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Effect of Heat Source on MHD Convective Flow of Non- Newtonian Fluid past an Oscillating Porous Plate in Porous Medium

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Abstract:
Introduction: An investigation was conducted to analyse the impact of unsteady flow caused by non-Newtonian fluid with magneto hydrodynamic properties on an oscillating plate made of porous material in a porous medium. The analysis of convective heat transfer included additional parameters such as electrically conducting unsteady flow-facing applied magnetic force. The Perturbation method was used to transform the governing equations into non-linear differential equations with utmost precision. Moreover, MATLAB was used to precisely evolve temperature and velocity profiles for different parameters such as Grashof number, Prandtl number, skin friction, magnetic parameter, and source/sink. The heat transfer rate was thoroughly evaluated and compiled using graphs and tables, guaranteeing the accuracy of the results.

Keywords: Unsteady, Fluid, Porous medium, Flow**,** Non-Newtonian fluid, Suction/Injection, Magneto-hydrodynamic.

1. Introduction

Universe is an un-parallel creation of nature. The earth we live on, the sky above our head studded with innumerable stars and planets. Moving around their own axis as well as the Sun, in a relative motion, under a force keeping all at a specific distance, maintaining the equilibrium. The Earth is abundant in several types of matter existing in solid, liquid, gaseous or plasma forms where Air and water are the most essential constituents of life for plants, trees, creatures, birds, animals including human beings and help their growth. A number of forces like gravitation, frictions etc. are regularly influencing the state of rest or of uniform motion of all elements in the system. Elements which exist in liquid, gaseous of plasma form and or can flow under certain conditions are said to be fluids. A substance which continues to deform and change its relative position under the effect of shear force can be termed as fluid. The study of their properties, characteristics as well as physical behaviour is a subject of science and comes under fluid mechanics known to be fluid dynamics.

Convection is considered to be the dominant mechanism. It has its application in chemical engineering, geothermal and oil reservoir modelling. There can be free or forced convection. Free convection flow is the outcome of the fluid body forces proportional to mass or density of fluid. Buoyant force determines the pattern of flow. Rise of smoke from fire is an example of such convection. In case the fluid motion is artificially created using external force like wind, fan, blower or pump the convection is forced one. On the other hand those found in nature are known to be free convections. Such flows are also experienced in industrial field like filtration process, food industries, metallurgical process, textile industries, nuclear reactors and solar energy collectors. Prior to the study of flows through porous medium, the study is carried out in the wake of a number of forces acting on the fluid, effecting monitoring and controlling its flow. Various concepts, principals, laws like conservation of energy, mass and momentum including law of viscosity, thermodynamics, mathematical applications, in addition boundary conditions and limitations are operative at appropriate situations. In present times multifarious problems and their complex nature poses new challenges in every field dealing with fluid flow, simulation of ocean currents, artificial kidney and heart mechanism and blood flow in veins and arteries. To solve fluid dynamics problems, numerous properties of fluid like temperature, pressure, velocity and density as functions of time and space are calculated in addition to some parameters and terms.

During the past few years, it's been noticed that fluids of non-Newtonian features have earned increasing significance for their application in several engineering and technical fields. More and more researchers have come to be fascinated to adopt studies pertaining to such form of fluids, which do not obey the laws of viscosity, moreover, show of a link between strain and stress rate of non-linear nature. Because of their complex nature, Various types of fluids together with ketchup, paints, shampoo, and polymer solutions have posed a project to engineers, mathematicians, and physicists to examine on those fluids. A number of fashions play fundamental position in their take a look at however no unmarried version has yet been determined that may show to be multi-function.

2. Literature Review

[1] This paper mainly aims at the combined impact of both free convective heating as well as mass transfer of stable two dimensional laminar, polar fluid through a Porous medium, with internal heat generation and first order chemical reaction. The highly non liner couple differential equations that control boundary layer flow are resolved using the two term (Eckert number E) method.[2] For investigating numerically the multiphase problem concerning both Newtonian as well as non-Newtonian flows, on improved ISPH algorithm has been utilised . [3] Investigation was made to know the result of free convection of conducting viscoelastic incompressible fluid in the matter of heat and mass transfer on vertical plate in Porous medium while a homogeneous transverse magnetic field was acting with heat source having time dependent permeability.[4] This paper studies Jeffrey's non-Newtonian incompressible fluid ,boundary layer flow of non- liner equilibrium nature across a porous plate in non-Darcy porous medium, using indirect Keller box finite difference technique. The transformed conservation equations are solved numerically using reverse plate under appropriate boundary conditions. [5]This paper carries out study into free convective MHD Casson fluid flow conducting electrically past an oscillating plate placed vertically in Porous medium under appropriate boundary conditions. The problem is solved analytically, solving the partial differential equations numerically .[6] Examination into free convective, unstable MHD Casson fluid flow across oscillating vertical plate has been conducted to know the thermal as well as viscous dissipation effect, while facing transverse magnetic field. Homotopy analysis method (HAM T is utilized to solve governing nonlinear boundary layer equations. [7] A Micro polar MHD fluid flow past an oscillating infinite vertical plate placed in porous medium has been analysed. Free convection takes place on the plate because of differences in temperature and concentration. As a result the effects of both radiant heat and mass transfer take place. [8] General solution for velocity, temperature, as well

as Nusselt number is obtained facing heat source and shear stress. Fluid velocity is taken to be the sum of mechanical and thermal components. They are capable of providing precise solution for any movement of this type of technical importance. Thermal and mechanical effects are considered important. The effect of physical parameters on temperature as well as velocity is underlined graphically for a slope-type heating plate which adds regular accelerating shear stress to the liquid. [9] Navier slip electrically conducting third order fluid through a porous medium, with convective boundary condition. Newton's law of cooling is observed in heat exchange of surfaces with the surrounding environment. Heat transfer characteristics are analysed graphically. A lower wall slip parameter is found to increase fluid velocity profile. The chemical kinetics of the flow system is assumed to be exothermic and asymmetric convective. The governing equations solved numerically with the help of semi-implicit finite difference scheme. [10] Optically thick radiation boundary is considered in this investigation in which the radiating heat flux expression is simplified by the use of Rosseland approximation. Entropy generation in magneto-hydrodynamic continuous flow of incompressible matter. Non-Newtonian Casson fluid conduction through horizontal pre-filled porous channel is materialised while a uniform transverse magnetic field along with constant pressure is acting with thermal radiation gradient. The fluid contains copper (Cu) Nano particles. [11] The study involves natural convective flow of MHD unsteady nature concerning rotating water-B fluid across oscillating plate facing fluctuating wall temperature and concentration difference, give rise to pressure field. [12] In this experiment MHD free convective flow pertaining to viscoelastic, electrically conducting fluid past a vertical porous plate placed in porous medium is under consideration. Continuous suction and heat source including transverse magnetic field was effective. Boussinesq approximation was utilised to derive governing equations of the flow field for energy dissipation. [13] The study involves heat transfer and fluid flow concerning pseudo plastic non-Newtonian Nano fluid using injection and suction through permeable surface. Similarity solution method is used to transfer governing partial differential equations into ordinary differential equations. These equations were further solved with the help of Runga-Kutta fehlbrg fourth and fifth order method (RKF-45). This system providing better heat transfer efficiency as compared to impermeable plate. [14] This study investigates heat source effect on MHD Casson fluid through vertically moving porous plate. Differential equations non-linearly related to dimensions are converted with the help of similarity variables. The transformed equations solved with the help of Glerkin finite element technique. The result shows that for heat source parameter increase velocity and temperature profile increase. [15] A study on effect of chemical reaction on laminar fluid cross flow and thermal mass transfer is conducted which reveals certain important characteristics of liquids namely variable thermal conductivity and its dependences on temperature field. Fluid movement is supposed to be caused by stress plate kept in porous medium. [16] Examined magneto hydrodynamic convective non-Newtonian Casson flow for mass and heat transfer. Fluid was made to flow across oscillating permeable straight up plate with thermal diffusion (Soret). To overcome the non-dimensional PDE'S in the fluid concentration. Velocity including temperature distribution, (FEM) finite element approach was adopted in the presence of different dimensional variables. [17] The study involves numerical approach in the matter of Newtonian MHD fluid flow across porous medium. A good number of investigation have been made in thermal equilibrium through in real applications the model exists without equilibrium state and local thermal imbalance accurately projects the thermal hydro flow. [18] Examined impact of Naviers slip on laminar boundary layer flow including thermal radiation on continuous non-Newtonian fluid flow related to viscous incompressible Nano fluid, through permeable, stretching/ shrinking plate.

3. Analytical solution of the problem

3.1 Mathematical interpretation of Flow

Study has been carried out to know the impact of electrically conducting , convective flow of Non-Newtonian viscous fluid, in porous medium, past a semi-infinite plate of porous nature placed in oscillating state, with transverse magnetic field B_0 acting on it. Velocity u is shown to have been taken along x axis and velocity v along the direction of y axis, both being perpendicular to the plate. Physical quantities involved are taken to be functions of y and t, with negligible Induced magnetic field. Pressure is considered constant and Reynolds number very small. Suction velocity at the plate taken as V_0 . Considering y=0, v=$-V_0$, continuity equation will be

$$\frac{\partial v}{\partial y}=0 \qquad \ldots\ldots. \qquad (1)$$

$$\frac{\partial u}{\partial t}+\mathrm{v}\frac{\partial u}{\partial y}=\vartheta\frac{\partial^2 u}{\partial y^2}+\frac{k_0}{\rho}\left(\frac{\partial^3 u}{\partial t\partial y^2}+\mathrm{v}\frac{\partial^3 u}{\partial y^3}\right)+g\beta(T-T_\infty)-\frac{\vartheta}{k_0}u-\frac{\sigma {B_0}^2 u}{\rho} \qquad \ldots\ldots \qquad \ldots.(2)$$

$$\frac{\partial T}{\partial t}+\frac{\partial T}{\partial y}=\mathrm{k}\;\frac{\partial^2 T}{\partial y^2}+\frac{k_0}{\rho}\frac{\partial^3 T}{\partial t\partial y^2}-\mathrm{S}[\mathrm{T}-T_\infty] \qquad \ldots\ldots\ldots \; . \qquad (3)$$

The related boundary conditions are

$$\mathrm{y}=0:\quad \mathrm{u}=V_0\; coswt,\;\; \mathrm{T}=T_\infty+(T_\omega-T_\infty)\,e^{-i\omega t}\;]$$

$$\mathrm{y}\to\infty:\quad \mathrm{u}\to 0,\quad \mathrm{T}\to T_\infty \qquad] \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; . \; (4)$$

here g is gravitational force, S heat source parameter, *u* the flow velocity component in the x-direction, k is thermal diffusivity, density ρ, T for temperature, v kinematic viscosity, β stands for volumetric heat transfer expansion coefficient, T_∞ stands for fluid temperature remote from the plate.

3. 2 Evolving the dimensionless quantities

$$t^*={V_0}^2\,\frac{t}{\vartheta} \quad , \quad {V_0}^*=\frac{V_0}{\vartheta} \quad , \quad \omega^*=\frac{\vartheta\omega}{{V_0}^2}$$

$$y^*=V_0\,\frac{y}{\vartheta} \quad , \quad u^*=\frac{u}{V_0} \quad , \quad S^*=\frac{\vartheta S}{{V_0}^2}$$

$$T^*=\frac{\mathrm{T}-T_\infty}{T_\omega-T_\infty} \quad , \quad \mathrm{M}=\frac{B_0}{V_0}\left(\frac{\vartheta\sigma}{\rho}\right)^{1/2} \quad , \quad P_r=\frac{\vartheta\rho C_p}{k}$$

$$G_r=\vartheta\mathrm{g}\beta\,\frac{T_\omega-T_\infty}{{V_0}^3} \quad , \quad K_p=\frac{k_0{V_0}^2}{\vartheta^2} \; , \quad R_c=\frac{k_0{V_0}^2}{\rho\vartheta^2}$$

Here P_r , G_r , M describe Prandtl number, Grashof number and Magnetic parameter respectively. Equations (2), (3) and related boundary condition (4) reduce to,

$$\frac{\partial u^*}{\partial t^*}-\frac{\partial u^*}{\partial y^*}=R_c\left(\frac{\partial^3 u^*}{\partial t^*\partial y^{*2}}-\frac{\partial^3 u^*}{\partial y^{*3}}\right)+\frac{\partial^2 u^*}{\partial y^{*2}}+G_r T^*-\left[\frac{1}{K_p}+M^2\right]u^* \qquad \ldots\ldots\ldots\;\ldots\;..(5)$$

$$\frac{\partial T^*}{\partial t^*}-\frac{\partial T^*}{\partial y^*}=\frac{1}{P_r}\frac{\partial^2 T^*}{\partial y^{*2}}-S^*T^*+R_c\frac{\partial^3 T^*}{\partial t^*\partial y^{*2}} \qquad \ldots\ldots\ldots\ldots \qquad (6)$$

(Dropping sign $*$ for convenience)

The conforming conditions of boundary are

u =cos wt , T = $e^{-i\omega t}$ at y = 0 ………… (7)

u→0, T→ 0 as y→∞

4. Solving the equations

Dropping ∗ and solving non-linear partial differential equation (5), (6) including boundary condition (7),

u = $u_o e^{i\omega t} + u_1 e^{-i\omega t}$ ………… (8)

T = $T_0 e^{i\omega t} + T_1 e^{-i\omega t}$ ………… (9)

Where u_i , T_i (i = 0 ,1) are functions of y only and u_0 u_1 , T_0, T_1 are unknown, to be determined.
Equating Harmonic and non-Harmonic terms and Putting the values of equations (5) and (6) from equations (8), (9)

Obtained Ordinary differential equations are given as:-

$$R_c \frac{d^3u_0}{dy^3} + (1 + R_c\omega i)\frac{d^2u_0}{dy^2} - \frac{du_0}{dy} + (\frac{1}{K_p} + M^2 + i\omega)\ u_0 = G_r\, T_{\ 0} \qquad \text{………}\ \text{…}\ (10)$$

$$R_c \frac{d^3u_1}{dy^3} - (1 - R_c\omega i)\frac{d^2u_1}{dy^2} - \frac{du_1}{dy} + (\frac{1}{K_p} + M^2 - i\omega)\ u_1 = G_r\, T_1 \qquad \text{……}\ \text{…}\ (11)$$

$$\frac{d^2T_0}{dy^2} + \frac{P_r}{1+P_rR_ci\omega}\frac{dT_0}{dy} - \frac{P_r\,(S+i\omega)}{1+P_rR_ci\omega}T_0 = 0 \qquad \text{………} \qquad (12)$$

$$\frac{d^2T_1}{dy^2} + \frac{P_r}{1-P_rR_ci\omega}\frac{dT_1}{dy} - \frac{P_r\,(S-i\omega)}{1-P_rR_ci\omega}T_1 = 0 \qquad \text{…} \qquad (13)$$

The resultant boundary conditions are,

at y = 0 $u_0 = \frac{1}{2}$ $u_1 = \frac{1}{2}$ ……… (14)

at Y→∞ $u_0 = 0$ $u_1 = 0$ ………. (15)

at y = 0 $T_0 = 0$ $T_1 = 1$ ………. (16)

at Y→∞ $T_0 = 0$ $T_1 = 0$ ……… (17)

From equation (12) and (13), obtained ordinary differential equations (16) and (17) with prescribed boundary conditions which are solved as under:

$T_0 = 0$ …… (18)

$T_1 = \exp(-m_3\, y)$ ……… (19)

Where $m_3 = \frac{A+\sqrt{A^2+4B}}{2}$ where $A = \frac{P_r}{1-P_rR_ci\omega}$, $B = \frac{P_r\,(S-i\omega)}{1-P_rR_ci\omega}$

5. Method Used

The Equations (10) and (11) are not solvable by using the given boundary conditions (14) and (15). Hence the perturbation method has been applied using *Rc* (*Rc* < 1), the elastic parameter as the perturbation parameter.

$u_0 = u_{00}(y) + R_c u_{01}(y)$ ……………………………… (20)

$u_1 = u_{10}(y) + R_c u_{11}(y)$ ……………………………… .(21)

Substituting Equation (20) and (21) into Equations (10) and (11) respectively , equating the coefficients of $R_c{}^0$ and $R_c{}^1$ to zero, we get the following set of equations.

Zeroth order equations

$u_{00}'' - u_{00}' - \left(\frac{1}{K_p} + M^2 + i\omega\right)u_{00} = 0$ ………………………………… …(22)

$u_{10}'' - u_{10}' - \left(\frac{1}{K_p} + M^2 - i\omega\right)u_{10} = -G_r exp.(-m_3 y)$ …………… …… (23)

First order equations

$u_{00}''' - u_{01}'' - i\omega u_{00}'' - u_{01}' + \left(\frac{1}{K_p} + M^2 + i\omega\right)u_{01} = 0$ ……….. …….(24)

$u_{10}''' - u_{11}'' + i\omega u_{10}'' - u_{11}' + \left(\frac{1}{K_p} + M^2 - i\omega\right)u_{11} = 0$……….. ……..(25)

Using the perturbation the boundary conditions are reduced as follows:

At y=0 $u_{00} = 0$ $u_{01} = 0$ $u_{10} = \frac{1}{2}$ $u_{11} = 0$ ……….. … (26)

At y→ ∞ $u_{00} = 0$ $u_{01} = 0$ $u_{10} = 0$ $u_{11} = 0$……… ………(27)

Solving these differential equations by using the boundary conditions we get the following results

u = (C_5 exp.(-m_5y)+ R_c C_9(exp.(-m_9y)-exp.(-m_5y))) $e^{i\omega t}$ +(C_7(exp.(-m_7y)-exp.(-m_3y)) +R_c(C_{11} exp.(-m_{11}y)+A_2 exp.(-m_7y)+A_3exp.(-m_3y))))$e^{-i\omega t}$……… .(28)

T = exp.(-m_3y)$e^{-i\omega t}$. …………………………… …………..(29)

Where $m_5 = \frac{1+\sqrt{\frac{1}{K_p}+M^2+i\omega}}{2}$, $m_7 = \frac{1+\sqrt{\frac{1}{K_p}+M^2-i\omega}}{2}$, $m_9 = m_5$, $m_{11} = m_7$

$$C_7 = \frac{-Gr}{m_3{}^2 - m_3 - (\frac{1}{K_p} + M^2 - i\omega)}$$

$$C_9 = \frac{m_5{}^2(m_5 + i\omega)}{2(m_5{}^2 - m_5 - (\frac{1}{K_p} + M^2 + i\omega)}$$

A_1=C_7, C11=-A_2-A_3

$$A_2 = \frac{A_1 m_7{}^2(-m_7 + i\omega)}{(m_7{}^2 - m_7 - (\frac{1}{K_p} + M^2 - i\omega)}$$

$$A_3 = \frac{A_1 m_3{}^2(m_3 - i\omega)}{(m_3{}^2 - m_3 - (\frac{1}{K_p} + M^2 - i\omega)}$$

6. Results Discussion

Present study involves Impact of heat source on convective MHD flow concerning viscous Non-Newtonian fluid. It is allowed to flow past a semi-infinite porous plate which is in oscillating position and acted upon by transverse magnetic field. Analytically derived mathematical expressions related to velocity and temperature, solving field equations. Figures 1-5 describe the velocity field u under the effect of parameters Gr, Kp, Pr, S and M .Figure 6-7 describe temperature field T under the effect of parameters S and Pr .

Figure -1, It depicts the variation in velocity field u due to varying values of Grashof number, Gr, velocity field u is taken against y which means velocity field u decreases sharply for increase in y approaches zero at the boundary. Figure -2, It depicts the variation in velocity field u due to varying values of Kp, Permeability parameter , velocity field u is plotted against y which means velocity field u increases sharply for increase in y approaches zero at the boundary. Figure- 3, It depicts the variation in velocity field u due to varying values of Prandtl number Pr, velocity field u is plotted against y which means velocity field u increases sharply for increase in y and tends to zero at the boundary. Figure- 4, It depicts the variation in velocity field u due to varying values of S, heat source parameter, Velocity field u is taken against y which means velocity field u increases sharply for increase in y and approaches zero at the boundary. Figure- 5, It depicts the variation in velocity field u due to varying values of M, Magnetic Parameter, Velocity field u is plotted against y which means velocity field u decreases sharply for increase in y and finally approaches zero at the boundary. Figure-6, It depicts the variation in temperature field u due to varying values of Heat Source parameter, S, temperature field T is plotted against y which means temperature field T decreases sharply for increase in value of y and approaches zero at the boundary. Figure-7, It depicts the variation in temperature field u due to varying values of Prandtl number, Pr, temperature field T is plotted against y which means temperature field T decreases sharply for increase in value of y and approaches zero at the boundary.

Table-1. It exhibits Skin-friction value C_f which increases with increase in heat source parameter S. Nusselt number decreases on increase of Heat Source Parameter, S.

Table-2. It shows an increase in Skin friction C_f for increase in Prandtl number but Nusselt number, Nu decreases with increase in Prandtl number Pr.

Table-3 shows that increase in (Kp) Permeability parameter brings about increase in Skin friction C_f but there occurs no change in value of Nusselt number due to increase in Kp.

Table-4. Increase in M, magnetic parameter causes decrease in skin-friction C_f but its increase brings about no change in Nusselt number.

Table-5. Increasing value of Grashof number causes decrease in C_f whereas no change occurs in case of Nusselt number.

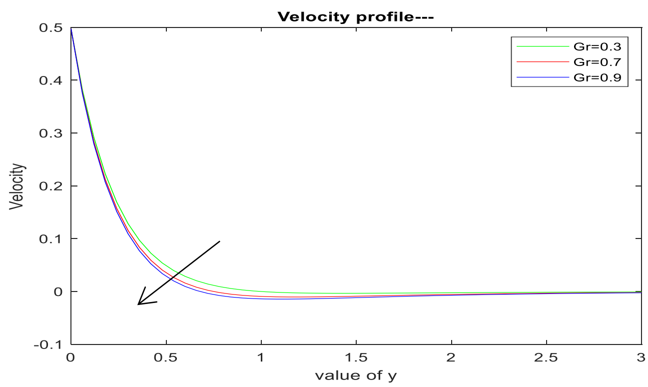

Figure-1. at S=0.1, M=2, Pr=1,Kp=0.1,w=0.2, exp=2.7183, t=0.5, Rc=0.1,Gr=0.3
Grashof number brings about variation in Velocity

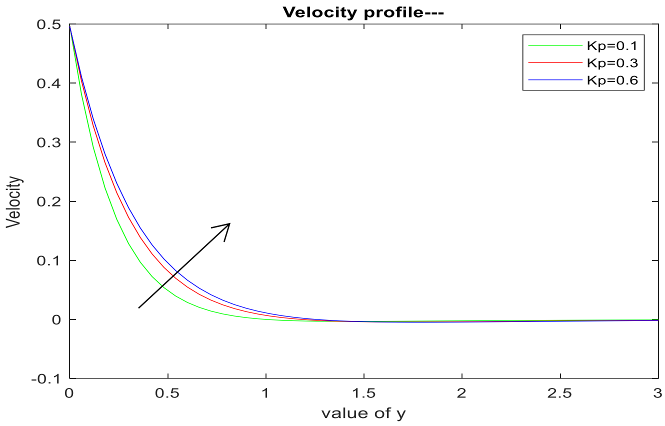

Figure-2. at S=0.1, M=2, Pr=1,Kp=0.1,w=0.2, exp=2.7183, t=0.5, Rc=0.1,Gr=0.3
Permeability parameter brings about variation in Velocity

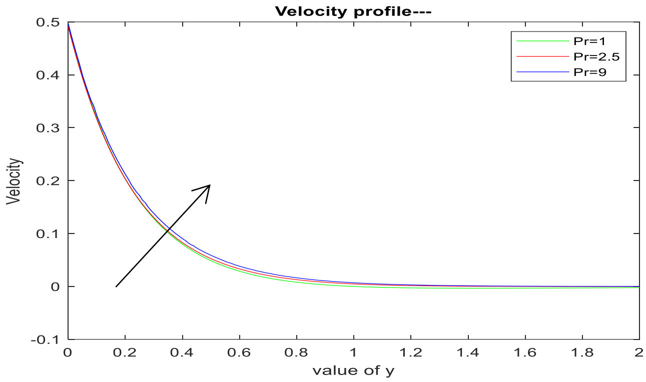

Figure-3. at S=0.1, M=2, Pr=1,Kp=0.1,w=0.2, exp=2.7183, t=0.5, Rc=0.1,Gr=0.3
Prandtl number brings about variation in Velocity

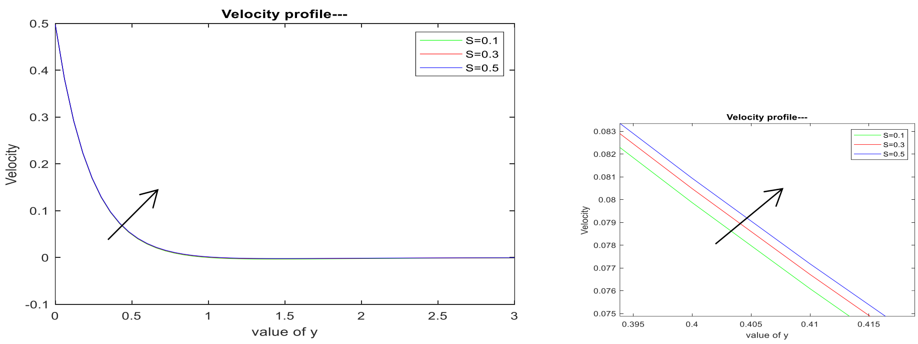

Figure-4. at S=0.1, M=2, Pr=1,Kp=0.1,w=0.2, exp=2.7183, t=0.5, Rc=0.1,Gr=0.3
Heat Source parameter brings about variation in Velocity

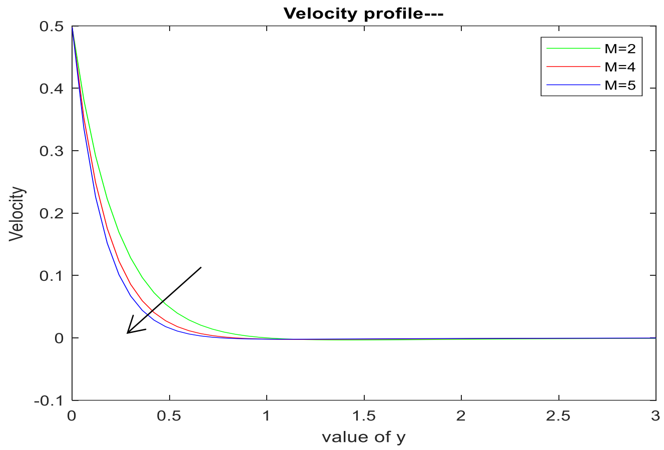

Figure-5. at S=0.1, M=2, Pr=1,Kp=0.1,w=0.2, exp=2.7183, t=0.5, Rc=0.1,Gr=0.3
Magnetic parameter brings about variation in Velocity

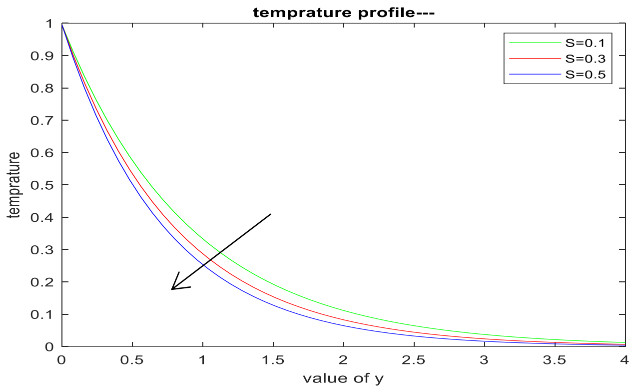

Figure-6. at S=0.1, M=2, Pr=1,Kp=0.1,w=0.2, exp=2.7183, t=0.5, Rc=0.1,Gr=0.3
Heat Source Parameter brings about variation in temperature

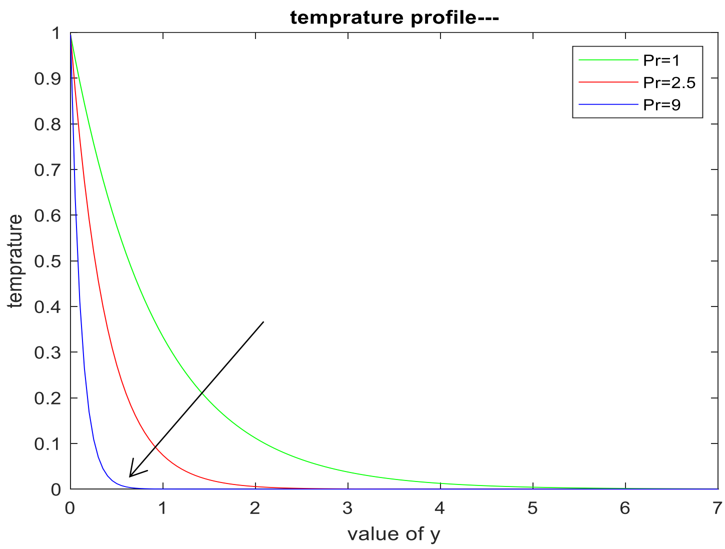

Figure-7. at S=0.1, M=2, Pr=1,Kp=0.1,w=0.2, exp=2.7183, t=0.5, Rc=0.1,Gr=0.3

Prandtl number brings about variation in temperature .

Table 1. Effect of S on Skin Friction , Nusselt Number, t= 0.5, exp=2.7183, w=0.2

S	Pr	Kp	M	Gr	Rc	Cf	Nu
0.1	1	0.1	2	0.3	0.1	-2.1949	-1.0891
0.2	1	0.1	2	0.3	0.1	-2.1936	-1.1675
0.3	1	0.1	2	0.3	0.1	-2.1925	-1.2377

Table 2. Effect of Pr on Skin Friction , Nusselt Number, t= 0.5, exp=2.7183, w=0.2

S	Pr	Kp	M	Gr	Rc	Cf	Nu
0.1	1	0.1	2	0.3	0.1	-2.1949	-1.0891
0.1	2.5	0.1	2	0.3	0.1	-2.1635	-2.5863
0.1	9	0.1	2	0.3	0.1	-2.1168	-8.8073

Table 3. Effect of Kp on Skin Friction , Nusselt Number, t= 0.5, exp=2.7183, w=0.2

S	Pr	Kp	M	Gr	Rc	Cf	Nu
0.1	1	0.1	2	0.3	0.1	-2.1949	-1.0891
0.1	1	0.3	2	0.3	0.1	-1.7072	-1.0891
0.1	1	0.6	2	0.3	0.1	-1.5562	-0.0891

Table 4. Effect of M on Skin Friction , Nusselt Number, t= 0.5, exp=2.7183, w=0.2

S	Pr	Kp	M	Gr	Rc	Cf	Nu
0.1	1	0.1	2	0.3	0.1	-2.1949	-1.0891
0.1	1	0.1	4	0.3	0.1	-2.8498	-1.0891
0.1	1	0.1	5	0.3	0.1	-3.2482	-1.0891

Table 5. Effect of Gr on Skin Friction , Nusselt Number, t= 0.5, exp=2.7183, w=0.2

S	Pr	Kp	M	Gr	Rc	Cf	Nu
0.1	1	0.1	2	0.3	0.1	-2.1949	-1.0891
0.2	1	0.1	2	0.7	0.1	-2.2857	-1.0891
0.3	1	0.1	2	0.9	0.1	-2.3311	-1.0891

7. Conclusion

- Velocity profile value decreases for increase in Grashof number.
- Velocity field value decreases for increase in Magnetic parameter (M) value.
- Velocity field increases for increasing Prandtl number (Pr), Heat Source (S) and Permeability (Kp).
- Skin-friction decreases for Increase of magnetic field, M and Grashof number, Gr.
- Nusselt number faces decrease due to increase in Prandlt number Pr and heat source parameter S.

- Skin friction increases as Heat Source (S), Prandtl number (Pr) and Permeability (Kp) increase.
- Temperature profile faces decrease due to increase in S, heat source parameter, Pr, Prandlt number.

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