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Magnetohydrodynamic Effect on 3D Nanofluid Flow via Stretching/Shrinking Surface

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Abstract:

This study presents a numerical technique to analyze the MHD effect on the motion of 3D NFs via shrinking or SS under convective BCs. The investigation incorporates convective and VSC. The gov. eqs. are transformed into a set of coupled NLODEs using appropriate ST. These transformed nonlinear equations are calculated using R-K-F method combined with the shooting technique. The influence of various physical parameters on , and distributions is illustrated graphically. Other that, the study evaluates SF coefficient and HTR for different NFs parameters.

Keywords: nanofluid, MHD, Shrinking or SS, VS.

Introduction:

The study of Casson NFs is a type of NNF that combines the viscoplastic properties of a CF with the enhanced thermal and physical characteristics of NFs, which are fluids containing suspended NPs. The Casson model describes the yield stress behavior, where the fluid begins to motion only after a certain threshold shear stress is exceeded, while the inclusion of NPs improves heat transfer, viscosity, and thermal conductivity. CNFs are widely studied for applications in biomedical engineering, such as blood flow modeling, as well as in HT systems like cooling technologies, microfluidics, and energy-efficient industrial processes. Their unique combination of properties makes them suitable for optimizing thermal performance and understanding complex flow behaviors under various conditions. Yousuf Ali et al. [1] examined the CNFs MHD unstable BL characteristics in the simultaneous transmission of thermoelectric and radiation on a SS. Suresh Kumar et al. [2] finding the motion velocity is a diminishing function of the TR parameter has observed in Hall parameter. Khan et al. [3] examines the MHD on CNFs in a porous medium .Shek Akbar et al. [4] consider the 2D Casson HNFs motion inside the channel. Rehman et al. [5] reported that, the MHD CSC motion of a HNFs, considering the impact of VD via SS is presented.

The MHD is behavior of ECF, such as plasmas, liquid metals, in the presence of MF and EF. Governed by modified Navier-Stokes and Maxwell's equations, MHD has wide-ranging applications, including the study of astrophysical phenomena (e.g., solar flares and planetary magnetospheres), fusion energy research, liquid metal cooling in reactors, MHD generators, and flow control in aerospace. It is crucial for understanding and engineering systems involving high-temperature plasmas or conductive fluids. Alamirew and Awgichew [6] examines the motion of MHD Casson NFs via vertically SS with effect of VD, MC in a spongy medium. Ismail et al. [7] developed the thermal instability of Tri-hybrid Casson NFs with TR PM. Khan et al. [8] present the constitution of MHD steady 3D CNFs motion containing gyrotactic microorganism via SS. Recently, some of authors developed numerical techniques applied in 3D NFs motion via SS was developed [9-11].

Mathematical Analysis:

Here, we consider the 3D CNFs motion with MHD effect via SS. the physical model of the problem as predicted in Fig. 1. It is considered that liquid motion taken by $x_1 y_1 -$ surface with VS. The z_1 direction has taken by negligible. Under these basic gov. eq's are shown below:

$$\frac{\partial u_1}{\partial x_1}+\frac{\partial v_1}{\partial y_1}+\frac{\partial w_1}{\partial z_1}=0, \tag{1}$$

$$u_1\frac{\partial u_1}{\partial x_1}+v_1\frac{\partial u_1}{\partial y_1}+w_1\frac{\partial u_1}{\partial z_1}=\upsilon_1\frac{\partial^2 u_1}{\partial\left(z_1\right)^2}-\frac{\sigma_1 B_0^2}{\rho_f}u_1, \tag{2}$$

$$u_1\frac{\partial v_1}{\partial x_1}+v_1\frac{\partial v_1}{\partial y_1}+w_1\frac{\partial v_1}{\partial z_1}=\upsilon_1\frac{\partial^2 v_1}{\partial\left(z_1\right)^2}-\frac{\sigma_1 B_0^2}{\rho_f}v_1, \tag{3}$$

$$u_1\frac{\partial T_1^*}{\partial x_1}+v_1\frac{\partial T_1^*}{\partial y_1}+w_1\frac{\partial T_1^*}{\partial z_1}=\alpha_m\left(\frac{\partial^2 T_1^*}{\partial\left(z_1\right)^2}\right)+\tau^*\left\{D_B\left(\frac{\partial C^*}{\partial z_1}\frac{\partial T_1^*}{\partial z_1}\right)+\frac{D_{T^*}}{T_\infty^*}\left(\frac{\partial T^*}{\partial z^*}\right)^2\right\}, \tag{4}$$

$$u_1\frac{\partial C_1^*}{\partial x_1}+v_1\frac{\partial C_1^*}{\partial y_1}+w_1\frac{\partial C_1^*}{\partial z_1}=D_B\left(\frac{\partial^2 C_1^*}{\partial\left(z_1\right)^2}\right)+\frac{D_{T^*}}{T_\infty^*}\left(\frac{\partial^2 T_1^*}{\partial\left(z_1\right)^2}\right), \tag{5}$$

Corresponding B.Cs are

$$\left.\begin{array}{ll}\left.\begin{array}{l}u_1=u_w^*(x_1)=\mathrm{U}_w(x_1)+\mathrm{N}_1\gamma_0\dfrac{\partial u_1}{\partial z_1},\ v_1=v_w^*(x_1)=V_w(x_1)+\mathrm{N}_2\gamma_0\dfrac{\partial v^*}{\partial z_1},\\ w_1=w_0,\ T_1^*=T_w^*,\ -k\dfrac{\partial C_1^*}{\partial z^*}=h_1(\mathrm{T}_f-\mathrm{T}_1^*),\end{array}\right\} & at \quad z_1=o\\ u_1\to 0,\ v_1\to 0,\ T_1^*\to T_\infty^*,\ C_1^*\to C_\infty^*, & as \quad z_1\to\infty\end{array}\right\} \tag{6}$$

The following dimensionless functions and the similarity variables are:

$$\left.\begin{array}{l}\eta = z_1\sqrt{\dfrac{a_1}{\upsilon_f}},\; u_1 = a_1 x_1 f'(\eta),\; v_1 = a_1 y_1 g'(\eta),\\ \mathrm{w}_1 = -\sqrt{a_1\upsilon_f}\left(f(\eta)+g(\eta)\right),\; \theta(\eta) = \dfrac{T_1^* - T_\infty^*}{T_w^* - T_\infty^*},\quad \phi(\eta) = \dfrac{C_1^* - C_\infty^*}{C_w^* - C_\infty^*}.\end{array}\right\} \quad (7)$$

Utilizing the above dimensions, Eq. (1) is identically satisfied and translate Eqs. (2)-(5)

$$f''' = -f''(f+\mathrm{g}) + \left(f'\right)^2 + Mf' - 1 \quad (8)$$

$$g''' = -g''(f+\mathrm{g}) + \left(g'\right)^2 + Mg' - 1 \quad (9)$$

$$\theta'' = -\Pr\left((f+g)\theta' + N_b\theta'\phi' + N_t\left(\theta'\right)^2\right) \quad (10)$$

$$\phi'' = -Le\Pr(f+g)\phi' - (\mathrm{N}_t/N_b)\theta'' \quad (11)$$

With subject to the boundary conditions are:

$$\left.\begin{array}{l} f(0) = \mathrm{S},\;\; \mathrm{g}(0) = 0,\;\; f'(0) = 1 + \mathrm{A}\, f''(0),\;\; \mathrm{g}'(0) = \lambda + \mathrm{Bg}''(0),\\ \phi(0) = 1,\;\; \theta'(0) + Bi(1-\theta(0)) = 0,\\ f'(\eta)\to 0,\;\; \mathrm{g}'(\eta)\to 0,\;\; \theta(\eta)\to 0,\;\; \phi(\eta)\to 0 \quad \text{as} \quad \eta\to\infty \end{array}\right\} \quad (12)$$

Results and Discussion

The physical effect of M ("Magnetic Parameter") on fluid motion component $g'(\eta)$ for the case $(\lambda > 0)$ and $(\lambda < 0)$ as predicts on **Fig. 2.** It is clear that the nanofluid flow velocity is slowly down along y-direction when the sheet is stretching $(\lambda > 0)$. Physically, a drag force like resistive type force is create disturbance by the fluid particles of the vertical MF to the electrically conducting liquid. This force has to reduce the motion of the fluid over a stretching surface.

Fig. 3 presented the S on liquid motion $\theta(\eta), \phi(\eta)$ for the cases of $(\lambda > 0)$, $(\lambda < 0)$. It is clear the liquid $\phi(\eta)$ is slow reduction via SS with various enlarge values of S. Physically, the larger values of mass flux effect in fluid particles and the liquid resistance slow down then its liquid motion BL thickness is reducing.

The impact of λ on velocity component $g'(\eta)$ along y^*- direction is explored through in **Fig. 4** for the cases of $(S > 0)$ and $(S < 0)$. It is clear the liquid motion is monotonically enhances via SS with various enlarge values of λ. Because, the liquid motion convergent to surface area very fast then the surface is injection case.

Fig. 5 depicts the physical parameter N_b on $\theta(\eta)$, $\phi(\eta)$. It is noticed that $\theta(\eta)$ of the NF enhances via surface while opposite motion of fluid $\phi(\eta)$ with higher values of N_b.

Conclusions: The main out comes of the present study are mentioned below:

- The temperature of Brownian motion parameter is declined while opposite trend follows concentration with higher statistical values of N_b.

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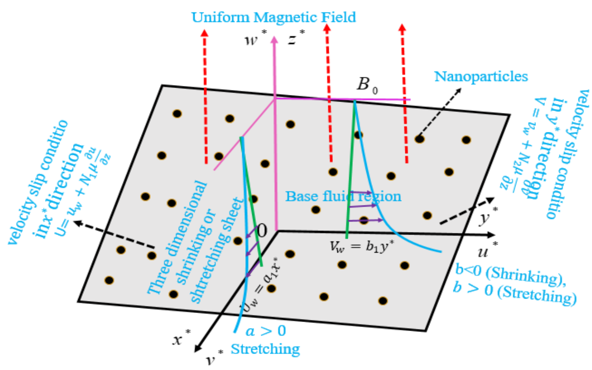

Fig. 1 Physical model of the problem

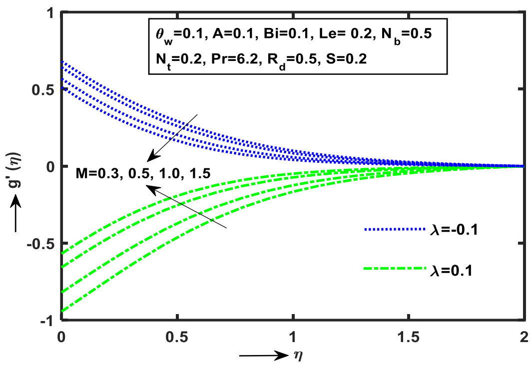

Fig. 2 Influence of M on $g'(\eta)$

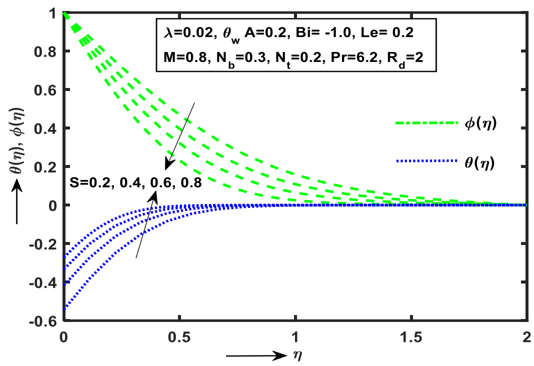

Fig. 3 Influence of S on $\theta(\eta)$, $\phi(\eta)$

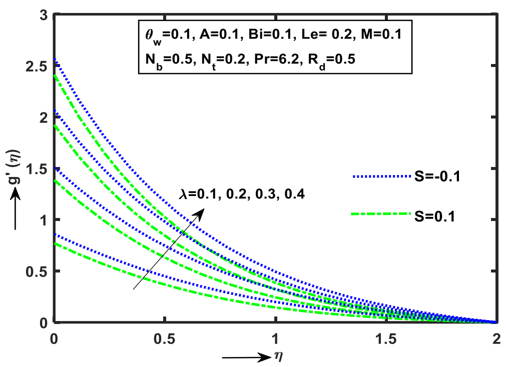

Fig. 4 Influence of λ on $g'(\eta)$

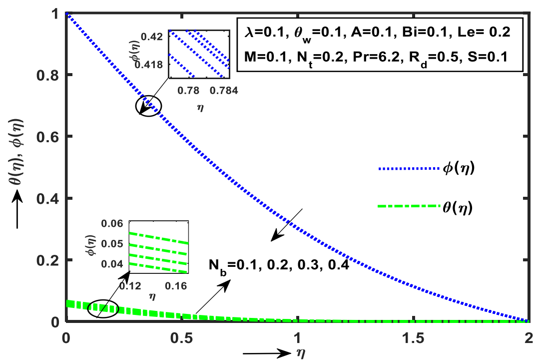

Fig. 5 Influence of N_b on $\theta(\eta)$, $\phi(\eta)$

Table. 1 Evaluation of Skin friction coefficient $-f''(0)$ for $A = B = Bi = R_d = 0$

M	Present study	Sarah et al. [12]	Nadeem et al. [13]	Gupta and Sharma [14]	Ahmad and Nazar [15]
0.0	1.000000	1.00000	1.0004	1.0003181	1.0042
10	3.316624	3.31662	3.3165	3.3165824	3.3165
100	10.04987	10.04987	10.049	10.049864	10.049

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Nomenclature	
(x^*, y^*) Cartesian coordinate's	T^* Temperature of the fluid
u_1, v_1, w_1 velocity components along x^*, y^*, z^*-axis	T_∞^* fluid temperature far away from the surface
A Velocity slip along x-axes $\sqrt{a\gamma_0}N_1$	T_w^* Constant fluid Temperature of the wall
B Velocity slip along y-axes $\sqrt{a\gamma_0}N_2$	U_w Stretching velocity
C^* Concentration	U_∞ Free stream velocity
C_f^* Skin friction coefficient	
c_p^* Specific heat	*Greek symbols*
C_∞^* Uniform ambient concentration	ρ Density
D_B Brownian diffusion	ϕ Dimensionless concentration
D_T Thermophoresis diffusion	σ_1 Boltzmann constant
f Dimensionless stream function	λ Constant stretching/shrinking parameter b_1/a_1
f' Dimensionless velocity	τ Ratio of the nanoparticle to the fluid $(\rho c)_p/(\rho c)_f$
S Constant mass flux parameter $w_0/\sqrt{a_1\gamma_0}$	υ_1 Kinematic viscosity of the fluid
k^* Thermal conductivity	σ^* Electrical conductivity
Le Lewis number $=\alpha_m^*/D_B$	θ Dimensionless temperature
M Magnetic field parameter $=\frac{\sigma_1 B_0^2}{a_1\rho_f}$	α_m^* Thermal diffusivity $=k/(\rho c_p)_f$
N_t Thermophoresis parameter $=\frac{\tau D_T}{\alpha_m^* T_\infty^*}(T_w^*-T_\infty^*)$	$(\rho c)_f$ Heat capacity of the fluid
N_b Brownian motion coefficient $=\frac{D_B\,\tau(C_w^*-C_\infty^*)}{\upsilon_f}$	$(\rho c)_p$ Heat capacity of the nanoparticle to the fluid
Pr Prandtl number $=\left(\frac{\upsilon_1}{\alpha_m^*}\right)$	ρ_f Fluid density
q_r Radiative heat flux	*Subscripts*
Re_x Reynolds number	∞ condition at free stream
R_d Radiation parameter $=\frac{16\sigma^* T^{*3}}{3\alpha_m^* k k^*}$	