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Geometrical Abstraction on VAN Hiele Levels Among Mathematics Undergraduates

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Abstract:

This research aims to analyze the geometrical abstraction on van Hiele levels among Mathematics Undergraduates at The State University of Makassar by utilizing an explanatory sequential mixed methods approach. Research subjects are selected by using a purposive sampling method from each van Hiele level to analyze their geometrical abstractions through geometrical abstraction tests and interviews. Data analysis includes: 1) statistical analysis, 2) qualitative data collection, 3) thematic analysis, 4) result interpretation, and 5) source triangulation. NVivo 12 software assists in coding qualitative data. Results indicate 27.7% of undergraduates are at the visualization level, 37.2% at the analysis level, 14.1% at the informal deduction level, 1.0% at the formal deduction level, and 2.6% at the rigor level. On the visualization level, abstraction involves understanding geometry concepts contextually. On the analysis level, it involves using concepts in complex contexts. On the informal deduction level, students connect geometry concepts, while on the formal deduction level, they deepen understanding through mathematical concepts. Furthermore, on the rigor level, students construct and connect various mathematical concepts broadly. This study also found higher van Hiele levels lead to more comprehensive geometrical abstraction. Future studies are recommended to explore the relationship between van Hiele levels and geometrical abstraction.

Keywords: Van Hiele Levels, Geometrical Abstraction, Mathematics Undergraduates.

Introduction

Geometry is a branch of Mathematics that explores the properties and relationships of points, lines, planes, solid objects, and higher-dimensional analogs which plays a vital role in developing students' spatial reasoning and problem-solving skills[1]. Chiossi (2021) asserts that Geometry holds significance for Mathematics undergraduates as it lays a strong foundation for mathematical thinking and essential visualization skills, crucial for success in higher-level Mathematics.

However, Geometry poses a challenge for many students worldwide, including those in Indonesia [2]. Students often struggle to grasp geometry concepts due to a lack of fundamental knowledge and limited geometry thinking skills [3]. According to data from the Program for International Student Assessment (PISA) 2022, Indonesia's Mathematics proficiency, particularly in Geometry, tends to be below the OECD average, with Indonesia ranking 63rd out of 77 assessed countries [4].

[1] Chiossi 2021; "Geometry" 2023
[2] Naufal et al. 2020; Purnomo 2017
[3] Ebissa 2020; Naufal et al. 2020
[4] OECD 2023

Another study by Putri and Nopriana (2019) reveals that based on the levels of geometric thinking using the van Hiele theory, approximately 15% of Mathematics undergraduates achieve pre-geometric thinking, 50% are at the visualization level, and 35% are at the analysis level in Plane Analytical Geometry, particularly concerning triangles. The highest attained level of geometric thinking is analysis. This is supported by observations conducted by Renanda et al. (2023), indicating that the van Hiele levels among Mathematics undergraduates are still primarily at the initial stages and have not fully developed.

Widada et al. (2019) stated that one contributing factor to geometric thinking ability is the abstraction process. Students struggle to grasp geometric concepts because they involve relatively challenging processes such as abstraction. Trimurtini et al. (2023) also found that the lack of geometric thinking ability is due to students' inability to think abstractly about geometric concepts and solve complex problems [5].

In geometry learning, Mitchelmore and White (2007) identified two main theories related to the abstraction process: empirical abstraction and theoretical abstraction. Empirical abstraction focus on recognizing similarities and then translating these similarities into new mental constructs, resulting in concept formation. Additionally, theoretical abstraction, focuses on forming concepts consistent with specific theories which requires a deeper understanding and generalization of concrete information [6].

Furthermore, based on preliminary observations conducted by the researchers, there are phenomena of empirical and theoretical abstractions playing significant roles in every level of van Hiele. The phenomena of empirical abstraction are evident in students' ability to observe, measure, and classify triangles based on their properties when working on van Hiele level tests. Additionally, the phenomena of theoretical abstraction are depicted in students' ability to explain and understand the conceptual relationships underlying geometric forms, also observed in the van Hiele level test process. This aligns with studies conducted by Nurhasanah et al. (2013), which explain the aspects of geometrical abstraction based on empirical and theoretical abstractions at each van Hiele level in Geometry learning.

Therefore, to gain a deeper and comprehensive understanding of Mathematics undergraduates' geometrical abstraction abilities at each van Hiele level, this research aims to analyze and provide a description of geometrical abstraction at each van Hiele level among Mathematics undergraduates.

Literature Review

Van Hiele Levels

In the late 1950s in the Netherlands, Dr. Pierre van Hiele and his wife, Dr. Dina van Hiele-Geldof, after studying the challenges students faced in learning geometry, introduced the hierarchical levels of geometric thinking known as the "van Hiele model" or "van Hiele theory" [7]. Van Hiele's theory outlines five levels of geometric understanding that describe the stages of students' geometric comprehension. These levels, as proposed by van Hiele (1999): 1) visualization: students can identify basic shapes like

[5] Pavlovičová & Bočková 2021
[6] Mitchelmore & White 2007; Nurhasanah et al. 2013
[7] Senk et al. 2022

circles or squares but may not have a clear understanding of their defining characteristics or the ability to distinguish them from similar shapes, 2) analysis: students begin to notice specific details of geometric shapes, such as the number of sides and angles in a triangle, but may not fully understand the relationships between different types of triangles, 3) informal Deduction: students start to define shapes accurately and understand how they relate to one another, 4) formal Deduction: students develop deductive reasoning skills, enabling them to understand and prove geometric results while recognizing initial assumptions, 5) rigor: involves a deep understanding of the role of axioms in systematically developing geometry. Van Hiele emphasized that student's progress through these levels hierarchically, although not necessarily at the same pace or age [8].

Geometrical Abstraction

Nurhasanah et al. (2013), building on Mitchelmore and White (2007), highlight two abstraction processes in geometry learning: empirical abstraction and theoretical abstraction. Empirical abstraction relies on direct experiences and interactions with geometric objects, making it a natural and intuitive process. On the other hand, theoretical abstraction involves a more deliberate and structured approach to concept formation[9]. It involves activities designed to promote concept formation within a specific theoretical framework.

Based on Mitchelmore and White (2007) theories of empirical and theoretical abstraction, Nurhasanah et al. (2013) stated that abstraction in geometry learning involves various aspects, including recognizing object characteristics through direct experience, forming generalizations based on observed patterns, and expressing mathematical objects using symbols or mathematical language. These aspects serve as indicators to guide researchers in exploring geometrical abstraction on each van Hiele level shown in Table 1.

Table 1. Indicators of Geometrical Abstraction

Indicator	**Aspect of Abstraction**
Direct Experience	Recognizing object characteristics through direct experience.
Reality or Imagination	Recognizing object characteristics, whether manipulated in real life or imagined.
Pattern-Based Generalization	Forming generalizations based on observed patterns or similarities.
Mathematical Representation	Expressing mathematical objects using symbols or mathematical language.
Integration of Mathematical Concepts	Building relationships between various mathematical processes or concepts to develop new understanding.
Real-World Relevance	Applying acquired concepts in relevant real-world contexts.
Manipulation of Abstract Concepts	Manipulating abstract mathematical concepts to solve problems or explore relationships.
Mathematical Idealization	Idealizing or abstracting physical properties of an object to focus on its mathematical essence.

[8] Senk et al. 2022
[9] Mitchelmore & White 2007

Research Method

This study aims to gain a deep understanding of the van Hiele levels and geometrical abstraction at each van Hiele level among Mathematics undergraduates. To enrich the interpretation of findings and achieve a comprehensive understanding of geometrical abstraction at each van Hiele level, an explanatory sequential mixed methods approach was employed. This approach sequentially combines quantitative and qualitative methods[10]. Initially, quantitative data is collected and analyzed to identify the van Hiele levels among the students. Subsequently, qualitative data is collected to explore geometrical abstraction at each identified level. This sequential approach allows for a deeper exploration of the research problem by using qualitative data to explain and elaborate on quantitative results.

Data Source

There are two primary data sources and one secondary data source in this study. The first primary data source focuses on assessing the levels of geometric thinking using van Hiele levels among Mathematics undergraduates at The State University of Makassar (UNM). The second primary data Source involves evaluating geometrical abstraction within the context of triangles in Cartesian coordinates among the Mathematics undergraduates. Moreover, secondary data sources include relevant theories and supporting arguments to provide a broader context for the research.

Data Collection Technique

Purposive sampling was used due to the specific goal of understanding geometrical abstraction at different van Hiele levels [11]. This sampling method maximizes the potential for gaining deep and relevant insights, thereby enhancing significant findings related to geometrical abstraction at each van Hiele levels. In this research, data collection processes through steps;

1. **Selection of Mathematics undergraduates:** Students who completed the Basic Geometry course were selected to ensure relevant background knowledge.

2. **Van Hiele level categorization:** Students are categorized their geometric thinking levels based on van Hiele.

3. **Subject Selection and Evaluation:** One student from each identified van Hiele level was selected for further evaluation through the Geometrical abstraction Test and interviews.

4. **Data Triangulation:** Multiple data sources were used to confirm findings and enhance validity.

Data Analysis Technique

Data was analyzed using NVivo 12, following the steps of explanatory sequential mixed methods [12];

1. **Statistical Analysis:** Quantitative data from the van Hiele level categorization was analyzed to provide initial insights into the distribution of van Hiele levels.

[10] Piccioli 2019; Plano Clark & Ivankova 2016
[11] Islam et al. 2022
[12] Plano Clark & Ivankova 2016

2. **Qualitative Data Collection:** Based on quantitative findings, qualitative data was gathered through the Geometrical abstraction Test and interviews.

3. **Thematic Analysis:** NVivo 12 was used to identify patterns and aspects of geometrical abstraction in the qualitative data.

4. **Interpretation of Results:** Qualitative results were interpreted to provide a comprehensive understanding of geometrical abstraction at each van Hiele level, relating findings to relevant literature and broader contexts

Result and Discussion

Van Hiele Levels of Mathematics Undergraduates

A total of 191 subjects from Mathematics Department of The State University of Makassar (UNM) were participated. The primary objective was to gather comprehensive data on the subjects' van Hiele level. Two subjects from each level were selected based on test results. The VHGT results are presented in Figure 1.

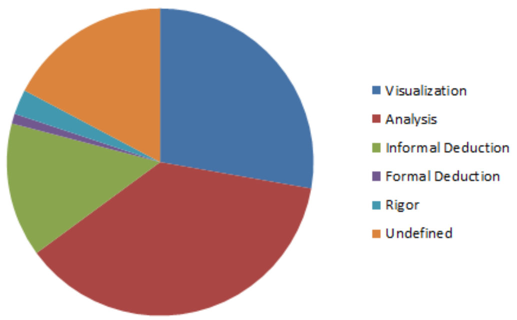

Figure 1. van Hiele Levels of Mathematics Undergraduates at UNM

Based on Figure 50, the data interpretation shows that the majority of students are at the visualization level (39%), followed by the analysis level (52%). This finding is consistent with Putri and Nopriana (2019), who also found that most students were at the visualization and analysis levels. Additionally, this study found that the highest level achieved by Mathematics undergraduates is the rigor level, with 3.62% of the total. This finding contrasts with Putri and Nopriana (2019), who stated that the highest level that could be achieved was the analysis level, but it aligns with Wicaksono and Juniati (2022), who revealed that Mathematics Education students reached all van Hiele levels, with the highest being the rigor level at 0.89% of the total.

The distribution in Table 2 further involves selecting two subjects from each level based on communicative criteria and material mastery for geometrical abstraction analysis, summarized in Table

Table 2. Selected Research Subjects

van Hiele Level	Selected Subjects
Visualization	SSV SSV01
Analysis	SSA SSA01

Informal Deduction	SID SID01
Formal Deduction	SFD SFD01
Rigor	SSR SSR01

Geometrical Abstraction on van Hiele Level

Based on the analysis of geometrical abstraction test data and interviews to ten selected subjects who represented each van Hiele level, geometrical abstraction on van Hiele level among Mathematics undergraduates is presented in Table 3.

Table 3. Geometrical Abstraction on van Hiele Levels

<table>
<tr><th>Indicator</th><th>Visualization</th><th>Analysis</th><th>Informal Deduction</th><th>Formal Deduction</th><th>Rigor</th></tr>
<tr><td>Direct Experience</td><td colspan="5">Identifying geometric characteristics of triangle ABC through visuals</td></tr>
<tr><td rowspan="2">Reality or Imagination</td><td colspan="5">Drawing a new triangle resulting from manipulating coordinate points</td></tr>
<tr><td>-</td><td colspan="4">Manipulating triangle variants</td></tr>
<tr><td rowspan="2">Mathematical Representation</td><td colspan="5">Representing coordinates A, B, and C in mathematical symbols</td></tr>
<tr><td>-</td><td>-</td><td>-</td><td>-</td><td>Formulating the side lengths of the triangle</td></tr>
<tr><td>Pattern-Based Generalization</td><td>-</td><td>-</td><td colspan="3">Understanding general characteristics of triangles</td></tr>
<tr><td>Mathematical Idealization</td><td colspan="5">Idealizing the shape of triangle ABC into the general form of a triangle</td></tr>
<tr><td rowspan="2">Real-World Relevance</td><td colspan="5">Providing examples of triangles in real-life situations visually</td></tr>
<tr><td>-</td><td colspan="4">Applying the triangle concept in real-life situations</td></tr>
<tr><td rowspan="2">Concept Manipulation</td><td>-</td><td colspan="4">Applying mathematical concepts to understand the triangle concept</td></tr>
<tr><td>-</td><td>-</td><td>-</td><td colspan="2">Manipulating triangle concepts to understand other math concepts</td></tr>
<tr><td>Integration of Mathematical Concepts</td><td>-</td><td>-</td><td>-</td><td colspan="2">Building relationships among Geometry, Trigonometry, and Cartesian Coordinates concepts</td></tr>
</table>

According to Mitchelmore and White (2007), empirical abstraction is the process of understanding geometric concepts through identifying similarities among geometric objects and their properties, direct experimentation, and constructing mental models based on direct experience. In this study, each van Hiele level involves empirical abstraction, including direct experience, reality or imagination, mathematical representation, mathematical idealization, and real-world relevance. This finding differs from Nurhasanah et al. (2013), who stated that empirical abstraction consists of two aspects:

recognizing object characteristics through direct experience and recognizing object characteristics through manipulation or imagination. This study found that empirical abstraction is also indicated through pattern-based generalization, mathematical representation, mathematical idealization, and real-world relevance visually.

Furthermore, theoretical abstraction is the process of forming concepts that involve applying these concepts within a specific theoretical [13]. In this study, theoretical abstraction is demonstrated through representation (formulating a formula), conceptual real-world relevance, manipulation of abstract concepts, and integration of mathematical concepts.

Based on the empirical and theoretical geometrical abstraction at each van Hiele level, the following describes geometrical abstraction on van Hiele level are follows:

Visualization Level

Geometrical abstraction at the visualization level involves the ability to understand, imagine, represent, idealize, and apply the triangle concept to real-world situations visually. On this level, the Mathematics undergraduates identify the geometric characteristics of triangle ABC through direct experience. For example, the Mathematics undergraduates can recognize that a triangle has three sides and three angles. Geometrical abstraction at the visualization level falls within empirical abstraction.

Analysis Level

Geometrical abstraction at the analysis level includes the ability to understand, imagine, represent, generalize, idealize, and apply the triangle concept to real-world situations conceptually. On this level, the Mathematics undergraduates draw a new triangle resulting from manipulating the positions of coordinate points. The Mathematics undergraduates also begin to develop the ability to manipulate triangle variants. Geometrical abstraction at the analysis level involves both empirical and theoretical abstraction.

Informal Deduction Level

Geometrical abstraction at the informal deduction level involves the ability to understand, imagine, represent, generalize, idealize, apply, and apply geometric concepts in various contexts. On this level, the Mathematics undergraduates manipulate triangle variants and idealize the shape of triangle ABC into a general form visually. The Mathematics undergraduates also begin to form relationships between the lengths of sides and angles of a triangle more formally. Geometrical abstraction at the informal deduction level involves both empirical and theoretical abstraction.

Formal Deduction Level

Geometrical abstraction at the formal deduction level involves the ability to understand, imagine, represent, generalize, idealize, apply, apply, and integrate geometric concepts in various contexts. On this level, the Mathematics undergraduates demonstrate a more formal understanding of geometry and manipulate triangle concepts to understand other mathematical concepts. The Mathematics undergraduates also build relationships among various mathematical concepts, such as geometry, trigonometry, and Cartesian coordinates, in understanding triangle ABC using a more formal and in-

[13] Mitchelmore & White 2007

depth understanding of mathematical concepts. Geometrical abstraction at the formal deduction level involves both empirical and theoretical abstraction.

Rigor Level

Geometrical abstraction at the rigor level involves the ability to understand, imagine, represent, formulate, generalize, idealize, apply, apply, and integrate geometric concepts in various contexts. On this level, the Mathematics undergraduates apply the triangle concept to real-world situations using practical geometric knowledge. Additionally, the Mathematics undergraduates integrate the mathematical concepts they have learned to build a broader understanding of geometry. Geometrical abstraction at the rigor level involves both empirical and theoretical abstraction.

Moreover, this study found a significant finding that the higher the van Hiele level possessed by the Mathematics undergraduates, the more comprehensive the aspects of geometrical abstraction they can achieve. This indicates a significant relationship between van Hiele levels and the Mathematics undergraduates ' geometrical abstraction.

Conclusion

In conclusion, the research conducted on Mathematics undergraduates at The State University of Makassar found several key findings in response to the formulated problem. Firstly, there are differences in the levels of van Hiele among Mathematics undergraduates at The State University of Makassar. Around 27.7% of Mathematics undergraduates demonstrate visualization skills, whereas 37.2% are at the analysis level. However, only a minority of undergraduates, about 14.1%, 1.0%, and 2.6% respectively, achieve the informal deduction, formal deduction, and rigor levels.

Secondly, the geometrical abstraction on the visualization level involves understanding and visually representing triangle concepts, primarily based on empirical abstraction. On the analysis level, students generalize triangle concepts conceptually, involving both empirical and early stages of theoretical abstraction. In addition, on the informal deduction level, students apply geometric concepts in different contexts, forming formal relationships between sides and angles based on experience and intuitive understanding, encompassing both empirical and deeper theoretical abstraction. Furthermore, formal deduction involves integrating triangle concepts into a broader mathematical context more formally, connecting triangles with other mathematical concepts like Trigonometry and Cartesian coordinates. This level consists of empirical and deep theoretical abstraction. Finally, at the rigor level, practical applications of triangle concepts in real-world situations and extensive integration with other mathematical concepts occur. This level involves strong empirical abstraction and broad theoretical understanding

Moreover, students with higher levels of van Hiele demonstrate better abilities to abstract geometric concepts, indicating a significant relationship between van Hiele levels and the aspects of geometrical abstraction achieved by students. These findings suggest the potential for further research activities to explore the application of van Hiele's theory in enhancing geometric understanding among Mathematics undergraduates, both in solving real-life problems and advancing scientific knowledge.

Eventually, based on the findings of this research, further research is suggested to explore the relationship between van Hiele levels and geometrical abstraction, other factors influencing the development of van Hiele levels and using longitudinal research designs for a deeper understanding of students' van Hiele levels. These recommendations aim to enhance the understanding of geometrical abstraction and its implications for Mathematics Education.

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