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An Interval approach to solve Fuzzy Fractional Programming Problem as a Fuzzy Linear Complementarity Problem

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Abstract:

An optimization problem where the goal function is a proportion between two functions is known as fractional programming, and the goal is to maximize or minimize the ratio. This paper discusses a methodology to resolve Fuzzy Fractional Programming Problem as a Fuzzy Linear Complementarity Problem. The study seeks to emphasize the key characteristics, make some new observation and motivate further application in linear complementarity problem. The constraints of the Fractional Programming problem is taken as a Fuzzy Linear Programming problem and further it transformed in to Fuzzy Linear Complementarity Problem by taking various α – cut levels. Utilizing Interval numbers operations the given problem is resolved. A numerical instance is provided to show how the suggested strategy works.

Keywords: Fuzzy Fractional Programming problem (FFPP), Linear programming problem (LPP), Fuzzy Linear Complementarity problem (FLCP), Fuzzy number (FN), Triangular fuzzy number (TFN), Interval number (IN).

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1. Introduction

A particular type of optimization problem known as fractional programming [1, 4, 27], aims to maximize or minimize a ratio between two functions, which is the objective function. It is an extension of LPP. In this case, the objective function is a linear function, this formulation allows for modelling a wide range of real-world problems where optimizing a ratio or a proportion is the primary goal. Unlike conventional optimization problems where the primary goal is to maximize or reduce a single objective function. The goal of fractional programming problems [9] is to maximize a relative measure, like a utility function, efficiency meter, or performance metric. These problems present unique hurdles because of this particular structure, necessitating the use of specialist methodologies and strategies.

Numerous practical situations, including resource allocation, portfolio optimization, performance optimization, and network optimization, are modelled by fractional programming problems. Charnes and Cooper (1962) [6] demonstrated that an LPP can be solved to optimize the linear fractional programming problem (LFPP). In 1967, R. L. Graves [24] put forth a Principle Pivoting method for Complementarity problems. Bimatrix games, linear and quadratic programs were developed together by Cottle and Danzig [8]. Bellman and Zadeh (1970) [3, 29, 30] proposed a concept for making decisions in fuzzy environments.

It can be difficult to solve FPP [25], although several approaches have been devised such as branch and bound algorithms, Dinkelbach's method, linearization techniques, heuristics, and approximations. Numerous disciplines, including computer science, engineering, economics, management, and finance, have used fractional programming. The LCP [10, 23, 28] is a fundamental problem in mathematics and computer science that combines elements of linear algebra and complementarity constraints. It is an effective framework for modelling and resolving a wide range of equilibrium issues, game-theoretic models, and optimization problems [12]. The LCP is useful in a wide range of applications: Optimization: Linear and quadratic programs, convex optimization, Game Theory, Economics, Physics, and Engineering: Structural analysis, mechanical systems.

A complementarity pivoting algorithm was suggested by Lemke et al. [2, 22] in 1968 to solve linear complementarity problems. K.G. Murthy et al. suggested a solution to parametric LCPs in 1997 [21]. Subsequently, a vast amount of research has been done on complementarity difficulties. Because of their complexity and the ensuing numerical challenges, enormous sized LP cannot be solved using the well-known simplex method. As a result, iterative methods designed for tackling LCPs hold considerable potential for handling these programs.

Let M be a given a real nxn square matrix and let q be a nx1 real vector, then the LCP (q, M) is to find real nx1 vector W, Z such that

$$W - MZ = q \qquad (1.1)$$

$$W_j \geq 0, Z_j \geq 0, Z_0 \geq 0, j = 1, \ldots, n \qquad (1.2)$$

$$W_j Z_j = 0, j = 1, \ldots, n \qquad (1.3)$$

the pair (W_j, Z_j) is a complementary variables.

Two variables in an LCP; one of the variables must always be zero. It is argued that each variable in a pair is complementary to the other. From the reference of Jenifer D H and R. Irene Hepzibah [13, 14, 15] discussed the conversion of solving Fuzzy Linear Programming problem as a Fuzzy Linear Complementarity problem by Maximum index method and also the Inverse of the basis method. Moumita and P.K. De [20] 2015 discussed Fuzzy Linear Fractional Programming Problem with triangular and trapezoidal fuzzy numbers. Chen S.H., Hsieh C.H., (1999) [7] proposed Graded Mean Integration Representation of generalized fuzzy number, this paper is extended to Fuzzy fractional programming problem. On account of, this method can be used in various types of FNs.

This work aims to address a certain kind of FFPP, in which all parameters as well as variables are TFN [11] and it can be solved with the proposed algorithm by taking levels of alpha-cuts [5]. An endeavour has been undertaken in this research to create an algorithm to execute FFPP as an FLCP in Interval numbers [26, 19]. The given problem can be first converted into FLPP further it transformed into FLCP. The aim is to explore the theoretical and computational aspects of fractional programming problems and develop a new algorithm for solving in terms of complementarity side.

The structure of the paper is as follows: The definitions, arithmetic operations of IN and Fuzzy Linear Complementarity problem are covered in section 2. An algorithm that provides the computing process for the best result is developed in section 3. Section 4 provides a numerical illustration to clarify the preceding method with different levels of $\alpha - cut$. Section 5 Concludes with main results.

2. Preliminaries

2.1 Triangular Fuzzy Number

A TFN $\tilde{A} = (a_1, a_2, a_3)$ such that $a_1 \le a_2 \le a_3$, with membership function defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & , a_1 \le x \le a_2 \\ \frac{(a_3-x)}{(a_3-a_2)} & , a_2 \le x \le a_3 \\ 0 & , \text{otherwise} \end{cases}$$

2.2 Fractional Programming problem

A FLP problem is defined as

Max Z = $\frac{ex+r}{fx+s}$

Subject to $AX \le B, X \ge 0$ where e = $(e_1, e_2, \dots, e_n)$, f = $(f_1, f_2, \dots, f_n)$, B = $(b_1, b_2, \dots, b_m)^T$, $X \epsilon \mathbb{R}^n$, $x \epsilon X$, r and s are scalar and A=$(a_{ij})_{nxm}$.

2.3 Interval Number

If $\tilde{T}^F$ is a TFN, let $\tilde{T}^F{}_\alpha = [T^-{}_\alpha, T^+{}_\alpha]$ be the closed interval, this represents an α - cut for $\tilde{T}^F$ with its left and right end points of $T^-{}_\alpha$ *and* $T^+{}_\alpha$ respectively. Let S and R are specified by ordered pairs of real numbers. S = [a,b], where a ≤ b, R = [c, d], where c ≤ d, when a = b and c = d, these interval numbers degenerate to a scalar real number.

2.4 Modified Arithmetic Operations on Interval Numbers

Any interval number $\tilde{\rho} = [\rho_1, \rho_2]$ is alternatively represented as $\tilde{\rho} = \langle t(\tilde{\rho}), s(\tilde{\rho})\rangle$, where $t(\tilde{\rho}) = \frac{\rho_1+\rho_2}{2}$ *and* $s(\tilde{\rho}) = \frac{\rho_1-\rho_2}{2}$ are the mid-point and half-width of the interval number $\tilde{\rho}$.

Let $\tilde{\rho} = [t, s]$ has the canonical expression $\tilde{\rho} = [t-s, t+s]$. $\tilde{r} = [r_1, r_2]$ *and* $\tilde{s} = [s_1, s_2]$ are two INs,

Addition: $\tilde{r} + \tilde{s}$ = $[r_1, r_2]$ + [s_1, s_2] = [(m($\tilde{r}$) + m($\tilde{s}$)) - k, (m($\tilde{r}$) + m($\tilde{s}$)) + k],

where k = $\left\{\frac{(s_2+r_2)-(s_1+r_1)}{2}\right\}$.

Subtraction: $\tilde{r} - \tilde{s}$ = $[r_1, r_2]$ - [s_1, s_2] = [(m($\tilde{r}$) - m($\tilde{s}$)) - k, (m($\tilde{r}$) - m($\tilde{s}$))+ k],

where k = $\left\{\frac{(s_2+r_2)-(s_1+r_1)}{2}\right\}$.

Scalar Multiplication: Let $\lambda \in \mathbb{R}$ then $\lambda\tilde{r}$ = $[\lambda_{r_1}, \lambda_{r_2}]$ for $\lambda \ge 0$, $[\lambda_{r_2}, \lambda_{r_1}]$ for $\lambda < 0$.

Multiplication: $\tilde{r}\tilde{s}$ = $[r_1, r_2]$ [s_1, s_2] = [(m($\tilde{r}$) m($\tilde{s}$)) - k, (m($\tilde{r}$) m($\tilde{s}$)) + k], where k = $\min\{(\mathrm{m}(\tilde{r})\mathrm{m}(\tilde{s})) - \alpha,\ \beta - (\mathrm{m}(\tilde{r})\mathrm{m}(\tilde{s}))\}$, $\alpha = \min(r_1s_1, r_1s_2, s_2r_1, r_2s_2)$ and

$\beta = \max(r_1s_1, r_1s_2, s_2r_1, r_2s_2)$.

Inverse: $\widetilde{r^{-1}} = [r_1, r_2]^{-1} = \frac{1}{[r_1,r_2]} = \left[\frac{1}{[r_1,r_2]} - k, \frac{1}{[r_1,r_2]} + \mathrm{k}\right]$,

where k = $\min\left\{\frac{1}{r_2}\left(\frac{r_2-r_1}{r_1+r_2}\right), \frac{1}{r_1}\left(\frac{r_2-r_1}{r_1+r_2}\right)\right\}$, $0 \in [r_1, r_2]$.

2.5 Graded Mean Integration Method [3]

A Triangular fuzzy number $\tilde{A} = (g_1, g_2, g_3)$, then the defuzzification of the FN is

$P(\tilde{A}) = (g_1 + 2g_2 + g_3) / 4$.

2.6 Fuzzy Linear Complementarity Problem

The parameters in (1.1) - (1.3) are fuzzy numbers. Then, by substituting fuzzy numbers for crisp parameters, the ensuing FLCP can be identified.

$$\tilde{W} - \tilde{M}\tilde{Z} = \tilde{q} \quad (2.6.1)$$

$$\tilde{W}_j \geq 0, \tilde{Z}_j \geq 0, \tilde{Z}_0 \geq 0, j = 1, \dots, n \quad (2.6.2)$$

$$\tilde{W}_j\tilde{Z}_j = 0, j = 1, \dots, n \quad (2.6.3)$$

The pair $(\tilde{W}_j, \tilde{Z}_j)$ is said to be a pair of Fuzzy Complementary variables.

3. Algorithm for the Fuzzy Fractional Linear Complementarity Problem

Step 1: Consider the constraints of the FPP as a Triangular fuzzy number.

Step 2: Rewrite the constraints in terms of linear programming problem.

Step 3: Using the conversion algorithm of LPP in to LCP [17], the given values can be converted into Complementarity values.

Step 4: Convert the parameters and variables in interval number by taking different levels of α – cut.

Step 5: The Converted systems can be solved according to the Lemke's algorithm [18].

Step 6: Consider the FLCP $(\tilde{q}, \tilde{M})$ of order n, and $q \epsilon \mathbb{R}^n$ satisfies the condition

$\tilde{w}$	$\tilde{Z}$	
$\tilde{I}$	$\tilde{M}$	$\tilde{q}$

Step 7: Identify t = $\frac{\tilde{q}_t}{\tilde{m}_{ts}} = minimum \left\{\frac{\tilde{q}_t}{\tilde{m}_{ts}}\ i = 1\ to\ n\right\}$ the column vector on the right side, Pivots and becomes nonnegative.

Step 8: The FLCP solution ends if s=t. If s ≠ t, then there is precisely one fuzzy complementarity pair in which both variables are contained, as well as exactly one basic variable and that contains both variables.

Step 9: Choose $\tilde{Z}_s$ as entering variable and it is ascertained by the complementary pivot rule.

Step 10: Finally, one variable from the complementary pair either drops out from the basic vector, equals zero, or, in the case of strictly positive in basic feasible solution.

4. Numerical Illustration

Consider the following linear fractional programming problem:

$$\text{Max } Z = \frac{5x_1+6x_2}{2x_2+7}$$

Subject to the constraints:

$$2x_1 + 3x_2 \le 6$$

$$2x_1 + x_2 \le 3$$

$$x_1, x_2 \ge 0.$$

The given problem can be developed by the above proposed algorithm and it is converted by means of conversion algorithm of Linear Programming problem [16]. So the following steps will be:

$$M = \begin{bmatrix} (-1,0,1) & (0,0,0) & (1,2,3) & (0,2,4) \\ (0,0,0) & (-1,0,1) & (2,3,4) & (0,1,2) \\ (-3,-2,-1) & (-4,-3,-2) & (-2,0,2) & (0,0,0) \\ (-3,-2,-1) & (-2,-1,0) & (0,0,0) & (-1,0,1) \end{bmatrix}, \quad q = \begin{bmatrix} (3,5,7) \\ (5,6,7) \\ (-8,-6,-4) \\ (-5,-3,-1) \end{bmatrix}$$

Therefore $x_1 = z_1$ and $x_2 = z_2$ will be an optimal solution to the given LPP. The constraints of the Fractional programming problem is taken as an interval number with different levels of α cut. After attempting these steps the following equations are given below,

When $\alpha = 1$, W - Mz = q, gives

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} - \begin{bmatrix} [0,0] & [0,0] & [2,2] & [2,2] \\ [0,0] & [0,0] & [3,3] & [1,1] \\ [-2,-2] & [-3,-3] & [0,0] & [0,0] \\ [-2,-2] & [-1,-1] & [0,0] & [0,0] \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} [5,5] \\ [6,6] \\ [-6,-6] \\ [-3,-3] \end{bmatrix}$$

The results are tabulated below,

Table 4.1 Values are taken as an Interval numbers, the following iterations are performed by Lemke's algorithm.

Bv	W1	W2	W3	W4	Z1	Z2	Z3	Z4	q
W1	[1,1]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[2,2]	[2,2]	[5,5]
W2	[0,0]	[1,1]	[0,0]	[0,0]	[0,0]	[0,0]	[3,3]	[1,1]	[6,6]
W3	[0,0]	[0,0]	[1,1]	[0,0]	[-2,-2]	[-3,-3]	[0,0]	[0,0]	[-6,-6]
W4	[0,0]	[0,0]	[0,0]	[1,1]	[-2,-2]	[-1,-1]	[0,0]	[0,0]	[-3,-3]
W1	[1,1]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[2,2]	[2,2]	[5,5]
W2	[0,0]	[1,1]	[0,0]	[0,0]	[0,0]	[0,0]	[3,3]	[1,1]	[6,6]
Z2	[0,0]	[0,0]	[-0.33,-0.33]	[0,0]	[0.666,0.666]	[1,1]	[0,0]	[0,0]	[2,2]
W4	[0,0]	[0,0]	[-0.33,-0.33]	[1,1]	[1.333,1.333]	[0,0]	[0,0]	[0,0]	[-1,-1]
W1	[1,1]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[2,2]	[2,2]	[5,5]
W2	[0,0]	[1,1]	[0,0]	[0,0]	[0,0]	[0,0]	[3,3]	[1,1]	[6,6]
Z2	[0,0]	[0,0]	[-0.5,-0.5]	[0.5,0.5]	[0,0]	[1,1]	[0,0]	[0,0]	[1.5,1.5]
Z1	[0,0]	[0,0]	[0.25,0.25]	[0.75,0.75]	[1,1]	[0,0]	[0,0]	[0,0]	[0.75,0.75]

Values are expressed by taking $\alpha = 1$ to TFNs, W and Z are complementary pair, q is a vector.

The optimal solution for $\alpha = 1$ (Table 4.1) is [$\mathbf{W_1}$, $\mathbf{W_2}$, $\mathbf{W_3}$, $\mathbf{W_4}$: $\mathbf{Z_1}$, $\mathbf{Z_2}$, $\mathbf{Z_3}$, $\mathbf{Z_4}$] = [[5,5], [6,6], [0,0],[0,0]: [0.75,0.75], [1.5,1.5], [0,0],[0,0]]

When $\alpha = 0$, W - Mz = q, gives

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} - \begin{bmatrix} [-1,1] & [0,0] & [1,3] & [0,4] \\ [0,0] & [-1,1] & [2,4] & [0,2] \\ [-3,-1] & [-4,-2] & [-2,2] & [0,0] \\ [-3,-1] & [-2,0] & [0,0] & [-1,1] \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} [3,7] \\ [5,7] \\ [-8,-4] \\ [-5,-1] \end{bmatrix}$$

The results are tabulated below

Table 4.2 The same procedure is followed in Table 4.2, where the following values are operated by the arithmetic operations of interval numbers.

Bv	W_1	W_2	W_3	W_4	Z_1	Z_2	Z_3	Z_4	q
W_1	[1,1]	[0,0]	[0,0]	[0,0]	[-1,1]	[0,0]	[1,3]	[0,4]	[3,7]
W_2	[0,0]	[1,1]	[0,0]	[0,0]	[0,0]	[-1,1]	[2,4]	[0,2]	[5,7]
W_3	[0,0]	[0,0]	[1,1]	[0,0]	[-3,-1]	[-4,-2]	[-2,2]	[0,0]	[-8,-4]
W_4	[0,0]	[0,0]	[0,0]	[1,1]	[-3,-1]	[-2,0]	[0,0]	[-1,1]	[-5,-1]
W_1	[1,1]	[0,0]	[0,0]	[0,0]	[-1,1]	[0,0]	[1,3]	[0,4]	[3,7]
W_2	[0,0]	[1,1]	[0,0]	[0,0]	[-1.25,1.25]	[-2.66,2.66]	[1.2,4.8]	[0,2]	[1.7,0.3]
Z_2	[0,0]	[0,0]	[-0.4167,0.2500]	[0,0]	[0.25,1.08]	[0.50,1.50]	[-0.83,0.83]	[0,0]	[1,2.996]
W_4	[0,0]	[0,0]	[-0.6667,0]	[1,1]	[-3.0,0.33]	[-2,2]	[-1.333,1.333]	[-1,1]	[-5,3]
W_1	[1,1]	[0,0]	[-3.02,3.02]	[-4.53,4.53]	[-14.58,14.58]	[-9.06,9.06]	[-5.02,9.02]	[-4.53,8.53]	[-19.64,29.64]
W_2	[0,0]	[1,1]	[-3.77,3.77]	[-5.66,5.66]	[-18.23,18.23]	[-13.98,13.98]	[-6.33,12.33]	[-5.66,7.66]	[-26.6,38.6]
Z_2	[0,0]	[0,0]	[-2.96,1.96]	[-3.3,4.3]	[-10.9,10.9]	[-8.11,10.11]	[-6.6,6.6]	[-4.3,4.3]	[-16.52,19.52]
Z_1	[0,0]	[0,0]	[-2.02,2.70]	[-4.8,3.3]	[-9.9,11.09]	[-9.6,9.6]	[-6.38,6.38]	[-4.8,4.8]	[-15.0,16.50]

Value of $\alpha = 0$. The optimal solutions are obtained when the component of q vector is positive.

The optimal solution for $\alpha = 0$ (Table 4.2) is [$\mathbf{W_1}$, $\mathbf{W_2}$, $\mathbf{W_3}$, $\mathbf{W_4}$: $\mathbf{Z_1}$, $\mathbf{Z_2}$, $\mathbf{Z_3}$, $\mathbf{Z_4}$] =

[[-19.64,29.64], [-26.6,38.6], [0,0],[0,0]: [-15.00,16.50], [-16.52,19.52], [0,0],[0,0]]

The other values are tabulated below,

$\alpha - values$	Optimal values od Z
$\alpha = 1$	[1.3 1.3]
$\alpha = 0.8$	[-11.3, 12.4]
$\alpha = 0.6$	[-20.2 23.3]
$\alpha = 0.4$	[-30.1, 33.2]
$\alpha = 0.2$	[-39, 42]
$\alpha = 0$	[-49, 51]

The Matlab output for the above example is:

Figure 4.1 denotes the different levels of $\alpha - cuts$, the values are taken in the Interval numbers and obtained from the Matlab output.

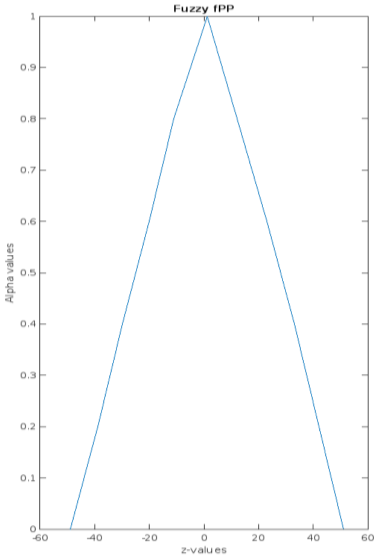

Fig 4.1

5. Conclusion

This study discusses an algorithm for solving Fuzzy fractional programming problem in a complementarity terms. In the proposed algorithm, firstly the FFPP is transformed into FLPP and then the resultant FLPP is converted in to FLCP under the interval numbers. Using this method, the output is easy and unmistakable. The design of this problem ensures that every α-cut of the interval is examined in order to get the best possible optimal solution. These methods provide additional concepts for creating new FLCP implementations.

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