, $\sigma_{11} \cdot \phi_{11}$), min (β, $\pi_{11} \cdot \chi_{11}$)] +[max (α, $\epsilon_{12} + \psi_{21} - \epsilon_{12} \cdot \psi_{21}$), max (α, $\theta_{12} + \rho_{21} - \theta_{12} \cdot \rho_{21}$)], [min (β, $\sigma_{12} \cdot \phi_{21}$), min (β, $\pi_{12} \cdot \chi_{21}$)] + [max (α, $\epsilon_{13} + \psi_{31} - \epsilon_{13} \cdot \psi_{31}$), max (α, $\theta_{13} + \rho_{31} - \theta_{13} \cdot \rho_{31}$)], [min (β, $\sigma_{13} \cdot \phi_{31}$), min (β, $\pi_{13} \cdot \chi_{31}$)] +[max (α, $\epsilon_{14} + \psi_{41} - \epsilon_{14} \cdot \psi_{41}$), max (α, $\theta_{14} + \rho_{41} - \theta_{14} \cdot \rho_{41}$)], [min (β, $\sigma_{14} \cdot \phi_{41}$), min (β, $\pi_{14} \cdot \chi_{41}$)]. Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

ω_{11} = max ([max (α, $\epsilon_{11} + \psi_{11} - \epsilon_{11} \cdot \psi_{11}$), max (α, $\theta_{11} + \rho_{11} - \theta_{11} \cdot \rho_{11}$)], [min (β, $\sigma_{11} \cdot \phi_{11}$), min (β, $\pi_{11} \cdot \chi_{11}$)],[max (α, $\epsilon_{12} + \psi_{21} - \epsilon_{12} \cdot \psi_{21}$), max (α, $\theta_{12} + \rho_{21} - \theta_{12} \cdot \rho_{21}$)], [min (β, $\sigma_{12} \cdot \phi_{21}$), min (β, $\pi_{12} \cdot \chi_{21}$)]) +[max (α, $\epsilon_{13} + \psi_{31} - \epsilon_{13} \cdot \psi_{31}$), max (α, $\theta_{13} + \rho_{31} - \theta_{13} \cdot \rho_{31}$)], [min (β, $\sigma_{13} \cdot \phi_{31}$), min (β, $\pi_{13} \cdot \chi_{31}$)] +[max (α, $\epsilon_{14} + \psi_{41} - \epsilon_{14} \cdot \psi_{41}$), max (α, $\theta_{14} + \rho_{41} - \theta_{14} \cdot \rho_{41}$)], [min (β, $\sigma_{14} \cdot \phi_{41}$), min (β, $\pi_{14} \cdot \chi_{41}$)]. Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

ω_{11} = max (max ([max (α, $\epsilon_{11} + \psi_{11} - \epsilon_{11} \cdot \psi_{11}$), max (α, $\theta_{11} + \rho_{11} - \theta_{11} \cdot \rho_{11}$)], [min (β, $\sigma_{11} \cdot \phi_{11}$), min (β, $\pi_{11} \cdot \chi_{11}$)],[max (α, $\epsilon_{12} + \psi_{21} - \epsilon_{12} \cdot \psi_{21}$), max (α, $\theta_{12} + \rho_{21} - \theta_{12} \cdot \rho_{21}$)], [min (β, $\sigma_{12} \cdot \phi_{21}$), min (β, $\pi_{12} \cdot \chi_{21}$)]),[max (α, $\epsilon_{13} + \psi_{31} - \epsilon_{13} \cdot \psi_{31}$), max (α, $\theta_{13} + \rho_{31} - \theta_{13} \cdot \rho_{31}$)], [min (β, $\sigma_{13} \cdot \phi_{31}$), min (β, $\pi_{13} \cdot \chi_{31}$)]) +[max (α, $\epsilon_{14} + \psi_{41} - \epsilon_{14} \cdot \psi_{41}$), max (α, $\theta_{14} + \rho_{41} - \theta_{14} \cdot \rho_{41}$)], [min (β, $\sigma_{14} \cdot \phi_{41}$), min (β, $\pi_{14} \cdot \chi_{41}$)]. Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

ω_{11} = max (max (max ([max (α, $\epsilon_{11} + \psi_{11} - \epsilon_{11} \cdot \psi_{11}$), max (α, $\theta_{11} + \rho_{11} - \theta_{11} \cdot \rho_{11}$)], [min (β, $\sigma_{11} \cdot \phi_{11}$), min (β, $\pi_{11} \cdot \chi_{11}$)],[max (α, $\epsilon_{12} + \psi_{21} - \epsilon_{12} \cdot \psi_{21}$), max (α, $\theta_{12} + \rho_{21} - \theta_{12} \cdot \rho_{21}$)], [min (β, $\sigma_{12} \cdot \phi_{21}$), min (β, $\pi_{12} \cdot \chi_{21}$)]),[max (α, $\epsilon_{13} + \psi_{31} - \epsilon_{13} \cdot \psi_{31}$), max (α, $\theta_{13} + \rho_{31} - \theta_{13} \cdot \rho_{31}$)], [min (β, $\sigma_{13} \cdot \phi_{31}$), min (β, $\pi_{13} \cdot \chi_{31}$)]),[max (α, $\epsilon_{14} + \psi_{41} - \epsilon_{14} \cdot \psi_{41}$), max (α, $\theta_{14} + \rho_{41} - \theta_{14} \cdot$

ρ_{41})], [min (β, $\sigma_{14} \cdot \phi_{41}$), min (β, $\pi_{14} \cdot \chi_{41}$)]). Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

Hence $\Pi_{\alpha,\beta}$(Ạ $\times_2$ Ḅ) and $\Omega_{\alpha,\beta}$(Ạ $\times_2$ Ḅ) is an interval valued intuitionistic fuzzy matrices.

Theorem 4.3. If Ạ $\times_3$ Ḅ are an interval valued intuitionistic fuzzy matrices, then $\Pi_{\alpha,\beta}$(Ạ $\times_3$ Ḅ) and $\Omega_{\alpha,\beta}$(Ạ $\times_3$ Ḅ) is also an interval valued intuitionistic fuzzy matrices.

Proof. Let us consider Ạ have 3×4 matrix and Ḅ have 4×3 matrix both are an interval valued intuitionistic fuzzy matrices. Then $\Pi_{\alpha,\beta}$(Ạ $\times_3$ Ḅ) = [(min [α, $\mu_A^L \cdot \mu_{\boldsymbol{B}}^L$], min [$\alpha, \mu_A^U . \mu_B^U$])], [(max [β, $\lambda_A^L + \lambda_B^L - \lambda_A^L . \lambda_B^L$], max [β, $\lambda_A^U + \lambda_B^U - \lambda_A^U . \lambda_B^U$])] and $\Omega_{\alpha,\beta}$(Ạ $\times_3$ Ḅ) = [(max [$\alpha, \mu_A^L \cdot \mu_{\boldsymbol{B}}^L$], max [$\alpha, \mu_A^U . \mu_B^U$])], [(min [β, $\lambda_A^L + \lambda_B^L - \lambda_A^L . \lambda_B^L$], min [β, $\lambda_A^U + \lambda_B^U - \lambda_A^U . \lambda_B^U$])]. Ạ $\times_3$ Ḅ = $\begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{pmatrix}$ and Ạ $\times_3$ Ḅ = $\begin{pmatrix} \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31} & \omega_{32} & \omega_{33} \end{pmatrix}$

$\Pi_{\alpha,\beta}$(Ạ$\times_3$Ḅ)=

$$\begin{pmatrix} [\epsilon_{11} & \theta_{11} & \sigma_{11} & \pi_{11}] & [\epsilon_{12} & \theta_{12} & \sigma_{12} & \pi_{12}] & [\epsilon_{13} & \theta_{13} & \sigma_{13} & \pi_{13}] & [\epsilon_{14} & \theta_{14} & \sigma_{14} & \pi_{14}] \\ [\epsilon_{21} & \theta_{21} & \sigma_{21} & \pi_{21}] & [\epsilon_{22} & \theta_{22} & \sigma_{22} & \pi_{22}] & [\epsilon_{23} & \theta_{23} & \sigma_{23} & \pi_{23}] & [\epsilon_{24} & \theta_{24} & \sigma_{24} & \pi_{24}] \\ [\epsilon_{31} & \theta_{31} & \sigma_{31} & \pi_{31}] & [\epsilon_{32} & \theta_{32} & \sigma_{32} & \pi_{32}] & [\epsilon_{33} & \theta_{33} & \sigma_{33} & \pi_{33}] & [\epsilon_{34} & \theta_{34} & \sigma_{34} & \pi_{34}] \end{pmatrix} \times_3$$

$$\begin{pmatrix} [\psi_{11} & \rho_{11} & \phi_{11} & \chi_{11}] & [\psi_{12} & \rho_{12} & \phi_{12} & \chi_{12}] & [\psi_{13} & \rho_{13} & \phi_{13} & \chi_{13}] \\ [\psi_{21} & \rho_{21} & \phi_{21} & \chi_{21}] & [\psi_{22} & \rho_{22} & \phi_{22} & \chi_{22}] & [\psi_{23} & \rho_{23} & \phi_{23} & \chi_{23}] \\ [\psi_{31} & \rho_{31} & \phi_{31} & \chi_{31}] & [\psi_{32} & \rho_{32} & \phi_{32} & \chi_{32}] & [\psi_{33} & \rho_{33} & \phi_{33} & \chi_{33}] \\ [\psi_{41} & \rho_{41} & \phi_{41} & \chi_{41}] & [\psi_{42} & \rho_{42} & \phi_{42} & \chi_{42}] & [\psi_{43} & \rho_{43} & \phi_{43} & \chi_{43}] \end{pmatrix}$$

τ_{11} = [min (α, $\epsilon_{11} \cdot \psi_{11}$), min (α, $\theta_{11} \cdot \rho_{11}$)], [max (β, $\sigma_{11} + \phi_{11} - \sigma_{11} \cdot \phi_{11}$), max (β, $\pi_{11} + \chi_{11} - \pi_{11} \cdot \chi_{11}$)] +[min (α, $\epsilon_{12} \cdot \psi_{21}$), min (α, $\theta_{12} \cdot \rho_{21}$)], [max (β, $\sigma_{12} + \phi_{21} - \sigma_{12} \cdot \phi_{21}$), max (β, $\pi_{12} + \chi_{21} - \pi_{12} \cdot \chi_{21}$)] +[min (α, $\epsilon_{13} \cdot \psi_{31}$), min (α, $\theta_{13} \cdot \rho_{31}$)], [max (β, $\sigma_{13} + \phi_{31} - \sigma_{13} \cdot \phi_{31}$), max (β, $\pi_{13} + \chi_{31} - \pi_{13} \cdot \chi_{31}$)] +[min (α, $\epsilon_{14} \cdot \psi_{41}$), min (α, $\theta_{14} \cdot \rho_{41}$)], [max (β, $\sigma_{14} + \phi_{41} - \sigma_{14} \cdot \phi_{41}$), max (β, $\pi_{14} + \chi_{41} - \pi_{14} \cdot \chi_{41}$)]. Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}.

τ_{11} = max ([min (α, $\epsilon_{11} \cdot \psi_{11}$), min (α, $\theta_{11} \cdot \rho_{11}$)], [max (β, $\sigma_{11} + \phi_{11} - \sigma_{11} \cdot \phi_{11}$), max (β, $\pi_{11} + \chi_{11} - \pi_{11} \cdot \chi_{11}$)],[min (α, $\epsilon_{12} \cdot \psi_{21}$), min (α, $\theta_{12} \cdot \rho_{21}$)], [max (β, $\sigma_{12} + \phi_{21} - \sigma_{12} \cdot \phi_{21}$), max (β, $\pi_{12} + \chi_{21} - \pi_{12} \cdot \chi_{21}$)]) +[min (α, $\epsilon_{13} \cdot \psi_{31}$), min (α, $\theta_{13} \cdot \rho_{31}$)], [max (β, $\sigma_{13} + \phi_{31} - \sigma_{13} \cdot \phi_{31}$), max (β, $\pi_{13} + \chi_{31} - \pi_{13} \cdot \chi_{31}$)] +[min (α, $\epsilon_{14} \cdot \psi_{41}$), min (α, $\theta_{14} \cdot \rho_{41}$)], [max (β, $\sigma_{14} + \phi_{41} - \sigma_{14} \cdot \phi_{41}$), max (β, $\pi_{14} + \chi_{41} - \pi_{14} \cdot \chi_{41}$)]. Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}.

τ_{11} = max (max ([min (α, $\epsilon_{11} \cdot \psi_{11}$), min (α, $\theta_{11} \cdot \rho_{11}$)], [max (β, $\sigma_{11} + \phi_{11} - \sigma_{11} \cdot \phi_{11}$), max (β, $\pi_{11} + \chi_{11} - \pi_{11} \cdot \chi_{11}$)],[min (α, $\epsilon_{12} \cdot \psi_{21}$), min (α, $\theta_{12} \cdot \rho_{21}$)], [max (β, $\sigma_{12} + \phi_{21} - \sigma_{12} \cdot \phi_{21}$), max (β, $\pi_{12} + \chi_{21} - \pi_{12} \cdot \chi_{21}$)]),[min (α, $\epsilon_{13} \cdot \psi_{31}$), min (α, $\theta_{13} \cdot \rho_{31}$)], [max (β, $\sigma_{13} + \phi_{31} - \sigma_{13} \cdot \phi_{31}$), max (β, $\pi_{13}+\chi_{31}-\pi_{13}\cdot\chi_{31}$)]) +[min (α, $\epsilon_{14}\cdot\psi_{41}$), min (α, $\theta_{14}\cdot\rho_{41}$)], [max (β, $\sigma_{14}+\phi_{41}-\sigma_{14}\cdot\phi_{41}$), max (β, $\pi_{14}+\chi_{41}-\pi_{14}\cdot\chi_{41}$)]. Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}.

τ_{11} = max (max (max ([min (α, $\epsilon_{11}\cdot\psi_{11}$), min (α, $\theta_{11}\cdot\rho_{11}$)], [max (β, $\sigma_{11}+\phi_{11}-\sigma_{11}\cdot\phi_{11}$), max (β, $\pi_{11}+\chi_{11}-\pi_{11}\cdot\chi_{11}$)],[min (α, $\epsilon_{12}\cdot\psi_{21}$), min (α, $\theta_{12}\cdot\rho_{21}$)], [max (β, $\sigma_{12}+\phi_{21}-\sigma_{12}\cdot\phi_{21}$), max (β, $\pi_{12}+\chi_{21}-\pi_{12}\cdot\chi_{21}$)]),[min (α, $\epsilon_{13}\cdot\psi_{31}$), min (α, $\theta_{13}\cdot\rho_{31}$)], [max (β, $\sigma_{13}+\phi_{31}-\sigma_{13}\cdot\phi_{31}$), max (β, $\pi_{13}+\chi_{31}-\pi_{13}\cdot\chi_{31}$)]),[min (α, $\epsilon_{14}\cdot\psi_{41}$), min (α, $\theta_{14}\cdot\rho_{41}$)], [max (β, $\sigma_{14}+\phi_{41}-\sigma_{14}\cdot\phi_{41}$), max (β, $\pi_{14}+\chi_{41}-\pi_{14}\cdot\chi_{41}$)]). Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}. And

ω_{11} = [max (α, $\epsilon_{11}\cdot\psi_{11}$), max (α, $\theta_{11}\cdot\rho_{11}$)], [min (β, $\sigma_{11}+\phi_{11}-\sigma_{11}\cdot\phi_{11}$), min (β, $\pi_{11}+\chi_{11}-\pi_{11}\cdot\chi_{11}$)] +[max (α, $\epsilon_{12}\cdot\psi_{21}$), max (α, $\theta_{12}\cdot\rho_{21}$)], [min (β, $\sigma_{12}+\phi_{21}-\sigma_{12}\cdot\phi_{21}$), min (β, $\pi_{12}+\chi_{21}-\pi_{12}\cdot\chi_{21}$)] +[max (α, $\epsilon_{13}\cdot\psi_{31}$), max (α, $\theta_{13}\cdot\rho_{31}$)], [min (β, $\sigma_{13}+\phi_{31}-\sigma_{13}\cdot\phi_{31}$), min (β, $\pi_{13}+\chi_{31}-\pi_{13}\cdot\chi_{31}$)] +[max (α, $\epsilon_{14}\cdot\psi_{41}$), max (α, $\theta_{14}\cdot\rho_{41}$)], [min (β, $\sigma_{14}+\phi_{41}-\sigma_{14}\cdot\phi_{41}$), min (β, $\pi_{14}+\chi_{41}-\pi_{14}\cdot\chi_{41}$)]. Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

ω_{11} = max ([max (α, $\epsilon_{11}\cdot\psi_{11}$), max (α, $\theta_{11}\cdot\rho_{11}$)], [min (β, $\sigma_{11}+\phi_{11}-\sigma_{11}\cdot\phi_{11}$), min (β, $\pi_{11}+\chi_{11}-\pi_{11}\cdot\chi_{11}$)],[max (α, $\epsilon_{12}\cdot\psi_{21}$), max (α, $\theta_{12}\cdot\rho_{21}$)], [min (β, $\sigma_{12}+\phi_{21}-\sigma_{12}\cdot\phi_{21}$), min (β, $\pi_{12}+\chi_{21}-\pi_{12}\cdot\chi_{21}$)]) +[max (α, $\epsilon_{13}\cdot\psi_{31}$), max (α, $\theta_{13}\cdot\rho_{31}$)], [min (β, $\sigma_{13}+\phi_{31}-\sigma_{13}\cdot\phi_{31}$), min (β, $\pi_{13}+\chi_{31}-\pi_{13}\cdot\chi_{31}$)] +[max (α, $\epsilon_{14}\cdot\psi_{41}$), max (α, $\theta_{14}\cdot\rho_{41}$)], [min (β, $\sigma_{14}+\phi_{41}-\sigma_{14}\cdot\phi_{41}$), min (β, $\pi_{14}+\chi_{41}-\pi_{14}\cdot\chi_{41}$)]. Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

ω_{11} = max (max ([max (α, $\epsilon_{11}\cdot\psi_{11}$), max (α, $\theta_{11}\cdot\rho_{11}$)], [min (β, $\sigma_{11}+\phi_{11}-\sigma_{11}\cdot\phi_{11}$), min (β, $\pi_{11}+\chi_{11}-\pi_{11}\cdot\chi_{11}$)],[max (α, $\epsilon_{12}\cdot\psi_{21}$), max (α, $\theta_{12}\cdot\rho_{21}$)], [min (β, $\sigma_{12}+\phi_{21}-\sigma_{12}\cdot\phi_{21}$), min (β, $\pi_{12}+\chi_{21}-\pi_{12}\cdot\chi_{21}$)]),[max (α, $\epsilon_{13}\cdot\psi_{31}$), max (α, $\theta_{13}\cdot\rho_{31}$)], [min (β, $\sigma_{13}+\phi_{31}-\sigma_{13}\cdot\phi_{31}$), min (β, $\pi_{13}+\chi_{31}-\pi_{13}\cdot\chi_{31}$)]) +[max (α, $\epsilon_{14}\cdot\psi_{41}$), max (α, $\theta_{14}\cdot\rho_{41}$)], [min (β, $\sigma_{14}+\phi_{41}-\sigma_{14}\cdot\phi_{41}$), min (β, $\pi_{14}+\chi_{41}-\pi_{14}\cdot\chi_{41}$)]. Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

ω_{11} = max (max (max ([max (α, $\epsilon_{11}\cdot\psi_{11}$), max (α, $\theta_{11}\cdot\rho_{11}$)], [min (β, $\sigma_{11}+\phi_{11}-\sigma_{11}\cdot\phi_{11}$), min (β, $\pi_{11}+\chi_{11}-\pi_{11}\cdot\chi_{11}$)],[max (α, $\epsilon_{12}\cdot\psi_{21}$), max (α, $\theta_{12}\cdot\rho_{21}$)], [min (β, $\sigma_{12}+\phi_{21}-\sigma_{12}\cdot\phi_{21}$), min (β, $\pi_{12}+\chi_{21}-\pi_{12}\cdot\chi_{21}$)]),[max (α, $\epsilon_{13}\cdot\psi_{31}$), max (α, $\theta_{13}\cdot\rho_{31}$)], [min (β, $\sigma_{13}+\phi_{31}-\sigma_{13}\cdot\phi_{31}$), min (β, $\pi_{13}+\chi_{31}-\pi_{13}\cdot\chi_{31}$)]),[max (α, $\epsilon_{14}\cdot\psi_{41}$), max (α, $\theta_{14}\cdot\rho_{41}$)], [min (β, $\sigma_{14}+\phi_{41}-\sigma_{14}\cdot\phi_{41}$), min (β, $\pi_{14}+\chi_{41}-\pi_{14}\cdot\chi_{41}$)]). Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

Hence $\Pi_{\alpha,\beta}(\underset{\cdot}{A} \times_3 \underset{\cdot}{B})$ and $\Omega_{\alpha,\beta}(\underset{\cdot}{A} \times_3 \underset{\cdot}{B})$ is an interval valued intuitionistic fuzzy matrices.

Theorem 4.4. If Ạ $\times_4$ Ḅ are an interval valued intuitionistic fuzzy matrices, then $\Pi_{\alpha,\beta}$(Ạ $\times_4$ Ḅ) and $\Omega_{\alpha,\beta}$(Ạ $\times_4$ Ḅ) is also an interval valued intuitionistic fuzzy matrices.

Proof. Let us consider Ạ have 3×4 matrix and Ḅ have 4×3 matrix both are an interval valued intuitionistic fuzzy matrices. Then $\Pi_{\alpha,\beta}$(Ạ $\times_4$ Ḅ) = [(min [min $(\alpha,\ \mu_A^L, \mu_{\boldsymbol{B}}^L)$], min [min $(\alpha, \mu_A^U, \mu_B^U)$])], [(max [max $(\beta, \lambda_A^L, \lambda_B^L)$], max [max $(\beta, \lambda_A^U, \lambda_B^U)$])] And $\Omega_{\alpha,\beta}$(Ạ $\times_4$ Ḅ) = [(max [max $(\alpha,\ \mu_A^L, \mu_{\boldsymbol{B}}^L)$], max [max $(\alpha, \mu_A^U, \mu_B^U)$])], [(min [min $(\beta, \lambda_A^L, \lambda_B^L)$], min [min $(\beta, \lambda_A^U, \lambda_B^U)$])]. Ạ $\times_4$ Ḅ = $\begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{pmatrix}$ and

Ạ $\times_4$ Ḅ = $\begin{pmatrix} \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31} & \omega_{32} & \omega_{33} \end{pmatrix}$

$\Pi_{\alpha,\beta}$(Ạ$\times_4$Ḅ)=

$$\begin{pmatrix} [\epsilon_{11} & \theta_{11} & \sigma_{11} & \pi_{11}] & [\epsilon_{12} & \theta_{12} & \sigma_{12} & \pi_{12}] & [\epsilon_{13} & \theta_{13} & \sigma_{13} & \pi_{13}] & [\epsilon_{14} & \theta_{14} & \sigma_{14} & \pi_{14}] \\ [\epsilon_{21} & \theta_{21} & \sigma_{21} & \pi_{21}] & [\epsilon_{22} & \theta_{22} & \sigma_{22} & \pi_{22}] & [\epsilon_{23} & \theta_{23} & \sigma_{23} & \pi_{23}] & [\epsilon_{24} & \theta_{24} & \sigma_{24} & \pi_{24}] \\ [\epsilon_{31} & \theta_{31} & \sigma_{31} & \pi_{31}] & [\epsilon_{32} & \theta_{32} & \sigma_{32} & \pi_{32}] & [\epsilon_{33} & \theta_{33} & \sigma_{33} & \pi_{33}] & [\epsilon_{34} & \theta_{34} & \sigma_{34} & \pi_{34}] \end{pmatrix} \times_4$$

$$\begin{pmatrix} [\Psi_{11} & \rho_{11} & \phi_{11} & \chi_{11}] & [\Psi_{12} & \rho_{12} & \phi_{12} & \chi_{12}] & [\Psi_{13} & \rho_{13} & \phi_{13} & \chi_{13}] \\ [\Psi_{21} & \rho_{21} & \phi_{21} & \chi_{21}] & [\Psi_{22} & \rho_{22} & \phi_{22} & \chi_{22}] & [\Psi_{23} & \rho_{23} & \phi_{23} & \chi_{23}] \\ [\Psi_{31} & \rho_{31} & \phi_{31} & \chi_{31}] & [\Psi_{32} & \rho_{32} & \phi_{32} & \chi_{32}] & [\Psi_{33} & \rho_{33} & \phi_{33} & \chi_{33}] \\ [\Psi_{41} & \rho_{41} & \phi_{41} & \chi_{41}] & [\Psi_{42} & \rho_{42} & \phi_{42} & \chi_{42}] & [\Psi_{43} & \rho_{43} & \phi_{43} & \chi_{43}] \end{pmatrix}$$

τ_{11} = [min (min (α, ϵ_{11}, ψ_{11})), min (min (α, θ_{11}, ρ_{11}))], [max (max (β, σ_{11}, ϕ_{11})), max (max (β, π_{11}, χ_{11}))] +[min (min (α, ϵ_{12}, ψ_{21})), min (min (α, θ_{12}, ρ_{21}))], [max (max (β, σ_{12}, ϕ_{21})), max (max (β, π_{12}, χ_{21}))] +[min (min (α, ϵ_{13}, ψ_{31})), min (min (α, θ_{13}, ρ_{31}))], [max (max (β, σ_{13}, ϕ_{31})), max (max (β, π_{13}, χ_{31}))] +[min (min (α, ϵ_{14}, ψ_{41})), min (min (α, θ_{14}, ρ_{41}))], [max (max (β, σ_{14}, ϕ_{41})), max (max (β, π_{14}, χ_{41}))]. Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}.

τ_{11} = max ([min (min (α, ϵ_{11}, ψ_{11})), min (min (α, θ_{11}, ρ_{11}))], [max (max (β, σ_{11}, ϕ_{11})), max (max (β, π_{11}, χ_{11}))],[min (min (α, ϵ_{12}, ψ_{21})), min (min (α, θ_{12}, ρ_{21}))], [max (max (β, σ_{12}, ϕ_{21})), max (max (β, π_{12}, χ_{21}))]) +[min (min (α, ϵ_{13}, ψ_{31})), min (min (α, θ_{13}, ρ_{31}))], [max (max (β, σ_{13}, ϕ_{31})), max (max (β, π_{13}, χ_{31}))] +[min (min (α, ϵ_{14}, ψ_{41})), min (min (α, θ_{14}, ρ_{41}))], [max (max (β, σ_{14}, ϕ_{41})), max (max (β, π_{14}, χ_{41}))]. Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}.

τ_{11} = max (max ([min (min (α, ϵ_{11}, ψ_{11})), min (min (α, θ_{11}, ρ_{11}))], [max (max (β, σ_{11}, ϕ_{11})), max (max (β, π_{11}, χ_{11}))],[min (min (α, ϵ_{12}, ψ_{21})), min (min (α, θ_{12}, ρ_{21}))], [max (max (β, σ_{12}, ϕ_{21})), max (max (β, π_{12}, χ_{21}))]),[min (min (α, ϵ_{13}, ψ_{31})), min (min (α, θ_{13}, ρ_{31}))], [max (max (β, σ_{13}, ϕ_{31})), max (max (β, π_{13}, χ_{31}))]) +[min (min (α, ϵ_{14}, ψ_{41})), min (min (α, θ_{14}, ρ_{41}))], [max (max (β, σ_{14}, ϕ_{41})), max (max (β, π_{14}, χ_{41}))]. Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}.

τ_{11} = max (max (max ([min (min (α, ϵ_{11}, ψ_{11})), min (min (α, θ_{11}, ρ_{11}))], [max (max (β, σ_{11}, ϕ_{11})), max (max (β, π_{11}, χ_{11}))],[min (min (α, ϵ_{12}, ψ_{21})), min (min (α, θ_{12}, ρ_{21}))], [max (max (β, σ_{12}, ϕ_{21})), max (max (β, π_{12}, χ_{21}))]),[min (min (α, ϵ_{13}, ψ_{31})), min (min (α, θ_{13}, ρ_{31}))], [max (max (β, σ_{13}, ϕ_{31})), max (max (β, π_{13}, χ_{31}))]),[min (min (α, ϵ_{14}, ψ_{41})), min (min (α, θ_{14}, ρ_{41}))], [max (max (β, σ_{14}, ϕ_{41})), max (max (β, π_{14}, χ_{41}))]). Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}. And

ω_{11} = [max (max (α, ϵ_{11}, ψ_{11})), max (max (α, θ_{11}, ρ_{11}))], [min (min (β, σ_{11}, ϕ_{11})), min (min (β, π_{11}, χ_{11}))] +[max (max (α, ϵ_{12}, ψ_{21})), max (max (α, θ_{12}, ρ_{21}))], [min (min (β, σ_{12}, ϕ_{21})), min (min (β, π_{12}, χ_{21}))] +[max (max (α, ϵ_{13}, ψ_{31})), max (max (α, θ_{13}, ρ_{31}))], [min (min (β, σ_{13}, ϕ_{31})), min (min (β, π_{13}, χ_{31}))] +[max (max (α, ϵ_{14}, ψ_{41})), max (max (α, θ_{14}, ρ_{41}))], [min (min (β, σ_{14}, ϕ_{41})), min (min (β, π_{14}, χ_{41}))]. Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

ω_{11} = max ([max (max (α, ϵ_{11}, ψ_{11})), max (max (α, θ_{11}, ρ_{11}))], [min (min (β, σ_{11}, ϕ_{11})), min (min (β, π_{11}, χ_{11}))],[max (max (α, ϵ_{12}, ψ_{21})), max (max (α, θ_{12}, ρ_{21}))], [min (min (β, σ_{12}, ϕ_{21})), min (min (β, π_{12}, χ_{21}))]) +[max (max (α, ϵ_{13}, ψ_{31})), max (max (α, θ_{13}, ρ_{31}))], [min (min (β, σ_{13}, ϕ_{31})), min (min (β, π_{13}, χ_{31}))] +[max (max (α, ϵ_{14}, ψ_{41})), max (max (α, θ_{14}, ρ_{41}))], [min (min (β, σ_{14}, ϕ_{41})), min (min (β, π_{14}, χ_{41}))]. Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

ω_{11} = max (max ([max (max (α, ϵ_{11}, ψ_{11})), max (max (α, θ_{11}, ρ_{11}))], [min (min (β, σ_{11}, ϕ_{11})), min (min (β, π_{11}, χ_{11}))],[max (max (α, ϵ_{12}, ψ_{21})), max (max (α, θ_{12}, ρ_{21}))], [min (min (β, σ_{12}, ϕ_{21})), min (min (β, π_{12}, χ_{21}))]),[max (max (α, ϵ_{13}, ψ_{31})), max (max (α, θ_{13}, ρ_{31}))], [min (min (β, σ_{13}, ϕ_{31})), min (min (β, π_{13}, χ_{31}))]) +[max (max (α, ϵ_{14}, ψ_{41})), max (max (α, θ_{14}, ρ_{41}))], [min (min (β, σ_{14}, ϕ_{41})), min (min (β, π_{14}, χ_{41}))]. Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

ω_{11} = max (max (max ([max (max (α, ϵ_{11}, ψ_{11})), max (max (α, θ_{11}, ρ_{11}))], [min (min (β, σ_{11}, ϕ_{11})), min (min (β, π_{11}, χ_{11}))],[max (max (α, ϵ_{12}, ψ_{21})), max (max (α, θ_{12}, ρ_{21}))], [min (min (β, σ_{12}, ϕ_{21})), min (min (β, π_{12}, χ_{21}))]),[max (max (α, ϵ_{13}, ψ_{31})), max (max (α, θ_{13}, ρ_{31}))], [min (min (β, σ_{13}, ϕ_{31})), min (min (β, π_{13}, χ_{31}))]),[max (max (α, ϵ_{14}, ψ_{41})), max (max (α, θ_{14}, ρ_{41}))], [min (min (β, σ_{14}, ϕ_{41})), min (min (β, π_{14}, χ_{41}))]). Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

Hence $\Pi_{\alpha,\beta}$(Ạ $\times_4$ Ḅ) and $\Omega_{\alpha,\beta}$(Ạ $\times_4$ Ḅ) is an interval valued intuitionistic fuzzy matrices.

Theorem 4.5. If Ạ $\times_5$ Ḅ are an interval valued intuitionistic fuzzy matrices, then $\Pi_{\alpha,\beta}$(Ạ $\times_5$ Ḅ) and $\Omega_{\alpha,\beta}$(Ạ $\times_5$ Ḅ) is also an interval valued intuitionistic fuzzy matrices.

Proof. Let us consider Ạ have 3×4 matrix and Ḅ have 4×3 matrix both are an interval valued intuitionistic fuzzy matrices. Then $\Pi_{\alpha,\beta}$(Ạ $\times_5$ Ḅ) = [(min [max (α, $\mu_A^L, \mu_{\boldsymbol{B}}^L$)], min [max (α, μ_A^U, μ_B^U)])], [(max [min (β, λ_A^L , λ_B^L)], max [min (β, λ_A^U, λ_B^U)])] and $\Omega_{\alpha,\beta}$(Ạ $\times_5$ Ḅ) = [(max [min (α, $\mu_A^L, \mu_{\boldsymbol{B}}^L$)], max [min (α, μ_A^U, μ_B^U)])], [(min [max (β, λ_A^L , λ_B^L)], min [max (β, λ_A^U, λ_B^U)])]. Ạ $\times_5$ Ḅ = $\begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{pmatrix}$ and

Ạ $\times_5$ Ḅ = $\begin{pmatrix} \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31} & \omega_{32} & \omega_{33} \end{pmatrix}$

$\Pi_{\alpha,\beta}(\dot{A}\times_5\dot{B})$=

$$\begin{pmatrix}[\epsilon_{11} & \theta_{11} & \sigma_{11} & \pi_{11}] & [\epsilon_{12} & \theta_{12} & \sigma_{12} & \pi_{12}] & [\epsilon_{13} & \theta_{13} & \sigma_{13} & \pi_{13}] & [\epsilon_{14} & \theta_{14} & \sigma_{14} & \pi_{14}]\\ [\epsilon_{21} & \theta_{21} & \sigma_{21} & \pi_{21}] & [\epsilon_{22} & \theta_{22} & \sigma_{22} & \pi_{22}] & [\epsilon_{23} & \theta_{23} & \sigma_{23} & \pi_{23}] & [\epsilon_{24} & \theta_{24} & \sigma_{24} & \pi_{24}]\\ [\epsilon_{31} & \theta_{31} & \sigma_{31} & \pi_{31}] & [\epsilon_{32} & \theta_{32} & \sigma_{32} & \pi_{32}] & [\epsilon_{33} & \theta_{33} & \sigma_{33} & \pi_{33}] & [\epsilon_{34} & \theta_{34} & \sigma_{34} & \pi_{34}]\end{pmatrix}\times_5$$

$$\begin{pmatrix}[\psi_{11} & \rho_{11} & \phi_{11} & \chi_{11}] & [\psi_{12} & \rho_{12} & \phi_{12} & \chi_{12}] & [\psi_{13} & \rho_{13} & \phi_{13} & \chi_{13}]\\ [\psi_{21} & \rho_{21} & \phi_{21} & \chi_{21}] & [\psi_{22} & \rho_{22} & \phi_{22} & \chi_{22}] & [\psi_{23} & \rho_{23} & \phi_{23} & \chi_{23}]\\ [\psi_{31} & \rho_{31} & \phi_{31} & \chi_{31}] & [\psi_{32} & \rho_{32} & \phi_{32} & \chi_{32}] & [\psi_{33} & \rho_{33} & \phi_{33} & \chi_{33}]\\ [\psi_{41} & \rho_{41} & \phi_{41} & \chi_{41}] & [\psi_{42} & \rho_{42} & \phi_{42} & \chi_{42}] & [\psi_{43} & \rho_{43} & \phi_{43} & \chi_{43}]\end{pmatrix}$$

τ_{11} = [min (max (α, ϵ_{11}, ψ_{11})), min (max (α, θ_{11}, ρ_{11}))], [max (min (β, σ_{11}, ϕ_{11})), max (min (β, π_{11}, χ_{11}))] +[min (max (α, ϵ_{12}, ψ_{21})), min (max (α, θ_{12}, ρ_{21}))], [max (min (β, σ_{12}, ϕ_{21})), max (min (β, π_{12}, χ_{21}))] +[min (max (α, ϵ_{13}, ψ_{31})), min (max (α, θ_{13}, ρ_{31}))], [max (min (β, σ_{13}, ϕ_{31})), max (min (β, π_{13}, χ_{31}))] +[min (max (α, ϵ_{14}, ψ_{41})), min (max (α, θ_{14}, ρ_{41}))], [max (min (β, σ_{14}, ϕ_{41})), max (min (β, π_{14}, χ_{41}))]. Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}.

τ_{11} = max ([min (max (α, ϵ_{11}, ψ_{11})), min (max (α, θ_{11}, ρ_{11}))], [max (min (β, σ_{11}, ϕ_{11})), max (min (β, π_{11}, χ_{11}))],[min (max (α, ϵ_{12}, ψ_{21})), min (max (α, θ_{12}, ρ_{21}))], [max (min (β, σ_{12}, ϕ_{21})), max (min (β, π_{12}, χ_{21}))]) +[min (max (α, ϵ_{13}, ψ_{31})), min (max (α, θ_{13}, ρ_{31}))], [max (min (β, σ_{13}, ϕ_{31})), max (min (β, π_{13}, χ_{31}))] +[min (max (α, ϵ_{14}, ψ_{41})), min (max (α, θ_{14}, ρ_{41}))], [max (min (β, σ_{14}, ϕ_{41})), max (min (β, π_{14}, χ_{41}))]. Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}.

τ_{11} = max (max ([min (max (α, ϵ_{11}, ψ_{11})), min (max (α, θ_{11}, ρ_{11}))], [max (min (β, σ_{11}, ϕ_{11})), max (min (β, π_{11}, χ_{11}))],[min (max (α, ϵ_{12}, ψ_{21})), min (max (α, θ_{12}, ρ_{21}))], [max (min (β, σ_{12}, ϕ_{21})), max (min (β, π_{12}, χ_{21}))]),[min (max (α, ϵ_{13}, ψ_{31})), min (max (α, θ_{13}, ρ_{31}))], [max (min (β, σ_{13}, ϕ_{31})), max (min (β, π_{13}, χ_{31}))]) +[min (max (α, ϵ_{14}, ψ_{41})), min (max (α, θ_{14}, ρ_{41}))], [max (min (β, σ_{14}, ϕ_{41})), max (min (β, π_{14}, χ_{41}))]. Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}.

τ_{11} = max (max (max ([min (max (α, ϵ_{11}, ψ_{11})), min (max (α, θ_{11}, ρ_{11}))], [max (min (β, σ_{11}, ϕ_{11})), max (min (β, π_{11}, χ_{11}))],[min (max (α, ϵ_{12}, ψ_{21})), min (max (α, θ_{12}, ρ_{21}))], [max (min (β, σ_{12}, ϕ_{21})), max (min (β, π_{12}, χ_{21}))]),[min (max (α, ϵ_{13}, ψ_{31})), min (max (α, θ_{13}, ρ_{31}))], [max (min (β, σ_{13}, ϕ_{31})), max (min (β, π_{13}, χ_{31}))]),[min (max (α, ϵ_{14}, ψ_{41})), min (max (α, θ_{14}, ρ_{41}))], [max (min (β, σ_{14}, ϕ_{41})), max (min (β, π_{14}, χ_{41}))]). Similarly, we can expand the expression of τ_{12},τ_{13},τ_{21},τ_{22},τ_{23},τ_{31},τ_{32},τ_{33}. And ω_{11} = [max (min (α, ϵ_{11}, ψ_{11})), max (min (α, θ_{11}, ρ_{11}))], [min (max (β, σ_{11}, ϕ_{11})), min (max (β, π_{11}, χ_{11}))] +[max (min (α, ϵ_{12}, ψ_{21})), max (min (α, θ_{12}, ρ_{21}))], [min (max (β, σ_{12}, ϕ_{21})), min (max (β, π_{12}, χ_{21}))] +[max (min (α, ϵ_{13}, ψ_{31})), max (min (α, θ_{13}, ρ_{31}))], [min (max (β, σ_{13}, ϕ_{31})), min (max (β, π_{13}, χ_{31}))] +[max (min (α, ϵ_{14}, ψ_{41})), max (min (α, θ_{14}, ρ_{41}))], [min (max (β, σ_{14}, ϕ_{41})), min (max (β, π_{14}, χ_{41}))]. Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

ω_{11} = max ([max (min (α, ϵ_{11}, ψ_{11})), max (min (α, θ_{11}, ρ_{11}))], [min (max (β, σ_{11}, ϕ_{11})), min (max (β, π_{11}, χ_{11}))],[max (min (α, ϵ_{12}, ψ_{21})), max (min (α, θ_{12}, ρ_{21}))], [min (max (β, σ_{12}, ϕ_{21})), min (max (β,

π_{12}, χ_{21}))]) +[max (min (α, ϵ_{13}, ψ_{31})), max (min (α, θ_{13}, ρ_{31}))], [min (max (β, σ_{13}, ϕ_{31})), min (max (β, π_{13}, χ_{31}))] +[max (min (α, ϵ_{14}, ψ_{41})), max (min (α, θ_{14}, ρ_{41}))], [min (max (β, σ_{14}, ϕ_{41})), min (max (β, π_{14}, χ_{41}))]. Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

ω_{11} = max (max ([max (min (α, ϵ_{11}, ψ_{11})), max (min (α, θ_{11}, ρ_{11}))], [min (max (β, σ_{11}, ϕ_{11})), min (max (β, π_{11}, χ_{11}))],[max (min (α, ϵ_{12}, ψ_{21})), max (min (α, θ_{12}, ρ_{21}))], [min (max (β, σ_{12}, ϕ_{21})), min (max (β, π_{12}, χ_{21}))]),[max (min (α, ϵ_{13}, ψ_{31})), max (min (α, θ_{13}, ρ_{31}))], [min (max (β, σ_{13}, ϕ_{31})), min (max (β, π_{13}, χ_{31}))]) +[max (min (α, ϵ_{14}, ψ_{41})), max (min (α, θ_{14}, ρ_{41}))], [min (max (β, σ_{14}, ϕ_{41})), min (max (β, π_{14}, χ_{41}))]. Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

ω_{11} = max (max (max ([max (min (α, ϵ_{11}, ψ_{11})), max (min (α, θ_{11}, ρ_{11}))], [min (max (β, σ_{11}, ϕ_{11})), min (max (β, π_{11}, χ_{11}))],[max (min (α, ϵ_{12}, ψ_{21})), max (min (α, θ_{12}, ρ_{21}))], [min (max (β, σ_{12}, ϕ_{21})), min (max (β, π_{12}, χ_{21}))]),[max (min (α, ϵ_{13}, ψ_{31})), max (min (α, θ_{13}, ρ_{31}))], [min (max (β, σ_{13}, ϕ_{31})), min (max (β, π_{13}, χ_{31}))]),[max (min (α, ϵ_{14}, ψ_{41})), max (min (α, θ_{14}, ρ_{41}))], [min (max (β, σ_{14}, ϕ_{41})), min (max (β, π_{14}, χ_{41}))]). Similarly, we can expand the expression of ω_{12},ω_{13},ω_{21},ω_{22},ω_{23},ω_{31},ω_{32},ω_{33}.

Hence $\Pi_{\alpha,\beta}(\dot{A} \times_5 \dot{B})$ and $\Omega_{\alpha,\beta}(\dot{A} \times_5 \dot{B})$ is an interval valued intuitionistic fuzzy matrices.

Conclusion: In this article some Cartesian product of intuitionistic fuzzy set and the versions of Cartesian product over interval valued intuitionistic fuzzy matrices is presented in this paper. And some properties, applications are discussed. Also, introduce different types of interval valued intuitionistic fuzzy sets, whether a membership function or a non-membership function, they must be a specific number or value, because they are specific in intuitionistic fuzzy sets. Also, we derive the operations cartesian product of interval valued intuitionistic fuzzy matrices. However, they become an interval or range, because of the uncertainty in interval valued intuitionistic fuzzy matrices. We have proved some type of Cartesian product of interval valued intuitionistic fuzzy matrices, which makes it match to some Cartesian product of interval valued intuitionistic fuzzy sets. We should try prove other Cartesian products of interval valued intuitionistic fuzzy sets shall be study in the future.

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