Developing Ethnomathematical-Based Learning Stages on Mathematical Communication Skills with Kastolan Error Analysis

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Abstract:
The ethnomathematics-based learning has been effective for students' mathematical communication skills, but it can still be optimized by minimizing common student errors. Therefore, the objectives of this study are to analyze the shortcomings of students to solve mathematical communication problems in terms of students' skill categories; 2) analyze the learning process based on ethnomathematics with the help of the geoGebra application; 3) Develop learning steps based on ethnomathematics, analyzing the shortcomings of the students. The study subjects were courageously selected, i.e. 36 students from the IX semester of the 2023/2024 academic year. Data were obtained through reported math skills, interest questionnaires, observation sheets, and dialogue sheets. Data analysis techniques begin with data analysis, testing students' mathematical communication skills, analyzing qualitative data, interpreting research results, and analyzing quantitative data. According to the analysis of the errors made by the students in solving mathematical communication problems, in terms of the categories of student ability, the students made more procedural errors in the categories of medium and low skills. At the same time, students in the high-ability category tend not to make significant mistakes. The results of the triangulation found two stages of learning necessary to minimize student errors, i.e., the exercise stage and the research stage. The stages of ethnomathematic learning are exploration, mapping, explanation, exercise, research, and reflection. The effectiveness of the learning model developed in the mathematical communicative skills of the students will be analyzed for future research.

Keywords: castellan analysis, ethnomathematics, geoGebra, exercise stage and research stage.

1. INTRODUCTION

Early mathematicians did not explicitly state that culture had nothing to do with mathematics. However, in the last two decades, D’Ambrosio and Milton Rosa of Brazil popularized the cultural link to mathematics. Initially, mathematics taught in schools was free from culture, meaning that mathematics was free from the cultural emphasis that occurred in the social dynamics of traditional or modern society (D’Ambrosio & Rosa, 2017; Rosa & Orey, 2011). Many people think that mathematics is kept away from culture, but this assumption is not all true because mathematics fundamentally contributes significantly to developing a culture that occurs in society (Freudenthal, 2006). For example, in Banten culture, various calculations are still carried out, which turns out to be closely related to mathematics itself. For instance, in Patingtung art culture, to measure sound, the terms "pa", "ting", and "tung" refer to logic and mathematical patterns in measuring Titi scale and rhythm, so from the statement that we should argue that between culture and mathematics, there is a relationship (Albanese & Perales Palacios, 2015; Rosa & Orey, 2011)
Ethnomathematics is the study of mathematics (mathematical ideas) and its relationship to culture in the context of social life (Gerdes, 2010). This coincides with the study that analyzes mathematical ideas or practices in the cultures of Albanese and Perales Palacios (2015) and Rosa and Orey (2011). Ethnomathematics is a view of the teaching and learning of mathematics, based on prior knowledge, background, the role of the environment in terms of content and methods, and on the past and present environment (D’Ambrosio, 1990). Therefore, the development of culture in mathematics is increasingly debatable, and some studies have applied ethnomatism to the concept of learning mathematics.

Ethnomathematical developments developed among them were introduced by Milton Rosa and Orey, namely ethnomodeling. Ethnomodeling is the study of a culture's ideas and procedures viewed from the mathematical modelling lens (Rosa & Orey, 2013). This is confirmed by Orey & Rosa (2011), who state that mathematical modelling involves practices that occur in the culture. Therefore, it is necessary to understand well that every culture that occurs in an area is not only extracting mathematical ideas related to that culture but also trying to connect about ethnomodeling, which can be seen from the perspective of academics.

The development of ethnomathematics develops in the process of learning mathematics. This is because ethnomathematical learning that develops in culture in an area will strengthen students with their customs and environment (D’Ambrosio, 2007; Knijnik, 2002; Mosimege, 2012; Prieto et al., 2015). This will make learning richer, and students can interpret the culture around them in the context of mathematics so that students' love for their area arises (D’Ambrosio, 1985; D’Ambrosio, 1999; D’Ambrosio & Rosa, 2017). Mathematics and culture will be an interesting scientific context because students will learn mathematics based on the cultures they understand that are relevant to their daily lives. Thus, students can more easily understand mathematics through acculturation between mathematics and culture. Banten is one of the regions that has cultural diversity that is still developing today, for example in traditional arts, traditional houses, typical foods, traditional clothing, and so on. The cultural potential in Banten must be explored optimally because Banten culture has its uniqueness and distinctiveness. This uniqueness can be an asset for the existence of Banten culture to be introduced to the general public.

In the past, there was a sultanate called the Banten Sultanate in Banten. After the Banten sultanate ended, what remains today are historical relics in the form of former royal palaces, the Great Mosque of Banten, Fort Speelwijk, Kasunyatan Mosque, and so on. These historical buildings belong to a cultural category of artefacts that have a basis in mathematical concepts (Anggraheni et al., 2020). In the Banten region, a community, namely the Baduy Tribe, still upholds the traditions inherited from its ancestors. According to Sutoto (2017), Baduy or kanekes are indigenous Sundanese people in Lebak Regency, Banten. The Baduy tribe has several types of culture, including the Baduy traditional house (Susilowati et al., 2020), motif batik Baduy, tenun Baduy (Namirah et al., 2019), system leuit Baduy (Iskandar & Iskandar, 2017), and totopong Baduy (Isnendes, 2014). Objects such as the roof of a traditional Baduy house, typical Banten batik motifs, typical Banten gates, Kasunyatan mosque gates, Baduy totopong, Banten tower pedestals, Baduy weaving ulurs, the roofs of the great mosque of Banten, and leuit Baduy contain various mathematical concepts. Many mathematical concepts can be studied in Banten culture, so much research is done to improve students' understanding of the material presented.
Various studies on Ethnomathematics in Banten culture have been carried out, including Karinawati et al. (2016), namely learning Sundanese ethnomathematics (Banten culture) look very interested so that students are active when working on the LKS given. One of the activities carried out in learning is using congklak and wayang Cepot games, which are very familiar to students, so that students are brave and confident to convey the results of their discussions or express their ideas. The research results (2021) show that one of the motifs of Banten batik is the philosophy of kahirupan baduy, which is the daily activity of the Baduy tribe community. There is a concept to find the area and circumference of the motif from the Kahuripan baduy motif in the form of an isosceles triangle. Furthermore, the lebak batik motif also contains geometric aspects and mathematical ideas in batik making activities such as symmetry at points, symmetry at lines, rectangles, triangles, and star graphs. Trisnawati (2022) also researched the ethnomathematics of Banten culture and it is known that an ethnomathematical approach based on local culture in Banten can be used as an alternative choice on the subject of rows and rows. The results of his research also show that students can build mathematical concepts and see abstract concepts becomes more accessible because it starts from concrete things they know. Furthermore the results of Nirmalsari et al. (2021) research, show that Banten culture has various Pythagorean concepts, besides that, according to him, with ethnomathematical studies of Banten culture as a form of strengthening character education on nationalist values by having competence, love for their own culture can grow and be owned by students as the identity of the nation. In the study that is carried out, ethnomathematics is used to transmit didactic material, so that it is a context and something habitual. Rosa and Orey (2021) pointed out that each region has a different culture, so each region has different ways of communicating, reading, and interpreting a plural world. In addition, ethnomathematics is used to translate mathematics into academic mathematics (Umbara et al., 2021). In this way, the learning design developed with Ethnomathematics will be maximized to streamline students' mathematical communication skills. On the other hand, learning math requires math communication skills. The National Council of Teachers of Mathematics (2000) and Baiduri et al. (2020) wrote that mathematical communication skills are a way of expressing mathematical ideas, such as pictures, diagrams, objects, algebraic presentations, or use of mathematical symbols. According to the NCTM, one of the basic math skills students should have in math is math skills. In addition, Baroody and Coslick (1993) pointed out that mathematical communication is fundamental, i.e. as a mathematical language (1), which means that mathematics not only helps to think, but mathematics helps to find models, solve problems, and communicate concrete and concrete ideas. (2) Mathematics is the learning of mathematics as a social activity, which means that mathematics is a social activity in the learning of mathematics, as well as the interaction between students and the teacher's communication with the students. Communication between teachers and students is essential for learning mathematics, to guide students to understand concepts or find solutions to problems. Through communication, students can exchange and explain their ideas or intelligences to their friends (Hendriana et al., 2013). A similar statement was expressed by the company Clark et al. (2005), who explained the importance of having mathematical communication skills, since mathematical communication is a way and understanding to construct meaning and explain one's own ideas. Mathematical communication is one of the standard processes for learning mathematics (CUMHUR & TEZER, 2020; Sumaji et al., 2019; Viseu and Oliveira, 2012). Mathematical communication is one of the skills that students need in real life and in learning mathematics. In the classroom, students always communicate with teachers and other students to solve mathematical problems and
present mathematical solutions or ideas to others. However, there are great difficulties in exchanging information or communicating with others, using the language of mathematics in the classroom. Therefore, communicative competence in mathematics is one of the competencies that students need. However, according to the results of the 2015 International Mathematics and Science Trends Study (TIMSS), it ranked students 45 out of 50 countries and the 2018 Programme for International Student Assessment (PISA) in 74 countries. In addition, from the results of previous research, we know that students' mathematical communication skills remain low. Österholm (2006) found that students' mathematical communication skills remained low, especially the ability and accuracy of theatre to observe or learn about a mathematical problem. The results of Österholm's (2006) research indicated that respondents had difficulty articulating the reasons for understanding a reading. In the Zulkarnain study (2013), low mathematical communication skills were shown and students were unable to communicate their ideas well. It is known that, from the results of the students' answers, there are still wrong answers to answer the given questions, and that the calculation steps performed by the students have not been well organized or very good. Students are not yet able to give arguments based on mathematical principles and concepts. Phuong and Tuyet (2018) wrote in their study that students are more interested in asking multilateral questions than in the questions they are asked, so students only give short answers, yes or no. In his opinion, students are fearful and lazy when it comes to writing down their ideas, as they are not so used to expressing their ideas in writing, but perform the same exercises as the students in the workbook so that the students do not know how to solve the different problems. When a learner can't logically describe their idea, they can't communicate their thoughts in words, so problems arise because the learner can't describe their mathematical idea coherently (Baxter et al., 2005). Undoubtedly, these problems require attention so that students can find solutions to improve their communication skills in mathematics. Improving students' mathematical communication skills must go hand in hand with the learning process. We can optimize communication skills by applying learning models that allow students to improve mathematical communication skills (Tinungki, 2015) and interact and interact. According to Pugalee (2001), in order for students to engage in mathematical communication skills, students must be accustomed to giving arguments for each answer and responding to the answers of others, so that what they are learning is more meaningful to them.

In this study, an analysis of student errors in solving mathematical communication problems was carried out after being given Ethnomathematics-based learning with the help of the GeoGebra application. By analyzing the tendency of mistakes made by students in solving mathematical communication problems after being given Ethnomathematical learning assisted by the GeoGebra application, it is then used as a reference to develop stages in an effective Ethnomathematics-based learning model so that these errors do not repeat themselves and can optimally improve students' mathematical communication skills. Kastolan stage is one of the tools to analyze the description answer errors students make in solving questions. Kastolan’s stages consist of conceptual, procedural, and technical errors (D et al., 2021; Najwa, 2021; Yarman et al., 2020). The mistakes made by the students can be information and reference on the progress and shortcomings to be made in the ethnomathematical learning process. In addition, the learning steps are designed based on these references to minimize student errors and improve student math communication skills. Therefore, the objective of this research is to analyze the errors that students make to solve mathematical communication problems, with the help of the GeoGebra application, after receiving learning based on ethnomathematics; 2) analyze the learning steps based on ethnomatetics, with the
help of the Geogebra application on the mathematical skills of the students. The aim of the research is to develop learning based on ethnomathematics, designing effective learning capable of training students’ communicative skills.

2. METHOD
This research uses quantitative and qualitative research methods in the first stage, using quantitative methods for obtaining quantitative data and, subsequently, in the second stage, using qualitative methods for the deepening, dissemination and testing of quantitative data. Quantitative research is based on the collection and analysis of numerical data to measure, descriptive, explain, or predict them, as well as to make broad generalizations (Mertler, 2020). Despite the fact that qualitative research is interactive research, researchers have had extensive experience with participants, but this participation will raise strategic, ethical, and personal issues in the qualitative research process (Creswell, 2015).

The study subjects were courageously selected, i.e. 36 students from the IX semester of the 2023/2024 academic year. The tools used in this study were mathematical communication skills, quizzes of interest, observation sheets, and dialogue sheets. The mathematical communication skills test is presented through descriptive questions, previously evaluated and assessed by experts. Tests of mathematical communication skills are based on indicators of mathematical communication skills, i.e., the ability to express mathematical ideas through writing, demonstration, and visual description; (2) the ability to understand, interpret, and evaluate mathematical ideas, orally, in writing, or in other visual forms; 3) the ability to use terms, mathematical notations, and structures to present ideas and describe national relationships and models (Niss & Højgaard, 2019). An interesting questionnaire for students in the form of 15 statements validated by experts, elaborated according to 5 indicators of interest in learning, that is: 1) feeling of pleasure in learning; 2) analyze attention and thinking in learning; 3) willing to learn; 4) the willingness to be active in learning; 5) the desire to learn has to do with self-schooling (Wulandari, 2021).

To carry out the questionnaire, the Likert scale is used with 5 answer options, that is, always, many times, few times and never. The students' dialogue sheets are not systematically organized, but are sketches of the problems they want to analyze, i.e. how they solve problems, the mistakes that students make to understand and solve the problems and the learning process in the learning process. The flow of the research is that learning is imparted in 8 encounters over 4 weeks on the geometric dimensions of transformation. The basic geometry competencies for the selected transformation are the explanation of geometric transformations associated with context problems (reflection, translation, rotation and diffusion) and the resolution of context problems related to geometric transformations. After learning, students receive a test of mathematical communication skills and form an interesting questionnaire for learning. The data from the math communication skills test is processed and ranked based on high, medium, low, and poor skills.

<table>
<thead>
<tr>
<th>Category</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>75 ≤ P ≤ 100</td>
</tr>
<tr>
<td>Medium</td>
<td>50 ≤ P &lt; 75</td>
</tr>
<tr>
<td>Low</td>
<td>25 ≤ P &lt; 50</td>
</tr>
<tr>
<td>Very Low</td>
<td>0 ≤ P &lt; 25</td>
</tr>
</tbody>
</table>

The score percentage formula used is:

\[
\text{Score percentage} = \frac{\text{number of scores obtained}}{\text{maximum number of scores}} \times 100%
\]
The data from the students' mathematical skills test are described in relation to the students' conceptual, procedural, and technical defects, according to Kastolan, that is, in terms of conceptual, pro-editorial, and technical errors, depending on the type of error corrected by each item of the question. In addition, students with high, medium and low communication skills chose two students to talk about the difficulty in facing the problem and the responses to the learning process that were given to them.

For data analysis, it is recommended to carry out qualitative data analysis techniques through tests of students' mathematical skills, the interpretation of research results and the analysis of quantitative data. The error assessment for each indicator of mathematical communication skills is reviewed based on error indicators adapted to Kastolan (D et al., 2021; 2021; 2021; Najwa, 2021; Yarman et al., 2020).

Table 2. Error Analysis Indicator According to Kastolan

<table>
<thead>
<tr>
<th>Error Type</th>
<th>Indicator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conceptual Errors</td>
<td>1. Errors in the use of formulas in answering questions</td>
</tr>
<tr>
<td></td>
<td>2. Use a formula that does not match the conditions or prerequisites</td>
</tr>
<tr>
<td></td>
<td>for the formula to apply</td>
</tr>
<tr>
<td>Procedural Errors</td>
<td>1. Inappropriate in performing troubleshooting steps</td>
</tr>
<tr>
<td></td>
<td>2. Not coherent in performing troubleshooting steps</td>
</tr>
<tr>
<td></td>
<td>3. Unable to solve the problem in its simplest form</td>
</tr>
<tr>
<td></td>
<td>4. Cannot manipulate troubleshooting steps</td>
</tr>
<tr>
<td>Technical Errors</td>
<td>1. Error in calculating the value of the calculate operation</td>
</tr>
<tr>
<td></td>
<td>2. Errors in writing, i.e. some constants or variables are miswritten or</td>
</tr>
<tr>
<td></td>
<td>missed or errors moving constants or variables from one step to the next</td>
</tr>
<tr>
<td></td>
<td>3. Error in changing values to variables</td>
</tr>
</tbody>
</table>

The research model used at the stage of concluding is the Miles and Huberman Model, which consists of (Miles & Huberman, 1994):

1. Data reduction

Research that has data obtained is written into reports or concrete data on data reduction. The reports generated on the basis of the obtained data are summarized, summarized, choose the main things, focus on essential things. Data results that are summarized and sorted according to concepts, subjects, and category-specific units will provide a clearer picture of the results of observations, and researchers will be able to search for data in addition to the data obtained if necessary.

2. Categorization

In qualitative analysis, data presentation techniques can be performed in different ways, such as tables, graphs, and the like. Data can be presented graphically, briefly, between camps, flowcharts, and the like. Therefore, narrative text is most often used to present data in qualitative research. The function of visualizing the data in this study is to facilitate and understand what happens when students' abilities to communicate in mathematics about students' abilities are analyzed.

3. Synthesis

Synthesizing is looking for links between one category and another. The association of one category with another category is named/labeled again.
4. Drawing up a working hypothesis
The conclusion of this study is to present the conclusions of the research results. The conclusions of the qualitative research are initially provisional and will be modified if there is no strong evidence of data protection at a post-collection stage. However, if the conclusions are protected by valuable and robust evidence, when the researchers receive the data, the conclusions presented are credible. The conclusions of qualitative research may be new discoveries that have never been. Findings can be a description or description of an object that was previously unclear or obscure, to clarify it as a relationship, hypothesis, or causal or interactive theories.

In this study, the researchers used a technique to verify the validity of credibility data with a triangulation model. Triangulation is, above all, the multimetric view that researchers collect in data collection and analysis. The fundamental idea is to be able to understand well the phenomenon that is being studied, so that, if it is analyzed from different points of view, the truth can reach a high level. Taking photos of a single phenomenon from different perspectives will allow you to reach a real and reliable level. Therefore, triangulation is an effort to check the accuracy of the data or information obtained by researchers from different points of view, reducing as much as possible the differences that occur at the time of data collection and analysis. Triangulation has four kinds, namely:

a. Triangulation method is done by comparing information or data in different ways.

b. Inter-researcher triangulation, which is triangulation is done by using more than one person in data collection and analysis.

c. Triangulation of data sources, which is to explore the truth of certain information through various methods and sources of data acquisition.

d. Theory triangulation is the final result of qualitative research in the form of an information formulation or thesis statement.

However, here researchers use the type of source triangulation because it is more suitable to find answers from the purpose of the research conducted. Source triangulation is done by exploring the truth of certain information through various data acquisition sources. In this study, the data sources used were tests of students' mathematical communication skills (high, medium, and low), interview results, interest questionnaire results, and observations during the learning process (Kelle et al., 2019).

3. RESULTS AND DISCUSSION
The following data on students' mathematical communication ability test results are presented in tables based on high, medium, low, and shallow criteria after being given Ethnomathematics-based learning with the help of the Geogebra application.

<table>
<thead>
<tr>
<th>Categories Mathematical Communication Skills</th>
<th>n</th>
<th>Percentage (%)</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>7</td>
<td>19.44</td>
<td>85.71</td>
</tr>
<tr>
<td>Medium</td>
<td>20</td>
<td>55.56</td>
<td>70.33</td>
</tr>
<tr>
<td>Low</td>
<td>9</td>
<td>25</td>
<td>42.22</td>
</tr>
<tr>
<td>Total</td>
<td>36</td>
<td>100</td>
<td>66.30</td>
</tr>
</tbody>
</table>

https://internationalpubls.com
The table above shows that of the 36 students who were treated with Ethnomathematics-based learning with the help of the geogebra application, more than 50% had moderate categories of mathematical communication skills. The rest is only 25% of students whose mathematical communication skills are in the low category and 19.44% of students whose mathematical communication skills are in the high category. The overall average is known to have exceeded the standard average of 65 which is 66.30. This data shows that ethnomathematics-based learning with the help of geogebra applications positively impacts students' mathematical communication skills. However, it is known that on average 55.56% of students in the medium category can still be improved to be able to reach the high category, as well as an average of 25% of students in the low category can still be improved again to be able to reach the medium or high category. In other words, the Ethnomathematics-based learning process with the help of the geogebra application can still be developed to be more optimal for students' mathematical communication skills. For this reason, further analysis of the stages during the learning process is carried out and analysis based on the types of mistakes students make, then triangulation is carried out based on the results of interviews, questionnaire results, and direct observation of the learning process that takes place. This is done to find the most optimal learning strategy for students' mathematical communication skills.

**Types of Student Errors in Solving Mathematical Communication Problems**

The communication skills test questions consist of three description questions based on three mathematical communication skill indicators. The following are the assessment results on each indicator of mathematical communication skills reviewed based on error indicators according to Kastolan.

Table 4. The results of the Mathematical Communication Skills Test are reviewed from the Indicators and Types of Errors According to Kastolan

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Mean</th>
<th>Number of Students who make a tendency to make mistakes (n = 36)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Conceptual Errors</td>
</tr>
<tr>
<td>Indicator 1</td>
<td>4.42</td>
<td>8 (22.22%)</td>
</tr>
<tr>
<td>the ability to express mathematical ideas through, writing and demonstrating and describing visually</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Indicator 2</td>
<td>2.11</td>
<td>15 (41.67%)</td>
</tr>
<tr>
<td>the ability to understand, interpret and evaluate mathematical ideas both in writing and in other visual forms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Indicator 3</td>
<td>3.42</td>
<td>11 (30.56%)</td>
</tr>
<tr>
<td>ability to use mathematical terms, notations and structures to present ideas and describe relationships and models of situations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>34</td>
</tr>
</tbody>
</table>
The data in table 4 shows that most students made procedural errors in solving mathematical communication problems of geometry transformation material. It is also known that most students make mistakes on question number 2 (indicator 2) with the lowest average score of mathematical communication skills at 2.11 out of a maximum score of 5. Question number 2 is a question that is classified as a problematic category when tested. The indicator in question 2 is the ability to understand, interpret and evaluate mathematical ideas in writing and other visual forms. The ability to interpret mathematical ideas is an ability that requires a deep understanding of concepts in order to be able to come up with ideas when facing mathematical problems (Rohid & Rusmawati, 2019). Further analysis is carried out based on student answer sheets and interviews with concerned students to obtain more in-depth information.

**Student Answer Sheet Analysis**

The following analysis is carried out on the answer sheet that students write in solving mathematical communication problems. The questions consist of 3 representing 3 indicators of students' communication skills. Each student's answer sheet was analyzed based on error indicators, namely conceptual errors, procedural errors, and technical errors and reviewed based on the student's mathematical communication skills category.

<table>
<thead>
<tr>
<th>Error Indicators</th>
<th>Categories Communication Skills</th>
<th>Mathematical Communication Skills</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High</td>
<td>Medium</td>
<td>Low</td>
</tr>
<tr>
<td>Konseptual</td>
<td>4</td>
<td>9</td>
<td>21</td>
</tr>
<tr>
<td>Prosedural</td>
<td>11</td>
<td>21</td>
<td>22</td>
</tr>
<tr>
<td>Technical</td>
<td>17</td>
<td>13</td>
<td>18</td>
</tr>
<tr>
<td>Sum</td>
<td>32</td>
<td>43</td>
<td>61</td>
</tr>
</tbody>
</table>

Table 5 shows that students with the lowest grades made the highest number of mistakes, with students with the highest grades being the least engaged. However, the main indicator of technical error shows that students in the math skills category make the same number of mistakes as in the lower levels. In addition, middle graders made fewer errors in solving math communication problems than upper-grade students. From these data, it can be concluded that students with high communication skills also tend to make more mistakes than students with low communication skills. Therefore, an additional analysis was performed of the students' response sheets, selected by category of students' mathematical communication skills.

The following is question number 1 which represents indicator 1, that is, the ability to visually express and test mathematical ideas through writing.
Jember fashion Carnaval (JFC) is a work of art born and developed in Jember City. JFC was first held on January 1, 2003 around Jember city square. The 12th JFC in 2013 carried the theme of defile "Artechsion" (Art meet Technology and Illusion). By combining technology and art, culture produces extraordinary works. Here is an example of a JFC defile Betawi costume image.

Betawi Parade

The question: The sketch on the left wing of the JFC costume looks like the following image. Draw the following image mirroring results!

Figure 1. Question Number 1 Indicator 1 Mathematical Communication Skills

Furthermore, the results of students' work in solving question 1 are presented based on the categories of student communication skills (high, medium, and low) randomly selected in the following picture.
Student A (Low category)

Error Description:
The picture on the side is an answer sheet of students who belong to the low category of mathematical communication skills. The mistake made by student A is a procedural error, which can be seen from the error in carrying out the problem-solving steps. The question given asks students to determine the results of mirroring based on the object and axis of the mirror given. Student A understands that mirroring an object will result in the same object. However, student A does not understand the rules in reflecting an object on the mirror axis. Student A's work results show that students are still shackled to the concept of mirroring objects on the mirror axis, which must be straight vertically parallel to the Y axis. So that the results of the reflection depicted seem to be forced parallel to the X axis but try to adjust the tilted mirror axis. As a result, the mirroring results made are incorrect. This fact can also be seen from the cartesian coordinate axis described by student A as the axis of help to solve the given problem. Even though the axis is not in the problem and does not have any impact on helping to solve the problem.

Results of the interview and its analysis:
Student A stated that he had no difficulty when solving question number 1. According to student A, the result of his work is already the correct answer. However, when asked, "if you look in the mirror is the exact mirror the same or maybe slightly different?" student A replied confidently, "definitely it should be the same mom". This explains that student A understands the mirroring concept but still has difficulty in the mirroring procedure. Referring to indicator 1, namely the ability to express mathematical ideas through writing and demonstrate and describe visually, student A has a proven ability that is still relatively low in describing his ideas to solve given problems. This fact shows that students need improvement in their skills in carrying out problem solving steps in order and correctly. Furthermore, student A should get used to double-checking his worksheet when solving problems.

Student B (Medium category)
Error Description:
Student B gives an answer that looks correct regarding the direction of the results of mirroring the object against the correct mirror axis. However, analyzed in more detail, it is known that there are still many gaps that show that the reflection results are still incorrect. Student B showed a technical error seen from the image of the reflecting object which was smaller than the original. The concept of mirroring and the steps taken to reflect the object against the inclined axis are correct. If you pay attention, some shapes of the reflected objects appear thinner than the actual object. Even though the results of mirroring an object should be precisely the same as the object being mirrored.

Results of the interview and its analysis:
The interview revealed that student B seemed hesitant about the answer, and had stated that the answer was still incorrect. Student B stated that it was more comfortable to solve the problem in a group way, because there was a lot of input and discussion towards improving the answers to the questions being done. Furthermore, student B stated, "I know that the mirrored image I made is not accurate and could still be better, but the location is correct, ma'am". This statement indicates that student B understands the concept of mirroring, but is less careful and neat in describing the problem.

Student C (High category)

Error Description:
Student C answers correctly and adds auxiliary lines when reflecting the problem object. Student C shows that he understands the concept and understands the procedure, as well as using the proper technique.
Results of the interview and its analysis:
The interview results found that student C was already very confident in his answer. Interestingly, student C mentioned that the mirroring image he made applies to flat mirrors but not to convex mirrors. In fact, the teacher has never mentioned flat mirrors and convex mirrors during the learning process. That is, student C has more knowledge than others. When asked, "do you know where the convex mirror comes from"? Student C replied "you know because at home I have a convex mirror". Student C stated that the mirroring material was entertaining, while doing homework student C shared that he tried to solve it using the mirror he had. However, the results of mirroring the object shown enlarged, apparently because the mirror used is convex. Student C's attitude shows high curiosity and willingness to explore. Willingness to explore and high curiosity are abilities that need to be instilled in other students to have better mathematical communication skills. Curiosity is significant in learning, and doing research in completing tasks makes learning more meaningful (Wagstaff et al., 2021).

Furthermore, question number 2 represents the second indicator, namely the ability to understand, interpret and evaluate mathematical ideas in writing and other visual forms. The problem is:

The following is an example image of one of the Tibet defile JFC costumes.

Question:
Consider the following sketch sketch of the wings of the Tibet defile costume.

If the c shape is rotated $270^\circ$ counterclockwise concerning the center of rotation, which shape results from the rotation?
Furthermore, the results of students' work in solving question 2 are presented based on the categories of student communication skills (high, medium, and low) randomly selected in the following picture.

### Student A (Low category)

Rotation 270° counterclockwise, so R = -240°
One box = 360°, mark 360° – 270° = 90°
Then the result of the rotation is at point g

**Error Description:**
Student A gives the correct answer result. Conceptually, students explain the answer correctly by showing the counterclockwise direction with the formula R = -240°. However, procedurally there is still a lack of detail. Referring to the expected achievement of the second indicator, it is known that student A has not achieved the indicator. Students can understand the ideas of the questions and develop them through answers. The drawback is that it does not explain the answer in detail or interpret in other visual forms to show "counterclockwise". The analogy is when asked for a home address and answered the address according to the one on the identity card (KTP). The answer is not wrong, but it has not clearly shown the location of the house's position. Question number 2 is given in the form of pictures, it will be better and clearer if explained the answer and a picture. This incident can be overcome by getting used to answering detailed questions every day.

### Results of the interview and its analysis:
Student A stated that the answer was correct based on the interview results. Students confidently explain the origin of the answer with the correct concept. Uniquely, when asked the same question, student A answered the teacher's question by pointing to the picture on the question. Students understand the intent and direction of the question, but on the answer sheet give a short answer because he thinks the answer is enough. When asked, "why not add a picture to the answer to question number 2", student A replied "because from the question there is no command to draw, and my answer has already answered the question". The results of students' answers are incorrect and still need an explanation confirming the answer.

### Student B (Medium category)
Error Description:
Student B gave the correct answer. On the answer sheet, student B does not write what the answer is but it is shown in the picture that the result of rotation of the C shape as far as 270° counterclockwise is the shape G. Conceptually, student B is correct in describing the rule of rotation counterclockwise as far as 270°. Procedurally, student B has correctly solved the problem in a simple form. Furthermore, technically also student B did not make a mistake. However, procedurally still incomplete, this error occurs because student B does not follow the steps of the procedure which usually includes ending the answer or explanation with a conclusion or summary of the information that has been submitted. They refer to the second indicator of mathematical communication skills, namely interpreting mathematical ideas in writing and other visual forms. Based on the results of students' answers, they still do not meet the point of "being able to interpret mathematical ideas in writing".

Results of the interview and its analysis:
Based on the interview results, student B stated that question number 2 was not too difficult and was sure that what was done was correct. Student B said he had done almost the same problem during practice at home. So question number 2 adds a line to show the size of the rotation. However, when asked "so what is the result of the rotation of the C shape?" the student immediately showed the picture he made. Then asked again "where? Why is it not written with the answer in question?" the student hesitated and expressed his shortcomings "oh yes yes ma'am, I forgot to write it". Student B did not make a mistake or give a final conclusion on the answer sheet. This deficiency of student B can undoubtedly be overcome if students are accustomed to solving story problems in a known writing order, which is asked, the answer, and the conclusion. Students can also be made into discussion groups to complement each other's shortcomings in the question answer results.
Student C (High category)

![Image of student's answer](image.png)

The c shape is rotated as far as 270° counterclockwise rotation to the center of rotation, so the result of rotation is the g shape.

**Error Description:**
Student C has correctly written the answer along with the complete description. Students with high communication skills seem accustomed to communicating their ideas through writing and pictures. The image created is also equipped with clear and easy-to-understand captions.

**Results of the interview and its analysis:**
Based on the interview results, student C stated that question number 2 is elementary and can quickly solve it. According to student C, questions like this increase their accuracy because they are done quickly so there is plenty of time to check back and tidy up the written answers. Uniquely, student C stated that he was worried if he finished quickly because other students asked a lot. This means that C students lack empathy and sharing, so it would be better to solve questions in groups. So that students who have high abilities can share with group members who have medium or low abilities.

Furthermore, question number 3 represents the third indicator: the ability to use terms, mathematical notations and structures to present ideas and describe relationships and models of situations. The problem is "If the sketch of the JFC costume wing defile Betawi is brought to the coordinate approach by suppose one of the triangular-shaped parts of the wing has coordinates points A(-5.11), B(-1.1), C(-2.12) reflected against the X axis. Determine the coordinates of the shadow of the ABC point!". The results of students' work solving question number 3 are presented based on the categories of student communication skills (high, medium, and low) randomly selected in the following figure.
Student A (Low category)

\[
\begin{align*}
A(x, y) & \rightarrow A'(x, -y) \\
B(x, y) & \rightarrow B'(-x, y) \\
C(x, y) & \rightarrow C'(-x, -y)
\end{align*}
\]

Error Description:
Student A writes the correct answer according to what is asked in question number 3 and uses a mirroring formula on the X axis. The hope in question number 3 is to see Student's ability to present ideas and describe relationships and situation models, mathematical notations, and structures. The facts written by student A have met the abilities in the third indicator, just not yet detailed adding other necessary explanations. This shows that students commit procedural mistakes that cannot solve problems in various simple forms and manipulate them. The third indicator has an expected point that has not been met from student A's answer, namely "also able to describe relationships and situation models". However, student A gives an answer that matches the question given.

Results of the interview and its analysis:
Based on the results of the interview, student A stated that it was difficult when doing question number 3. The difficulty is caused by the mirroring formula against the X and Y axes, which are almost the same, so they are sometimes reversed. Some students express minor difficulty if the question is negative because sometimes they forget to operate it. However, based on the results of the students' responses, the students have fulfilled the third indicator of mathematical communication skills, i.e., the ability to use terms, mathematical notations, and structures to present ideas and describe relationships and situation models. It lacks the ability point in "describing relationships and situation models". According to students, the questions given have no drawing commands, so students only write down formulas that are remembered when facing problems. This shows that students lack the initiative to double-check the correctness of the answers. So that learning that trains students to recheck the answer sheet can familiarize students to provide several answers or checking steps when solving questions.

Student B (Medium category)
**Error Description:**
Student B gives the correct answer and is the same as the answer written by student A. So it is known that student B made a procedural error because it was incomplete in describing relationships and situation models.

**Results of the interview and its analysis:**
Based on the results of the same answer between student A and student B, the description that can be explained is the same. Student B's answer already showed that the student almost met the third indicator, only lacking in the point of "describing relationships and situation models". However, based on the results of the interview, it was found that student B felt that the answer was not entirely correct. Student B is not yet convinced but is reluctant to double-check with pictures or try to do it differently. Student B stated that if this problem were solved by group discussion then the final answer written would be more convincing. This statement is an input that group discussion is one of the necessary learning stages.

**Student C (High category)**

Error Description:
Student C writes down the answers correctly and clearly. Question number 3 is based on the third indicator, meaning that student C has fulfilled the third indicator: Ability to use terms.
mathematical notations, and structures to present ideas and describe relationships and models of the situation. Written answers are in the form of a correct formula to solve the given problem and are used correctly so that the written results are correct. Student C then gives a picture as further explanation, as well as a cross-check to make sure the written answer is correct. The drawings provided accurate explanations and showed the correct questions and answers.

Results of the interview and its analysis:
Based on the interview results, student C stated he was enthusiastic about doing question number 3. This is because in addition to the easy problem, student C also stated that he likes problems that can be solved with pictures. Uniquely, student C stated that he forgot the mirroring formula for the X axis. So the first step taken to solve the problem is to draw it. Student C then annotates with a formula based on the drawing he made. This can also be seen from the work of students who draw first and then rewrite the results of the answer along with the formula. This fact shows that student C, who have high communication skills, does mathematical communication by presenting mathematical ideas through pictures by looking at relationships and situation models.

Figure 5. Description of Student Answers Question Number 3 is reviewed from the Student Communication Skills Category

Overall, analysis of student response sheets and interview results showed that one way to improve student math communication skills is to minimize the mistakes that students often make. Most mistakes are made at low and medium levels, so they are solutions to minimize errors made by students at low and medium levels. Students with low communication skills make some mistakes they are unaware of and tend to make conceptual and procedural mistakes. Students believe the answer is correct and tend not to want to check again. Even though when interviewed and asked to double-check the answer, students realized where they were wrong. This needs a solution to minimize the mistakes students make when solving problems so that it influences the improvement of the mathematical skills taught by the students. It is known that some mistakes made by students with low communication skills on average start from conceptual errors that impact procedural errors and continue to technical errors. This certainly impacts the overall wrong answer in solving mathematical communication problems. From the analysis above, one of the steps that can be used is to do repeated exercises, group discussions, provide feedback, get used to double-checking answer sheets, and use concrete material, one of which is using culture during the learning process.

Furthermore, students with moderate communication skills categories were found to be prone to technical mistakes. They mostly expressed doubts about the answers written, so they gave incomplete answers. The results of the analysis of the responses found that the responses of the students with categories of communication skills gave correct answers, but the answers were written irregularly and in less detail. So students like this need an active learning environment doing repetitive and complementary exercises. One solution in learning is to hold group discussions, complete contextual
exercises, then provide consistent space to encourage students to check back with the results of the answers.

Observation Results of Ethnomathematically Based Learning Steps assisted by Geogebra Application

The role of ethnomathematics in learning is that students can recognize and use connections between mathematical ideas in solving project problems, relate mathematical and mathematical ideas to disciplines outside mathematics, and mathematics with the natural world in everyday life (Mania & Alam, 2021; Nur et al., 2020). One of the cultures used in learning is the motif on batik. The material chosen is transformation geometry material for 8 meetings (one initial test, one final test, and 6 learning times). The syntax of Ethnomathematics-based learning assisted by the Geogebra application applied in transformation geometry learning is presented in the following figure.

<table>
<thead>
<tr>
<th>Stages</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exploration stage</td>
<td>Student activities explore mathematical ideas in culture. Students are given material about culture</td>
</tr>
<tr>
<td>Mapping stage</td>
<td>With the assistance of the teacher, students make a map of the relationship between mathematical concepts (transformation geometry) and ethnomathematics</td>
</tr>
<tr>
<td>Explanatory stage</td>
<td>Students learn mathematical concepts (transformation geometry) with the help of the Geogebra application and communicate what is learned, share, appreciate what is learned in various forms</td>
</tr>
<tr>
<td>Reflection stage</td>
<td>Summarizing what is learned both knowledge of mathematical concepts (geometry of transformations) and cultural values developed in the learning process</td>
</tr>
</tbody>
</table>

From the results of the learning notes, it is known that all students are very satisfied with each stage of learning taught, especially in the mapping period.

At this mapping stage, the individual learning process becomes a discussion forum. Students are engrossed in asking each other to discuss and draw the relationship between batik motifs and transformation geometry material presented on worksheets. From the observations, students mentioned a lot about extracurricular batik activities that are usually done every Friday. Discussions become more attractive for students and the material delivered is connected to their daily lives so that it becomes...
more meaningful not just memorizing. Contextual learning and discussion invite students to understand the material not just memorizing (Ozdem-Yilmaz & Bilican, 2020; Sung et al., 2022). Furthermore, the explanation stage is also a stage that significantly impacts students’ communication skills.

Figure 7. Students understand the Material with the Geogebra Application

Accurate visuals inspire students to find connections between concepts and objects displayed. Other visual representations help students understand mathematical abstractions better, which in turn can improve their ability to explain those concepts to others. The help of the Geogebra application makes learning mathematics more interactive. Students can interact with mathematical objects in this case batik patterns, then change parameters, and see how those changes affect the results. This helps them feel and understand concepts more deeply, which can translate into better communication skills. Furthermore, the GeoGebra application allows students to collaborate in understanding mathematical concepts (Celen, 2020).

Students share their work with classmates during learning, stimulating them to discuss and exchange ideas. The ability to communicate and convey their understanding to others can develop in this context. Use of the Geogebra application, designed by limited computer resources in the school in which the study is carried out, i.e. that the teacher gives explanations about the class, that some students can practice directly in front of the class and use the mobile phones of the students who have installed the GeoGebra application before studying. At the first meeting, time was offered to compare it to the course plan, as students were adjusting to using the Geogebra app to understand the material and solve geometry problems. At the next meeting, however, the students acted enthusiastically and initiated a curriculum. The students like this type of learning, the results of the interviews with the students are known, as the students are tired of their usual learning. The students say that the use of the apps is not so dreamy and very interesting. In addition, learning is more interactive and students can practice it directly.

Nevertheless, some students mention that expect other variations in learning such as games or group discussions. Students also mentioned that the questions done were less numerous and less varied so that when faced with different questions they became confused and complicated. According to students, sometimes when they have completed a question and are still unsure about the results, students feel reluctant to ask questions or check again. This is undoubtedly an input that needs to be considered to develop an ethnomathematics-based learning model assisted by geogebra applications.

In short, based on the results of the interviews and the analysis of the students’ response sheets, reinforced by the observations made throughout the learning, we know that learning based on ethnomathematics has been effective for the mathematical communication skills of the students. However, minimizing mistakes made by students will further improve students’ mathematical
communication skills more appropriately. As for some of the stages developed, they appear in the following table.

Table 7. Development of Ethnomathematical Based Learning Syntax assisted by Geogebra Application

<table>
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</tr>
<tr>
<td>Tahap exercise</td>
<td>Using group discussions, students are given practice questions of various types to strengthen understanding of concepts and practice skills</td>
</tr>
<tr>
<td>Stages of investigation by way of exchange of group members</td>
<td>Students are directed to re-investigate the results of answers discussed by exchanging group members.</td>
</tr>
<tr>
<td>Reflection stage</td>
<td>Summarizing what is learned both knowledge of mathematical concepts (geometry of transformations) and cultural values developed in the learning process</td>
</tr>
</tbody>
</table>

The table above shows two stages aggregated in two stages, i.e. from the results of the data triangulation based on the exercise stage and the research phase. The exercise stage consists of giving practical questions from different types of groups. The results of the previous analysis were known to students in the middle and lower levels of mathematics.

the majority understood the concept of the problem at hand, but most made procedural errors and technical problems based on lack of practice. In the learning process, it is not uncommon to give practice questions to their students, but it is not a mandatory learning stage so that time runs out with material exposure and a little practice. Through practice, students can better understand how they think and structure their mathematical thinking (Hwang et al., 2021). According to students, practice can help students develop presentation skills, especially if asked to present a solution or problem-solving in front of the class. This is supported by Alam & Mohanty (2023), that teachers can provide constructive feedback regarding students' mathematical communication skills through exercises. Feedback helps students understand areas for improvement and improve the quality of their communication. So, doing exercises needs to be a priority stage by making it one of the stages that must appear in every learning process with a measurable period.

The second stage developed is the investigation stage, which asks students to exchange their group members with other group members, then investigate the results of the exercise discussion. This is done as a step of cross-checking answers but still in discussion. Investigations conducted after
exchanging group members become more effective as they are faced with new members and new ideas to investigate the results of answers. Activities involve investigating and comparing students' answers to ascertain correctness or error in solving mathematical problems (Mellone et al., 2020). Cross-checking helps students become more aware of any mistakes or mistakes they may make during the completion process.

Further, this awareness can help students correct their understanding and avoid similar mistakes in the future. According to Reid O’Connor & Norton (2022), by examining the answers thoroughly, students are more likely to focus on the completion process rather than just looking for the final answer. It helps students in the development of logical thinking skills and systematic steps. Re-checking activities can involve cooperation between students in learning. Students can also learn math and thinking from their peers, enriching the experience of their peers. With the previous accreditation, the system developed from this learning improves students' mathematical communication skills and promotes more meaningful learning behaviors in mathematics. With a deeper understanding and better problem-solving, students can gain confidence in the future to do similar math work.

4. CONCLUSION
It was concluded that students of medium and low mathematical communication skills made more procedural errors. By minimizing errors, it directly improves students' mathematical communication skills. The solution found to minimize student errors is to make the exercise stage and the investigation stage as stages that must appear every Ethnomathematics-based learning. So that the stages of Ethnomathematics-based learning developed consist of exploration, mapping, explanation, exercise, investigation, and practical reflection on students' mathematical communication skills. For future research, it is necessary to carry out experimental studies that demonstrate the effectiveness of the ethnological model developed in the mathematical communicative skills of students.

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