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Three Persons Satisfactory Roommates Problem with Incomplete List

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Abstract:
The Satisfactory Roommates Problem (SFRP) is the problem of finding satisfactory matching between any pair of roommates. In the complete list (SFRP) each person in the set of even cardinality $2n$ ranks the $2n-1$ others in order of preference. In the incomplete list (SFRPI) there is some member in the group prefers less than $2n-1$ members. There are $3n$ people in this Three Persons Satisfactory Roommates Problem with incomplete list (TPSRPI), and each person has a preference list for their two companions. Some people have preferences that are fewer than $3n-1$. A set of triples is referred to as a matching. A new, complex algorithm for locating perfect triples in rooms is presented in this study.

Keywords: Incomplete list, preference value, satisfactory value matrix, perfect matching, three person rooms, modified satisfactory value matrix.

1. Introduction

The Stable Marriage with Incomplete Preference Lists (SMI) is a generalization of the SM. In the event that an individual decides that one or more members of the set are not suitable as a mate, those individuals are removed from their list of preferences, resulting in an incomplete preference list. The preferences of agents over other agents as roommates, where the preferences may have ties or be incomplete, characterize the Stable Roommates problem with Ties and Incomplete lists (SRTI), a matching problem.

The analysis was further developed by Brio et al. [1] to include the Stable Roommates problem with incomplete lists. Two variations of the traditional Stable Roommates (SR) problem with incomplete (but strictly ordered) preference lists (SRI) that are degree-constrained—that is, preference lists with restricted length—were examined by Cseh et al. [2]. In the presence of ties and incomplete lists, Adil et al. [3] examined the parameterized complexity of NP-hard optimization variants of stable roommates and stable matching. Fidan and Erdem [4] presented a knowledge-based approach to SRTI that takes domain-specific knowledge into account and looks into its practical use for matching up students in a university dorm. A formal framework known as SRTI-ASP was presented by Erdem and Fidan [5]. It makes use of the Answer Set Programming logic programming paradigm, which is proven and sufficiently generic to handle a large number of these SR variations.

An undirected non-bipartite graph $G=(V,E)$ is a perfect illustration of the highly stable roommates problem with incomplete lists and ties (SRTI). An adjacency list is a list of ties, or vertices that are equally excellent for a given vertex. The set of all highly stable matchings can be represented by a partial order with $O(m)$ elements, as demonstrated by Kunysz [6] and provided an $O(\text{nm})$ strategy for creating this kind of representation. The algorithms are predicated on a straightforward reduction to the problem's bipartite form. Bredereck et al. [7] examined the boundary between W[1]-hardness and fixed-parameter tractability of stable roommates with ties and incomplete lists from the perspective of parameterized graph algorithmics.

Irving [8] provided an $O(m)$ algorithm that, given an instance I of SR, finds a stable matching or states that none exists. The approach is predicated on the assumptions that all preference lists in I are complete (that is, $A_i = A \setminus \{a_i\}$ for each $a_i \in A$) and n is even. However, it is simple to apply the algorithm to the problem model that is described here, which is the incomplete list problem [9]. The problem of finding large weakly stable matchings in the presence of incomplete preference lists containing onesided ties was examined by Lam and Plaxton [10]. Additionally shown a polynomial-time approach that produces a $1 + 1/e$ better approximation ratio. The approach is based on a proposal process wherein numerical priorities are utilized for tiebreaking and are modified based on the outcome of a linear program.

SR was expanded to three-person rooms by Iwama et al. [11], this system is known as 3D-SR (3-Dimensional Stable Roommates). The three-person satisfying roommate dilemma is made up of $3n$ person (3D-SR) who may have a list of preferences over the other three persons $(3n - 1)$. Every individual has a fully ordered preference list that includes every other person ranked from 1 to $3n - 1$ based on his preferences. This study uses the same TPSR approach to discover a satisfactory match for the roommate's problem with an incomplete list, which is an extension of the three-person roommate's problem.

2. Satisfactory Roommate's Problem with Incomplete list in two roommates

A traditional Satisfactory Roommate's Problem (SFRP) where each group member will receive a complete preference list. The term SFRPI refers to the Satisfactory Roommates Problem, which is characterized by an incomplete preference list [12]. In other words, some members of the group of $2n$ members are preferred above members of $2n - 1$. The SMAR procedure, which is detailed in the SFRPI, can be used to produce satisfactory matching for this particular type of SFRPI problem. Participant p is acceptable to participant q if it appears on q's preference list and undesirable otherwise.

Displaying the members' preference lists as a satisfactory value matrix. $SVM = [a_{ij}]$, where $[a_{ij}]$ is equal to the sum of the preference values of the j^{th} member with regard to the ith member, the preference value of the i^{th} member with respect to the j^{th} member, and the preference value of the i^{th} member with respect to the j^{th} member. If $i = j$, then $[a_{ij}]$ is not specified. This equal $S_{ij} = P_{ij} + P_{ji}$. P stands for the preference value, which is defined as the value that is assigned to each member in the preference list based on their relative preference. For example, the first member is represented as $\frac{n-1}{n-1}$, the second as $\frac{n-2}{n-1}$, the third as $\frac{n-3}{n-1}$ and so on [13]. The assignment technique is used to determine a satisfactory fit between roommates. Using the Hungarian algorithm, SVM on one-to-one optimum matching is obtained.

3. Three Person Satisfactory Matching (TPSM)

In this paper, we propose an algorithm to find triple roommates from a group of 3n person based on their preference lists and some members prefer less than 3n-1 persons. This algorithm provides a 3n set of triple roommates based on the individual satisfactory level.

Preference value (TPSRP) is defined as the value assigned to the members in the preference list according to the order of preference with respect to the persons by considering the first person as $\frac{3n-2}{3n-2}$, the second person as $\frac{3n-2}{3n-2}$, the third person as $\frac{3n-3}{3n-2}$, the fourth person as $\frac{3n-4}{3n-2}$ and so on. If the person *i* did not prefer *j*, then j^{th} place in i^{th} list must be considered as infinity.

$P:[P_{ij}]$ defines the preference value matrix.

$$P_{ij} = \begin{cases} \frac{3n-2}{3n-2} & \text{if j is the first or second preference of i} \\ \frac{3n-k}{3n-2} & \text{if j is } k^{\text{th}} \text{ preference of i and } k = 3,4,5,6 \ldots\ldots \text{upto } (3n-1) \\ - & \text{if } i = j \end{cases}$$

The modified preference value matrix is defined as follows:

$$m_{(ij),k} = \begin{cases} - & \text{if } k = i \text{ and } k = j \\ P_{ik} + P_{ji} & k = 1,2,\ldots.3n \neq i,j \end{cases}$$

3.1 Algorithm (TPSMA)

1. Get the preference lists from each person.

2. Form a preference value matrix based on their preference lists.

3. Considering the preference value matrix as a Maximization assignment problem.

4. Construct a Minimized Preference Value Matrix and applying Hungarian algorithm for that matrix.

5. The Resultant pairs must be the optimum pairs like *(i,j), (k,l), (m,n)* and so on obtained.

6. Construct the Modified Preference Value matrix by considering the pairs *(i,j), (k,l), (m,n)* … as rows and 1,2,….3n members as column. By using MPVM definition which is given above.

7. Considering the Modified preference value matrix as a minimized assignment problem and apply Hungarian Algorithm for getting the optimum triples.

8. List out all the triples and let it be *(i,j,k), (l,m,n), (i,l,n)*…..

9. From the obtained triples, choose one by one and find the satisfactory value of each member of a group.

10. If *(i,j,k)* be the first triples, find the preference value of *i* with respect to *j* and *k*, then adding the preference values. Now we get the overall preference value and multiply the value by 50.

It gives a satisfactory value of *i* with respect to *j* and *k*. similarly find the satisfactory value of *j* w.r.to *i & k*, also for *k* w.r.to *i & j*.

11. After getting these three satisfactory values, find overall satisfaction for the triple *(i,j,k)*. In the same manner repeat the process for all the remaining triples.

12. Now choose mutually exclusive and exhaustive triples which achieves the maximum level of satisfaction that is the optimum triples.

4. Example : Consider the problem instance of TPSRP based on order of preference

1 3 4 2 5 6

2 5 3 1 6 4

3 2 4 6 5 1

4 3 1 6 2

5 2 6 3 1

6 3 5 4 2 1

Solution:

The given preference list can be constructed as a preference value matrix by considering first person as $\frac{3n-2}{3n-2}$, second person as $\frac{3n-2}{3n-2}$, third person as $\frac{3n-3}{3n-2}$, , fourth person as $\frac{3n-4}{3n-2}$, fifth person as $\frac{3n-5}{3n-2}$. The preference value of the persons 3,4,2,5,6 is $\frac{4}{4}$, $\frac{4}{4}$, $\frac{3}{4}$, $\frac{2}{4}$ and $\frac{1}{4}$ with respect to person 1. The preference value of persons 5,3,1,6,4 is $\frac{4}{4}$, $\frac{4}{4}$, $\frac{3}{4}$, $\frac{2}{4}$, and $\frac{1}{4}$ with respect to person 2. The preference value for persons 2,4,6,5,1 is $\frac{4}{4}$, $\frac{4}{4}$, $\frac{3}{4}$, $\frac{2}{4}$, and $\frac{1}{4}$ with respect to person 3. The preference value for persons 3,1,6,2 is $\frac{4}{4}$, $\frac{4}{4}$, $\frac{3}{4}$, $\frac{2}{4}$, with respect to person 4. The preference value for persons 2,6,3,1 is $\frac{4}{4}$, $\frac{4}{4}$, $\frac{3}{4}$, $\frac{2}{4}$ with respect to person 5. The preference value for members 3,5,4,2,1 is $\frac{4}{4}$, $\frac{4}{4}$, $\frac{3}{4}$, $\frac{2}{4}$ and $\frac{1}{4}$ with respect to person 6. The preference values are presented in the form of matrix which is given below

The preference value matrix

$$PVM = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} - & \frac{3}{4} & \frac{4}{4} & \frac{4}{4} & \frac{2}{4} & \frac{1}{4} \\ \frac{3}{4} & - & \frac{4}{4} & \frac{1}{4} & \frac{4}{4} & \frac{2}{4} \\ \frac{1}{4} & \frac{4}{4} & - & \frac{4}{4} & \frac{2}{4} & \frac{3}{4} \\ \frac{4}{4} & \frac{2}{4} & \frac{4}{4} & - & - & \frac{3}{4} \\ \frac{2}{4} & \frac{4}{4} & \frac{3}{4} & - & - & \frac{4}{4} \\ \frac{1}{4} & \frac{2}{4} & \frac{4}{4} & \frac{3}{4} & \frac{4}{4} & - \end{pmatrix} \end{array}$$

From the preference value matrix to construct the minimized preference value matrix by subtracting all the elements in the matrix from the highest element in the matrix. Then the Minimized preference value matrix given below

The minimized preference value matrix

$$mPVM = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} - & \frac{1}{4} & 0 & 0 & \frac{2}{4} & \frac{3}{4} \\ \frac{1}{4} & - & 0 & \frac{3}{4} & 0 & \frac{2}{4} \\ \frac{3}{4} & 0 & - & 0 & \frac{2}{4} & \frac{1}{4} \\ 0 & \frac{2}{4} & 0 & - & - & \frac{1}{4} \\ \frac{2}{4} & 0 & \frac{1}{4} & - & - & 0 \\ \frac{3}{4} & \frac{2}{4} & 0 & \frac{1}{4} & 0 & - \end{pmatrix} \end{array}$$

Considering the mPVM as the assignment problem and applying Hungarian algorithm for finding the optimum pairs.

(1,4),(2,3),(3,2),(4,1),(5,6),(6,5) be the optimum pairs.

To find the optimum triples, from the mPVM choose (1,4) pair, the first row fourth column corresponding element 0 and add with entire first row and assign that particular element place with -. Likewise, choose the next pair (2,3), the second row third column element 0 and add with entire second row and assign that particular element place with -. Similarly, repeat the process for all optimum pairs. Then the resultant matrix will be a Modified Minimum Preference Value Matrix (MmPVM).

The modified minimum preference value matrix is given below,

$$MmPVM = \begin{array}{c} \\ (1,4) \\ (2,3) \\ (3,2) \\ (4,1) \\ (5,6) \\ (6,5) \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} - & \frac{1}{4} & 0 & - & \frac{2}{4} & \frac{3}{4} \\ \frac{1}{4} & - & - & \frac{3}{4} & 0 & \frac{2}{4} \\ \frac{3}{4} & - & - & 0 & \frac{2}{4} & \frac{1}{4} \\ - & \frac{2}{4} & 0 & - & - & \frac{1}{4} \\ \frac{2}{4} & 0 & \frac{1}{4} & - & - & - \\ \frac{3}{4} & \frac{2}{4} & 0 & \frac{1}{4} & - & - \end{pmatrix} \end{array}$$

Considering this above matrix as the assignment problem and apply Hungarian algorithm then get the optimum triples.

The triples are (1,4,2), (2,3,5), (3,2,4) (4,1,6), (5,6,1), (6,3,5).

To obtain a best three persons as a roommate, by calculate an individual preference value and overall satisfaction.

Choose the first triple (1,4,2), to get 1st person satisfactory value, by adding the 4th person's preference value in the preference value matrix with respect to 4th person and 3rd person and an individual satisfaction level can be obtained to multiplying by 50.

That is $1 \rightarrow \frac{4}{4} + \frac{3}{4} = \frac{7}{4} \times 50 = 87.5\%$.

$4 \rightarrow \frac{4}{4} + \frac{2}{4} = \frac{6}{4} \times 50 = 75\%$.

$2 \rightarrow \frac{3}{4} + \frac{1}{4} = 1 \times 50 = 50\%$.

Find the average for the individual satisfaction for getting the overall satisfaction level. That is, $87.5+75+50 = \frac{212.5}{3} = 70.8\ \%$.

9970.8% will be the overall satisfaction for the 1st, 4th, 2nd person as the roommates.

Similarly this process can be applied for all the triples which we have.

Now choose mutually exclusive and exhaustive triples,

Then the result as shown in the table 1,

Table 1. Three person individual and overall satisfactory matching

S.No	Possible Triples	Individual Satisfaction	Overall Satisfaction	
1	(1,4,2)	1 → 2 & 4 = 87.5%.	70.8%	77
		2 → 1 & 4 = 50%.		
		4 → 1 & 2 = 75%.		
2	(3,5,6)	3 → 5 & 6 = 62.5%.	83.33%	
		5 → 3 & 6 = 87.5%.		
		6 → 3 & 5 = 100%.		
3	(2,3,5)	2 → 3 & 5 = 100%.	87.5%	77
		3 → 2 & 5 = 75%.		
		5 → 2 & 3 = 87.5%.		
4	(4,1,6)	4 → 1 & 6 = 87.5%.	66.66%	
		1 → 4 & 6 = 62.5%.		
		6 → 1 & 4 = 50%.		
5	(5,6,1)	5 → 1 & 6 = 75%.	58.33%	69
		6 → 1 & 5 = 62.5%.		
		1 → 5 & 6 = 37.5%.		
6	(3,2,4)	3 → 2 & 4 = 100%.	79.16%	
		2 → 3 & 4 = 62.5%.		
		4 → 2 & 3 = 75%.		

From the above table, the overall satisfaction of the triples (1,2,4), (3,5,6) and (2,3,5), (1,4,6) are 77 percentage, that is both have the same overall satisfaction percentage. Then we have to move the other hand for finding the best triples as the roommates.

Consider the individual satisfaction percentage for possible triples, and calculate the range of them. We know that, the lowest range is the best one. The range can be calculated by subtracting the lowest value from the highest value.

In the possible triple (1,2,4), the highest percentage is 87.5 and the lowest percentage is 50. Then the range is 87.5 – 50 =37.5%

In the possible triple (3,5,6), the highest percentage is 100 and the lowest percentage is 62.5. Then the range is 100 – 62.5 =37.5%

In the possible triple (2,3,5), the highest percentage is 100 and the lowest percentage is 75. Then the range is 100 – 75 =25%

In the possible triple (1,4,6), the highest percentage is 87.5 and the lowest percentage is 50. Then the range is 87.5 – 50 =37.5%

By examine the range of the triples, (2,3,5) have the lowest range percentage. So, we conclude that, the triples (2,3,5) and (1,4,6) best triples as the roommates.

5. Conclusion

In this paper, we proposed a preference value for the preferred incomplete lists of roommate's instances with three person rooms, and described an Algorithm TPSMA. This algorithm yields a triple matching in addition to a perfect, satisfactory matching pair in its initial place. We determine that the acquired matching yields flawless triple matching after looking over the data. Here, we've demonstrated the speed and effectiveness of TPSMA. This algorithm produces a matching where every pair reaches the highest level of satisfaction possible. Thus, roommates with incomplete lists are satisfactorily matched according to TPSMA results.

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