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Point Set Neutrosophic Domination in Single Valued Neutrosophic Graph

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Abstract:

In this paper, we demonstrate a concept of point set neutro-sophic domination, 2-point set neutrosophic domination, connected point set neutrosophic domination, point set tree neutrosophic domination with appropriate example. Some of their theoretical properties are investigates.

Keywords: Neutrosophic graph, Dominance in neutrosophic graph, point set neutrosophic dominance, point set tree neutrosophic dominance, connected point set neutrosophic dominance.

1. Introduction

In 1965, L.A. Zadeh [21] gave initial proposal for occurrence of uncertainty in real life situation of mathematical framework. Rosenfeld [13] developed the idea of fuzzy networks with membership value in [0, 1] after noticed Zadeh fuzzy function on fuzzy batches. Idea of expanding fuzzy network to intuition-istic fuzzy networks by K.T. Atanassov [1] and introduced additional level of indeterminacy in intuitionistic fuzzy relationships. Florentine Smarandache *et al.* [15, 19, 20] gave an idea for neutrosophic network & single valued neutrosophic network or graphs as an extension of K.T. Atanassov concept on the fuzzy network and the intuitionistic fuzzy network. The concept of Single valued neutrosophic graph and its additives was introduced by Said Broumi *et al.* [3]. Orge [13] and Berge [2] was introduced domination in graphs and In 1977, a study on domination number was begun by Cockayne & Hedetniemi [5]. A. Somasundaram and S. Somasundaram [16] introduced domination in fuzzy network. Domination in fuzzy graph using strong arcs is discussed by A. Nagoorgani V.T. Chandrasekaran [12]. An idea of point set domination in graphs are introduced by Sampathkumar and Pushpalatha [14]. S. Kaspar & B. Gayathri [11] introduced few results on point set tree domination of graphs. V. Swaminathan and R. Poovazhaki [17] introduced the idea of point set domination with reference to degree and also discussed connected point set domination of graph. Idea of connected point set domination of fuzzy graph was discussed by S. Vimala & J.S. Sathya [18].

In this paper Section 2 contains preliminary, section 3 defines point set neutrosophic dominance number, point set tree neutrosophic dominance number, connected point set neutrosophic dominance number in neutrosophic network and their bounds has been formulated and Section 4 concludes the paper.

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2. Preliminaries

Definition 2.1 (8). *A Pair* $G = (A, B)$ *is known as single valued neutrosophic graph with the underlying set* V .

1. The functions $TA \to [0, 1], IA : V \to [0, 1]$ *and* $FA : V \to [0, 1]$ *denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element* $vi \in V$ *respectively and* $0 \leq TA(vi) + IA(vi) + FA(vi) \leq 3$ *for all* $vi \in V$.

2 . The functions $TB : E \subseteq V \times V \to [0, 1], IB : E \subseteq V \times V \to [0, 1]$ *and*

$FB : E \subseteq V \times V \to [0, 1]$ *are defined by truth-membership, indeterminacy-membership and falsity-membership of the* $TA(vi, vj) \leq TA(vi) \wedge TA(vj)$, $IA\,(vi, vj) \geq IA(vi) \vee IA(vj)$, $FA(vi, vj) \geq FA(vi) \vee FA(vj)$, *denotes the degree of edge* $(vi, vj) \in E\ (i, j = 1, 2, \ldots, n)$.

Definition 2.2 (4). *Let* $G = (A, B)$ *be a SVNG, G is said to be strong SVNG if* $TB(u, v) = TA(u) \wedge TA(v)$, $IB(u, v) = IA(u) \vee TA(v), FB(u, v) = FA(u) \vee FA(v)$ *for every* $(u, v) \in E$.

Definition 2.3 (6). *Let* $G = (A, B)$ *be a SVNG, G is said to be complete*

SVNG if $TB(u, v) = TA(u) \wedge TA(v)$, $IB(u, v) = IA(u) \vee TA(v)$, $FB(u, v) = FA(u) \vee FA(v)$ *for every* $u, v \in E$.

Definition 2.4 (7). *Let* $G = (A, B)$ *be a SVNG on* V *, then the neutrosophic*

vertex cardinality of G is defined by $|V| = \sum_{V\,(u,v)\in V} \frac{1 + TB(u,v) + IB(u,v) - FB(u,v)}{2}$

Definition 2.5 (7). *Let* $G = (A, B)$ *be a SVNG on* E *, then the neutrosophic*

edge cardinality of G is defined by $|E| = \sum_{(u,v)\in E} \frac{1 + TB(u,v) + IB(u,v) - FB(u,v)}{2}$

Definition 2.6 (12). *An arc* (u, v) *of a SVNG* G *is called strong arc if* $TB(u, v) = TA(u) \wedge TA(v), I\,B(u, v) = IA(u) \vee IA(v), FB(u, v) = FA(u) \vee FA(v)$.

Definition 2.7 (7). Let $G = (A, B)$ be a SVNG on. Let $(u, v) \in V$, we say that u dominates v in G, if there exist a strong arc between them.

Definition 2.8 (7). *Given* $S \subset V$ *is dominating set in G if for every vertex* $v \in V - S$ *there exist a vertex* $u \in S$ *such that u dominates* v, *for all* $e \in A, u, v \in V$.

Definition 2.9 (10). *Let* $G = (A, B)$ *be a fuzzy graph. Let* $u, v \in V$ *and we say that* u *dominates* v *in* G *if* $\mu(u, v) = \sigma(u) \vee \sigma(u)$. *A subset* S *of* V *is called dominance set in* G *if for every* $v \in V - S$ *there exist* $u \in S$ *such that* u *dominates* v. *The minimum fuzzy cardinality of a dominating set in* G *is called the dominance number of* G *and is denoted by* $\gamma(G)$

Definition 2.10 (18). *A dominating set* $D \subseteq V$ *of a fuzzy graph G is said to be a point set dominating set of* G *if for every* $S \subseteq V - D$ *there exist a node* $d \in D$ *such that* $< S \cup \{d\} >$ *is a connected fuzzy graph. The minimum cardinality taken over all minimal connected point set is called the point set domination number of the fuzzy graph* G *and it is denoted by* $\gamma p(G)$

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Definition 2.11 (18)**.** *A point set dominating set $D \subseteq V$ of any fuzzy graph G is a connected point set dominating set of G if the subgraph $< D >$ induced by D is a connected fuzzy graph. The minimum cardinality taken over all minimal connected point set dominating set is called the connected point set domination number $\gamma cp(G)$.*

Definition 2.12 (16)**.** *Let $G = (X, Y)$ be a single valued neutrosophic network.consider a subset S of V such that $u \in S$ dominating v for every $v \in V - S$, then that subset is known to be a neutrosophic dominance set in G is given by $\gamma nd(G)$.*

3. Point set domination in neutrosophic graph

In this paper we use some basic notation, $G = (X, Y)$ is neutrosophic network or graph, X be a Vertex set, Y be a edge set, $T_X(v)$, $I_X(v)$ $F_X(v)$ be truth, indeterminacy and falsity membership values of vertices in graph G. $T_Y(u, v)$, , $I_Y(u, v)$, , $F_Y(u, v)$ is truth, indeterminacy and falsity membership value of the edge (u, v) of G.

Definition 3.1. *Let $G = (X, Y)$ be a single valued neutrosophic network, if for every $A \subseteq X - B$ there exist a vertices $b \in B$ such that $< A \cup \{b\} >$ is connected neutrosophic set in G then the subset $B \subseteq X$ is called point set neutrosophic dominance set (D^{psn}). Point set domination number of G is the number with the minimum vertex cardinality in all point set domination set of G and it is denoted by $\gamma_{pnd}(G)$.*

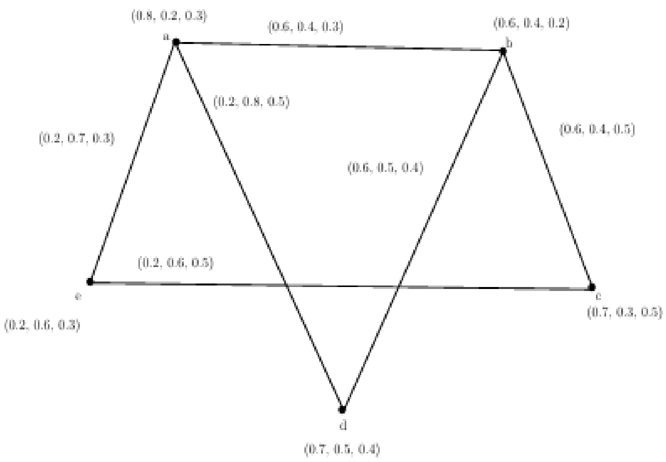

Figure 1: $D^{psn} = \{b, e\}, \{a, d, c\}, \{d, c\}, \{b, d, e\}, \{a, b, c, d\}$ are few points set neutrosophic domination & $\gamma_{pnd}(G) = 1.65$

Definition 3.2. *Let $G = (X, Y)$ be a single valued neutrosophic network, if the subset $< B >$ induced by B is a connected neutrosophic graph then the point set domination $B \subseteq X(G)$ of any neutrosophic graph G is connected point set domination set (D^{cpsn}).*

Connected point set domination number of G is the number with the minimum vertex cardinality in all connected point set domination set of G and it is denoted by $\gamma_{cpnd}(G)$.

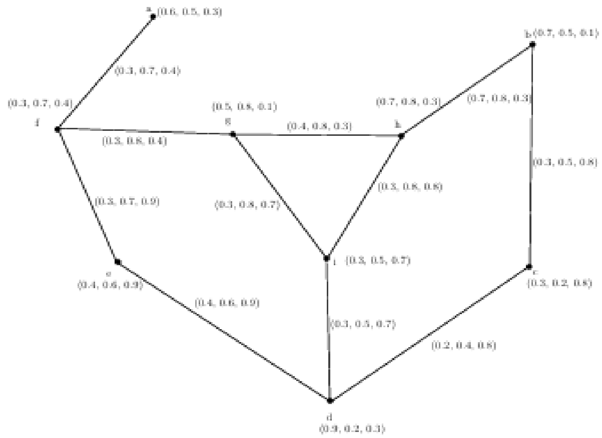

Figure 2: $D^{cpsn} = \{b, c, d, e, f\}, \{b, f, g, i, d, c\}, \{b, i, d, e, f\}$ are few connected point set neutrosophic domination & $\gamma_{cpnd}(G) = 3.65$

Definition 3.3. *Let $G = (X, Y)$ be single valued neutrosophic network, a set $B \subseteq X(G)$ is called 2-point set neutrosophic domination set (D^{2psn}) of G if for every set $T \subseteq X - B$ there exists a non-empty set $S \subseteq B$ containing at-most two vertices such that the induced subgraph $< S \cup T >$ is connected. 2-point set neutrosophic domination number of G is the number with theminimum vertex cardinality in all 2-point set domination set of G and it is denoted by $\gamma_{2psnd}(G)$.*

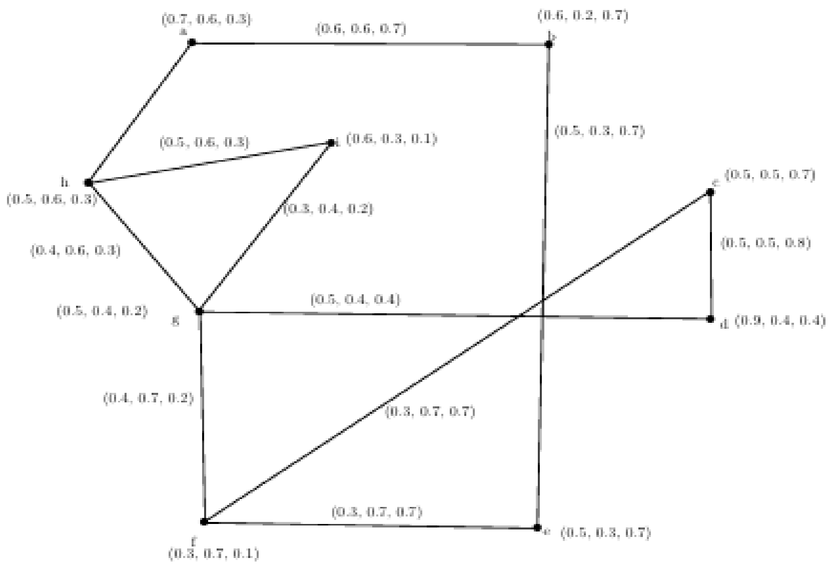

Figure 3: D^{2psn}= *{a, f, g, h}, {b, f, g, h}, {b, f, d, h}, {b, f, g, i}, {b, c, e, g, h}* are few 2-point set neutrosophic domination & $\gamma_{2psnd}(G)$ = 2.95**.**

Definition 3.4. *Let G = (X, Y) be single valued neutrosophic network, if for each subset $A \subseteq X - B$ there exists a vertex $b \in B$ such that subgraph < $A \cup \{b\}$ > induced by the vertices of $A \cup \{b\}$ is tree then a subset B ⊆ X(G) of any graph G is a point set tree neutrosophic domination (D^{pstn}). point set tree neutrosophic domination number of G is the number with the minimum vertex cardinality in all point set tree domination set of G and it is denoted by $\gamma_{pstnd}(G)$.*

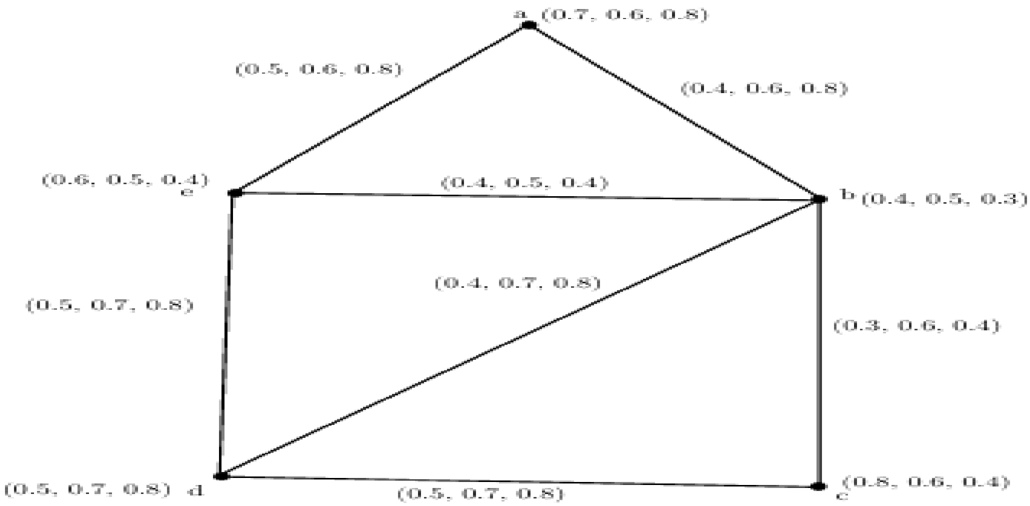

Figure 4: D^{pstn}= *{b, c}, {b, a, c}, {b, a, e, c}, {b, d, c}, {a, b, c, d}, {a, b, d}* are point set tree neutrosophic domination & $\gamma_{pstnd}(G)$. = 1.8.

Theorem 3.5. *In Single valued neutrosophic network G* = (*X*, *Y*),

δ(*G*) ≤ *O*(D^{psn}) & Δ(*G*) ≤ *O*(D^{psn}, *Where O*(D^{psn}) *is point set neutrosophic domination.*

Proof. From fig:1

The Maximum degree of *G* : Δ*T*(*G*) = *max dT*(*vi/vi* ∈*V*) = 1.1, Δ*I*(*G*) = *max dI*(*vi/vi* ∈*V*) = 1.2, Δ*F*(*G*) = *max* *dF*(*vi/vi* ∈*V*) = 1.5.
The maximum degree of *G* is Δ(*G*) = *max{dT*(*vi*), *dI*(*vi*), *dF*(*vi*)*}* = (1.1, 1.2, 1.5)

The Minimum degree of *G* : *δT*(*G*) = *min dT*(*vi/vi* ∈*V*) = 0.8, *δI*(*G*) = *min dI*(*vi/vi* ∈*V*) = 0.9, *δF*(*G*) = *min* *dF*(*vi/vi* ∈*V*) = 0.5.
The minimum degree of *G* is *δ*(*G*) = *max{dT*(*vi*), *dI*(*vi*), *dF*(*vi*)*}* = (0.8, 0.9, 0.5)

OT(*D*) = *OT*(*b, c, d, i*) = 2.6, *OI*(*D*) = *OI*(*b, c, d, i*) = 1.4, *OF*(*D*) = *OF*(*b, c, d, i*) = 1.9,

O(D^{psn}) = (2.6, 1.4, 1.9)

Therefore *δ*(*G*) ≤ *O*(D^{psn}) & Δ(*G*) ≤ *O*(D^{psn})

Theoren 3.6. Let G be a complete neutrosophic graph and ${D_K}^{psn}$ is a point *set neutrosophic domination set then V* $-{D_K}^{psn}$ *has a point set neutrosophic domination set.*

Proof. Given *G* is a complete neutrosophic graph then every edge $e \in Y(G)$ is an effective edge and each vertex $v \in X(G)$ is dominating all others. Thus a minimum point set neutrosophic dominance set ${D_K}^{psn}$ contains only one vertex, then $V-{D_K}^{psn}$ set domination set. is point set domination set. Consequently $V-{D_K}^{psn}$ has point set domination set.

Theorem 3.7. *In Single valued neutrosophic network* $G = (X, Y)$, *complement of the complete graph can not point set neutrosophic domination.*

Theorem 3.8. *For single valued Neutrosophic network $G = (X, Y)$, any point set domination set is 2-point set domination set of G*

Proof. Consider $G = (X, Y)$ is neutrosophic network, Let G be a 2-point set neutrosophic dominance set of G. If for every set $T \subseteq X - D$, there exist a non empty set $S \subseteq D$ containing at most two vertices such that the induced subgraph $< S \cup T >$ is connected. Since it is 2-point set neutrosophic dominance set of G. If for each $T \subseteq X - D$ there exist $u \in S$, then $< \{u\} \cup T >$ is connected. Therefore G is point set neutrosophic dominance.

Theorem 3.9. *For SVNG, $\gamma_{2psnd}(G) \leq \gamma_{psnd}(G) \leq \gamma_{pstnd}(G) \leq \gamma_{cpsnd}(G)$.*

Proof. Let A be a least point set neutrosophic dominance set of neutrosophic network G and $\gamma_{psnd}(G) = s$. Every point set domination is a 2-point set domination in neutrosophic network so, $\gamma_{2psnd}(G)) = s$.

$(i.e)$ $\gamma_{2psnd}(G) \leq \gamma_{psnd}(G)$

Suppose A is not a least point set neutrosophic domination set and if A' is least point set neutrosophic domination set then $\gamma_{psnd}(G) > s$.

Then $\gamma_{2psnd}(G) \leq \gamma_{psnd}(G)$

Let B be a least point set tree neutrosophic domination set of neutrosophic graph G then $\gamma_{psntd}(G) = t$. If B be a least connected point set neutrosophic domination set of neutrosophic graph G then $\gamma_{psntd}(G) = t$.

$(i.e)$ $\gamma_{psntd}(G) \leq \gamma_{cpsnd}(G)$

Suppose B is not a least connected point set neutrosophic domination and if B' is a least connected point set neutrosophic dominance set then $\gamma_{cpsnd}(G) > t$. Thus $\gamma_{psntd}(G) \leq \gamma_{cpsnd}(G)$.

Let C be a least point set neutrosophic domination set of neutrosophic graph and $\gamma_{psnd}(G) = u$ if C is also least point set tree neutrosophic domination of neutrosophic graph G then $\gamma_{psntd}(G) = u$. (i.e) $\gamma_{psnd}(G) \leq \gamma_{psntd}(G)$

Suppose C is not a least point set tree neutrosophic dominance set and if C' is a least point set tree neutrosophic dominance set then $\gamma_{psntd}(G) > u$. Then $\gamma_{psnd}(G) \leq \gamma_{psntd}(G)$.

From (1), (2), (3) we get, $\gamma_{2psnd}(G) \leq \gamma_{psnd}(G) \leq \gamma_{psntd}(G) \leq \gamma_{cpsnd}(G)$.

Theorem 3.10. *A domination set neutrosophic graph $G = (X, Y)$ is point set domination if for each vertex $u \in X - D$ satisfies one of the following conditions,*

1. $< X - D >$ is connected
2. If there does not exist $u - v$ path between at most any two vertices of $X - D$ then there exist $d \in D$ such that $N(u) \cap N(v) = d$.

$\therefore D$ is point set neutrosophic domination set, which satisfies one of the above conditions

Definition 3.12. *If no proper subset of point set domination set D is point set domination set, then D is said to be minimal point set domination in neutrosophic graph G*

Theorem 3.13. *In a neutrosophic graph G, a point set domination set is minimal if and only if for each vertex in dominating set $b \in D$ one of the following conditions holds.*

1. b is independent vertices in D

2. There is a vertex $u \in V - D$ such that $N(u) \cap D = b$

Proof. Assume that D is a minimal point set domination of G. Then for every vertex $b \in D, D - b$ is not a point set dominating set and hence there exists $w \in V - (D - b)$ which is not dominated by the vertex in $D - b$. If $w = b$, w is not a strong neighbour of any vertex in D. If $w \neq b$, w is not dominated by $D - w$, but is dominated by D, then the vertex w is a strong neighbor only to b in D. That is $N(w) \cap D = b$. conversely, Assume that D is a point set neutrosophic domination set for each vertex $b \in D$, one of the two condition holds Suppose D is not a minimal point set neutrosophic domination set, then there exist a vertex $b \in D, D - b$ is a

point set neutrosophic dominating set. Hence b is a strong neighbor to at least one vertex in $D - b$, the condition one does not hold. If $D - b$, the condition one does not hold. If $D - b$ is a point set then every vertex in $V - D$ is a strong neighbor to at least one vertex in $D - b$, the second condition does not hold which is contradiction to our assumption that at least one of the conditions.

Definition 3.14. *Lower point set dominating number (d^{psn}) of neutrosophic graph G is minimum cardinality of all minimal point set dominating number.*

Definition 3.15. *upper point set dominating number (D^{psn}) of neutrosophic graph G is maximum cardinality of all minimal point set dominating number.*

4. Conclusion

Neutrosophic point set domination set gives more efficient results than other existing point set domination sets. In this proposed work, the definition of point set neutrosophic domination number is defined with appropriate examples and few theorems and bounds on point set domination in neutrosophic graph are developed. In future, the concept of point set domination in neutrosophic graphs will be extended and applied to many real-life situation problems.

5. Refrences

[1] Atanassov, *Intuitionistic fuzzy sets*, Fuzzy sets and systems,20, pp.87-96, 1986. beginthebibliography99

[2] Berge. C,*Graphs and hypergraphs*, North Holland Amsterdam, 1973.

[3] Broumi, S., R. Sundareswaran., M. Shanmugapriya., Assia Bakali., Mo-hamed Talea., *Theory and applications of Fermatean Neutrosophic graphs,* Neutrosophic sets and systems, Vol.50, pp.286, 2002.

[4] Broumi,S., Smarandache,F., Talea, M. and Bakali, A.*Single valued neu-trosophic graphs: degree,order and size.*, EEE international conference on fuzzy systems, pp.2444-2451, 2016.

[5] E.J. Cockayne, S.T. Hedetniemi,*Theory of domination in graphs Net-works*, Vol.7, 247-261, 1977.

[6] Huang, L., Hu,Y., Li,Y., Kumar, P.K., Koley, D and Dey, A.*A study of regular and irregular neutrosophic graphs with real life applications.*, Mathematics, 7,6, pp.551, 2019.

[7] Hussain, S.S., Hussain, R., and Smarandache, F.,*Domination number in neutrosophic soft graphs*, Neutrosophic sets and systems, 28, pp.228-244 (2019).

[8] Kandasamy Vasantha, K. Ilanthendral and Floretin Smarandache., *Neu-trosophic graphs: a new dimension to graph theory*. Infinite study, 2015.

[9] Karunambigai, M,G, Sivasankar. S., and Palanivel .K.,*Secure domination in fuzzy graphs and intuitionistic fuzzy graphs*, Annals of fuzzy mathe-matics and informatics, 4, 14, pp.419-43, 2017.

[10] Karunambigai .M.G., M. Akram., Sivasankar .S., Palanivel .K., *Domina-tion in bipolar fuzzy graph*, 10.1109/FUZZ-IEEE.2013.6622326.

[11] S. Kaspar and B. Gayathri,*some results on point set tree domination in graphs*, International journal of pure and applied mathematics.

[12] A. Nagoorgani, V.T. Chandeasekaran, *A first look at fuzzy graph theory*, Allied Publishers Pvt Ltd.(2010).

[13] Rosenfeld,*A fuzzy graphs: Fuzzy sets & their applications to cognitive & decision processes*, Acadamic press, pp.77-95, 1975.

[14] E. Sampathkumar, L. Pushpalatha, *Point set domination number of graph*, Indian. J. Pure appl. Math., 24(4)(1993), 225-229.

[15] E. Sampathkumar, L. Pushpalatha, *Point set domination number of graph*, Indian. J. Pure appl. Math., 24(4) (1993), 225-229.

[16] Sivasankar .S., Said Broumi,Secure domination in Neutrosophic graphs.Neutrosophic sets and systems, Vol.56, 2023.

[17] A. Somasundaram, S. Somasundaram,*Domination in fuzzy graphs-I*, Pat-tern Recognition Letters 19(1998), 787-791.

[18] V. Swaminathan, R. Poovazhaki, *Connected point set domination in graphs*, Int. J. Alg. comp and Mathematics, Vol.3 No.1 (2010), 17-22.

[19] S. Vimala & J.S. Sathya, *connected point set domination in fuzzy graph*, International Journal of Mathematics and soft computing Vol.2, No.2(2012), 75-78.

[20] Wang, H., Smarandache, F, ;Zhang., Y.Q. ;Sunderraman,*R. Single valued neutrosophic sets*. Multispace, Multistruct, 4, pp.410-413, 2010.

[21] Wang,H.; Smarandache,F.; Zhang,Y.Q.; Sunderraman,*R. Interval Neutrosophic sets & logic: theory and Applications in computing, Phoenix, AZ:Hexis, 2005*, Allied Publishers Pvt Ltd.(2010).