Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 31 No. 1 (2024)

Bi-Magic and Anti-Magic Labeling on M-Fuzzy Graph

Suruthi. S[1], Shanmuga Sundari. M[2], Sivakami. L[3*]
[1, 3]Department of Mathematics and Statistics, Faculty of Science and Humanities, SRM Institute of Science and Technology, Kattankulathur- 603203, Chengalpattu District, Tamil Nadu, India
[2]Department of Mathematics, Sir Theagaraya College, Chennai- 600021, Tamil Nadu, India
[1]E mail id: ss5112@srmist.edu.in
[2]E mail id: shanmugasundarim@stcchennai.edu.in
[3]*Corresponding author E mail id: sivakaml@srmist.edu.in

Article History:
Received: *30-09-2023*
Revised: *04-11-2023*
Accepted: *25-11-2023*

Abstract:
The conceptions of M-fuzzy anti-magic and M-fuzzy bi-magic labeling are developed by the proposed algorithms. M-fuzzy anti-magic labeling for standard graphs such as star and path graph explained. M-fuzzy bi-magic labeling for a few graphs such as star and bi-star is determined. Also, we explored the characteristics of such labeling applied to these graphs.

Keywords: M-fuzzy graph, M-fuzzy labeling, M-fuzzy edge anti-magic graph, M-fuzzy vertex anti-magic graph.

1. Introduction

Real world problems are often represented with graph theory. Because of uncertainties or haziness in system parameters, graphs do not properly represent all systems nowadays. Graphs can represent social networks, for example, by representing the accounts (people, institutions, etc.) as vertices and the relations between the accounts as edges. The accuracy of representation should be enhanced by fuzziness if the relations among accounts are measured by their frequency of contact. Rosenfeld [2] introduced fuzzy graphs for the first time. A wealth of research is conducted in fuzzy graph theory after that. Crisp graphs and fuzzy graphs have similar structural features. There is a separate importance to fuzzy graphs when there is uncertainty about vertices and edges. In real life, fuzzy graphs are found in many situations since the world is filled with uncertainty. In fuzzy graph theory, there are a variety of branches that can be explored.

Fuzzy labeling graph (FLG) is a rapidly evolving area of research in the field of graph theory that has procured considerable notice in recent years because of its remarkable real-world applications. In essence, it involves assigning numerical values, or labels, to the edges or vertices of a fuzzy graph (FG), where the labels are allowed to take on imprecise or uncertain values. This enables fuzzy graph labeling to model complex systems that are inherently uncertain, such as social networks, biological systems, and computer networks. Gallian [1] discussed more graph labeling in his dynamic survey of graph labeling. An anti-magic graph was first introduced in 1990 by Hartsfield and Ringel [13]. Nagoorgani et al., [4] presented the novel idea of fuzzy labeling, while Basker Babujee [8] presented the concept of bi-magic labeling, which involves the identification of two magic values. Prathap et al., [15] also postulated bi magic labeling for cycle related graph. Bibi et al., [6] discussed the fuzzy vertex graceful labeling for some family of graphs. Devi et al., [7] developed the idea of fuzzy bi-magic

(FBM) labeling. Devi et al., [5] postulated the idea of fuzzy anti- magic labeling. Sujatha N, Dharman, Thirusangu [9] proposed Triangular anti-magic fuzzy labeling in an algorithmic approach for some standard graphs. Thirusangu et al., [10] developed fuzzy bi- magic and anti- magic labeling for star graphs. Bharathi et al., [14] developed the fuzzy proper bi magic labeling for tadpole graph. Jahir Hussain et al., [11] proposed a new class of fuzzy graph. Shyamala Malini and Prathika [12] developed the idea of M-fuzzy magic labeling for special graphs. In this article, we present the existence of M-fuzzy edge(vertex) anti- magic labeling (M-FEAM and M-FVAM) of path and star, M- fuzzy bi- magic labeling (M-FBML) of star and bi- star. The end of the article will present the conclusion.

All the basic definitions are followed, as in [3], [4], [5], [7], [8], [9], [10], and [12].

2. Preliminaries

Definition 2.1. [5] A FLG is called as FEAM graph if $\mu_1(u)+\rho_1(u,v)+\mu_1(v)$ has different value for all $u,v \in V$.

Definition 2.2: [9] "A FLG is called an FVAM graph if" G is a 1-1 correspondence $f: E(G) \to \{1,2,3,\cdots,|E(G)|\}$ in which for any two different vertices v and w, the total of the labels on "edges incident to v distinct from the total of labels on edges incident to w."

Definition 2.3. [10] "An FG is said to admit edge bi-magic labeling if the total of membership values of edges and vertices incident at the vertices are either k_1 or k_2 where k_1 and k_2 are constants and denoted by $\tilde{B}m_0(G)$. A FLG which admits an edge bi- magic labeling is called a FEBM labeling graph."

Definition 2.4. [12] "A graph $G(\mu_1,\rho_1)$ is said to be a M-fuzzy labeling graph if $\mu_1 : V \to [0,1]$ and $\rho_1 : V \times V \to [0,1]$ is bijective such that the membership value of edges and vertices are distinct and satisfied the following conditions $\rho_1(u,v) \le \mu_1(u)$ or $\mu_1(u) \le \rho_1(u,v) \le \mu_1(v)$, then $\rho_1(u,v) \le \mu_1(u) \wedge \mu_1(v) \forall u,v \in V$ ".

3. M-Fuzzy Anti-magic Graphs

Definition 3.1: A M- FLG is said to be a M-FEAM graph if $\mu_1(u)+\rho_1(u,v)+\mu_1(v)$ has different value for all $u,v \in V$, which is denoted by $\tilde{A}m_0(G)$.

Definition 3.2: A M- FLG is said to be a M-FVAM graph if G is 1-1 correspondence $f: E(G) \to \{1,2,3,\cdots,|E(G)|\}$ in which for any two different vertices v and w, the total of labels on edges incident to v distinct from the total of labels on edges incident to w.

M-Fuzzy Labeled Path graph P_n :

Algorithm 1

M-FL of edges and vertices of P_n with, where n is the path length and $z \in (0,1]$

1. **Input:** Enter values for n and z .
2. Maximum number of vertices are $n+1$.
3. Vertex labeling be,
   for $i=1$ to $n+1$
   {
   $\mu_1(v_i)=(n+i)z$ where $1 \le i \le n+1$
   }
4. Edge labeling be,
   for $i=1$ to $n+1$
   if $1 \le i \le n$
   {
   $\rho_1(v_i, v_{i+1}) = \max\{\mu_1(v_i), \mu_1(v_{i+1})\} - \min\{\mu_1(v_i), \mu_1(v_{i+1})\} + (i-1)z$
   }
5. **Output:** Print the values of vertices and edges.

Theorem 3.1. Let $P_n(\mu_1, \rho_1)$ be the M-fuzzy labeled path graph for all $n \ge 2$ then P_n admits M-FEAM labeling.

Proof. Let $z \to (0,1]$ such that

$$z = \begin{cases} \dfrac{1}{10^2}, & 1 < n \le 49 \\ \dfrac{1}{10^{k+2}}, & 49 < n \le 49 + \sum\limits_{\substack{t=1 \\ 1\le i\le k}}^{i} (45\times 10^t), where k = 1 \\ \dfrac{1}{10^{k+3}}, & 49 + \sum\limits_{\substack{t=1 \\ 1\le i\le k}}^{i} (45\times 10^t) < n \le 49 + \sum\limits_{\substack{t=1 \\ 1\le i\le k+1}}^{i} (45\times 10^t), where k = 1,2,3,\ldots \end{cases}$$

The vertex membership values and edge membership values are defined using algorithm 1.

$\mu_1 : V \to [0,1] \ni \mu_1(v_i) = (n+i)z \forall v_i \in V, 1 \le i \le n+1$

$\rho_1 : V \times V \to [0,1]$ such that

$$\rho_1(v_i, v_{i+1}) = \max\{\mu_1(v_i), \mu_1(v_{i+1})\} - \min\{\mu_1(v_i), \mu_1(v_{i+1})\} + (i-1)z, 1 \le i \le n$$

We discuss M-FEAM labeling of path graph,

$$\tilde{A}m_0(P_n) = \mu_1(v_i) + \rho_1(v_i, v_{i+1}) + \mu_1(v_{i+1}), 1 \le i \le n$$

$$= (n+i)z + \max\{\mu_1(v_i), \mu_1(v_{i+1})\} - \min\{\mu_1(v_i), \mu_1(v_{i+1})\} + (i-1)z + (n+i+1)z$$

$$= (2n+3i+1)z$$

From the above, it is evident that the M-fuzzy labeled path graph P_n admits M-FEAM labeling.

Example 3.1.

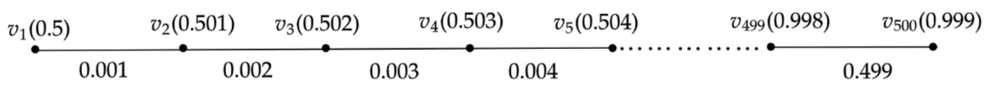

Fig. 1. P_{499} M-FEAM path graph

Theorem 3.2. Let $P_n(\mu_1, \rho_1)$ be the M-fuzzy labeled path graph for all $n \ge 2$ and if n is even then P_n admits M-FVAM labeling.

Proof. Given $P_n(\mu_1, \rho_1)$ be the fuzzy labeled path graph.

To prove that M-fuzzy labeled path graph $P_n(\mu_1, \rho_1)$ satisfies the condition of M-FVAM labeling.

That is to prove that for any two vertices u and v in P_n, the total of the grade membership on the edges incident at the vertex u is distinct from the total of the grade membership on the edges incident at the vertex v.

The vertex membership values and edge membership values are defined using algorithm 1.

Let $z \to (0,1]$ such that

$$z = \begin{cases} \dfrac{1}{10^2}, & 1 < n \le 49 \\ \dfrac{1}{10^{k+2}}, & 49 < n \le 49 + \sum\limits_{\substack{t=1 \\ 1 \le i \le k}}^{i} (45 \times 10^t), where k = 1 \\ \dfrac{1}{10^{k+3}}, & 49 + \sum\limits_{\substack{t=1 \\ 1 \le i \le k}}^{i} (45 \times 10^t) < n \le 49 + \sum\limits_{\substack{t=1 \\ 1 \le i \le k+1}}^{i} (45 \times 10^t), where k = 1, 2, 3, \ldots \end{cases}$$

$$\mu_1 : V \to [0,1] \ni \mu_1(v_i) = (n+i)z \forall v_i \in V, 1 \le i \le n+1 \tag{3.1}$$

$\rho_1 : V \times V \to [0,1]$ such that

$$\rho_1(v_i, v_{i+1}) = \max\{\mu_1(v_i), \mu_1(v_{i+1})\} - \min\{\mu_1(v_i), \mu_1(v_{i+1})\} + (i-1)z, 1 \le i \le n \tag{3.2}$$

Also, the total of the edge labels incident at

$$v_i = Wt(v_i) = \sum_{u \in N(v_i)} \rho_1(u, v_i) \tag{3.3}$$

Where $N(v_i)$ be the neighbourhood vertices of v_i for all $i=1$ to $n+1$ and $Wt(v_i)$ be the weight of v_i. From equation (3.1), (3.2) and (3.3) for any two vertices v_p and v_q with $p \ne q$, $Wt(v_p)$ and $Wt(v_q)$ have distinct values.

Thus, a path graph of even length admits M-FVAM labeling.

Example 3.2.

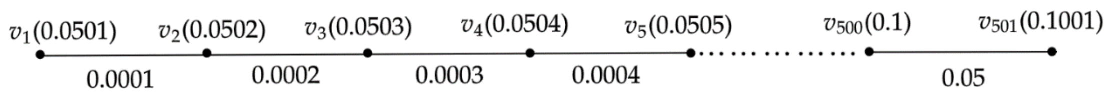

Fig. 2. P_{500} M-FVAM path graph

M-Fuzzy Labeled Star graph $S_{1,n}$:

Algorithm 2

M-Fuzzy labeling of edges and vertices of $S_{1,n}$ with $n \ge 3$, n is the no. of pendent edges and $z \in (0,1]$

1. **Input:** Enter values for n and z.
2. Maximum number of vertices are $n+1$.
3. Vertex labeling be,

   for $i=1$ to $n+1$

   {

   if $i=1$ or $i=2$

   $\mu_1(v_i) = (n+i)z$

   if $3 \le i \le n$

   $\mu_1(v_i) = (n+i+1)z$

   }

   if $n \ge 3$

   $\mu_1(w) = (n+3)z$

4. Edge labeling be,
   for $i=1$ to $n+1$
   {
   if $i=1$
   $\rho_1(w,v_i)=\max\{\mu_1(w),\mu_1(v_i)\}-\min\{\mu_1(w),\mu_1(v_i)\}-z$
   if $i=2$
   $\rho_1(w,v_i)=\max\{\mu_1(w),\mu_1(v_i)\}-\min\{\mu_1(w),\mu_1(v_i)\}+z$
   if $3\le i\le n$
   $\rho_1(w,v_i)=\max\{\mu_1(w),\mu_1(v_i)\}-\min\{\mu_1(w),\mu_1(v_i)\}+2z$
   }
5. **Output:** Print the values of vertices and edges.

Theorem 3.3. Let $S_{1,n}(\mu_1,\rho_1)$ be the M-fuzzy labeled star graph for all $n \geq 3$. Then $S_{1,n}$ is a M-FEAM labeling.

Proof. Let $z \to (0,1]$ such that

$$z=\begin{cases} \dfrac{1}{10^2}, & 1<n\le 49 \\ \dfrac{1}{10^{k+2}}, & 49<n\le 49+\sum\limits_{\substack{t=1 \\ 1\le i\le k}}^{i}(45\times 10^t), wherek=1 \\ \dfrac{1}{10^{k+3}}, & 49+\sum\limits_{\substack{t=1 \\ 1\le i\le k}}^{i}(45\times 10^t)<n\le 49+\sum\limits_{\substack{t=1 \\ 1\le i\le k+1}}^{i}(45\times 10^t), wherek=1,2,3,\dots \end{cases}$$

The vertex membership values and edge membership values are defined using algorithm 2.

$\mu_1 : V \to [0,1]$ such that

$\mu_1(v_i)=(n+i)z, i=1,2$

$\mu_1(v_i)=(n+i+1)z, 3\le i\le n$

$\mu_1(w)=(n+3)z, \forall n\ge 3$

$\rho_1 : V\times V \to [0,1]$ such that

$\rho_1(w,v_i)=\max\{\mu_1(w),\mu_1(v_i)\}-\min\{\mu_1(w),\mu_1(v_i)\}-z, i=1$

$\rho_1(w,v_i)=\max\{\mu_1(w),\mu_1(v_i)\}-\min\{\mu_1(w),\mu_1(v_i)\}+z, i=2$

$$\rho_1(w,v_i)=\max\{\mu_1(w),\mu_1(v_i)\}-\min\{\mu_1(w),\mu_1(v_i)\}+2z, 3\le i\le n$$

We discuss M-FEAM labeling in three cases,

Case 1:

$$\tilde{A}m_0(S_{1,n})=\mu_1(w)+\rho_1(w,v_i)+\mu_1(v_i), i=1$$

$$=(n+3)z+\max\{\mu_1(w),\mu_1(v_i)\}-\min\{\mu_1(w),\mu_1(v_i)\}-z+(n+i)z$$

$$=(2n+4+i)z$$

Case 2:

$$\tilde{A}m_0(S_{1,n})=\mu_1(w)+\rho_1(w,v_i)+\mu_1(v_i), i=2$$

$$=(n+3)z+\max\{\mu_1(w),\mu_1(v_i)\}-\min\{\mu_1(w),\mu_1(v_i)\}+z+(n+i)z$$

$$=(2n+5+i)z$$

Case 3:

$$\tilde{A}m_0(S_{1,n})=\mu_1(w)+\rho_1(u,v_i)+\mu_1(v_i), 3\le i\le n$$

$$=(n+3)z+\max\{\mu_1(w),\mu_1(v_i)\}-\min\{\mu_1(w),\mu_1(v_i)\}+2z+(n+1+i)z$$

$$=(2n+4+2i)z$$

Hence, in Case 1, Case 2, and Case 3, the M-fuzzy labeled star graph admits M-FEAM labeling.

Theorem 3.4. Let $S_{1,n}(\mu_1,\rho_1)$ be the M-fuzzy labeled star graph for all $n\ge 3$. Then $S_{1,n}$ is a M-FVAM labeling.

Proof. Given $S_{1,n}(\mu_1,\rho_1)$ be the fuzzy labeled star graph.

To prove that fuzzy labeled $S_{1,n}(\mu_1,\rho_1)$ satisfies the condition of M-FVAM labeling.

That is to prove that for any two vertices u and v in $S_{1,n}$, the total of the grade membership on the edges incident at the vertex u is distinct from the total of the grade membership on the edges incident at the vertex v.

Let $z\to(0,1]$ such that

$$z=\begin{cases}\dfrac{1}{10^{2}}, & 1<n\le 49\\ \dfrac{1}{10^{k+2}}, & 49<n\le 49+\sum\limits_{\substack{t=1\\1\le i\le k}}^{i}(45\times 10^{t}), where\, k=1\\ \dfrac{1}{10^{k+3}}, & 49+\sum\limits_{\substack{t=1\\1\le i\le k}}^{i}(45\times 10^{t})<n\le 49+\sum\limits_{\substack{t=1\\1\le i\le k+1}}^{i}(45\times 10^{t}), where\, k=1,2,3,\dots\end{cases}$$

The vertex membership values and edge membership values are defined using algorithm 2.

$\mu_1 : V \to [0,1]$ such that

$$\mu_1(v_i)=(n+i)z, i=1,2 \tag{3.4}$$

$$\mu_1(v_i)=(n+i+1)z, 3\le i\le n \tag{3.5}$$

$$\mu_1(w)=(n+3)z, \forall n\ge 3 \tag{3.6}$$

$\rho_1 : V\times V \to [0,1]$ such that

$$\rho_1(w,v_i)=\max\{\mu_1(w),\mu_1(v_i)\}-\min\{\mu_1(w),\mu_1(v_i)\}-z, i=1 \tag{3.7}$$

$$\rho_1(w,v_i)=\max\{\mu_1(w),\mu_1(v_i)\}-\min\{\mu_1(w),\mu_1(v_i)\}+z, i=2 \tag{3.8}$$

$$\rho_1(w,v_i)=\max\{\mu_1(w),\mu_1(v_i)\}-\min\{\mu_1(w),\mu_1(v_i)\}+2z, 3\le i\le n \tag{3.9}$$

Also, the total of the edge labels incident at

$$v_i = Wt(v_i)=\sum_{w\in N(v_i)}\rho_1(w,v_i) \tag{3.10}$$

Where $N(v_i)$ be the neighbourhood vertices of v_i for all $i=1$ to $n+1$ and $Wt(v_i)$ be the weight of v_i.

From equation (3.4), (3.5) (3.6), (3.7), (3.8), (3.9) and (3.10) for any two vertices v_p and v_q with $p\ne q$, $Wt(v_p)$ and $Wt(v_q)$ have distinct values.

Thus, a star graph $S_{1,n}\forall n\ge 3$ admits M-FVAM labeling.

Example 3.3.

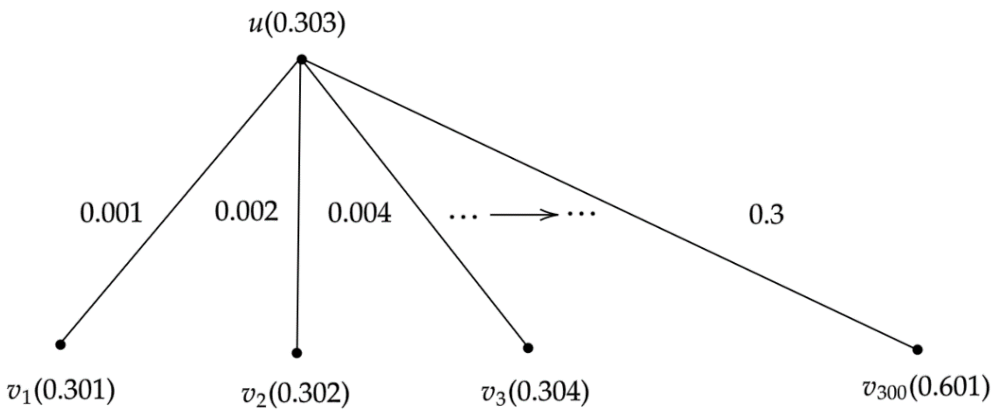

Fig. 3. $S_{1,300}$ M-FEAM and M-FVAM star graph

4. M-Fuzzy Bi-magic Graph

Definition 4.1: A M-FLG is said to be a M-FEBM graph if $\mu_1(u)+\rho_1(u,v)+\mu_1(v)$ have either k_1 or k_2 where k_1 and k_2 are constants.

M-fuzzy labeled Star $S_{1,n}$ **:**

Algorithm 3

M-Fuzzy labeling of edges and vertices of $S_{1,n}$ with $n \geq 4$, n is the no. of pendent edges and $z \in (0,1]$

1. **Input:** Enter values for n and z .
2. Maximum number of vertices are $n+1$.
3. Vertex labeling be,
   for $i=1$ to $n+1$
   {
   if $i=1$ **or** $i=2$ **or** $i=3$
   $\mu_1(v_i)=(2n+2-i)z$
   if $4 \leq i \leq n$
   $\mu_1(v_i)=(2n+1-i)z$
   }
   if $n \geq 4$
   $\mu_1(d)=(2n-2)z$
4. Edge labeling be,
   for $i=1$ to $n+1$

{
if $i=1$
$\rho_1(d,v_i)=\max\{\mu_1(d),\mu_1(v_i)\}-\min\{\mu_1(d),\mu_1(v_i)\}-2z$
if $i=2$
$\rho_1(d,v_i)=\max\{\mu_1(d),\mu_1(v_i)\}-\min\{\mu_1(d),\mu_1(v_i)\}$
if $i=3$
$\rho_1(d,v_i)=\max\{\mu_1(d),\mu_1(v_i)\}-\min\{\mu_1(d),\mu_1(v_i)\}+2z$
if $4\le i\le n$
$\rho_1(d,v_i)=\max\{\mu_1(d),\mu_1(v_i)\}-\min\{\mu_1(d),\mu_1(v_i)\}+3z$
}

5. **Output:** Print the values of vertices and edges.

Theorem 4.1. Let $S_{1,n}(\mu_1,\rho_1)$ be the M-fuzzy labeled star graph for all $n\ge 4$. Then $S_{1,n}$ is a M-FBML.

Proof. Let $z\to(0,1]$ such that

$$z=\begin{cases}\dfrac{1}{10^2}, & 1<n\le 49\\ \dfrac{1}{10^{k+2}}, & 49<n\le 49+\sum\limits_{\substack{t=1\\1\le i\le k}}^{i}(45\times 10^t), where\, k=1\\ \dfrac{1}{10^{k+3}}, & 49+\sum\limits_{\substack{t=1\\1\le i\le k}}^{i}(45\times 10^t)<n\le 49+\sum\limits_{\substack{t=1\\1\le i\le k+1}}^{i}(45\times 10^t), where\, k=1,2,3,\ldots\end{cases}$$

The vertex membership values and edge membership values are defined using algorithm 3.

$\mu_1:V\to[0,1]$ such that

$\mu_1(v_i)=(2n+2-i)z, i=1,2,3$

$\mu_1(v_i)=(2n+1-i)z, 4\le i\le n$

$\mu_1(d)=(2n-2)z, \forall n\ge 4$

$\rho_1:V\times V\to[0,1]$ such that

$\rho_1(d,v_i)=\max\{\mu_1(d),\mu_1(v_i)\}-\min\{\mu_1(d),\mu_1(v_i)\}-2z, i=1$

$\rho_1(d,v_i)=\max\{\mu_1(d),\mu_1(v_i)\}-\min\{\mu_1(d),\mu_1(v_i)\}, i=2$

$\rho_1(d,v_i) = \max\{\mu_1(d), \mu_1(v_i)\} - \min\{\mu_1(d), \mu_1(v_i)\} + 2z, i = 3$

$\rho_1(d,v_i) = \max\{\mu_1(d), \mu_1(v_i)\} - \min\{\mu_1(d), \mu_1(v_i)\} + 3z, 3 \le i \le n$.

For all edges dv_i in $S_{1,n}$, $\mu_1(d) + \rho_1(d,v_i) + \mu_1(v_i) = 4nz$ or $(4n-1)z$. Fix $k_1 = 4nz$; and $k_2 = (4n-1)z$. So, M-fuzzy $S_{1,n}$ satisfies all conditions of a bi-magic graph, and M-fuzzy star graph $S_{1,n}$ is bi-magic.

Example 4.1.

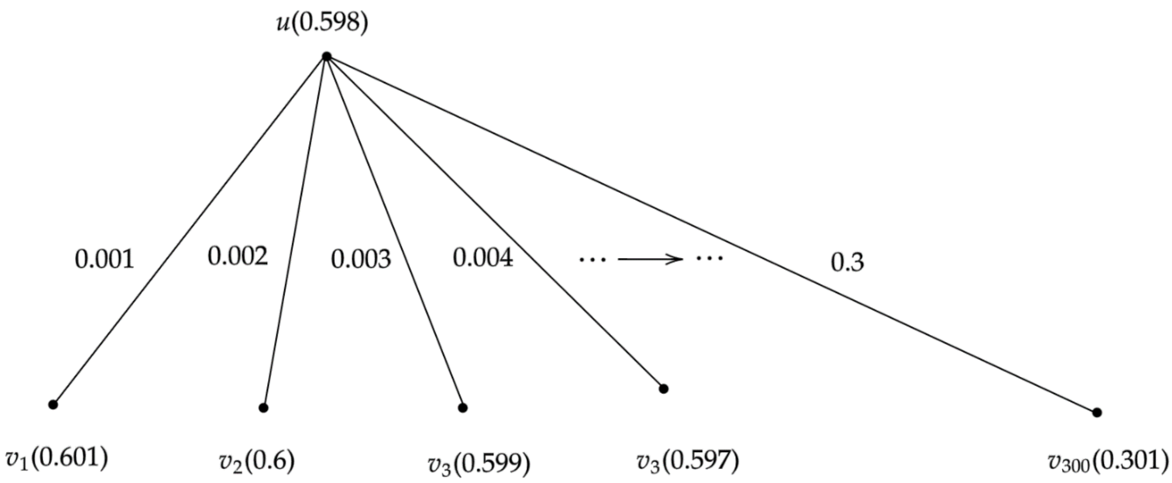

Fig. 4. $S_{1,300}$ M-FBM star graph

M-Fuzzy Labeled Bi-star graph $B_{n,n}$:

Algorithm 4

M-fuzzy labeling of edges and vertices of bi-star graph $B_{n,n}$ with $n \ge 1$ where n is the number of pendent edges and $z \in (0,1]$

1. **Input:** Enter values for n and z .
2. Maximum number of vertices are $2n+2$.
3. Vertex labeling be,

   for $i=1$ to $2n$

   {
   if $1 \le i \le n$
   $\mu_1(v_i) = (n+i)z$
   if $n+1 \le i \le 2n$
   $\mu_1(v_i) = (2n+1+i)z$

}

if $n \geq 1$

$\mu_1(u) = (2n+1)z$

if $n \geq 1$

$\mu_1(w) = (4n+3)z$

4. Edge labeling be,

for $i = 1$ to $2n$

{

if $1 \leq i \leq n$

$\rho_1(u, v_i) = \max\{\mu_1(u), \mu_1(v_i)\} - \min\{\mu_1(u), \mu_1(v_i)\}$

if $n+1 \leq i \leq 2n$

$\rho_1(w, v_i) = \max\{\mu_1(w), \mu_1(v_i)\} - \min\{\mu_1(w), \mu_1(v_i)\} + 2nz$

}

if $n \geq 1$

$\rho_1(u, w) = \max\{\mu_1(u), \mu_1(w)\} - \min\{\mu_1(u), \mu_1(w)\} + 2nz$

5. **Output:** Print the values of vertices and edges.

Theorem 4.2. Let $B_{n,n}(\mu_1, \rho_1)$ be the M-fuzzy labeled star graph for all $n \geq 1$. Then $B_{n,n}$ is a M-FBML.

Proof. Let $z \to (0,1]$ such that

$$z = \begin{cases} \dfrac{1}{10^2}, & 1 < n \leq 24 \\ \dfrac{1}{10^{k+3}}, & 24 < n \leq 24 + \sum\limits_{\substack{t=0 \\ 0 \leq i \leq k}}^{i} (225 \times 10^t), where\, k = 0 \\ \dfrac{1}{10^{k+4}}, & 24 + \sum\limits_{\substack{t=0 \\ 0 \leq i \leq k}}^{i} (225 \times 10^t) < n \leq 24 + \sum\limits_{\substack{t=0 \\ 0 \leq i \leq k+1}}^{i} (225 \times 10^t), where\, k = 0,1,2,3,\ldots \end{cases}$$

The vertex membership values, and edge membership values are defined using algorithm 4.

$\mu_1 : V \to [0,1]$ such that

$\mu_1(v_i) = (n+i)z, 1 \leq i \leq n$

$\mu_1(v_i) = (2n+1+i)z, n+1 \leq i \leq 2n$

$\mu_1(u) = (2n+1)z, n \geq 1$

$\mu_1(w) = (4n+3)z, n \geq 1$

$\rho_1 : V \times V \rightarrow [0,1]$ such that

$\rho_1(u, v_i) = \max\{\mu_1(u), \mu_1(v_i)\} - \min\{\mu_1(u), \mu_1(v_i)\}, 1 \leq i \leq n$

$\rho_1(w, v_i) = \max\{\mu_1(w), \mu_1(v_i)\} - \min\{\mu_1(w), \mu_1(v_i)\} + 2nz, n+1 \leq i \leq 2n$

$\rho_1(u, w) = \max\{\mu_1(u), \mu_1(w)\} - \min\{\mu_1(u), \mu_1(w)\} + 2nz, n \geq 1$

For all edges uv_i in $B_{n,n}$, $\mu_1(u) + \rho_1(u, v_i) + \mu_1(v) = (4n+2)z$ or $(10n+6)z$. Fix $k_1 = (4n+2)z$ and $k_2 = (10n+6)z$. So, M-fuzzy $B_{n,n}$ satisfies all conditions of a bi-magic graph, and the M-fuzzy bistar graph $B_{n,n}$ is bi-magic.

Example 4.2.

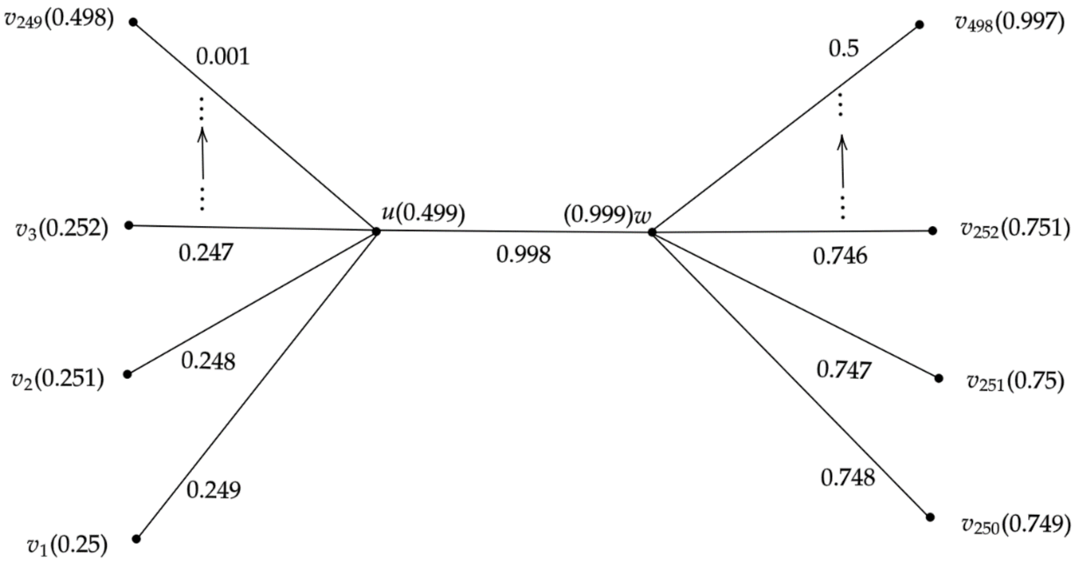

Fig.5. $B_{249,249}$ M-FBM bi-star graph

5. Findings

1. Every M-FBM graph is a FLG, but the converse is not true.
2. Every M-FEAM and M-FVAM graph is a FLG, but the converse is not true.
3. If G is a M-FBM graph then $d(u) \neq d(v)$ for any pair of vertices $u, v \in V(G)$.
4. If G is a M-FEAM and M-FVAM path graph of even length then $d(u) \neq d(v)$ for any pair of vertices $u, v \in V(G)$.

5. Every M-FEAM path graph of odd length must have one pair of vertices whose degrees are same.

6. Conclusion

The proposed article introduces the M-FBM, M-FEAM, and M-FVAM labeling concepts, discussing M-FBM labeling for bi-star and star graphs, as well as M-FEAM and M-FVAM labeling for star and path graphs. Additionally, we examined the properties of M-FBML, M-FEAM, and M-FVAM graphs, and will expand our analysis to include some other specialised graph classes.

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