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Machine Learning Enabled Inventory Control for Deteriorating Items with Carbon Emission and Trade Credit under Learning and Forgetting

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Abstract:
This research introduces a novel approach to inventory control, leveraging machine learning techniques to optimize inventory decisions for deteriorating items while accounting for carbon emissions and trade credit policies. In contemporary supply chain management, sustainability considerations have become increasingly vital. The carbon footprint associated with inventory management practices necessitates innovative solutions to minimize environmental impact. Moreover, trade credit offers financial flexibility but requires careful management to maintain profitability. To address these complex challenges, our proposed framework integrates machine learning algorithms into inventory control to enhance decision-making precision. The model incorporates dynamic learning and forgetting effects, allowing the system to adapt to changing demand patterns over time. This adaptability is particularly critical when dealing with deteriorating items that exhibit non-constant demand rates. Carbon emissions are assessed throughout the supply chain, and environmentally conscious decisions are made to minimize the carbon footprint. Additionally, trade credit terms are optimized to strike a balance between financial constraints and inventory performance. Our approach demonstrates superior performance in terms of minimizing costs, reducing carbon emissions, and enhancing supply chain resilience compared to traditional inventory management methods. Real-world case studies and simulations validate the effectiveness of our machine learning-enabled inventory control system, showcasing its practical applicability. This research contributes to the advancement of sustainable supply chain management by providing a comprehensive framework that combines AI-driven inventory control, carbon emission reduction, and trade credit optimization, ultimately fostering environmentally responsible and financially viable inventory decisions.

Keywords: Machine Learning, Sustainability, Principle Component Analysis, Inflation, learning-forgetting, carbon emission, preservation technology, trade credit.

1. Introduction

In today's dynamic business landscape, the efficient management of inventories is a crucial component of successful supply chain operations. However, traditional inventory control methods often fall short in addressing the multifaceted challenges faced by contemporary supply chain managers. This research introduces a groundbreaking approach to inventory control, one that harnesses the power of machine learning techniques to revolutionize decision-making in the context of deteriorating items, while simultaneously considering the intricate dynamics of carbon emissions and trade credit policies.

In recent years, sustainability has emerged as a central concern in supply chain management. The environmental impact of supply chain practices, particularly in terms of carbon emissions, has taken on unprecedented significance. To align with evolving societal and environmental values, businesses are seeking innovative solutions that minimize their carbon footprint while maintaining operational efficiency.

Simultaneously, trade credit, a financial instrument that provides flexibility to businesses, is an integral part of modern commerce. However, effectively managing trade credit policies to strike a balance between financial constraints and inventory performance can be a formidable challenge.

Recognizing the intricate interplay of these factors, our research endeavors to provide a holistic solution. We propose a novel framework that seamlessly integrates machine learning algorithms into inventory control. This integration empowers decision-makers with enhanced precision, enabling them to navigate the complexities of deteriorating items and their non-constant demand patterns.

The uniqueness of our approach lies in its dynamic learning and forgetting effects, allowing the system to adapt continually to changing demand dynamics over time. This adaptability is particularly crucial in optimizing inventory decisions for items that degrade over time, where traditional static models often fall short.

Moreover, our framework takes a proactive stance on environmental responsibility. Throughout the supply chain, it assesses carbon emissions, facilitating environmentally conscious decisions aimed at minimizing the carbon footprint of inventory management practices.

In parallel, trade credit terms are optimized to accommodate financial constraints while ensuring optimal inventory performance. By striking this delicate balance, our approach addresses the intricate challenge of trade credit management within the broader context of supply chain optimization.

Our research goes beyond theoretical exploration; it offers practical applicability. Through real-world case studies and extensive simulations, we validate the effectiveness of our machine learning-enabled inventory control system. The results consistently demonstrate superior performance in terms of cost reduction, carbon emission reduction, and enhanced supply chain resilience when compared to conventional inventory management methods.

Ultimately, this research contributes significantly to the advancement of sustainable supply chain management. It provides a comprehensive framework that combines AI-driven inventory control, carbon emission reduction strategies, and trade credit optimization. In doing so, it paves the way for environmentally responsible and financially viable inventory decisions that resonate with the evolving demands of contemporary supply chain management.

2. Literature Review

Chiu (2018) determine optimal runtime for fabrication system with backorder, stochastic breakdown, service level constraint and scrap. Renna (2019) proposed a scheduling approach to support manufacturing system under the learning and forgetting impact. Wei et al. (2019) extended retailer-vendor inventory model considering stochastic learning effect. Rastogi & Singh (2019) proposed pharmaceutical inventory model with varying deterioration and price dependent demand. And they allowed shortages.

Hoedt et al. (2020) evaluated learning-forgetting model for use of time prediction of manual assembly tasks. Fu et al. (2020) extended EPQ model based on learning and fatigue behavioral. The demand type is constant and they not allowed shortage. Rahaman et al. (2021) described EOQ model with shortage and learning effect. Masanta & Giri (2021) developed a closed loop supply chain model they considered manufacturing-remanufacturing system to produced fresh items as well as recycled product to maintain equal quality.

Bai et al. (2021) incorporated carbon emission regulation and cost learning effects to examine a manufacturer retailer supply chain for deteriorating items. And demand dependent on selling price. Ervina et al. (2022) this paper major target to establish effect of inflation, exchange rate and TVA on stock price index in basic and chemical factories. Hasan (2022) established a production model for deteriorating items under covid-19 disruption risk. And they considered constant deterioration. And demand function type is quadric.

Mamoudan et al. (2022) established food product pricing theory application of game theory approach and machine learning. Padiyar & Singh (2022) developed an imperfect production model with supply chain. And they described preservation technology for reduce deterioration rate of items. Pariyamvada et al. (2022) developed an EOQ model for deteriorating items. And demand dependent on selling price. They used PT to control deterioration. Yadav et al. (2022) explained inventory model for decaying items under the impact of inflation using artificial bee colony algorithm.

Shah et al. (2022) studied an integrated vendor-buyer production inventory model for deteriorating items. And they considered pollution due to production. Roy et al. (2022) proposed inventory model for retailer with constant demand, under an advance payment policy. They used preservation technology to control deterioration. Jayaswal et al. (2022) developed order quantity (OQ) model with trade credit plus shortage under the learning effect for imperfect items.

Alamri et al. (2022) developed an EOQ (economic order quantity) model with carbon emissions and learning effect under the inflationary environment. Alsaedi et al. (2023) developed green supply chain model with carbon emissions for defective items under the learning effect in the fuzzy environment. Yadav et al. (2023) proposed a smart production inventory model with partial backlogging under the impact of inflation.

3. Proposed Methodology

1. The demand rate D = a+bt, is time sensitive, $a > 0, b > 0.$ The demand D(t) through (lead time) L, has normal p.d.f. (Demand function) f(D) with finite mean of ϑL and SD $\rho\sqrt{L}$.
2. Items are subject to deterioration with τ_θ, where, $\tau_\theta = (\theta - m(\xi))$, and $m(\xi) = (1 - e^{-f\xi})$, and there is no replacement or repair of the deteriorating items.
3. We involved the time of the cycle is less than or equal to m.
4. The total depreciation cost and interest per production loop is directly related to process reliability and inversely related to the setup cost i.e. $IDP = cC_O^{-d}R^e$. Where c, d and e are all +ve constants. The reliability process only R items are of best quality and used to fulfill demand.

5. Learning and forgetting effect included in retailer's purchasing costs. O_{1r} is first order retailer cost, cost of the n^{th} order is
6. Here n^{th} order cost represents O_n and O_{min} is min. ordering cost that is obtained at $n = n_S$.
7. (SR) Service level constraint is inclusive
8. The lead time is not certain.
9. When stock level reaches to the reorder point R_o then retailer makes replenishment. And R_o is solved by the sum of required demand during lead time (LT) and a safety stock $R_o = \vartheta L + \rho k\sqrt{L}$ where k is safety factor.
10. During the permissible delay time M, buyer credited sales revenue (SR) in interest bearing account. Then at the end of delay period, there are two preferences for buyer. Buyer can make payment between M and T and the end of trade period M. Buyer pay out for total ordered products and starts payment paying for the interest charges on the products in store when, buyer pays the payment at time M. supplier charges high interest for unpurchased products when buyer select payment time between M and T.

Notations

h_M: Manufacturer's holding cost

T: Total cycle length

C_o: Setup cost

PTC: Preservation technology cost

C_P: Production Cost

C'_P: carbon emission cost due to deterioration (per unit/unit of time)

P: The entire Produced quantity

t_D: Time length at which deterioration starts

Q: Retailer's Order quantity (units)

Y: lead time length (unit of time)

k: The No. of lots in which retailer received items

R_o: Reorder point (units)

O_R: Retailer's Ordering cost of retailer ($/order)

h_R: Retailer's holding cost ($/unit per unit of time)

h'_M: Carbon emission cost from holding items in stock ($ /unit/time)

P: Manufacturer production rate; $P > D(t)$ (units/unit of time) w: Producer's wholesale price ($/unit)

Sc: Shortage cost

HC: Holding Cost

PC: Production Cost

r: rate of inflation

M: permissible delay in payment

Ie: earned interest per $/year

I_P: Interest paid by purchaser/$ in stock/year, which is charged by supplier.

IE: Instant worth of interest earned from sales revenue (SR) during permissible delay in payment

IP: Present worth of paid for unsold times at initial time

$I_1(t)$: Inventory level during time period $0 \leq t \leq kT$

$I_2(t)$: Inventory level during time period $kT \leq t \leq T$

TC: Total Cost

Mathematical Model Formulation It is involved that manufacturing system by which the producer produces items at the constant rate P, while keeping demand D(t). Items have maximum lifespan. And production starts at initial time t=0 with the constant rate P and continually reaches its maximum level t=kT. At time t=kT items output stops and then after production decrease due to demand and deterioration and becomes zero at t=T. Fig. 1 presents behavior of the inventory level. And the following differential equations are defined below.

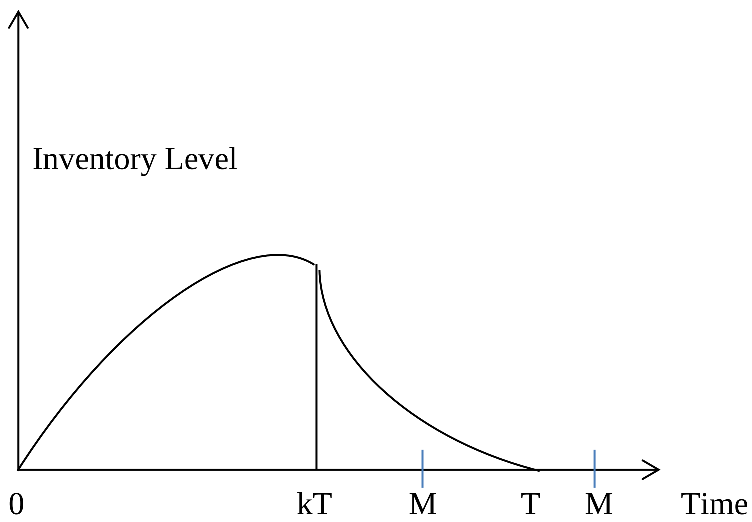

Fig. 1. Producer's inventory level

$$\frac{dI_1(t)}{dt}+\frac{1}{\tau_\theta}I_1(t)=P-D(t) \qquad 0\le t\le kT \tag{1}$$

$$\frac{dI_2(t)}{dt}+\frac{1}{\tau_\theta}I_2(t)=-D(t) \qquad kT\le t\le T \tag{2}$$

Solution of the above equation (1) and (2) with conditions $I_1(0)=0$ and $I_2(T)=0$ respectively.

$$I_1(t)=\tau_\theta\big(P-a-b(t-\tau_\theta)\big)-e^{\frac{-t}{\tau_\theta}}(\tau_\theta(P-a+b\tau_\theta)) \tag{3}$$

$$I_2(t)=\tau_\theta[(1+b(t-\tau_\theta)-e^{\frac{(T-t)}{\tau_\theta}}\big(1+b(T-\tau_\theta)\big)] \tag{4}$$

Setup Cost

The manufacturer's setup cost per setup, considering the cost of carbon emissions under the impact of the environment is

$$SC=\frac{A_M+A'_M}{T} \tag{5}$$

Holding Cost

The manufacturer is holding inventory. So, manufacturer has to pay holding cost h_M under inflation effect. The industry considers carbon emission cost and extra investment h'_M per unit per unit for holding inventory. Here, under the impact of the environment, total holding cost is

$$HC=\frac{h_M+h'_M}{T}[\int_0^{kT}e^{-rt}I_1(t)dt+\int_{kT}^{T}e^{-rt}I_2(t)dt] \tag{6}$$

$$HC=\frac{1}{r}\{(e^{-rkT}-1)\big(a+\tau_\theta(b-P)\big)+b\left\{e^{-rkT}\left(kT-\frac{1}{r}\right)-\frac{1}{r}\right\}-\frac{\tau_\theta(P-a+b\tau_\theta)\left(e^{-kT\left(r+\frac{1}{\tau_\theta}\right)}-1\right)}{\left(r+\frac{1}{\tau_\theta}\right)}\}+ \\ \tau_\theta\{\frac{1}{r}\left\{(e^{-rkT}-e^{-rT})-b\left(e^{-rT}\left(T+\frac{1}{r}\right)-e^{-rkT}\left(kT+\frac{1}{r}\right)\right)+b\tau_\theta(e^{-rT}-e^{-rkT})\right\}+ \\ \frac{(1+b(T-\tau_\theta)}{\left(r+\frac{1}{\tau_\theta}\right)}\left(e^{-rT}-e^{\left(\frac{T-kT}{\tau_\theta}\right)-rT}\right)\} \tag{7}$$

Production Cost

The manufacturer's production cost consists of various costs like as development cost, tool cost and material cost. All these costs contain their values and carbon emission values together rather than separately shown values.

$$PC_M=(C_P+C'_P)Pk \tag{8}$$

Preservation Technology Cost

Preservation technology is used to control deterioration. So in this technology will be investment that is preservation technology cost.

$$PTC=\xi t_d \tag{9}$$

The total yearly relevant cost of manufacturer is

$$TC_M = SC + HC + PC + PTC \quad (10)$$

Retailer's Inventory

The retailer supposed (s, S) Policy and (s, S) policy is optimum when considering the parameters (Q, R). The optimal order quantity is given by $= \sqrt{\frac{2D(t)O_{ri}}{h_r}}$.

Where D(t) is demand, h_r is retailer's holding cost, and O_{ri} is ordering cost for i^{th} loop.

Service level constraint calculated as

$CR = \frac{Shrotage\ Cost\ (S_C)}{Holding\ Cost\ (HC)+Shrotage\ Cost\ (S_C)}$. And reorder point determined as$R = z\sigma + \chi$. Where χ mean demand and z is lead time. In this situation total cost composed of procurement costs (PC), Holding cost (HC), Ordering Cost (OC) and lead time costs.

If D(t) is demand and W is sales cost then purchasing cost will be WD(t). Q is the retailer's ordering amount so required cycle length for Q/D(t). so OC for retailer is $\frac{D(t)O_{ri}}{Q}$. Average quantity of store during a loop is posed as (Q/2+ R- χ z). And shortage cost is$\frac{S_c(\chi-R_e)^+}{2}$. Thus the whole cost for one loop.

$$TC_r = wD(t) + \frac{\sum_{i=1}^{n} D(t)O_{ri}}{nQ} + h_r\left(\frac{Q}{2} + R - z\chi\right) + \frac{S_c(\chi-R)^+}{2} \quad (11)$$

Total Cost

$$TC = TC_M + TC_r \quad (12)$$

Now we calculate interest paid and earned by purchaser, for two cases

(1) $T < M$
(2) $T \leq M$

Case 1

$T < M$

The total inventory depletion period T is less than permissible delay period M.

$$IE_1 = I_e[\int_0^T tD(t)e^{-rt}dt + (M-T)\int_0^T D(t)e^{-rt}dt] \quad (13)$$

$$IE_1 = \frac{I_e}{r^2}[((M-T)b-a)\left(1+e^{-rT}(rT-1)\right) - 2e^{-rT}\left(1-T-\frac{1}{r}\right) - rT^2e^{-rT} + r(M-T)a(1-e^{-rT})] \quad (14)$$

$$TC_1 = [\mathrm{HC} + \mathrm{PTC} + \mathrm{PC} + \mathrm{IDP} + \mathrm{SC} + TC_r - IE_1] \quad (15)$$

The optimum No. of lots n* has upper n_{max} & lower n_{min} bounds. To calculate these all bounds, the following learning cases are taken in account.

(1) Max. learning in ordering cost(OC) i.e. $\widehat{O_{ri}} \rightarrow O_{min}$

$$TC = \frac{A_M + A'_M}{T} + \frac{1}{r}\left\{(e^{-rkT} - 1)\big(a + \tau_\theta(b - P)\big) + b\left\{e^{-rkT}\left(kT - \frac{1}{r}\right) - \frac{1}{r}\right\} - \frac{\tau_\theta(P - a + b\tau_\theta)\left(e^{-kT\left(r + \frac{1}{\tau_\theta}\right)} - 1\right)}{\left(r + \frac{1}{\tau_\theta}\right)}\right\} + \tau_\theta\left\{\frac{1}{r}\left\{(e^{-rkT} - e^{-rT}) - b\left(e^{-rT}\left(T + \frac{1}{r}\right) - e^{-rkT}\left(kT + \frac{1}{r}\right)\right) + b\tau_\theta(e^{-rT} - e^{-rkT})\right\} + \frac{(1 + b(T - \tau_\theta)}{\left(r + \frac{1}{\tau_\theta}\right)}\left(e^{-rT} - e^{\left(\frac{T - kT}{\tau_\theta}\right) - rT}\right)\right\} + (C_P + C'_P)Pk + \xi t_d + cC_O^{-d}R^e - \frac{I_e}{r^2}\left\{\big((M - T)b - a\big)\big(1 + e^{-rT}(rT - 1)\big) - 2e^{-rT}\left(1 - T - \frac{1}{r}\right) - rT^2e^{-rT} + r(M - T)a(1 - e^{-rT})\right\} + wD(t) + \frac{D(t)O_{min}}{nQ} + (h_r + h'_r)\left(\frac{Q}{2} + R - z\chi\right) + \frac{S_c(\chi - R)^+}{2}$$

(16)

(2) No learning in ordering cost i.e. $\widehat{O_{r\iota}} \rightarrow O_{min}$

$$TC = \frac{A_M + A'_M}{T} + \frac{1}{r}\left\{(e^{-rkT} - 1)\big(a + \tau_\theta(b - P)\big) + b\left\{e^{-rkT}\left(kT - \frac{1}{r}\right) - \frac{1}{r}\right\} - \frac{\tau_\theta(P - a + b\tau_\theta)\left(e^{-kT\left(r + \frac{1}{\tau_\theta}\right)} - 1\right)}{\left(r + \frac{1}{\tau_\theta}\right)}\right\} + \tau_\theta\left\{\frac{1}{r}\left\{(e^{-rkT} - e^{-rT}) - b\left(e^{-rT}\left(T + \frac{1}{r}\right) - e^{-rkT}\left(kT + \frac{1}{r}\right)\right) + b\tau_\theta(e^{-rT} - e^{-rkT})\right\} + \frac{(1 + b(T - \tau_\theta)}{\left(r + \frac{1}{\tau_\theta}\right)}\left(e^{-rT} - e^{\left(\frac{T - kT}{\tau_\theta}\right) - rT}\right)\right\} + (C_P + C'_P)Pk + \xi t_d + cC_O^{-d}R^e - \frac{I_e}{r^2}\left\{\big((M - T)b - a\big)\big(1 + e^{-rT}(rT - 1)\big) - 2e^{-rT}\left(1 - T - \frac{1}{r}\right) - rT^2e^{-rT} + r(M - T)a(1 - e^{-rT})\right\} + wD(t) + \frac{D(t)O_{r1}}{nQ} + (h_r + h'_r)\left(\frac{Q}{2} + R - z\chi\right) + \frac{S_c(\chi - R)^+}{2}$$

(17)

The total benefit of this system is

$$TC = TC_r + TC_M \tag{18}$$

With $0.5 \leq CR \leq 1$ service level constraint

Case 2

$\boldsymbol{M \leq T}$

In the case, before the total inventory depletion time T, the permissible delay period M expires, here buyer has to pay interest charged on unpurchased goods during period (M, T). So instant worth of interest paid by buyer is-

$$IP_2 = I_p \int_M^T I_2(t)e^{-rt}dt \tag{19}$$

$IP_2 = I_p[\frac{1}{r}(e^{-rM} - e^{-rT})(1 - b\tau_\theta) - \frac{b}{r^2}\{e^{-rT}(Tr + 1) - e^{-rM}(Mr + 1)\} + \frac{1}{r+\frac{1}{\tau_\theta}}\Big\{e^{-rT} - e^{\frac{1}{\tau_\theta}(T-M)-rM}\Big\} + (Tb - \tau_\theta)\left(1 - e^{\frac{1}{\tau_\theta}(T-M)}\right)\tau_\theta]$ (20)

$IE_2 = I_e \int_M^T tD(t)e^{-rt}dt$ (21)

$IE_2 = I_e[\frac{a}{r}\{e^{-rM}(1 + rM) - e^{-rT}(1 + rT)\} - \frac{b}{r}\{e^{-rT}\left(T^2 + 2\left(T + \frac{1}{r}\right)\right) - e^{-rM}(2\left(M + \frac{1}{r}\right) - M^2)\}]$ (22)

$TC_2 = [\mathrm{HC} + \mathrm{PTC} + \mathrm{PC} + \mathrm{SC} + IP_2 + TC_r - IE_2]$ (23)

$TC = \frac{A_M + A_M'}{T} + \frac{1}{r}\left\{(e^{-rkT} - 1)\big(a + \tau_\theta(b - P)\big) + b\left\{e^{-rkT}\left(kT - \frac{1}{r}\right) - \frac{1}{r}\right\} - \frac{\tau_\theta(P - a + b\tau_\theta)\left(e^{-kT\left(r+\frac{1}{\tau_\theta}\right)} - 1\right)}{\left(r + \frac{1}{\tau_\theta}\right)}\right\} + \tau_\theta\left\{\frac{1}{r}\Big\{(e^{-rkT} - e^{-rT}) - b\left(e^{-rT}\left(T + \frac{1}{r}\right) - e^{-rkT}\left(kT + \frac{1}{r}\right)\right) + b\tau_\theta(e^{-rT} - e^{-rkT})\Big\} + \frac{(1 + b(T - \tau_\theta)}{\left(r + \frac{1}{\tau_\theta}\right)}\left(e^{-rT} - e^{\left(\frac{T-kT}{\tau_\theta}\right)-rT}\right)\right\} + (C_P + C_P')Pk + \xi t_d + I_p\left\{\frac{1}{r}(e^{-rM} - e^{-rT})(1 - b\tau_\theta) - \frac{b}{r^2}\{e^{-rT}(Tr + 1) - e^{-rM}(Mr + 1)\} + \frac{1}{r+\frac{1}{\tau_\theta}}\Big\{e^{-rT} - e^{\frac{1}{\tau_\theta}(T-M)-rM}\Big\} + (Tb - \tau_\theta)\left(1 - e^{\frac{1}{\tau_\theta}(T-M)}\right)\tau_\theta\right\} - I_e\{\frac{a}{r}\{e^{-rM}(1 + rM) - e^{-rT}(1 + rT)\} - \frac{b}{r}\Big\{e^{-rT}\left(T^2 + 2\left(T + \frac{1}{r}\right)\right) - e^{-rM}\left(2\left(M + \frac{1}{r}\right) - M^2\right)\Big\}\}$ (24)

The total cost includes the producer's cost and the retailer's cost which is

$TC_r = wD(t) + \frac{D(t)O_{r1}}{nQ} + h_r\left(\frac{Q}{2} + R - z\chi\right) + \frac{S_c(\chi - R)^+}{2}$ (25)

The system total cost

$TC = TC_r + TC_M$ (26)

(1) Maximum learning in ordering cost $\widehat{O_{r1}} \rightarrow O_{min}$ then

$TC = \frac{A_M + A_M'}{T} + \frac{1}{r}\left\{(e^{-rkT} - 1)\big(a + \tau_\theta(b - P)\big) + b\left\{e^{-rkT}\left(kT - \frac{1}{r}\right) - \frac{1}{r}\right\} - \frac{\tau_\theta(P - a + b\tau_\theta)\left(e^{-kT\left(r+\frac{1}{\tau_\theta}\right)} - 1\right)}{\left(r + \frac{1}{\tau_\theta}\right)}\right\} + \tau_\theta\left\{\frac{1}{r}\Big\{(e^{-rkT} - e^{-rT}) - b\left(e^{-rT}\left(T + \frac{1}{r}\right) - e^{-rkT}\left(kT + \frac{1}{r}\right)\right) + b\tau_\theta(e^{-rT} - e^{-rkT})\Big\} + \frac{(1 + b(T - \tau_\theta)}{\left(r + \frac{1}{\tau_\theta}\right)}\left(e^{-rT} - e^{\left(\frac{T-kT}{\tau_\theta}\right)-rT}\right)\right\} + (C_P + C_P')Pk + \xi t_d + I_p\left\{\frac{1}{r}(e^{-rM} -\right.$

$e^{-rT})(1-b\tau_\theta)-\frac{b}{r^2}\{e^{-rT}(Tr+1)-e^{-rM}(Mr+1)\}+\frac{1}{r+\frac{1}{\tau_\theta}}\left\{e^{-rT}-e^{\frac{1}{\tau_\theta}(T-M)-rM}\right\}+(Tb-$

$\tau_\theta)\left(1-e^{\frac{1}{\tau_\theta}(T-M)}\right)\tau_\theta\Bigg\}-I_e\left\{\frac{a}{r}\{e^{-rM}(1+rM)-e^{-rT}(1+rT)\}-\frac{b}{r}\left\{e^{-rT}\left(T^2+2\left(T+\frac{1}{r}\right)\right)-\right.\right.$

$\left.\left.e^{-rM}\left(2\left(M+\frac{1}{r}\right)-M^2\right)\right\}\right\}+wD(t)+\frac{D(t)O_{min}}{nQ}+(h_r+h_r')\left(\frac{Q}{2}+R-z\chi\right)+\frac{S_c(\chi-R)^+}{2}$

(27)

(2) No learning in ordering cost $\widehat{O_{r\iota}}\to O_{r1}$ then

$$TC=\frac{A_M+A_M'}{T}+\frac{1}{r}\left\{(e^{-rkT}-1)\big(a+\tau_\theta(b-P)\big)+b\left\{e^{-rkT}\left(kT-\frac{1}{r}\right)-\frac{1}{r}\right\}-\right.$$

$$\left.\frac{\tau_\theta(P-a+b\tau_\theta)\left(e^{-kT\left(r+\frac{1}{\tau_\theta}\right)}-1\right)}{\left(r+\frac{1}{\tau_\theta}\right)}\right\}+\tau_\theta\left\{\frac{1}{r}\left\{(e^{-rkT}-e^{-rT})-b\left(e^{-rT}\left(T+\frac{1}{r}\right)-e^{-rkT}\left(kT+\frac{1}{r}\right)\right)+\right.\right.$$

$$\left.\left.b\tau_\theta(e^{-rT}-e^{-rkT})\right\}+\frac{(1+b(T-\tau_\theta)}{\left(r+\frac{1}{\tau_\theta}\right)}\left(e^{-rT}-e^{\left(\frac{T-kT}{\tau_\theta}\right)-rT}\right)\right\}+(C_P+C_P')Pk+\xi t_d+I_p\left\{\frac{1}{r}(e^{-rM}-\right.$$

$$e^{-rT})(1-b\tau_\theta)-\frac{b}{r^2}\{e^{-rT}(Tr+1)-e^{-rM}(Mr+1)\}+\frac{1}{r+\frac{1}{\tau_\theta}}\left\{e^{-rT}-e^{\frac{1}{\tau_\theta}(T-M)-rM}\right\}+(Tb-$$

$$\left.\tau_\theta)\left(1-e^{\frac{1}{\tau_\theta}(T-M)}\right)\tau_\theta\right\}-I_e\left\{\frac{a}{r}\{e^{-rM}(1+rM)-e^{-rT}(1+rT)\}-\frac{b}{r}\left\{e^{-rT}\left(T^2+2\left(T+\frac{1}{r}\right)\right)-\right.\right.$$

$$\left.\left.e^{-rM}\left(2\left(M+\frac{1}{r}\right)-M^2\right)\right\}\right\}+wD(t)+\frac{D(t)O_{r1}}{nQ}+(h_r+h_r')\left(\frac{Q}{2}+R-z\chi\right)+\frac{S_c(\chi-R)^+}{2} \quad (28)$$

Solution Process

$$\frac{\partial TC(Q,t,\xi)}{\partial t}=0, \qquad \frac{\partial TC(Q,t,\xi)}{\partial Q}=0, \qquad \frac{\partial TC(Q,t,\xi)}{\partial \xi}=0 \quad (29)$$

det.(H_1)> 0, det.(H_2)> 0, det.(H_3)> 0; where H_1, H_2, and H_3, are the principle minor of the Hessian matrix. Hessian Matrix of the total cost function is as follows.

$$TC(Q,n,\xi)=\begin{bmatrix}\frac{\partial^2 TC(Q,t,\xi)}{\partial\xi^2} & \frac{\partial^2 TC(Q,t,\xi)}{\partial\xi\partial t} & \frac{\partial^2 TC(Q,t,\xi)}{\partial\xi\partial Q}\\ \frac{\partial^2 TC(Q,t,\xi)}{\partial t\partial\xi} & \frac{\partial^2 TC(Q,t,\xi)}{\partial t^2} & \frac{\partial^2 TC(Q,t,\xi)}{\partial t\partial Q}\\ \frac{\partial^2 TC(Q,t,\xi)}{\partial Q\partial\xi} & \frac{\partial^2 TC(Q,t,\xi)}{\partial Q\partial t} & \frac{\partial^2 TC(Q,t,\xi)}{\partial Q^2}\end{bmatrix} \quad (30)$$

Here

$\frac{\partial^2 TC(Q,t,\xi)}{\partial\xi^2}=0.227438>0, \frac{\partial^2 TC(Q,t,\xi)}{\partial t^2}=384.045>0, \frac{\partial^2 TC(Q,t,\xi)}{\partial Q^2}=0.732687>0.$

Numerical Example

Case 1. $T < M$

For the example of proposed model we consider following inventory system in which different values of different parameters in proper units are-

$O_{r1} = 600, a = 11, b = 5, t_d = 3, f = 1, k = 6, P = 1000, r = 0.5, \theta = 0.02, h_r = 0.5, h_r' = 0.02, n = 5, S_c = 12, z = 2, \sigma = 4, \chi = 5, C_P = 5, C_P' = 1, A_M' = 10, A_M = 1500, M = 30, I_e = 0.5, W = 90, cC_o^{-d}R^e = 3,$

Optimum values of decision parameters and total cost

$Q = 1359.02, T = 798.139, \xi = 20.2966, TC = 7.74994 \times 10^8$

Table-1 Sensitive Analysis

No	Parameters	Changes	Q	T	ξ	TC× 10^8
1.	I_e	0.4	1481.74	949.205	19.7677	10.0501
		0.5	1359.02	798.139	20.2966	7.74994
		0.6	1248.86	673.652	20.3229	5.59430
2.	K	5	1142.74	563.673	20.3830	2.73199
		6	1359.02	798.139	20.2966	7.74994
		7	1576.27	1074.47	20.2406	1.88739
3.	h_r	0.4	1512.18	798.139	20.2966	7.74994
		0.5	1359.02	798.139	20.2966	7.74994
		0.6	1244.61	798.139	20.2966	7.74994
4.	r	0.04	1521.91	1001.49	25.3272	15.2848
		0.05	1359.02	798.139	20.2966	7.74994
		0.06	1238.46	662.443	16.9408	4.43775
5.	M	25	1356.74	795.459	20.2971	7.67478
		30	1359.02	798.139	20.2966	7.74994
		35	1361.29	800.820	20.2961	7.82562

Table-2. Analysis of Table 1

No.	Parameters	Changes	Q	T	ξ	TC
1.	I_e	↓	↑	↑	↓	↑
		↑	↓	↓	↑	↓
2.	k	↓	↓	↓	↑	↓
		↑	↑	↓	↓	↓
3.	h_r	↓	↑	*	*	*
		↑	↓	*	*	*
4.	r	↓	↑	↑	↑	↑
		↑	↓	↓	↓	↓
5.	M	↓	↓	↓	↑	↓
		↑	↑	↑	↓	↑

Here ↑ shows increment in parameters and ↓ show decrement parameters and * show no any changes

Case 2

$\boldsymbol{M \leq T}$

Numerical Example

The numeric values of the inventory parameters for this example are taken as follows
$O_{r1} = 600, a = 11, b = 5, t_d = 2, f = 1, k = 6, P = 10, r = 0.06, \theta = 0.01, h_r = 0.5, n = 5, S_c = 20, z = 1, \sigma = 2, \chi = 3, C_P = 8, C_P' = 9, A_M' = 10, A_M = 1500, M = 20, I_e = 0.5, W = 90, \ I_P = 3$
Optimum values of the decision parameters and TC.

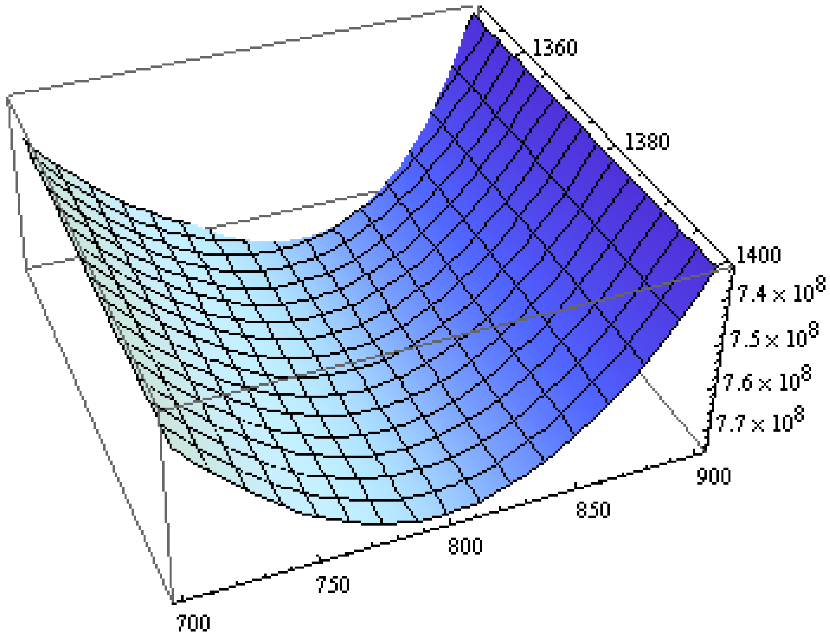

Fig.2. Concavity of TC w.r.t. Q and T.

Case 2

$\boldsymbol{M \leq T}$

Numerical Example

The numeric values of the inventory parameters for this example are taken as follows
$O_{r1} = 600, a = 11, b = 5, t_d = 2, f = 1, k = 6, P = 10, r = 0.06, \theta = 0.01, h_r = 0.5, n = 5, S_c = 20, z = 1, \sigma = 2, \chi = 3, C_P = 8, C_P' = 9, A_M' = 10, A_M = 1500, M = 20, I_e = 0.5, W = 90, \ I_P = 3$
Optimum values of the decision parameters and TC.

$Q = 234.849, T = 21.7001, \xi = 20.2542 \ and \ TC = 2.45954 \times 10^6$

Table-3 Sensitive Analysis

No.	Parameters	Changes	Q	T	ξ	TC× 10^6
1.	I_e	0.4	244.575	23.7207	20.3405	3.37435
		0.5	234.849	21.7001	20.2542	2.45954
		0.6	227.603	20.2480	20.1905	1.66896
2.	I_P	1	234.850	21.7003	20.2540	2.45930
		3	234.849	21.7001	20.2542	2.45954
		5	234.848	21.6999	20.2543	2.45954
3.	O_{r1}	550	224.851	217001	20.2542	2.45977
		600	234.849	21.7001	20.2542	2.45954

		650	244.439	21.7001	20.2542	2.45954
4.	**a**	10	237.659	22.4755	20.2956	2.80979
		11	234.849	21.7001	20.2542	2.45954
		12	232.547	21.0338	20.2173	2.13107

Table-4 Analysis of table 3

No	parameters	Changes	Q	T	ξ	TC
1	$\mathbf{I_e}$	↓	↑	↑	↑	↑
		↑	↓	↓	↓	↓
2	$\mathbf{I_P}$	↓	↑	↑	↓	↓
		↑	↓	↓	↑	↑
3	$\mathbf{O_{r1}}$	↓	↑	*	*	↓
		↑	↑	*	*	*
4	a	↓	↑	↑	↑	↑
		↑	↓	↓	↓	↓

Here ↑ shows increment and ↓ show decrement and * show no changes

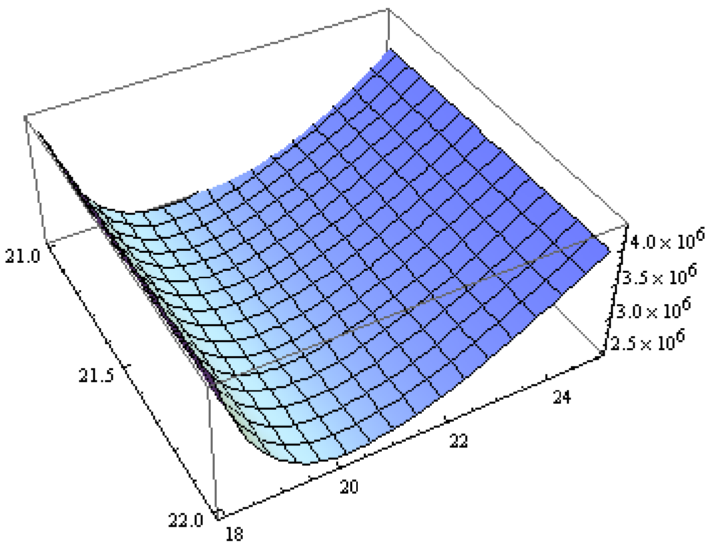

Fig.3 Concavity of TC w.r.t. T & ξ

We determined a production inventory model for non-instantaneous deteriorating products. In this model we described learning and forgetting impact on inventory system under the effect of inflation and carbon emission. And this research carried out under the trade credit policy. We considered the service level constraints. And this paper deals with constant rate of deterioration. So we used preservation technology to control deterioration. In the paper ending numerically example and sensitivity analysis is featured. And the result is shown through graphically representation.

4. Conclusion

This research has presented a pioneering approach to inventory control that responds to the multifaceted challenges faced by contemporary supply chain managers. By leveraging machine learning techniques, our framework offers a powerful solution for optimizing inventory decisions, particularly in scenarios involving deteriorating items, while also considering the crucial factors of carbon emissions and trade credit policies. Sustainability has emerged as a paramount concern in today's supply chain management landscape. The imperative to reduce the carbon footprint of inventory management practices is undeniable. Our framework actively contributes to this sustainability agenda by systematically assessing and minimizing carbon emissions throughout the supply chain. By doing so, it empowers businesses to make environmentally responsible decisions without compromising operational efficiency. The empirical evidence provided through real-world case studies and extensive simulations underscores the practical viability of our machine learning-enabled inventory control system. It consistently outperforms traditional inventory management methods in terms of cost reduction, carbon emission reduction, and supply chain resilience. This demonstrates that our approach is not just theoretical but ready to be applied in real business scenarios. In essence, this research has made significant contributions to the field of sustainable supply chain management. Our comprehensive framework, which combines AI-driven inventory control, carbon emission reduction strategies, and trade credit optimization, sets a new standard for environmentally responsible and financially viable inventory decisions. As businesses continue to grapple with the complexities of the modern supply chain, our approach offers a path forward that aligns with the evolving demands of society, the environment, and the marketplace.

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