Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 4s (2025)

Hybrid Minkowski-Based Fuzzy AHP with MCDM Optimization for Green Supply Chain Transportation

Hasim Khan
Department of Mathematics, College of Science, Jazan University, P.O. Box 114 Jazan, 45142, Kingdom of Saudi Arabia
hhanif@jazanu.edu.sa

Article History:
Received: *02-10-2024*
Revised: *30-11-2024*
Accepted: *08-12-2024*

Abstract:
In this research, a Hybrid Minkowski Consistency Operator is introduced for the application of Green Supply Chain (GSC) transportation problem based on Fuzzy Analytic Hierarchy Process (AHP)-Multi Criteria Decision Making (MCDM) techniques. Consequently, the model primarily focuses on enhancing efficiency in transportation processes within green supply chain logistics while minimizing its environmental impact. This is enabled by taking into account these multiple criteria often in conflict with one another: cost, emission, and delivery time. It combines fuzzy logic with the Hybrid Minkowski Consistency Operator for managing uncertainties that may usually come up in a real-world supply chain system, hence making the decisions more precise. This research demonstrates the proposed model's potential to produce optimal and sustainable logistics solutions, a real-case study is employed for validation. In addition, besides this optimization of transport, the paper discusses a hybrid multi-objective optimization methodology that involves Ant Colony Optimization (ANT) and Firefly Algorithm (FA). This hybrid optimization enhances overall performance in this model and provides an overall framework whereby all existing challenges are solved and a foundation for future developments is laid regarding green supply chain management.

Keywords: Hybrid Minkowski Consistency, Fuzzy AHP, Green Supply Chain Management (GSCM), Transportation Problem, MCDM

1. Introduction

Reducing environmental effects without sacrificing operational efficiency has been a primary goal for firms in today's globalized and ecological marketplace, according to GSCM [1]. The most noteworthy revelations on the intricacies of supply networks concerned transportation, which encompasses Delivery time, carbon emission, and multi-objective cost [2]. There is unprecedented pressure on decision-makers due to ever-tighter surroundings and sustainability goals, which necessitates finding strategies to balance conflicting agendas [3].

A general term for a collection of methods that identify the best course of action when it is difficult to evaluate options by several intricate and contradictory criteria is MCDM [4]. Since the 1960s, MCDM has become a more recognized branch of study that works with the development of mathematical and computational tools to help decision-makers select the optimal course of action in complicated situations [5]. The use of MCDM approaches increased rapidly after the 1990s. Many MCDM techniques have been developed over time; some examples are as follows: The techniques that have been employed are as follows: the PROMETHEE, the BWM, the TOPSIS, ELECTRE, the AHP, SAW, the DEMATEL, the ANP, the LINMAP [6], [7]. Green transportation planning involves inherent conflicts and uncertainties, which are effectively managed by combining fuzzy logic and

MCDM methodologies [8]. One might list AHP as one of the most useful MCDM approaches. However conventional AHP lacks the adaptability needed to capture uncertainty in real-world issues [9]. In line with this, fuzzy AHP has emerged as a reliable method for incorporating imprecise data into decision-making procedures [10].

Thomas L. Saaty developed the AHP model, which is frequently used for rating, analyzing, prioritizing, and reviewing decision possibilities [11]. As a result, the AHP technique also divides problems into hierarchies according to the evaluations of those in charge. The challenges of the task are indicated by the number of tiers in a hierarchy [12]. Since Zadeh's explanation of OFS, they have been extensively employed in practically all study domains. Extensions to conventional fuzzy sets, such as HFS, IFS, T2FS, PFS, and NS, have been found in several investigations [13]. Pythagorean and Neutrosophic fuzzy sets combine to create three-dimensional SFS. AHP-based SFSs have been used in a few studies.

It is suggested that green supply chain transportation issues be resolved using the Hybrid Distance-Based Aggregation Operator with Fuzzy AHP and MCDM Optimization that is provided here [14]. Fuzzy logic is combined with a hybrid distance-based technique to capture the uncertainties and interdependencies across parameters like cost, environmental impact, and delivery performance. This approach hasn't been matched up till now. When various criteria are taken into account throughout the process of making decisions, an optimal model is produced, which enables the creation of a suggested model to show the solution modeling of green transportation planning robustly and flexibly [15]. The remainder of the article will address the methodology, use of case studies, and possible drawbacks of this hybrid approach to provide a reliable and effective framework for transportation optimization in green supply chains.

The following is the paper's contribution.

1) In this research, we theoretically suggested a new consistency formula in the fuzzy AHP methodology is called Minkowski consistency, which is founded on the Minkowski distance.
2) The proposed method offers an improved outcome by applying it to a green supply chain challenge based on a transportation issue.
3) The application of a Green Supply Chain is demonstrated as a transportation problem to minimize the transportation cost, emission of carbon footprints, and delivery time.

2. Related Work

Demircan and Yetilmezsov, 2023 [16] evaluated four distinct intelligent disposal of waste solutions making use of a mixed fuzzy MCDM method to solve the shortage in research. The efficiency of the suggested approach substitutes was evaluated using research and fifteen sub-criteria set up into four primary groups. Fuzzy AHP, or fuzzy analytic hierarchy procedure, was paired with a fuzzy preference for order by resemblance method to generate fuzzy TOPSIS, which offered the most effective response.

Demir *et.al.*, 2023 [17] developed a two-stage fuzzy set for use in MCDM investigations about transportation. The proposed strategy is a special tactic that expands on Pythagorean fuzzy sets and combines the AHP and VIKOR approaches. In this way, the suggested method makes use of databases from the assessment of three train lines in Antalya, one of the biggest cities in Turkey. One popular MCDM method has the FAHP, which weights thirteen sub-factors in addition to four primary criteria. AHP has been supported by IVPFNs.

Du *et.al.*, 2021 [18] addressed the TSP, a modified ACO algorithm called AHACO—which is recommended to rely on an adaptive heuristic element. To enhance the algorithm's performance, the AHACO proposes three primary enhancements. First, a city classification technique called k-means is presented. To increase population diversity and enhance the algorithm's search capabilities, the AHACO offers various mobility techniques for various city classes. To further fine-tune the solution, a changed 2-opt local optimizer is suggested. To prevent the algorithm from stagnating, a mechanism for exiting the local optimum is finally included.

Fidanova *et.al.*, 2020 [19] discovered an algorithm based on the Ant Colony Optimization approach that has been suggested for passenger flow and transit modeling. The problem has been referred to as multi-objective optimization. Minimal transportation time and minimum cost are the two optimization goals. There have been some hazy elements in it. The price has been regarded as equal when it falls within a predetermined range. In the same way as the initial journey time. The percentage of passengers who have preferred employing a train over a bus due to factors like price, time, or preference has the target.

Calik, 2021 [20] developed a unique approach to group decision-making that uses Industry 4.0 components to combine AHP and TOPSIS approaches in a Pythagorean fuzzy environment to choose the top green vendor. The suggested method states that many experts' opinions are expressed using linguistic phrases generated by PFN. The criteria weights have been determined using the interval-valued Pythagorean Fuzzy AHP approach. To rank and choose the best fit, the Pythagorean Fuzzy TOPSIS method has been applied, taking into consideration the distances between the providers. A genuine case study on an agricultural machinery and equipment company has been included at the conclusion to demonstrate the efficacy and precision of the suggested selection procedure.

Tripathi *et.al.*, 2021 [21] revealed eleven factors that fall under three categories: environmental, geographic, and socioeconomic. The study's purpose has been regulated as to whether the MCDA approach would be best for choosing new hospital locations. AHP and FAHP have two MCDA techniques employed here. This study also presented an MCDA approach based on GIS. A comparison was made between the outcomes of the AHP and FAHP techniques.

Abdullah *et.al.*, 2023 [22] identified 21 social, economic, and technical factors that have been taken into attention when developing the best site development for Indonesian nuclear power plants (NPPs). These factors have included the operating costs, transmission network, geology, geotechnics, seismology, economic impact, environment, meteorology, proximity to hazardous facilities, population density, hydrology, cooling water, topography, evacuation route, proximity to wetland, transportation network, security, the impact of tourism, legal considerations, public acceptance, historical sites, and land ownership.

Rouyendegh and Savalan, 2022 [23] have shown how the complexity of agricultural production has increased the need for more sophisticated agricultural production techniques (APTs). Using organic management techniques, these lacks downsides consequences associated with conventional and genetically engineered production, organic agriculture seeks to safeguard the environment while also enhancing consumer pleasure. In the meantime, goods made by genetic engineering and conventional methods are more affordable. This challenge serves as the dataset for the causal agent to demonstrate the superiority of the suggested fuzzy MCDM hybrid model. The evaluation approach has increased the ratio of input data to output data because the challenge contains a lot of contradictory quantitative and qualitative criteria. It has been crucial to regulate agricultural productivity comprehensively as a result.

Nguyen *et.al*., 2022 [24] suggested integrating SF-AHP, the SF-WASPAS, and DEA to have been capable of finding a maintainable supplier for Vietnam's steel manufacturing sector. This research considers both quantitative and qualitative criteria through expert interviews and a comprehensive literature analysis. DEA has been used in the initial step to verify high-efficiency suppliers according to several quantitative standards. In the second phase, these suppliers were assessed further based on qualitative standards like social, environmental, and economic aspects. While the SF-WASPAS was used to find maintainable suppliers, the SF-AHP was used to determine the importance of the criterion.

Nguyen *et.al*., 2022 [25] advised methods include the PF-AHP and PF-CoCoSo. Benefit expectations (BEs) have been ranked according to AI adoption by PF-CoCoSo, whereas PF-AHP establishes the important element weighting criteria. The accuracy of the suggested technique in tackling the issue of AI technology adoption is attested to by the study's findings and comparisons between the output of the recommended methodology and other MCDM methodologies. These contributions, which offer methods and recommendations for analyzing AI applications, will be helpful to practitioners and researchers in the field.

3. The Fuzzy AHP Method

The final result of integrating the method of AHP with a fuzzy concept method is fuzzy AHP. For example, selecting a new park's site. In pairs, assess every criterion about of the objective. For instance, how significant are the costs? and access. The 1–9 scale displayed in Table 1 is usually employed for this. 9 implies that one criterion is most important, whereas 1 signifies two equally important factors.

Table 1: Noteworthy nine-point intensity scale

Scale Value	Vocal Judgment
1	Equal Importance
3	Moderate Importance
5	Strong Importance
7	Very Strong Importance
9	Extreme Importance
2, 4, 6, 8	Intermediate Values

The fuzzy AHP value was calculated using the TFN. In linguistics, the fuzzy theory of associations, or TFN, is used to explain how people impartially evaluate one another. Hierarchical sets with several membership levels are called fuzzy sets. The weight from the AHP scale could be transformed into a fuzzy number using a triangle fuzzy number. Using the values q, r, and s which have the most potential but the lowest probability - the TFN has been found. The TFN and its grade are shown in Table 2.

Table 2: The TFN and its grade

Grade	TFN
(1,1,1)	Equivalence level
(1,1,3)	Lower level
(1,3,5)	Average level
(3,5,7)	Better level
(5,7,9)	Excellence level

4. Methodology

The current paper reflects the concepts of transportation problems based on fuzzy AHP. Consider source $S_i \ \forall\ i = 1,2,\dots,n$ and demand $D_j \ \forall\ j = 1,2,\dots,m$ for the transportation. Hence the objective function for the transportation is given below.

$$\min Z = \sum(w_{TC}.TC + w_{CF}.CF + w_{DT}.DT) \tag{1}$$

Subject to

$$TC = \sum_{i=1}^{n}\sum_{j=i}^{m} c_{ij} \tag{2}$$

$$CF = \sum_{i=1}^{n}\sum_{j=i}^{m} e_{ij} \tag{3}$$

$$DT = \sum_{i=1}^{n}\sum_{j=i}^{m} t_{ij}\ ,\ x_{ij} \geq 0 \ \forall\ i,j \tag{4}$$

Here, TC denotes the transportation cost, CF represents the emission of carbon footprint for the transportation and DT specifies the delivery time taken to reach the destination. The notations of the transportation problem are as follows.

1) S_i are the suppliers (source).
2) D_j are the customers (destination).
3) x_{ij} be the amount of goods transported (capacity) from the supplier i to destination j.
4) c_{ij} be the cost of transportation per unit of goods from the supplier i to destination j.
5) e_{ij} be the emission of carbon footprint per unit of goods transported from the supplier i to destination j.
6) t_{ij} be the delivery time per unit of goods transported from the supplier i to destination j.
7) w_{TC} is the weight for transportation costs.
8) w_{CF} is the weight of carbon footprints.
9) w_{DT} is the weight for delivery time.

Step 1:

It requires a decision matrix to be set up first, representing a hierarchy of criteria pertinent to the problem. Criteria $S_i \ \forall\ i = 1,2,\dots,n$ refer to different sources or criteria to be analyzed within the problem. On the one side, alternatives $D_j \ \forall\ j = 1,2,\dots,m$ refer to options or decisions that need to be ranked or decided. The decision matrix shows how each option D_j fares against every criterion S_i; hence, it provides the ground for pairwise comparisons.

Step 2:

Decision takers establish fuzzy pairwise contrast matrices corresponding to the following criterion utilizing linguistic values for TFNs. The fuzzy pairwise comparison matrix reflects subjective judgments by the decision-makers, encapsulating uncertainty and imprecision in human judgment.

Step 3:
Given the availability of the fuzzy pairwise comparison matrix, then the next step involves normalizing every element of that matrix, with each of the fuzzy elements divided by the column sum in the matrix to which it belongs. After this, a normalized fuzzy matrix will have been obtained, where each element will be a relative weight.
Step 4:
By the centroid method, we convert the normalized fuzzy matrix into crisped values through the process of defuzzification. Thus, the defuzzified value of each criterion is obtained. The result is a crisp value for each criterion, which facilitates comparison and ranking of the alternatives.

$$Centroid\ formula = \frac{x_1+x_2+x_3}{3} \tag{5}$$

Step 5:
Next, sum up the defuzzified values obtained from the pairwise comparison. Then, we calculate the final weight by separating each defuzzified value by its sum of the defuzzified values corresponding to its rows. Thus, the fuzzy weight of the criteria is obtained whose sum is one.
Step 6:
By contemplating the random distribution of the decision makers as [03, 0.4, 0.3]. To determine the criteria's ultimate weight, we shall multiply the random distribution of the decision makers by each element of fuzzy weight. Thus, results from the final weight of the criteria which has the total one.
Step 7: ***Proposed validation***
It is more important to check the consistency in fuzzy AHP. For consistency checking, we have proposed a new formula using Minkowski distance that will provide a successful result.
Step 7.1:
Firstly, change the fuzzy pairwise comparison matrix into hard values using the centroid formula of equation (5). Let us denote the crisp values as A. The general Minkowski distance is given by $D_p(A,B) = \left(\sum_{i,j=1}^{n}\left(x_{ij} - y_{ij}\right)^p\right)^p$. In the proposed methodology we shall modify the Minkowski distance as $P = 1$ and fixing x_{ij} as the Centre matrix element. Thus, the newly proposed Minkowski consistency ratio is as follows.

$$Minkowski\ CR\ = sin\left(\frac{\sum_{i,j=1}^{n}(x_{ij}-y_{ij})}{\det A}\right) \tag{6}$$

Thus, 0.1 is the maximum CR value. Hence the newly proposed Minkowski consistency ratio reveals the superficial consistency of the fuzzy AHP.

4.1 Application of the study

A vital component of the economy and daily life is public transportation. Different forms of public transportation provide various benefits. The transportation problem is one of the optimization problems, which requires the transportation of goods from various sources to different destinations in the most efficient way. A green supply chain is one in which the supply chain incorporates environmentally friendly practices in its operation, reducing ecological impact along all steps from production to delivery. In focus will come such issues as sustainability, waste minimization, and reduction of carbon footprints, while sustaining efficiency and profitability. We consider an application of green supply chain-based transportation problems using Fuzzy AHP. A company needs to ship goods from a collection of suppliers to a collection of destinations. Let the suppliers be $S_i \ \forall\ i = 1,2,3$. The destinations of the suppliers are named as customers and denoted as $D_j \ \forall\ j = 1,2,3,4$. The company aims to minimize the total transportation cost, emission of carbon footprints of the transportation, and the delivery time taken by the supplier to reach the customer. The capacity of the suppliers $S_i \ \forall\ i = 1,2,3$ are 70, 100, and, 90 respectively. The measuring units of the transportation cost, emission of carbon footprints, and delivery time are to be rupees, kilograms, and hours respectively. Table 4 depicts the dataset of the suppliers in the company.

Table 4: Dataset

	c_{ij} (rupees)				e_{ij} (kg)				t_{ij} (hrs)			
	D_1	D_2	D_3	D_4	D_1	D_2	D_3	D_4	D_1	D_2	D_3	D_4
S_1	5	4	6	7	4	6	8	7	3	4	5	4
S_2	8	5	7	6	9	5	6	8	6	3	4	7
S_3	6	9	5	8	4	7	3	6	5	6	5	4

As aspects of the decision maker, we obtain a pairwise comparison matrix based on the TFN. Table 5 contains the tabulation of the pairwise comparison.

Table 5: Pairwise comparison

	c_{ij}	e_{ij}	t_{ij}
c_{ij}	(1,1,1)	(2,3,4)	(1/5,1/4,1/3)
e_{ij}	(1/6,1/5,1/4)	(1,1,1)	(3,4,5)
t_{ij}	(5,6,7)	(1/7,1/6,1/5)	(1,1,1)

Each fuzzy element can be divided by the total of the corresponding column elements to form a normalized matrix, which can be found in Table 6 as the pairwise comparison matrix.

Table 6: Normalized matrix

	c_{ij}	e_{ij}	t_{ij}
c_{ij}	(0.162,0.138,0.12)	(0.636,0.720,0.769)	(0.047,0.047,0.052)
e_{ij}	(0.027,0.027,0.030)	(0.318,0.240,0.192)	(0.714,0.761,0.789)
t_{ij}	(0.811,0.83,0.848)	(0.045,0.039,0.038)	(0.238,0.190,0.157)

Using the centroid formula as mentioned in equation (5), we get the defuzzified value of each criterion and thus tabulated in Table 7.

Table 7: Defuzzified values

	c_{ij}	e_{ij}	t_{ij}	sum
c_{ij}	0.14	0.708	0.048	0.896
e_{ij}	0.028	0.25	0.754	1.032
t_{ij}	0.830	0.038	0.195	1.063

To obtain the fuzzy weight in Table 8, we regularize them by separating each defuzzified value by all its corresponding sum of the rows.

Table 8: Fuzzy weight

	c_{ij}	e_{ij}	t_{ij}
c_{ij}	0.156	0.790	0.053
e_{ij}	0.027	0.242	0.730
t_{ij}	0.780	0.036	0.188

The graph that compares fuzzy weights of the criteria is shown in Figure 1.

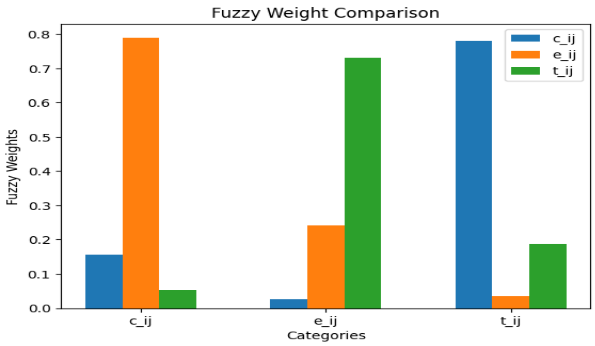

Figure 1: Fuzzy weights

According to step 6 to find the final weight of the criteria the random distribution [0.3 0.4 0.3] is multiplied by the elements of Table 8. The method is as follows.

$$[0.3 \quad 0.4 \quad 0.3] \times \begin{bmatrix} 0.156 & 0.790 & 0.053 \\ 0.027 & 0.242 & 0.730 \\ 0.780 & 0.036 & 0.188 \end{bmatrix} = \begin{matrix} 0.2916 \\ 0.3446 \\ 0.3643 \end{matrix}$$

As a result, the criteria's ultimate weight is reached. The weight for the transportation cost is 0.2916. The weight for the carbon footprints is 0.3446. The weight for the delivery time is 0.3646.

From equations (2), (3) and (4), we have

$TC = \sum_{i=1}^{3} \sum_{j=i}^{4} c_{ij} = 5 + 4 + 6 + 7 + 8 + 5 + 7 + 6 + 6 + 9 + 5 + 8 = 76.$

$CF = \sum_{i=1}^{3} \sum_{j=i}^{4} e_{ij} = 4 + 6 + 8 + 7 + 9 + 5 + 6 + 8 + 4 + 7 + 3 + 6 = 73.$

$DT = \sum_{i=1}^{3} \sum_{j=i}^{4} t_{ij} = 3 + 4 + 5 + 4 + 6 + 3 + 4 + 7 + 5 + 6 + 5 + 4 = 56.$

Hence equation (1) becomes $\min Z = \sum (w_{TC}.TC + w_{CF}.CF + w_{DT}.DT) = 0.2916 \times 76 + 0.3446 \times 73 + 0.3643 \times 56 = 22.1616 + 25.1558 + 20.4008 = 67.7182.$

Hence the minimum transportation cost of the company is 67.7182, respectively.

To find the consistency for the required fuzzy AHP, we convert the values of fuzzy pairwise comparison into crisp value matrix A. Thus, the crisp value matrix

$A = \begin{bmatrix} 1 & 3 & 0.26 \\ 0.21 & 1 & 4 \\ 6 & 0.169 & 1 \end{bmatrix}$. calculating the determinant of A,

$det\, A = 1 \times \begin{vmatrix} 1 & 4 \\ 0.169 & 1 \end{vmatrix} - 3 \times \begin{vmatrix} 0.21 & 4 \\ 6 & 1 \end{vmatrix} + 0.26 \times \begin{vmatrix} 0.21 & 1 \\ 6 & 0.169 \end{vmatrix} = 70.143.$

$$\sum_{i,j=1}^{n} (x_{ij} - y_{ij}) = \sum_{i,j=1}^{3} (x_{22} - y_{ij}) = 0 + 2 + 0.74 + 0.6241 + 0 + 3 + 5 + 0.690561 + 0$$
$$= 12.054661.$$

We use the new proposed Minkowski consistency ratio to validate the fuzzy AHP problem.

$$Minkowski\ CR\ = sin\left(\frac{\sum_{i,j=1}^{n}\left(x_{ij}-y_{ij}\right)}{\det A}\right)$$
$$= sin\left(\frac{12.054661}{70.143}\right) = sin(0.17185) = 0.002999 < 0.1.$$

As the consequence is less than 0.1, the matrix is considered consistent, and the required fuzzy AHP yields superficial outcomes.

5. Ant Colony Optimization

Swarm intelligence, which derives inspiration from nature, represents one of the most efficient methods for solving optimization problems in an array of situations. Ant Colony Optimization (ACO) is a stochastic optimization technique that mimics the behaviours of real ants as they forage for food. It was created by Dorigo. By collaborating and transferring data, it calculates the quickest path an ant colony takes for reaching sources of food. Because they happen to be going along a common roadway, the ants are following each other. This is caused by the reality that, when they adhered to the path, every worker ant left a substance known as pheromone. After establishing the potency of the pheromone, the remaining ants pursue the route with the greatest intensity. They are trying to figure out the best route in this way. Traveling randomly is an expression employed to define an ant colony's effort to arrive at its objective first. During their return trip, the ants assess the strength of the pheromones and choose the route with the greatest concentration of those pheromones. It ought to be highlighted that pheromone dissipates with time; hence, the briefest roadway has more pheromone as it requires a shorter period to arrive there than the remaining routes. Because of this, almost every one of the ants would pick the most advantageous route since the shortest route would entice them through greater pheromone strength. The theoretical basis of ant behaviours can be described below: The kth ant at the location i requires the subsequent probability formula as per equation (7) to move on to the next node, j.

$$p_{ij}^{k}(t) = \begin{cases} \frac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}(t)]^{\beta}}{\sum_{s\in J_k(i)}[\tau_{is}(t)]^{\alpha}[\eta_{is}(t)]^{\beta}}, j \in J_k(i) \\ 0, \qquad otherwise \end{cases} \tag{7}$$

where α and β represent the relative importance of the pheromones and heuristic function, accordingly, τ_{ij} denotes the pheromone attention on the pathway between i and j, and η_{ij} indicates the heuristic function, i.e., which corresponds to the reciprocal of the distance between the i and j positions.

After ants finish their tour, the pheromone trial values are rationalized according to the equation (8).

$$\tau_{ij}(t+1) = (1-\rho)\left[\tau_{ij}(t)\right] + \sum_{k=1}^{m}\Delta\tau_{ij}^{k} \tag{8}$$

where $\rho(0 < \rho \leq 1)$ implies the evaporation rate, $\Delta\tau_{ij}^{k}$ signifies the extra pheromone left in the path (i,j) by the k-th ant. $\Delta\tau_{ij}^{k}$ signifies the extra pheromone left in the path (i,j) by the k-th ant. $\Delta\tau_{ij}^{k}$ could be considered through the equation (9).

$$\Delta\tau_{ij}^{k}(t) = \begin{cases} \frac{Q}{L_k}, if\ ant\ k\ passes\ the\ path\ (i,j) \\ \qquad 0,\ otherwise \end{cases} \tag{9}$$

where $\Delta\tau_{ij}^{k}$ specifies the number of pheromones added by the kth ant, Q is the pheromone update constant, N is the total number of ants in the nest, and L_k is the length of the path traveled by the kth ant.

5.1 Case Study

The study ACO is put into an application of green supply chain. Here is a company which is named Eco Mobility. It is a manufacturing company of electric vehicles. With a motive of reducing environmental footprint, Eco Mobility is committed to this production for the adaption of Green Supply Chain management. Eco Mobility has many different suppliers that are spread across various locations. Eco Mobility has many suppliers who provide raw materials, which should be transferred to its main production facility. Each supplier provides components that are essential, such as batteries, electric motors, and control units. So, to maximize the supply chain of the corporation, the components of electric vehicles must be distributed and transported from the suppliers to its manufacturing plant. The key objective of the company is to decrease carbon emissions, fuel consumption, and delivery time. The distances covered between the routes chosen between suppliers and plant, time consumed, and environmental impact are not the same for each route. This makes the optimization problem complex. Eco Mobility strives to find the best routes with the ACO algorithm.

Firstly, the transportation network can be represented as a graph in figure 2. In this model, each node represents a location, namely suppliers and the manufacturing plant.

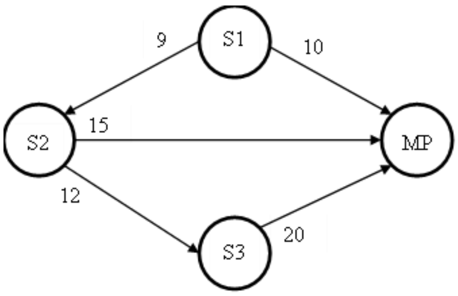

Figure 2: Transportation network graph

Each edge in the graph represents every potential route between any two of the nodes. Each edge is associated with certain attributes, such as carbon emissions, fuel consumption, and delivery time. Let the alternatives be supplier 1 (S1), supplier 2 (S2), supplier 3 (S3), and the manufacturing plant (MP). The attributes are measured in certain units followed by distance in kilometers (km), carbon emission in grams of CO_2 per km (g), fuel consumption in liters per kilometer (l/km), and the delivery time in hours (hrs.). Table 8 reflects the dataset of the company.

Table 8: Dataset

Routes	Distance (km)	carbon emission (g)	Fuel consumption(l/km)	Delivery time (hrs)
S1-MP	100	50	0.2	1.5
S2-MP	150	60	0.25	2
S3-MP	200	70	0.3	3
S1-S2	90	45	0.22	1.8
S2-S3	125	55	0.18	1.2

The cost of the transportation problem depends on the route they follow. Thus, the cost of the route is calculated by the equation (10).

$$cost\ of\ route = distance \times fuel\ consumption + penalty\ emission \tag{10}$$

Where penalty emission is proportional to emissions that are 0.001×g CO_2/km.

Now we can calculate the total cost of each route in Table 9 using the equation (10) as follows.

Table 9: Total cost

Routes	Distance (km)	Fuel consumption(l/km)	penalty emission	Total cost
S1-MP	100	0.2	5	25
S2-MP	150	0.25	9	46.5
S3-MP	200	0.3	14	74
S1-S2	90	0.22	4.05	23.85
S2-S3	125	0.18	6.86	29.36

The heuristic is inversely proportional to the cost and calculated by the equation (11) as follows.

$$\eta_{ij} = \frac{1}{total\ cost\ of\ route} \tag{11}$$

We can evaluate the heuristic information for each route of the company's transportation cost using equation (11) as follows.

For the route S1-MP, $\eta_{ij} = \frac{1}{25} = 0.04$

For the route S2-MP, $\eta_{ij} = \frac{1}{46.5} = 0.0215$

For the route S3-MP, $\eta_{ij} = \frac{1}{74} = 0.01351$

For the route S1-S2, $\eta_{ij} = \frac{1}{23.85} = 0.0419$

For the route S2-S3, $\eta_{ij} = \frac{1}{29.36} = 0.0340$

A supplier chooses a path based on the heuristic information (η_{ij}) and pheromone level (τ_{ij}). The probability p_{ij} for each route that a supplier selects the route ij from a starting node is calculated by equation (7). Let's assume that the initial pheromone level τ_{ij} of each route to be 1. Assume the weight of the pheromone and heuristic values as 1 (that is $\alpha = 1$ and $\beta = 1$). The possible routes from S1 are MP or S2.

For the route S1-MP,

$$P_{\mathrm{S1-MP}} = \frac{[\tau_{\mathrm{S1-MP}}(t)]^{\alpha}[\eta_{\mathrm{S1-MP}}(t)]^{\beta}}{[\tau_{\mathrm{S1-MP}}(t)]^{\alpha}[\eta_{\mathrm{S1-MP}}(t)]^{\beta} + [\tau_{\mathrm{S1-S2}}(t)]^{\alpha}[\eta_{\mathrm{S1-S2}}(t)]^{\beta}} = \frac{[1]^1[0.04]^1}{[1]^1[0.04]^1 + [1]^1[0.0419]^1}$$
$$= \frac{0.04}{0.04 + 0.0419} = \frac{0.04}{0.0819} \approx 0.4885$$

For the route S1-S2,

$$P_{\mathrm{S1-S2}} = \frac{[\tau_{\mathrm{S1-S2}}(t)]^{\alpha}[\eta_{\mathrm{S1-S2}}(t)]^{\beta}}{[\tau_{\mathrm{S1-MP}}(t)]^{\alpha}[\eta_{\mathrm{S1-MP}}(t)]^{\beta} + [\tau_{\mathrm{S1-S2}}(t)]^{\alpha}[\eta_{\mathrm{S1-S2}}(t)]^{\beta}} = \frac{[1]^1[0.0419]^1}{[1]^1[0.04]^1 + [1]^1[0.0419]^1}$$
$$= \frac{0.0419}{0.04 + 0.0419} = \frac{0.0419}{0.0819} \approx 05115$$

The next possible route from the node S2 is MP and S3.

For the route S2-MP,

$$P_{\text{S2}-\text{MP}} = \frac{[\tau_{\text{S2}-\text{MP}}(t)]^{\alpha}[\eta_{\text{S2}-\text{MP}}(t)]^{\beta}}{[\tau_{\text{S2}-\text{MP}}(t)]^{\alpha}[\eta_{\text{S2}-\text{MP}}(t)]^{\beta} + [\tau_{\text{S2}-\text{S3}}(t)]^{\alpha}[\eta_{\text{S2}-\text{S3}}(t)]^{\beta}} = \frac{[1]^{1}[0.0215]^{1}}{[1]^{1}[0.0215]^{1} + [1]^{1}[0.0340]^{1}}$$
$$= \frac{0.0215}{0.0215 + 0.0340} = \frac{0.0215}{0.0555} \approx 0.3874$$

For the route S2-S3,

$$P_{\text{S2}-\text{S3}} = \frac{[\tau_{\text{S2}-\text{S3}}(t)]^{\alpha}[\eta_{\text{S2}-\text{S3}}(t)]^{\beta}}{[\tau_{\text{S2}-\text{MP}}(t)]^{\alpha}[\eta_{\text{S2}-\text{MP}}(t)]^{\beta} + [\tau_{\text{S2}-\text{S3}}(t)]^{\alpha}[\eta_{\text{S2}-\text{S3}}(t)]^{\beta}} = \frac{[1]^{1}[0.0340]^{1}}{[1]^{1}[0.0215]^{1} + [1]^{1}[0.0340]^{1}}$$
$$= \frac{0.0340}{0.0215 + 0.0340} = \frac{0.0340}{0.0555} \approx 0.6126$$

The only possible route from S3 is MP. Thus, the probability is $P_{\text{S3}-\text{MP}} = 1$.

After the suppliers complete their paths, pheromone levels are updated. Consider the pheromone update constant $Q = 100$ and the evaporation rate $\rho = 0.1$. For our convenience assume that the supply starts from S1 to MP that covers the distance 100 km. To calculate the additional pheromone level $\Delta\tau_{ij}^{k}$ deposits of each route, we use equation (9).

For the route S1-MP, $\Delta\tau_{\text{S1}-\text{MP}}^{k} = \frac{Q}{L_k} = \frac{100}{100} = 1$

For the route S2-MP, $\Delta\tau_{\text{S2}-\text{MP}}^{k} = \frac{Q}{L_k} = \frac{100}{150} \approx 0.67$

For the route S3-MP, $\Delta\tau_{\text{S3}-\text{MP}}^{k} = \frac{Q}{L_k} = \frac{100}{200} = 0.5$

For the route S1-S2, $\Delta\tau_{\text{S1}-\text{S2}}^{k} = \frac{Q}{L_k} = \frac{100}{90} \approx 1.11$

For the route S2-S3, $\Delta\tau_{\text{S2}-\text{S3}}^{k} = \frac{Q}{L_k} = \frac{100}{125} = 0.8$

Now we calculate the update pheromone level $\tau_{ij}(t+1)$ of each route as per the equation (8).

For the route S1-MP, $\tau_{\text{S1}-\text{MP}}(t+1) = (1-0.1).1 + 1 = 0.9 + 1 = 1.9$

For the route S2-MP, $\tau_{\text{S2}-\text{MP}}(t+1) = (1-0.1).1 + 0.67 = 0.9 + 0.67 = 1.57$

For the route S3-MP, $\tau_{\text{S3}-\text{MP}}(t+1) = (1-0.1).1 + 0.5 = 0.9 + 0.5 = 1.4$

For the route S1-S2, $\tau_{\text{S1}-\text{S2}}(t+1) = (1-0.1).1 + 1.11 = 0.9 + 1.11 = 2.01$

For the route S2-S3, $\tau_{\text{S2}-\text{S3}}(t+1) = (1-0.1).1 + 0.8 = 0.9 + 0.8 = 1.7$

Hence the initial pheromone level, additional pheromone level and update pheromone level are tabulated in the table 10 as follows.

Table 10: Pheromone level

Routes	$\boldsymbol{\tau_{ij}(t)}$	$\boldsymbol{\Delta\tau_{ij}^{k}}$	$\boldsymbol{\tau_{ij}(t+1)}$
S1-MP	1	1	1.9
S2-MP	1	0.67	1.57
S3-MP	1	0.5	1.4
S1-S2	1	1.11	2.01
S2-S3	1	0.8	1.7

After one cycle of the Ant Colony Optimization algorithm, the concentrations of pheromone on all routes have increased. The intensity of pheromone concentration is high on smaller routes such as

S1-S2 because it is normal considering the short distances that are involved hence making them more attractive in the optimization process. Iterating over many such iterations, the pheromone levels would continue to change; more suppliers will continue to trace higher pheromone levels' paths and converge upon routes that are optimized for consuming less fuel, fewer emissions of carbon, and the minimum delivery time. Considering the new level of pheromone, the developed routes, like S1 → S2, are allocated an increasing attractiveness to the future suppliers. These will most likely be the most optimal routes in the green supply chain of Eco Mobility.

6. Firefly Algorithm

Three rules form the basis of the Firefly algorithm:

- Because they are all unisex, fireflies will always be drawn to one another regardless of gender.
- Their brightness determines how attractive they are; the dimmer will gravitate toward the brig hter one haphazard motion when they are evenly illuminated.
- Firefly brightness is correlated with the objective function's value.

The relationship between the light intensity I and the distance r, is expressed as $I = I_0 \, exp(-\gamma r)$, where I_0 is the original light intensity, and γ is the absorption coefficient that ranges between 0 and 1.

Attractiveness is given by $\beta = \beta_0$ exp $(-\gamma r)$, where β_0 is the attractiveness when $r = 0$.

The distance in d-dimensions between two fireflies i and j at x_i and x_j, respectively, is given by $r_{ij} = \|x_i - x_j\| = \sqrt{\sum_{k=1}^{d}\left(x_{i,k} - x_{j,k}\right)^2}$, and it is $r_{ij} = \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2}$ in two dimensions.

The movement of firefly i attracted to a brighter firefly j is given by

$$x_i = x_i + \beta_0 e^{-\gamma r_{ij}{}^2}\left(x_j - x_i\right) + \alpha\left(rand - \frac{1}{2}\right), \tag{12}$$

where the following term is the outcome of randomization with a parameter for control α between 0 and 1, and the second term is the consequence of attraction. An FF's flashing behaviour is employed to figure out an appropriate route for action when the robot meets with both static as well as moving things. For a robot to achieve the specific performance the need of the tasks that follow: identifying obstacles, avoidance of obstacles, conquering trap-like circumstances, preventing random excursions, and appropriate qualities should be recorded in a feedback-based function with objectives for route organizing. This enables the robot to produce an optimal path. Robots with sensors attached can determine their position in a new region and gain knowledge concerning their surroundings. In basic terms, navigation is a reduction issue or a route optimization issue. Considering the goal and the position of the obstacle, the function of objectives for the same could represent an appropriate parameter. Each time, a significantly stronger FF's location is determined, and the robot travels in an order of stages toward there it. In the scenario that the robot stumbles through an obstacle on its way toward accomplishing its current goal, the function calls are made itself. Based on the robot's sensor inputs, a set quantity of randomized fireflies appears around a challenge when a robot recognizes it. Each FF's distance from Euclid to the object being targeted and the obstacle is calculated. The robot selects the most vibrant firefly (FF) and progresses toward it, maintaining its distance from impediments and an appropriate distance from its final location. The optimization of the path planning problem could be articulated as:

$$f_i = K_1 \frac{1}{\min_{o_n \in o_s} \|D_{fo}\|} + K_2 \|D_{fg}\| \quad (13)$$

where K_1 is the fitting parameter for the path safety and K_2 defines the maximum and minimum path length of the navigation. D_{fo} is the distance of the FF f_i from the obstacle, and D_{fg} is its distance from the objective. The robot remains secure by circumventing the obstruction when K_1 reaches its maximum value. The path length is maximized at the minimum value of K_2 and minimized at the maximum value of K_2. The optimum value of the function with objectives is established by correctly selecting the appropriate parameters, an objective that could be achieved through trial and error.

7. Conclusion

The hybrid model proposed here solves the green supply chain transportation problem through the integrated use of the Hybrid Minkowski Consistency Operator, Fuzzy AHP, and MCDM techniques in a powerful and novel manner. The model has further proceeded into the multi-conflicting of these criteria of cost, emission, and delivery time to provide a more comprehensive and balanced optimization approach toward transportation efficiency in green supply chain management. It enabled the treatment of uncertainties, which has been quite common in a real-world supply chain, and reached more accurate and reliable decision-making processes with the utilization of fuzzy logic. In addition, the Hybrid Minkowski Consistency Operator has reinforced the model for complex criteria processing, while the multi-objective optimization technique combines ACO with FA, which further refined the overall performance. This hybrid optimization strategy strongly enhanced the model's capability to find a balance between competing objectives while furthering sustainability goals. Further validation of the model has been done through the case study that practically shows how its application results in effective outcomes for any real-world scenario and, therefore, provided an effective framework for optimizing logistics with reduced environmental impact. In the process, it has been shepherded in new dimensions in the current best practices of green supply chain management and also provided a sound foundation for future research and development related to sustainable logistics and transportation optimization.

References

[1] Asim, Z., Shamsi, I.R.A., Wahaj, M., Raza, A., Abul Hasan, S., Siddiqui, S.A., Aladresi, A., Sorooshian, S. and Seng Teck, T., (2022). Significance of sustainable packaging: A case-study from a supply chain perspective. *Applied System Innovation*, *5*(6), pp117.

[2] Bekrar, A., Ait El Cadi, A., Todosijevic, R. and Sarkis, J., (2021). Digitalizing the closing-of-the-loop for supply chains: A transportation and blockchain perspective. *Sustainability*, *13*(5), pp2895

[3] Woo, E.J. and Kang, E., (2020). Environmental issues as an indispensable aspect of sustainable leadership. *Sustainability*, *12*(17), pp7014.

[4] Alvarez, P.A., Ishizaka, A. and Martínez, L., (2021). Multiple-criteria decision-making sorting methods: A survey. *Expert Systems with Applications*, *183*, pp115368.

[5] Abdelli, A., Mokdad, L. and Hammal, Y., (2020). Dealing with value constraints in decision making using MCDM methods. *Journal of Computational Science*, *44*, pp101154.

[6] Liu, P.C., Lo, H.W. and Liou, J.J., (2020). A combination of DEMATEL and BWM-based ANP methods for exploring the green building rating system in Taiwan. *Sustainability*, *12*(8), pp3216.

[7] Abdel-Basset, M., Gamal, A., Chakrabortty, R.K. and Ryan, M., (2021). Development of a hybrid multi-criteria decision-making approach for sustainability evaluation of bioenergy production technologies: A case study. *Journal of Cleaner Production*, *290*, pp.125805.

[8] Hamurcu, M. and Eren, T., (2020). Strategic planning based on sustainability for urban transportation: An application to decision-making. *Sustainability*, *12*(9), pp.3589.

[9] Zhu, G.N., Hu, J. and Ren, H., (20200. A fuzzy rough number-based AHP-TOPSIS for design concept evaluation under uncertain environments. *Applied Soft Computing*, *91*, pp.106228.

[10] Taylan, O., Alamoudi, R., Kabli, M., AlJifri, A., Ramzi, F. and Herrera-Viedma, E., (2020). Assessment of energy systems using extended fuzzy AHP, fuzzy VIKOR, and TOPSIS approaches to manage non-cooperative opinions. *Sustainability*, *12*(7), pp.2745.

[11] Aliyev, R., Temizkan, H. and Aliyev, R., (2020). Fuzzy analytic hierarchy process-based multi-criteria decision making for universities ranking. *Symmetry*, *12*(8), pp.1351.

[12] Chen, T., (2021). A diversified AHP-tree approach for multiple-criteria supplier selection. *Computational Management Science*, *18*(4), pp.431-453.

[13] Işık, G., (2023). A framework for choosing an appropriate fuzzy set extension in modeling. *Applied Intelligence*, *53*(11), pp.14345-14370.

[14] Kieu, P.T., Nguyen, V.T., Nguyen, V.T. and Ho, T.P., (2021). A spherical fuzzy analytic hierarchy process (SF-AHP) and combined compromise solution (CoCoSo) algorithm in distribution center location selection: A case study in agricultural supply chain. *Axioms*, *10*(2), pp.53.

[15] Ahmed, S. and Dey, K., (2020). Resilience modeling concepts in transportation systems: a comprehensive review based on mode, and modeling techniques. *Journal of Infrastructure Preservation and Resilience*, *1*, pp.1-20.

[16] Demircan, B.G. and Yetilmezsoy, K., (20230. A hybrid fuzzy AHP-TOPSIS approach for implementation of smart sustainable waste management strategies. *Sustainability*, *15*(8), pp.6526.

[17] Demir, E., Ak, M.F. and Sari, K., (2023). Pythagorean fuzzy based AHP-VIKOR integration to assess rail transportation systems in Turkey. *International Journal of Fuzzy Systems*, *25*(2), pp.620-632.

[18] Du, P., Liu, N., Zhang, H. and Lu, J., (2021). An improved ant colony optimization based on an adaptive heuristic factor for the traveling salesman problem. *Journal of Advanced Transportation*, *2021*(1), pp.6642009.

[19] Fidanova, S., Roeva, O. and Ganzha, M., (2020), September. Ant colony optimization algorithm for fuzzy transport modelling. In *2020 15th Conference on Computer Science and Information Systems (FedCSIS)* (pp. 237-240). IEEE.

[20] Çalık, A., (2021). A novel Pythagorean fuzzy AHP and fuzzy TOPSIS methodology for green supplier selection in the Industry 4.0 era. *Soft Computing*, *25*(3), pp.2253-2265.

[21] Tripathi, A.K., Agrawal, S. and Gupta, R.D., (2021). Comparison of GIS-based AHP and fuzzy AHP methods for hospital site selection: a case study for Prayagraj City, India. *GeoJournal*, pp.1-22.

[22] Abdullah, A.G., Shafii, M.A., Pramuditya, S., Setiadipura, T. and Anzhar, K., (2023). Multi-criteria decision making for nuclear power plant selection using fuzzy AHP: Evidence from Indonesia. *Energy and AI*, *14*, pp.100263.

[23] Rouyendegh, B.D. and Savalan, Ş., (2022). An integrated fuzzy MCDM hybrid methodology to analyze agricultural production. *Sustainability*, *14*(8), pp.4835.

[24] Nguyen, T.L., Nguyen, P.H., Pham, H.A., Nguyen, T.G., Nguyen, D.T., Tran, T.H., Le, H.C. and Phung, H.T., (2022). A novel integrating data envelopment analysis and spherical fuzzy MCDM approach for sustainable supplier selection in steel industry. *Mathematics*, *10*(11), pp.1897.

[25] Nguyen, T.M.H., Nguyen, V.P. and Nguyen, D.T., (2022). A new hybrid Pythagorean fuzzy AHP and COCOSO MCDM based approach by adopting artificial intelligence technologies. *Journal of Experimental & Theoretical Artificial Intelligence*, pp.1-27.