Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 3s (2025)

Aggregation operators applied to cotangent trigonometric spherical fuzzy sets and its generalization

M. Prakash[1], **S.V. Manemaran**[(2,*)], **Abdallah Al-Husban**[3,4]
[1]Research Scholar, Department of Mathematics, Bharath Institute of Higher Education and Research, Chennai, Tamilnadu, India.
[2]Department of Mathematics, Bharath Institute of Higher Education and Research, Chennai, Tamilnadu, India.
[3]Department of Mathematics, Faculty of Science and Technology, Irbid, National University, P.O. Box: 2600 Irbid, Jordan.
[4]Jadara Research Center, Jadara University, Irbid 21110, Jordan.
E-mails:[1]mprakashmagi@gmail.com, [2]svmanemaran@gmail.com,
[3]dralhosban@inu.edu.jo
[*]Corresponding author: S.V. Manemaran.
Received: 24-09-2024 Revised: 08-11-2024 Accepted: 27-11-2024.

Abstract

This paper presents a new method to generate cotangent trigonometric (α, β) spherical fuzzy sets (SFS). The SFS and (α, β) SFS are extended by this. This article will lead to a discussion of the concepts of of COTT spherical fuzzy weighted averaging (SFWA), COTT spherical fuzzy weighted geometric (SFWG), generalized COTT spherical fuzzy weighted averaging (GCOTTSFWA) and generalized COTT spherical fuzzy weighted geometric (GCOTTSFWG). Furthermore, idempotency, boundedness, commutativity, and monotonicity characterize the COTTSFN method.

Keywords: COTTSFWA; COTTSFWG; GCOTTSFWA and GCOTTSFWG.

1 Introduction

Numerous uncertain theories, such as fuzzy set (FS),[1] intuitionistic FS (IFS),[2] Pythagorean FS (PFS)[3] and spherical FS (SFS),[4] have been proposed to deal with the ambiguities. Subsequently, Atanassov introduced the idea of an IFS that is separated into categories using non-membership grade (NMD), which cannot exceed one.[2] An FS is a set of objects with membership grade (MD) in the provided set values from zero to one. When both the MD and NMD scores are larger than one, only one problem may be communicated to the decision-making (DM). According to Yager,[3] PFS is defined as an IFS with a value less than one and a square sum of MD and NMD less than one. The three parameters that Cuong et al.[5] established comprise the picture FS concept: positive MD, neutral MD, and negative MD. It offers some benefits over PFS and IFS as a consequence. Liu et al.[6] investigated an extension of picture FS with AOs. Liu et al.[7] have presented a generalized PFS based on AO and its applications. PFS and interval values-based AO characteristics.[8] Liu et al.[6] introduced an AO-based image FS. The total of the positive, neutral, and negative MD values in the

DM approach challenge rarely ever goes beyond one. Ashraf et al.[4] introduce the idea of SFS, which ensures that the square sum of the positive, neutral, and negative grades does not exceed 1. Fatmaa et al. investigated the idea of SFS.[9]
Aggregation operators are essential to solving MADM problems (AOs). According to Xu et al.,[10] there are IFS averaging operators that may be used to average IFS data. Recently many authors discussed the new research and its aggregating operators[11]-.[20] Furthermore, weighted, ordered weighted, and hybrid operators are examples of geometric operators derived from IFSs that were developed by Xu et al.[21] GOWs, or generalized ordered weighted averaging operators, were suggested by Li et al.[22] in 2002. Zeng et al.[23] explained how AOs and distance measurements are used to compute ordered weighted distances. Under PFS weighted, ordered weighted, and weighted power circumstances, Yager[3] produced several averaging and geometric AOs. Peng et al. examined a basic PFS based on the characteristics of AOs.[24] Liu et al. developed a generalized PFS under AOs.[7] Fuzzy spherical Dombi AOs were established by Ashraf et al.[25][26,27] has further details on SFSs and T-SFSs. Temel et al. (2022)[28] discussed how to apply it to MADM using Muirhead power normal SFS. Al Husband et al.[29]-[38] deals that concepts of Fuzzy logical and its generalization. Peng et al.[39] use TOPSIS and MABAC techniques to investigate neutrosophic sets using MADM. Zhang et al. discussed the TOPSIS-based generalization of PFS.[40] Numerous algebraic structures and aggregation operations with applications were recently covered by Palanikumar et al.[41][46] Recently, many authors interacted the concept including neutrosophic sets and its extension.[47]-[51] For the rest of this work, I will hold myself to the format indicated below. Section 1 for an introduction. Section 2 included a discussion of PFS and SFS. Part of the operations on (α, β) SFNs are covered in Section 3. Thus, the paper's primary conclusions are as follows:

1. To define the ED, HD, and score values for COTTSFSs.
2. By using COTT (α, β) SFN, COTT (α, β) SFWA, COTT (α, β) SFWG, GCOTT (α, β) SFWA and GCOTT (α, β) SFWG operators are developed.

2 Background

This section contains a number of important definitions that we must review for our further learning.

Definition 2.1. [8] Let $\mathcal{X}$ be an universal. The PIVFS $\ell = \Big\{\wp, \Big\langle \Xi_\ell^T(\wp), \Xi_\ell^F(\wp) \Big\rangle \Big| \wp \in \mathcal{X} \Big\}$, where $\Xi_\ell^T, \Xi_\ell^F : \mathcal{X} \to Int([0,1])$ denote the MD and NMD of $\wp \in \mathcal{X}$ to the set ℓ, respectively, and $0 \leq (\Xi_\ell^T(\wp))^2 + (\Xi_\ell^F(\wp))^2 \leq 1$. For convenience, $\ell = \Big\langle \Big[\Xi_\ell^T, \Xi_\ell^T\Big], \Big[\Xi_\ell^F, \Xi_\ell^F\Big] \Big\rangle$ is called a Pythagorean interval-valued fuzzy number (PyIVFN).

Definition 2.2. The NS $\ell = \Big\{x, \langle \Xi_\ell^T(\wp), \Xi_\ell^I(\wp), \Xi_\ell^F(\wp) \rangle \big| \wp \in \mathcal{X} \Big\}$, where $\Xi_\ell^T, \Xi_\ell^I, \Xi_\ell^F : \mathcal{X} \to [0,1]$ is denote the positive MD, neutral MD and negative MD of $\wp \in \mathcal{X}$, respectively and $0 \leq (\Xi_\ell^T(\wp)) + (\Xi_\ell^I(\wp)) + (\Xi_\ell^F(\wp)) \leq 2$. For $M = \langle \Xi_\ell^T, \Xi_\ell^I, \Xi_\ell^F \rangle$ is called a neutrosophic number (SFN).

Definition 2.3. The Pythagorean NS $\ell = \Big\{\wp, \langle \Xi_\ell^T(\wp), \Xi_\ell^I(\wp), \Xi_\ell^F(\wp)\rangle \big| \wp \in \mathcal{X}\Big\}$, where $\Xi_\ell^T, \Xi_\ell^I, \Xi_\ell^F : \mathcal{X} \to [0,1]$ is denote the positive MD, neutral MD and negative MD of $\wp \in \mathcal{X}$, respectively and $0 \le (\Xi_\ell^T(\wp))^2 + (\Xi_\ell^I(\wp))^2 + (\Xi_\ell^F(\wp))^2 \le 2$. For $M = \langle \Xi_\ell^T, \Xi_\ell^I, \Xi_\ell^F\rangle$ is called a Pythagorean neutrosophic number (PyNSN).

Definition 2.4. [9] The SFS ℓ in $\mathcal{X}$ is given by $\ell = \Big\{\wp, \langle \Xi_\ell^T(\wp), \Xi_\ell^I(\wp), \Xi_\ell^F(\wp)\rangle \big| \wp \in \mathcal{X}\Big\}$, where $\Xi_\ell^T, \Xi_\ell^I, \Xi_\ell^F : \mathcal{X} \to [0,1]$ denote the truth, indeterminacy and falsity membership grade of $\wp \in \mathcal{X}$ to ℓ, respectively and $0 \le (\Xi_\ell^T(\wp))^2 + (\Xi_\ell^I(\wp))^2 + (\Xi_\ell^F(\wp))^2 \le 1$. For all $\wp \in \mathcal{X}$, $\sqrt{1 - ((\Xi_\ell^T(\wp))^2 + (\Xi_\ell^I(\wp))^2 + (\Xi_\ell^F(\wp))^2)}$ is called the grade of refusal of membership of $\wp$ in ℓ. For convenience, $\ell = \langle \Xi_\ell^T, \Xi_\ell^I, \Xi_\ell^F\rangle$ is called a spherical fuzzy number (SFN).

Definition 2.5. [10] Let $\ell_1 = (a_1, b_1) \in N$and $\ell_2 = (a_2, b_2) \in N$. Then the distance between ℓ_1 and ℓ_2 is defined as $\mathcal{D}(\ell_1, \ell_2) = \sqrt{(a_1 - a_2)^2 + \frac{1}{2}(b_1 - b_2)^2}$, where N is a natural number.

3 Operations for COTT (α, β) SFN

We discuss the concept of cotangent trigonometric (α, β) spherical fuzzy number (COTT (α, β) SFN). As a result, the COTT (α, β) SFN and its operations were defined.

Definition 3.1. The (α, β) SFS $\ell = \Big\{\wp, \Big\langle \Big(\cot^2(\pi/4 \cdot \Xi_\ell^T)(\wp), \cot^2(\pi/4 \cdot \Xi_\ell^I)(\wp), \cot^2(\pi/4 \cdot \Xi_\ell^F)(\wp)\Big), (\alpha, \beta)\Big\rangle \Big| \wp \in \mathcal{X}\Big\}$, where $\cot^2(\pi/4 \cdot \Xi_\ell^T), \cot^2(\pi/4 \cdot \Xi_\ell^I), \cot^2(\pi/4 \cdot \Xi_\ell^F) : \mathcal{X} \to [0,1]$ denote the PMD, neutral MD and NMD of $\wp \in \mathcal{X}$ to ℓ, respectively and $0 \le (\cot^2(\pi/4 \cdot \Xi_\ell^T)(\wp))^\alpha + (\cot^2(\pi/4 \cdot \Xi_\ell^I)(\wp))^{lcm(\alpha,\beta)} + (\cot^2(\pi/4 \cdot \Xi_\ell^F)(\wp))^\beta \le 1$. For convenience, $\ell = \Big\langle \Big(\cot^2(\pi/4 \cdot \Xi_\ell^T), \cot^2(\pi/4 \cdot \Xi_\ell^I), \cot^2(\pi/4 \cdot \Xi_\ell^F)\Big), (\alpha, \beta)\Big\rangle$ is represent a COTT (α, β) SFN.

Definition 3.2. Let $\ell = \langle(\cot^2(\pi/4 \cdot \Xi^T), \cot^2(\pi/4 \cdot \Xi^I), (\cot^2(\pi/4 \cdot \Xi^F))), (\alpha, \beta)\rangle$, $\ell_1 = \langle(((\cot^2 \pi/4 \cdot \Xi_1^T), (\cot^2 \pi/4 \cdot \Xi_1^I), (\cot^2 \pi/4 \cdot \Xi_1^F)), (\alpha, \beta)\rangle$ and $\ell_2 = \langle(((\cot^2 \pi/4 \cdot \Xi_2^T), (\cot^2 \pi/4 \cdot \Xi_2^I), (\cot^2 \pi/4 \cdot \Xi_2^F)), (\alpha, \beta)\rangle$ be any three COTT (α, β) SFNs, and $\mho > 0$. Then

$$1.\ \ \ell_1 \bigoplus \ell_2 = \begin{bmatrix} \sqrt[\alpha]{\begin{matrix} ((\cot^2 \pi/4 \cdot \Xi_1^T))^\alpha + ((\cot^2 \pi/4 \cdot \Xi_2^T))^\alpha \\ -((\cot^2 \pi/4 \cdot \Xi_1^T))^\alpha \cdot ((\cot^2 \pi/4 \cdot \Xi_2^T))^\alpha \end{matrix}}, \\ \sqrt[lcm(\alpha,\beta)]{\begin{matrix} ((\cot^2 \pi/4 \cdot \Xi_1^I))^{lcm(\alpha,\beta)} + ((\cot^2 \pi/4 \cdot \Xi_2^I))^{lcm(\alpha,\beta)} \\ -((\cot^2 \pi/4 \cdot \Xi_1^I))^{lcm(\alpha,\beta)} \cdot ((\cot^2 \pi/4 \cdot \Xi_2^I))^{lcm(\alpha,\beta)} \end{matrix}}, \\ ((\cot^2 \pi/4 \cdot \Xi_1^F))^{1_3} \cdot ((\cot^2 \pi/4 \cdot \Xi_2^F))^{1_3} \end{bmatrix},$$

$$2.\ \ \ell_1 \oslash \ell_2 = \begin{bmatrix} ((\cot^2 \pi/4 \cdot \Xi_1^T))^{1_1} \cdot ((\cot^2 \pi/4 \cdot \Xi_2^T))^{1_1}, \\ \sqrt[lcm(\alpha,\beta)]{\begin{matrix} ((\cot^2 \pi/4 \cdot \Xi_1^I))^{lcm(\alpha,\beta)} + ((\cot^2 \pi/4 \cdot \Xi_2^I))^{lcm(\alpha,\beta)} \\ -((\cot^2 \pi/4 \cdot \Xi_1^I))^{lcm(\alpha,\beta)} \cdot ((\cot^2 \pi/4 \cdot \Xi_2^I))^{lcm(\alpha,\beta)} \end{matrix}}, \\ \sqrt[\beta]{\begin{matrix} ((\cot^2 \pi/4 \cdot \Xi_1^F))^\beta + ((\cot^2 \pi/4 \cdot \Xi_2^F))^\beta \\ -((\cot^2 \pi/4 \cdot \Xi_1^F))^\beta \cdot ((\cot^2 \pi/4 \cdot \Xi_2^F))^\beta \end{matrix}} \end{bmatrix}$$

3. $$\mho \cdot \ell = \begin{bmatrix} \sqrt[\alpha]{1-\left(1-(\cot^2\pi/4\cdot(\Xi^T)^\alpha\right)^\mho}, \\ \sqrt[lcm(\alpha,\beta)]{1-\left(1-(\cot^2\pi/4\cdot(\Xi^I)^{lcm(\alpha,\beta)}\right)^\mho}, \\ ((\cot^2\pi/4\cdot(\Xi^F)^\beta)^\mho \end{bmatrix},$$

4. $$\ell^\mho = \begin{bmatrix} ((\cot^2\pi/4\cdot(\Xi^T)^\alpha)^\mho, \\ \sqrt[lcm(\alpha,\beta)]{1-\left(1-(\cot^2\pi/4\cdot(\Xi^I)^{lcm(\alpha,\beta)}\right)^\mho}, \\ \sqrt[\beta]{1-\left(1-(\cot^2\pi/4\cdot(\Xi^F)^\beta\right)^\mho} \end{bmatrix}.$$

4 AOs based on COTT (α,β) SFN

Here we describe the AOs using COTT (α,β) SFWA, COTT (α,β) SFWG, GCOTT (α,β) SFWA, and GCOTT (α,β) SFWG.

4.1 COTT $(\alpha,\beta)SFWA$

Definition 4.1. Let $\ell_i = \langle(((\cot^2\pi/4\cdot\Xi_i^T),(\cot^2\pi/4\cdot\Xi_i^I),(\cot^2\pi/4\cdot\Xi_i^F))),(\alpha,\beta)\rangle$ be the COTT (α,β) SFNs, $W=(\varpi_1,\varpi_2,...,\varpi_n)$ be the weight of ℓ_i, $\varpi_i\geq 0$ and $\bigoplus_{i=1}^n\varpi_i=1$. Then COTT (α,β) SFWA $(\ell_1,\ell_2,...,\ell_n)=\bigoplus_{i=1}^n\varpi_i\ell_i$.

Theorem 4.2. *Let* $\ell_i = \left\langle\left(((\cot^2\pi/4\cdot\Xi_i^T),(\cot^2\pi/4\cdot\Xi_i^I),(\cot^2\pi/4\cdot\Xi_i^F))\right),(\alpha,\beta)\right\rangle$ *be the COTT* (α,β) *SFNs. Then COTT* $(\alpha,\beta)SFWA(\ell_1,\ell_2,...,\ell_n)$

$$= \begin{bmatrix} \sqrt[\alpha]{1-\bigodot_{i=1}^n\left(1-((\cot^2\pi/4\cdot\Xi_i^T))^\alpha\right)^{\varpi_i}}, \\ \sqrt[lcm(\alpha,\beta)]{1-\bigodot_{i=1}^n\left(1-((\cot^2\pi/4\cdot\Xi_i^I))^{lcm(\alpha,\beta)}\right)^{\varpi_i}}, \\ \bigodot_{i=1}^n(((\cot^2\pi/4\cdot\Xi_i^F))^\beta)^{\varpi_i} \end{bmatrix}.$$

Proof. If $n=2$, then COTT (α,β) SFWA$(\ell_1,\ell_2)=\varpi_1\ell_1\bigoplus\varpi_2\ell_2$, where

$$\varpi_1\ell_1 = \begin{bmatrix} \sqrt[\alpha]{1-\left(1-((\cot^2\pi/4\cdot\Xi_1^T))^\alpha\right)^{\varpi_1}}, \\ \sqrt[lcm(\alpha,\beta)]{1-\left(1-((\cot^2\pi/4\cdot\Xi_1^I))^{lcm(\alpha,\beta)}\right)^{\varpi_1}}; \\ (((\cot^2\pi/4\cdot\Xi_1^F))^\beta)^{\varpi_1} \end{bmatrix}$$

$$\varpi_2\ell_2 = \begin{bmatrix} \sqrt[\alpha]{1-\left(1-((\cot^2\pi/4\cdot\Xi_2^T))^\alpha\right)^{\varpi_2}}, \\ \sqrt[lcm(\alpha,\beta)]{1-\left(1-((\cot^2\pi/4\cdot\Xi_2^I))^{lcm(\alpha,\beta)}\right)^{\varpi_2}}; \\ (((\cot^2\pi/4\cdot\Xi_2^F))^\beta)^{\varpi_2} \end{bmatrix}.$$

Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 3s (2025)

Now,

$$\varpi_1\ell_1 \bigoplus \varpi_2\ell_2 = \left[\begin{array}{c} \sqrt[\alpha]{\begin{array}{c}\left(1-\left(1-((\cot^2\pi/4\cdot\Xi_1^T))^\alpha\right)^{\varpi_1}\right)+\\ \left(1-\left(1-((\cot^2\pi/4\cdot\Xi_2^T))^\alpha\right)^{\varpi_2}\right)\\ -\left(1-\left(1-((\cot^2\pi/4\cdot\Xi_1^T))^\alpha\right)^{\varpi_1}\right)\cdot\\ \left(1-\left(1-((\cot^2\pi/4\cdot\Xi_2^T))^\alpha\right)^{\varpi_2}\right),\end{array}} \\ \sqrt[lcm(\alpha,\beta)]{\begin{array}{c}\left(1-\left(1-((\cot^2\pi/4\cdot\Xi_1^I))^{lcm(\alpha,\beta)}\right)^{\varpi_1}\right)+\\ \left(1-\left(1-((\cot^2\pi/4\cdot\Xi_2^I))^{lcm(\alpha,\beta)}\right)^{\varpi_2}\right)\\ -\left(1-\left(1-((\cot^2\pi/4\cdot\Xi_1^I))^{lcm(\alpha,\beta)}\right)^{\varpi_1}\right)\cdot\\ \left(1-\left(1-((\cot^2\pi/4\cdot\Xi_2^I))^{lcm(\alpha,\beta)}\right)^{\varpi_2}\right),\end{array}} \\ (((\cot^2\pi/4\cdot\Xi_1^F))^\beta)^{\varpi_1}\cdot(((\cot^2\pi/4\cdot\Xi_2^F))^\beta)^{\varpi_2} \end{array}\right]$$

$$= \left[\begin{array}{c} \sqrt[\alpha]{1-\left(1-((\cot^2\pi/4\cdot\Xi_1^T))^\alpha\right)^{\varpi_1}\left(1-((\cot^2\pi/4\cdot\Xi_2^T))^\alpha\right)^{\varpi_2}}, \\ \sqrt[\alpha]{1-\left(1-((\cot^2\pi/4\cdot\Xi_1^I))^{lcm(\alpha,\beta)}\right)^{\varpi_1}\left(1-((\cot^2\pi/4\cdot\Xi_2^I))^{lcm(\alpha,\beta)}\right)^{\varpi_2}}, \\ (((\cot^2\pi/4\cdot\Xi_1^F))^\beta)^{\varpi_1}\cdot(((\cot^2\pi/4\cdot\Xi_2^F))^\beta)^{\varpi_2} \end{array}\right]$$

Hence, COTT $(\alpha,\beta)SFWA(\ell_1,\ell_2)$

$$= \left[\begin{array}{c} \sqrt[\alpha]{1-\bigodot_{i=1}^2\left(1-((\cot^2\pi/4\cdot\Xi_i^T))^\alpha\right)^{\varpi_i}}, \\ \sqrt[lcm(\alpha,\beta)]{1-\bigodot_{i=1}^2\left(1-((\cot^2\pi/4\cdot\Xi_i^I))^{lcm(\alpha,\beta)}\right)^{\varpi_i}}; \\ \bigodot_{i=1}^2(((\cot^2\pi/4\cdot\Xi_i^F))^\beta)^{\varpi_i} \end{array}\right].$$

It valid for $n \geq 3$,
Thus, COTT $(\alpha,\beta)SFWA(\ell_1,\ell_2,...,\ell_l)$

$$= \left[\begin{array}{c} \sqrt[\alpha]{1-\bigodot_{i=1}^l\left(1-((\cot^2\pi/4\cdot\Xi_i^T))^\alpha\right)^{\varpi_i}}, \\ \sqrt[lcm(\alpha,\beta)]{1-\bigodot_{i=1}^l\left(1-((\cot^2\pi/4\cdot\Xi_i^I))^{lcm(\alpha,\beta)}\right)^{\varpi_i}}; \\ \bigodot_{i=1}^l(((\cot^2\pi/4\cdot\Xi_i^F))^\beta)^{\varpi_i} \end{array}\right].$$

If $n = l + 1$, then COTT (α, β) SFWA $(\ell_1, \ell_2, ..., \ell_l, \ell_{l+1})$

$$= \left[\begin{array}{c} \sqrt[\alpha]{\begin{array}{r} \bigoplus_{i=1}^{l} \left(1 - \left(1 - ((\cot^2 \pi/4 \cdot \Xi_i^T))^{\alpha}\right)^{\varpi_i}\right) + \\ \left(1 - \left(1 - (\Xi_{l+1}^T)^{\alpha}\right)^{\varpi_{l+1}}\right) \\ - \bigodot_{i=1}^{l} \left(1 - \left(1 - ((\cot^2 \pi/4 \cdot \Xi_i^T))^{\alpha}\right)^{\varpi_i}\right) \cdot \\ \left(1 - \left(1 - (\Xi_{l+1}^T)^{\alpha}\right)^{\varpi_{l+1}}\right), \end{array}} \\ \sqrt[lcm(\alpha,\beta)]{\begin{array}{r} \bigoplus_{i=1}^{l} \left(1 - \left(1 - ((\cot^2 \pi/4 \cdot \Xi_i^I))^{lcm(\alpha,\beta)}\right)^{\varpi_i}\right) + \\ \left(1 - \left(1 - (\Xi_{l+1}^I)^{lcm(\alpha,\beta)}\right)^{\varpi_{l+1}}\right) \\ - \bigodot_{i=1}^{l} \left(1 - \left(1 - ((\cot^2 \pi/4 \cdot \Xi_i^I))^{lcm(\alpha,\beta)}\right)^{\varpi_i}\right) \\ \cdot \left(1 - \left(1 - (\Xi_{l+1}^I)^{lcm(\alpha,\beta)}\right)^{\varpi_{l+1}}\right), \end{array}} \\ \bigodot_{i=1}^{l} (((\cot^2 \pi/4 \cdot \Xi_i^F))^{\beta})^{\varpi_i} \cdot ((\Xi_{l+1}^F)^{\beta})^{\varpi_{l+1}}, \end{array} \right]$$

$$= \left[\begin{array}{c} \sqrt[\alpha]{1 - \bigodot_{i=1}^{l+1} \left(1 - ((\cot^2 \pi/4 \cdot \Xi_i^T))^{\alpha}\right)^{\varpi_i}}, \\ \sqrt[lcm(\alpha,\beta)]{1 - \bigodot_{i=1}^{l+1} \left(1 - ((\cot^2 \pi/4 \cdot \Xi_i^I))^{lcm(\alpha,\beta)}\right)^{\varpi_i}}, \\ \bigodot_{i=1}^{l+1} (((\cot^2 \pi/4 \cdot \Xi_i^F))^{\beta})^{\varpi_i} \end{array} \right].$$

□

Theorem 4.3. *Let* $\ell_i = \left\langle \left(((\cot^2 \pi/4 \cdot \Xi_i^T), (\cot^2 \pi/4 \cdot \Xi_i^I), (\cot^2 \pi/4 \cdot \Xi_i^F))\right), (\alpha, \beta) \right\rangle$ *be the COTT* (α, β) *SFNs. Then COTT* (α, β) *SFWA* $(\ell_1, \ell_2, ..., \ell_n) = \ell$ *(idempotency property).*

Proof. Since $(\cot^2 \pi/4 \cdot \Xi_i^T) = (\cot^2 \pi/4 \cdot \Xi^T)$, $(\cot^2 \pi/4 \cdot \Xi_i^I) = (\cot^2 \pi/4 \cdot \Xi^I)$ and

$(\cot^2 \pi/4 \cdot \Xi_i^F) = (\cot^2 \pi/4 \cdot \Xi^F)$ and $\bigoplus_{i=1}^{n} \varpi_i = 1$. Now, COTT $(\alpha, \beta)SFWA(\ell_1, \ell_2, ..., \ell_n)$

$$
\begin{aligned}
&= \left[\begin{array}{c}
\sqrt[\alpha]{1 - \bigodot_{i=1}^{n} \left(1 - ((\cot^2 \pi/4 \cdot \Xi_i^T))^{\alpha}\right)^{\varpi_i}}, \\
\sqrt[lcm(\alpha,\beta)]{1 - \bigodot_{i=1}^{n} \left(1 - ((\cot^2 \pi/4 \cdot \Xi_i^I))^{lcm(\alpha,\beta)}\right)^{\varpi_i}}; \\
\bigodot_{i=1}^{n}(((\cot^2 \pi/4 \cdot \Xi_i^F))^{\beta})^{\varpi_i}
\end{array}\right] \\
&= \left[\begin{array}{c}
\sqrt[\alpha]{1 - \left(1 - (\cot^2 \pi/4 \cdot (\Xi^T)^{\alpha}\right)^{\bigoplus_{i=1}^{n} \varpi_i}}, \\
\sqrt[lcm(\alpha,\beta)]{1 - \left(1 - (\cot^2 \pi/4 \cdot (\Xi^I)^{lcm(\alpha,\beta)}\right)^{\bigoplus_{i=1}^{n} \varpi_i}}; \\
((\cot^2 \pi/4 \cdot (\Xi^F)^{\beta})^{\bigoplus_{i=1}^{n} \varpi_i},
\end{array}\right] \\
&= \left[\begin{array}{c}
\sqrt[\alpha]{1 - \left(1 - (\cot^2 \pi/4 \cdot (\Xi^T)^{\alpha}\right)}, \\
\sqrt[lcm(\alpha,\beta)]{1 - \left(1 - (\cot^2 \pi/4 \cdot (\Xi^I)^{lcm(\alpha,\beta)}\right)}, \\
(\cot^2 \pi/4 \cdot (\Xi^F)^{\beta}
\end{array}\right] \\
&= \ell.
\end{aligned}
$$

□

Theorem 4.4. *Let* $\ell_i = \left\langle \left(((\cot^2 \pi/4 \cdot \Xi_i^T), (\cot^2 \pi/4 \cdot \Xi_i^I), (\cot^2 \pi/4 \cdot \Xi_i^F))\right), (\alpha, \beta) \right\rangle$ *be the COTT* (α, β) *SFNs. Then COTT* (α, β) *SFWA*$(\ell_1, \ell_2, ..., \ell_n)$
where $\underbrace{\Xi^T}_{} = \min(\cot^2 \pi/4 \cdot \Xi_{ij}^T)$, $\overbrace{(\cot^2 \pi/4 \cdot \Xi^T)}^{} = \max(\cot^2 \pi/4 \cdot \Xi_{ij}^T)$, $\underbrace{(\cot^2 \pi/4 \cdot \Xi^I)}_{} = \min(\cot^2 \pi/4 \cdot \Xi_{ij}^I)$, $\overbrace{(\cot^2 \pi/4 \cdot \Xi^I)}^{} = \max(\cot^2 \pi/4 \cdot \Xi_{ij}^I)$, $\underbrace{(\cot^2 \pi/4 \cdot \Xi^F)}_{} = \min(\cot^2 \pi/4 \cdot \Xi_{ij}^F)$, $\overbrace{(\cot^2 \pi/4 \cdot \Xi^F)}^{} = \max(\cot^2 \pi/4 \cdot \Xi_{ij}^F)$ *and where* $1 \leq i \leq n,\ j = 1, 2, ..., i_j$. *Then,*
$\left\langle \underbrace{(\cot^2 \pi/4 \cdot \Xi^T)}_{}, \underbrace{(\cot^2 \pi/4 \cdot \Xi^I)}_{}, \overbrace{(\cot^2 \pi/4 \cdot \Xi^F)}^{} \right\rangle$

$$
\begin{aligned}
&\leq COTT(\alpha, \beta)SFWA(\ell_1, \ell_2, ..., \ell_n) \\
&\leq \overbrace{(\cot^2 \pi/4 \cdot \Xi^T)}^{}, \overbrace{(\cot^2 \pi/4 \cdot \Xi^I)}^{}, \underbrace{(\cot^2 \pi/4 \cdot \Xi^F)}_{}] \Big\rangle.
\end{aligned}
$$

(Boundedness property).

Proof. Since, $\underbrace{(\cot^2 \pi/4 \cdot \Xi^T)}_{} = \min(\cot^2 \pi/4 \cdot \Xi_{ij}^T)$, $\overbrace{(\cot^2 \pi/4 \cdot \Xi^T)}^{} = \max(\cot^2 \pi/4 \cdot \Xi_{ij}^T)$ and $\underbrace{(\cot^2 \pi/4 \cdot \Xi^T)}_{} \leq (\cot^2 \pi/4 \cdot \Xi_{ij}^T) \leq \overbrace{(\cot^2 \pi/4 \cdot \Xi^T)}^{}$.
Now, $\underbrace{(\cot^2 \pi/4 \cdot \Xi^T)}_{} = \sqrt[\alpha]{1 - \bigodot_{i=1}^{n} \left(1 - (\underbrace{(\cot^2 \pi/4 \cdot \Xi^T)}_{})^{\alpha}\right)^{\varpi_i}}$
$\leq \sqrt[\alpha]{1 - \bigodot_{i=1}^{n} \left(1 - ((\cot^2 \pi/4 \cdot (\cot^2 \pi/4 \cdot \Xi_{ij}^T)))^{\alpha}\right)^{\varpi_i}}$
$\leq \sqrt[\alpha]{1 - \bigodot_{i=1}^{n} \left(1 - (\overbrace{(\cot^2 \pi/4 \cdot \Xi^T)}^{})^{\alpha}\right)^{\varpi_i}} = \overbrace{(\cot^2 \pi/4 \cdot \Xi^T)}^{}$.

Since, $\underbrace{(\cot^2\pi/4\cdot\Xi^I)} = \min(\cot^2\pi/4\cdot\Xi^I_{ij})$, $\overbrace{(\cot^2\pi/4\cdot\Xi^I)} = \max(\cot^2\pi/4\cdot\Xi^I_{ij})$ and $\underbrace{(\cot^2\pi/4\cdot\Xi^I)} \leq (\cot^2\pi/4\cdot\Xi^I_{ij}) \leq \overbrace{(\cot^2\pi/4\cdot\Xi^I)}$.

Now, $\underbrace{(\cot^2\pi/4\cdot\Xi^I)} = \sqrt[lcm(\alpha,\beta)]{1-\bigodot_{i=1}^{n}\left(1-((\underbrace{\cot^2\pi/4\cdot\Xi^I}))^{lcm(\alpha,\beta)}\right)^{\varpi_i}}$
$\leq \sqrt[lcm(\alpha,\beta)]{1-\bigodot_{i=1}^{n}\left(1-((\cot^2\pi/4\cdot\Xi^I_{ij}))^{lcm(\alpha,\beta)}\right)^{\varpi_i}}$
$\leq \sqrt[lcm(\alpha,\beta)]{1-\bigodot_{i=1}^{n}\left(1-(\overbrace{(\cot^2\pi/4\cdot\Xi^I)})^{lcm(\alpha,\beta)}\right)^{\varpi_i}}$
$= \overbrace{(\cot^2\pi/4\cdot\Xi^I)}$.

Since, $\underbrace{(\cot^2\pi/4\cdot(\Xi^F)^\beta} = \min((\cot^2\pi/4\cdot\Xi^F_{ij}))^\beta$, $\overbrace{(\cot^2\pi/4\cdot(\Xi^F)^\beta} = \max((\cot^2\pi/4\cdot\Xi^F_{ij}))^\beta$ and $\underbrace{(\cot^2\pi/4\cdot(\Xi^F)^\beta} \leq ((\cot^2\pi/4\cdot\Xi^F_{ij}))^\beta \leq \overbrace{(\cot^2\pi/4\cdot(\Xi^F)^\beta}$.

We have, $\underbrace{(\cot^2\pi/4\cdot(\Xi^F)^\beta} = \bigodot_{i=1}^{n}(\underbrace{(\cot^2\pi/4\cdot(\Xi^F)^\beta})^{\varpi_i} \leq \bigodot_{i=1}^{n}(((\cot^2\pi/4\cdot\Xi^F_{ij}))^\beta)^{\varpi_i} \leq \bigodot_{i=1}^{n}(\overbrace{(\cot^2\pi/4\cdot(\Xi^F)^\beta})^{\varpi_i} = \overbrace{(\cot^2\pi/4\cdot(\Xi^F)^\beta}$.

Therefore,

$$\begin{bmatrix} \left(\sqrt[\alpha]{1-\bigodot_{i=1}^{n}\left(1-((\underbrace{\cot^2\pi/4\cdot\Xi^T}))^{\alpha}\right)^{\varpi_i}}\right)^2 \\ -\left(\sqrt[lcm(\alpha,\beta)]{1-\bigodot_{i=1}^{n}\left(1-((\underbrace{\cot^2\pi/4\cdot\Xi^I}))^{lcm(\alpha,\beta)}\right)^{\varpi_i}}\right)^2 \\ +1-\left(\bigodot_{i=1}^{n}((\overbrace{(\cot^2\pi/4\cdot\Xi^F)})^\beta)^{\varpi_i}\right)^2 \end{bmatrix}$$

$$\leq \begin{bmatrix} \left(\sqrt[\alpha]{1-\bigodot_{i=1}^{n}\left(1-((\cot^2\pi/4\cdot(\cot^2\pi/4\cdot\Xi^T_{ij})))^{\alpha}\right)^{\varpi_i}}\right)^2 \\ -\left(\sqrt[lcm(\alpha,\beta)]{1-\bigodot_{i=1}^{n}\left(1-((\cot^2\pi/4\cdot\Xi^I_{ij}))^{lcm(\alpha,\beta)}\right)^{\varpi_i}}\right)^2 + \\ +1-\left(\bigodot_{i=1}^{n}(((\cot^2\pi/4\cdot\Xi^F_{ij}))^\beta)^{\varpi_i}\right)^2 \end{bmatrix}$$

$$\leq \begin{bmatrix} \left(\sqrt[\alpha]{1-\bigodot_{i=1}^{n}\left(1-(\overbrace{(\cot^2\pi/4\cdot\Xi^T)})^{\alpha}\right)^{\varpi_i}}\right)^2 \\ -\left(\sqrt[lcm(\alpha,\beta)]{1-\bigodot_{i=1}^{n}\left(1-(\overbrace{(\cot^2\pi/4\cdot\Xi^I)})^{lcm(\alpha,\beta)}\right)^{\varpi_i}}\right)^2 + \\ +1-\left(\bigodot_{i=1}^{n}((\underbrace{\cot^2\pi/4\cdot\Xi^F}))^\beta)^{\varpi_i}\right)^2 \end{bmatrix}.$$

Hence,
$\left\langle \underbrace{(\cot^2\pi/4\cdot\Xi^T)}, \underbrace{(\cot^2\pi/4\cdot\Xi^I)}, \overbrace{(\cot^2\pi/4\cdot\Xi^F)} \right\rangle$
$\leq COTT(\alpha,\beta)SFWA(\ell_1,\ell_2,...,\ell_n)$
$\leq \langle \overbrace{(\cot^2\pi/4\cdot\Xi^T)}, \overbrace{(\cot^2\pi/4\cdot\Xi^I)}, \underbrace{(\cot^2\pi/4\cdot\Xi^F)} \rangle$. □

Theorem 4.5. *Let* $\ell_i = \langle((\cot^2\pi/4 \cdot \Xi^T_{t_{ij}}), (\cot^2\pi/4 \cdot \Xi^I_{t_{ij}}), (\cot^2\pi/4 \cdot \Xi^F_{t_{ij}})), (\alpha, \beta)\rangle$ *and* $W_i = \langle((\cot^2\pi/4 \cdot \Xi^T_{h_{ij}}), (\cot^2\pi/4 \cdot \Xi^I_{h_{ij}}), (\cot^2\pi/4 \cdot \Xi^F_{h_{ij}})), (\alpha, \beta)\rangle$, *be the COTT* (α, β) *SFWAs. For any i, if there is* $(\cot^2\pi/4 \cdot \Xi^T_{t_{ij}})^2 \leq (\cot^2\pi/4 \cdot \Xi^T_{h_{ij}})^2$ *and* $(\cot^2\pi/4 \cdot \Xi^I_{t_{ij}})^2 \leq (\cot^2\pi/4 \cdot \Xi^I_{h_{ij}})^2$ *and* $(\cot^2\pi/4 \cdot \Xi^F_{t_{ij}})^2 \geq (\cot^2\pi/4 \cdot \Xi^F_{h_{ij}})^2$ *or* $\ell_i \leq W_i$. *Prove that COTT* $(\alpha, \beta)SFWA(\ell_1, \ell_2, ..., \ell_n) \leq COTT(\alpha, \beta)SFWA(W_1, W_2, ..., W_n)$, *where* $(i = 1, 2, ..., n); (j = 1, 2, ..., i_j)$ *(monotonicity property).*

Proof. For any i, $(\cot^2\pi/4 \cdot \Xi^T_{t_{ij}})^2 \leq (\cot^2\pi/4 \cdot \Xi^T_{h_{ij}})^2$.
Therefore, $1 - ((\cot^2\pi/4 \cdot \Xi^T_{t_i}))^2 \geq 1 - ((\cot^2\pi/4 \cdot \Xi^T_{h_i}))^2$.
Hence, $\bigodot_{i=1}^n \left(1 - \left((\cot^2\pi/4 \cdot \Xi^T_{t_i})\right)^2\right)^{\varpi_i} \geq \bigodot_{i=1}^n \left(1 - \left((\cot^2\pi/4 \cdot \Xi^T_{h_i})\right)^2\right)^{\varpi_i}$
and $\sqrt[\alpha]{1 - \bigodot_{i=1}^n \left(1 - \left((\cot^2\pi/4 \cdot \Xi^T_{t_i})\right)^\alpha\right)^{\varpi_i}} \leq \sqrt[\alpha]{1 - \bigodot_{i=1}^n \left(1 - \left((\cot^2\pi/4 \cdot \Xi^T_{h_i})\right)^\alpha\right)^{\varpi_i}}$.
For any i, $(\cot^2\pi/4 \cdot \Xi^I_{t_{ij}})^{lcm(\alpha,\beta)} \leq \left((\cot^2\pi/4 \cdot \Xi^I_{h_{ij}})\right)^{lcm(\alpha,\beta)}$.
Therefore, $1 - \left((\cot^2\pi/4 \cdot \Xi^I_{t_i})\right)^{lcm(\alpha,\beta)} \geq 1 - \left((\cot^2\pi/4 \cdot \Xi^I_{h_i})\right)^{lcm(\alpha,\beta)}$.
Hence, $\bigodot_{i=1}^n \left(1 - \left((\cot^2\pi/4 \cdot \Xi^I_{t_i})\right)^{lcm(\alpha,\beta)}\right)^{\varpi_i} \geq \bigodot_{i=1}^n \left(1 - \left((\cot^2\pi/4 \cdot \Xi^I_{h_i})\right)^{lcm(\alpha,\beta)}\right)^{\varpi_i}$.
This implies that $\sqrt[lcm(\alpha,\beta)]{1 - \bigodot_{i=1}^n \left(1 - \left((\cot^2\pi/4 \cdot \Xi^I_{t_i})\right)^{lcm(\alpha,\beta)}\right)^{\varpi_i}}$
$\leq \sqrt[lcm(\alpha,\beta)]{1 - \bigodot_{i=1}^n \left(1 - \left((\cot^2\pi/4 \cdot \Xi^I_{h_i})\right)^{lcm(\alpha,\beta)}\right)^{\varpi_i}}$.
For any i, $\left((\cot^2\pi/4 \cdot \Xi^F_{t_{ij}})\right)^2 \geq \left((\cot^2\pi/4 \cdot \Xi^F_{h_{ij}})\right)^2$ and
$\left((\cot^2\pi/4 \cdot \Xi^F_{t_{ij}})\right)^\beta \geq \left((\cot^2\pi/4 \cdot \Xi^F_{h_{ij}})\right)^\beta$.
Therefore, $1 - \left(\bigodot_{i=1}^n (\cot^2\pi/4 \cdot \Xi^F_{t_{ij}})\right)^\beta \leq 1 - \left(\bigodot_{i=1}^n (\cot^2\pi/4 \cdot \Xi^F_{h_{ij}})\right)^\beta$.

$$\begin{bmatrix} \left(\sqrt[\alpha]{1 - \bigodot_{i=1}^n \left(1 - ((\cot^2\pi/4 \cdot \Xi^T_{ti}))^\alpha\right)^{\varpi_i}}\right)^2 \\ -\left(\sqrt[lcm(\alpha,\beta)]{1 - \bigodot_{i=1}^n \left(1 - ((\cot^2\pi/4 \cdot \Xi^I_{t_i}))^{lcm(\alpha,\beta)}\right)^{\varpi_i}}\right)^2 \\ +1 - \left(\bigodot_{i=1}^n ((\cot^2\pi/4 \cdot \Xi^F_{t_i}))^\beta\right)^2 \end{bmatrix}$$

$$\leq \begin{bmatrix} \left(\sqrt[\alpha]{1 - \bigodot_{i=1}^n \left(1 - ((\cot^2\pi/4 \cdot \Xi^T_{hi}))^\alpha\right)^{\varpi_i}}\right)^2 \\ -\left(\sqrt[lcm(\alpha,\beta)]{1 - \bigodot_{i=1}^n \left(1 - ((\cot^2\pi/4 \cdot \Xi^I_{h_i}))^{lcm(\alpha,\beta)}\right)^{\varpi_i}}\right)^2 \\ +1 - \left(\bigodot_{i=1}^n ((\cot^2\pi/4 \cdot \Xi^F_{h_i}))^\beta\right)^2 \end{bmatrix}.$$

Hence, COTT $(\alpha, \beta)SFWA\,(\ell_1, \ell_2, ..., \ell_n) \leq COTT(\alpha, \beta)SFWA\,(W_1, W_2, ..., W_n)$. □

4.2 COTT (α, β) SFWG

Definition 4.6. Let $\ell_i = \left\langle \left(((\cot^2 \pi/4 \cdot \Xi_i^T), (\cot^2 \pi/4 \cdot \Xi_i^I), (\cot^2 \pi/4 \cdot \Xi_i^F))\right) \right\rangle$ be the COTT (α, β) SFNs. Then (α, β)SFWG $(\ell_1, \ell_2, ..., \ell_n) = \bigodot_{i=1}^n \ell_i^{\varpi_i}$.

Theorem 4.7. *Let* $\ell_i = \left\langle \left(((\cot^2 \pi/4 \cdot \Xi_i^T), (\cot^2 \pi/4 \cdot \Xi_i^I), (\cot^2 \pi/4 \cdot \Xi_i^F))\right), (\alpha, \beta) \right\rangle$ *be the COTT* (α, β) *SFNs. Then COTT* (α, β) *SFWG* $(\ell_1, \ell_2, ..., \ell_n)$

$$= \begin{bmatrix} \bigodot_{i=1}^n (((\cot^2 \pi/4 \cdot \Xi_i^T))^\alpha)^{\varpi_i}; \\ \sqrt[lcm(\alpha,\beta)]{1 - \bigodot_{i=1}^n \left(1 - ((\cot^2 \pi/4 \cdot \Xi_i^I))^{lcm(\alpha,\beta)}\right)^{\varpi_i}}, \\ \sqrt[\beta]{1 - \bigodot_{i=1}^n \left(1 - ((\cot^2 \pi/4 \cdot \Xi_i^F))^\beta\right)^{\varpi_i}} \end{bmatrix}.$$

Proof. Based on Theorem 4.2, the following results follow. □

Theorem 4.8. *Let* $\ell_i = \left\langle \left(((\cot^2 \pi/4 \cdot \Xi_i^T), (\cot^2 \pi/4 \cdot \Xi_i^I), (\cot^2 \pi/4 \cdot \Xi_i^F))\right), (\alpha, \beta) \right\rangle$ *be the COTT* (α, β) *SFNs and all are equal. Then* (α, β)*SFWG*$(\ell_1, \ell_2, ..., \ell_n) = L$.

Proof. Based on Theorem 4.3, the following results follow. □

Remark 4.9. It has other properties, including boundedness and monotonicity, as well as having (α, β)SFWG.

Proof. Based on Theorem 4.4 and 4.5, the following results follow. □

4.3 Generalized COTT (α, β) SFWA (GCOTT(α, β)SFWA)

Definition 4.10. Let $\ell_i = \left\langle \left(((\cot^2 \pi/4 \cdot \Xi_i^T), (\cot^2 \pi/4 \cdot \Xi_i^I), (\cot^2 \pi/4 \cdot \Xi_i^F))\right), (\alpha, \beta) \right\rangle$ be the COTT (α, β) SFN. Then GCOTT (α, β) SFWA $(\ell_1, \ell_2, ..., \ell_n) = \left(\bigoplus_{i=1}^n \varpi_i \ell_i^{\mho}\right)^{1/\mho}$.

Theorem 4.11. *Let* $\ell_i = \left\langle \left(((\cot^2 \pi/4 \cdot \Xi_i^T), (\cot^2 \pi/4 \cdot \Xi_i^I), (\cot^2 \pi/4 \cdot \Xi_i^F))\right), (\alpha, \beta) \right\rangle$ *be the COTT* (α, β) *SFNs. Then GCOTT* (α, β) *SFWA* $(\ell_1, \ell_2, ..., \ell_n)$

$$= \begin{bmatrix} \left(\sqrt[\alpha]{1 - \bigodot_{i=1}^n \left(1 - \left(((\cot^2 \pi/4 \cdot \Xi_i^T))^\alpha\right)^\alpha\right)^{\varpi_i}}\right)^{1/\alpha}, \\ \left(\sqrt[lcm(\alpha,\beta)]{1 - \bigodot_{i=1}^n \left(1 - \left(((\cot^2 \pi/4 \cdot \Xi_i^I))^{lcm(\alpha,\beta)}\right)^{lcm(\alpha,\beta)}\right)^{\varpi_i}}\right)^{1/lcm(\alpha,\beta)}, \\ \sqrt[\beta]{1 - \left(1 - \left(\bigodot_{i=1}^n \left(\sqrt[\beta]{1 - \left(1 - ((\cot^2 \pi/4 \cdot \Xi_i^F))^\beta\right)^\beta}\right)^{\varpi_i}\right)^\beta\right)^{1/\beta}} \end{bmatrix}.$$

Proof. We can prove this first by demonstrating that,

$$\bigoplus_{i=1}^{n}\varpi_i\ell_i^{\alpha}=\left[\begin{array}{c}\sqrt[\alpha]{1-\bigodot_{i=1}^{n}\left(1-\left(((\cot^2\pi/4\cdot\Xi_i^T))^{\alpha}\right)^{\alpha}\right)^{\varpi_i}};\\ \sqrt[lcm(\alpha,\beta)]{1-\bigodot_{i=1}^{n}\left(1-\left(((\cot^2\pi/4\cdot\Xi_i^I))^{lcm(\alpha,\beta)}\right)^{lcm(\alpha,\beta)}\right)^{\varpi_i}};\\ \bigodot_{i=1}^{n}\left(\sqrt[\beta]{1-\left(1-((\cot^2\pi/4\cdot\Xi_i^F))^{\beta}\right)^{\beta}}\right)^{\varpi_i},\end{array}\right].$$

Put $n=2$, $\varpi_1\ell_1\bigoplus\varpi_2\ell_2$

$$=\left[\begin{array}{c}\sqrt[\alpha]{\begin{array}{c}\left(\sqrt[\alpha]{1-\left(1-\left(((\cot^2\pi/4\cdot\Xi_1^T))^{\alpha}\right)^{\alpha}\right)^{\varpi_1}}\right)^{\alpha}+\\ \left(\sqrt[\alpha]{1-\left(1-\left(((\cot^2\pi/4\cdot\Xi_2^T))^{\alpha}\right)^{\alpha}\right)^{\varpi_1}}\right)^{\alpha},\\ -\left(\sqrt[\alpha]{1-\left(1-\left(((\cot^2\pi/4\cdot\Xi_1^T))^{\alpha}\right)^{\alpha}\right)^{\varpi_1}}\right)^{\alpha}\cdot\\ \left(\sqrt[\alpha]{1-\left(1-\left(((\cot^2\pi/4\cdot\Xi_2^T))^{\alpha}\right)^{\alpha}\right)^{\varpi_1}}\right)^{\alpha}\end{array}}\\ ,\\ \sqrt[lcm(\alpha,\beta)]{\begin{array}{c}\left(\sqrt[lcm(\alpha,\beta)]{1-\left(1-\left(((\cot^2\pi/4\cdot\Xi_1^I))^{lcm(\alpha,\beta)}\right)^{lcm(\alpha,\beta)}\right)^{\varpi_1}}\right)^{lcm(\alpha,\beta)}+\\ \left(\sqrt[lcm(\alpha,\beta)]{1-\left(1-\left(((\cot^2\pi/4\cdot\Xi_2^I))^{lcm(\alpha,\beta)}\right)^{lcm(\alpha,\beta)}\right)^{\varpi_1}}\right)^{lcm(\alpha,\beta)},\\ -\left(\sqrt[lcm(\alpha,\beta)]{1-\left(1-\left(((\cot^2\pi/4\cdot\Xi_1^I))^{lcm(\alpha,\beta)}\right)^{lcm(\alpha,\beta)}\right)^{\varpi_1}}\right)^{lcm(\alpha,\beta)}\cdot\\ \left(\sqrt[lcm(\alpha,\beta)]{1-\left(1-\left(((\cot^2\pi/4\cdot\Xi_2^I))^{lcm(\alpha,\beta)}\right)^{lcm(\alpha,\beta)}\right)^{\varpi_1}}\right)^{lcm(\alpha,\beta)}\end{array}}\\ \left(\sqrt[\beta]{1-\left(1-((\cot^2\pi/4\cdot\Xi_1^F))^{\beta}\right)^{\beta}}\right)^{\varpi_1}\cdot\left(\sqrt[\beta]{1-\left(1-((\cot^2\pi/4\cdot\Xi_2^F))^{\beta}\right)^{\beta}}\right)^{\varpi_1},\end{array}\right]$$

$$=\left[\begin{array}{c}\sqrt[\alpha]{1-\bigodot_{i=1}^{2}\left(1-\left(((\cot^2\pi/4\cdot\Xi_1^T))^{\alpha}\right)^{\alpha}\right)^{\varpi_i}};\\ \sqrt[lcm(\alpha,\beta)]{1-\bigodot_{i=1}^{2}\left(1-\left(((\cot^2\pi/4\cdot\Xi_1^I))^{lcm(\alpha,\beta)}\right)^{lcm(\alpha,\beta)}\right)^{\varpi_i}};\\ \bigodot_{i=1}^{2}\left(\sqrt[\beta]{1-\left(1-((\cot^2\pi/4\cdot\Xi_i^F))^{\beta}\right)^{\beta}}\right)^{\varpi_i};\end{array}\right].$$

Hence,

$$\bigoplus_{i=1}^{l}\varpi_i\ell_i^{\mho}=\left[\begin{array}{c}\sqrt[\alpha]{1-\bigodot_{i=1}^{l}\left(1-\left(((\cot^2\pi/4\cdot\Xi_1^T))^{\alpha}\right)^{\alpha}\right)^{\varpi_i}};\\ \sqrt[lcm(\alpha,\beta)]{1-\bigodot_{i=1}^{l}\left(1-\left(((\cot^2\pi/4\cdot\Xi_1^I))^{lcm(\alpha,\beta)}\right)^{lcm(\alpha,\beta)}\right)^{\varpi_i}};\\ \bigodot_{i=1}^{l}\left(\sqrt[\beta]{1-\left(1-((\cot^2\pi/4\cdot\Xi_i^F))^{\beta}\right)^{\beta}}\right)^{\varpi_i};\end{array}\right].$$

If $n=l+1$, then $\bigoplus_{i=1}^{l}\varpi_i\ell_i^{\mho}+\varpi_{l+1}\ell_{l+1}^{\mho}=\bigoplus_{i=1}^{l+1}\varpi_i\ell_i^{\mho}$.
Now,$\bigoplus_{i=1}^{l}\varpi_i\ell_i^{\mho}+\varpi_{l+1}\ell_{l+1}^{\mho}=\varpi_1\ell_1^{\mho}\bigoplus\varpi_2\ell_2^{\mho}\bigoplus...\bigoplus\varpi_l\ell_l^{\mho}\bigoplus\varpi_{l+1}\ell_{l+1}^{\mho}$

$$=\left[\begin{array}{c}\sqrt[\alpha]{\begin{array}{c}\left(\sqrt[\alpha]{1-\bigodot_{i=1}^{l}\left(1-\left(((\cot^2\pi/4\cdot\Xi_i^T))^{\alpha}\right)^{\alpha}\right)^{\varpi_i}}\right)^{\alpha}+\left(\sqrt[\alpha]{1-\left(1-\left((\Xi_{l+1}^T)^{\alpha}\right)^{\alpha}\right)^{\varpi_1}}\right)^{\alpha},\\ -\left(\sqrt[\alpha]{1-\bigodot_{i=1}^{l}\left(1-\left(((\cot^2\pi/4\cdot\Xi_i^T))^{\alpha}\right)^{\alpha}\right)^{\varpi_i}}\right)^{\alpha}\cdot\left(\sqrt[\alpha]{1-\left(1-\left((\Xi_{l+1}^T)^{\alpha}\right)^{\alpha}\right)^{\varpi_1}}\right)^{\alpha}\end{array}}\\ \sqrt[lcm(\alpha,\beta)]{\begin{array}{c}\left(\sqrt[lcm(\alpha,\beta)]{1-\bigodot_{i=1}^{l}\left(1-\left(((\cot^2\pi/4\cdot\Xi_i^I))^{lcm(\alpha,\beta)}\right)^{lcm(\alpha,\beta)}\right)^{\varpi_i}}\right)^{lcm(\alpha,\beta)}+\\ \left(\sqrt[lcm(\alpha,\beta)]{1-\left(1-\left((\Xi_{l+1}^I)^{lcm(\alpha,\beta)}\right)^{lcm(\alpha,\beta)}\right)^{\varpi_1}}\right)^{lcm(\alpha,\beta)},\\ -\left(\sqrt[lcm(\alpha,\beta)]{1-\bigodot_{i=1}^{l}\left(1-\left(((\cot^2\pi/4\cdot\Xi_i^I))^{lcm(\alpha,\beta)}\right)^{lcm(\alpha,\beta)}\right)^{\varpi_i}}\right)^{lcm(\alpha,\beta)}\cdot\\ \left(\sqrt[lcm(\alpha,\beta)]{1-\left(1-\left((\Xi_{l+1}^I)^{lcm(\alpha,\beta)}\right)^{lcm(\alpha,\beta)}\right)^{\varpi_1}}\right)^{lcm(\alpha,\beta)}\end{array}}\\ \bigodot_{i=1}^{l}\left(\sqrt[\beta]{1-\left(1-((\cot^2\pi/4\cdot\Xi_i^F))^{\beta}\right)^{\beta}}\right)^{\varpi_i}\cdot\left(\sqrt[\beta]{1-\left(1-(\Xi_{l+1}^F)^{\beta}\right)^{\beta}}\right)^{\varpi_1},\end{array}\right]$$

$$\bigoplus_{i=1}^{l+1}\varpi_i\ell_i^{\alpha}=\left[\begin{array}{c}\sqrt[\alpha]{1-\bigodot_{i=1}^{l+1}\left(1-\left(((\cot^2\pi/4\cdot\Xi_1^T))^{\alpha}\right)^{\alpha}\right)^{\varpi_i}};\\ \sqrt[lcm(\alpha,\beta)]{1-\bigodot_{i=1}^{l+1}\left(1-\left(((\cot^2\pi/4\cdot\Xi_1^I))^{lcm(\alpha,\beta)}\right)^{lcm(\alpha,\beta)}\right)^{\varpi_i}};\\ \bigodot_{i=1}^{l+1}\left(\sqrt[\beta]{1-\left(1-((\cot^2\pi/4\cdot\Xi_i^F))^{\beta}\right)^{\beta}}\right)^{\varpi_i}\end{array}\right].$$

$$\left(\bigoplus_{i=1}^{l+1}\varpi_i\ell_i^{\mho}\right)^{1/\mho}=$$

$$\left[\begin{array}{c} \left(\sqrt[\mho]{1-\bigodot_{i=1}^{l+1}\left(1-\left(((\cot^2\pi/4\cdot\Xi_i^T))^{\alpha}\right)^{\alpha}\right)^{\varpi_i}}\right)^{1/\alpha}; \\ \left(\sqrt[lcm(\alpha,\beta)]{1-\bigodot_{i=1}^{l+1}\left(1-\left(((\cot^2\pi/4\cdot\Xi_i^I))^{lcm(\alpha,\beta)}\right)^{lcm(\alpha,\beta)}\right)^{\varpi_i}}\right)^{1/lcm(\alpha,\beta)}; \\ \sqrt[\beta]{1-\left(1-\left(\bigodot_{i=1}^{l+1}\left(\sqrt[\beta]{1-\left(1-((\cot^2\pi/4\cdot\Xi_i^F))^{\beta}\right)^{\beta}}\right)^{\varpi_i}\right)^{2}\right)^{1/\beta}}, \end{array}\right]$$

□

Remark 4.12. An operator modified from the GCOTT (α,β) SFWA operator to the COTT (α,β) SFWA operator is performed if $\mho = 1$.

Theorem 4.13. *If all* $\ell_i = \left\langle \left(((\cot^2\pi/4\cdot\Xi_i^T), (\cot^2\pi/4\cdot\Xi_i^I), (\cot^2\pi/4\cdot\Xi_i^F))\right), (\alpha,\beta)\right\rangle$ *and all are equal. Then GCOTT* (α,β) *SFWA*$(\ell_1,\ell_2,...,\ell_n) = \ell$.

Proof. There is a proof based on the Theorem 4.3. □

Remark 4.14. In the GCOTT (α,β) SFWA operator, boundedness and monotonicity are satisfied.

Proof. There is a proof based on the Theorem 4.4 and Theorem 4.5. □

4.4 Generalized COTT (α,β)SFWG (GCOTT (α,β)SFWG)

Definition 4.15. Let $\ell_i = \left\langle \left(((\cot^2\pi/4\cdot\Xi_i^T), (\cot^2\pi/4\cdot\Xi_i^I), (\cot^2\pi/4\cdot\Xi_i^F))\right), (\alpha,\beta)\right\rangle$ be the COTT (α,β) SFNs. Then GCOTT(α,β)SFWG $(\ell_1,\ell_2,...,\ell_n) = \frac{1}{\mho}\left(\bigodot_{i=1}^{n}(\mho\ell_i)^{\varpi_i}\right)$.

Theorem 4.16. *Let* $\ell_i = \left\langle \left(((\cot^2\pi/4\cdot\Xi_i^T), (\cot^2\pi/4\cdot\Xi_i^I), (\cot^2\pi/4\cdot\Xi_i^F))\right), (\alpha,\beta)\right\rangle$ *be the COTT* (α,β) *SFNs. Then GCOTT*(α,β)*SFWG*$(\ell_1,\ell_2,...,\ell_n)$

$$= \left[\begin{array}{c} \sqrt[\alpha]{1-\left(1-\left(\bigodot_{i=1}^{n}\left(\sqrt[\alpha]{1-\left(1-((\cot^2\pi/4\cdot\Xi_i^T))^{\alpha}\right)^{\alpha}}\right)^{\varpi_i}\right)^{\alpha}\right)^{1/\alpha}}; \\ \left(\sqrt[lcm(\alpha,\beta)]{1-\bigodot_{i=1}^{n}\left(1-\left(((\cot^2\pi/4\cdot\Xi_i^I))^{lcm(\alpha,\beta)}\right)^{lcm(\alpha,\beta)}\right)^{\varpi_i}}\right)^{1/lcm(\alpha,\beta)}; \\ \left(\sqrt[\beta]{1-\bigodot_{i=1}^{n}\left(1-\left(((\cot^2\pi/4\cdot\Xi_i^F))^{\beta}\right)^{\beta}\right)^{\varpi_i}}\right)^{1/\beta} \end{array}\right].$$

Proof. Based on Theorem 4.11 the following results follow. □

Remark 4.17. There is a conversion that takes place when $\mho = 1$, which converts the GCOTT (α, β) SFWG into the (α, β)SFWG.

Remark 4.18. Boundness and monotonicity properties that are satisfied by GCOTT(α, β)SFWG operators.

Proof. Based on Theorem 4.4 and 4.5, the following results follow. □

Theorem 4.19. *If all* $\ell_i = \left\langle \left(((\cot^2 \pi/4 \cdot \Xi_i^T), (\cot^2 \pi/4 \cdot \Xi_i^I), (\cot^2 \pi/4 \cdot \Xi_i^F))\right), (\alpha, \beta) \right\rangle$ *are equal. Then GCOTT*(α, β)*SFWG*$(\ell_1, \ell_2, ..., \ell_n) = \ell$.

References

[1] L. A. Zadeh, Fuzzy sets, Information and control, 8(3), (1965), 338–353.

[2] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and Systems, 20(1), (1986), 87–96.

[3] R. R. Yager, Pythagorean membership grades in multi criteria decision-making, IEEE Trans. Fuzzy Systems, 22, (2014), 958–965.

[4] S. Ashraf, S. Abdullah, T. Mahmood, F. Ghani and T. Mahmood, Spherical fuzzy sets and their applications in multi-attribute decision making problems, Journal of Intelligent and Fuzzy Systems, 36, (2019), 2829–284.

[5] B.C. Cuong and V. Kreinovich, Picture fuzzy sets a new concept for computational intelligence problems, in Proceedings of 2013 Third World Congress on Information and Communication Technologies (WICT 2013), IEEE, (2013), 1–6.

[6] P. Liu, G. Shahzadi, M. Akram, Specific types of picture fuzzy Yager aggregation operators for decision-making, International Journal of Computational Intelligence Systems, 13(1), (2020), 1072–1091.

[7] W.F. Liu, J. Chang, X. He, Generalized Pythagorean fuzzy aggregation operators and applications in decision making, Control Decis. 31, (2016), 2280–2286.

[8] X. Peng, and Y. Yang, Fundamental properties of interval valued Pythagorean fuzzy aggregation operators, International Journal of Intelligent Systems, (2015), 1–44.

[9] K.G. Fatmaa, K. Cengiza, Spherical fuzzy sets and spherical fuzzy TOPSIS method, Journal of Intelligent and Fuzzy Systems, 36(1), (2019), 337–352.

[10] R.N. Xu and C.L. Li, Regression prediction for fuzzy time series, Appl. Math. J. Chinese Univ., 16, (2001), 451–461.

[11] Abu, Ibraheem. , Al-Husban, Abdallah. , J., Lejo. , J., Jamil. , Palanikumar, M., Balaji, G..(2024) Selection process real-life application for new type complex neutrosophic sets using various aggregation operators. International Journal of Neutrosophic Science, 23(4) 136 153.

[12] Selvaraj, S.. , Gharib, Gharib. , Al-Husban, Abdallah. , Al, Maha. , Lenin, K.. , Palanikumar, Murugan. , Sundareswari, K.. (2024) New algebraic approach towards interval-valued neutrosophic cubic vague set based on subbisemiring over bisemiring. International Journal of Neutrosophic Science, 23(4), 272-292.

[13] C. Sivakumar, Mowafaq Omar Al-Qadri, Abdallah shihadeh, Ahmed Atallah Al-saraireh, Abdallah Al-Husban, P. Maragatha Meenakshi, N. Rajesh, M. Palanikumar. (2024). q-rung square root interval-valued sets with respect to aggregated operators using multiple attribute decision making. Journal of, 23(3), 154-174.

[14] Abu, Ibraheem.,T., T.,Al-Husban, Abdallah.,Alahmade, Ayman.,Azhaguvelavan, S.. , Palanikumar, Murugan. (2024) Computer purchasing using new type neutrosophic sets and its extension based on aggregation operators. International Journal of Neutrosophic Science, 24(1), 171-185.

[15] Raja, K., Meenakshi, P. M., Al-Husban, A., Al-Qadri, M. O., Rajesh, N., & Palanikumar, M. (2024). Multi-criteria group decision making approach based on a new type of neutrosophic vague approach is used to select the shares of the companies for purchase. Full Length Article, 23(3), 296-96.?

[16] J., Lejo. , Damrah, Sadeq. , M., Mutaz. , Al-Husban, Abdallah. , Palanikumar, M.. (2024). Type-I extension Diophantine neutrosophic interval valued soft set in real life applications for a decision making. International Journal of Neutrosophic Science, 24(4), 151-164.

[17] Abu, Ibraheem. , Al-Husban, Abdallah. , J., Lejo. , J., Jamil. , Palanikumar, M.. , Balaji, G.. (2024). Selection process real-life application for new type complex neutrosophic sets using various aggregation operators. International Journal of Neutrosophic Science, 23(4), 136-153.

[18] Shihadeh, Abdallah. , Mahmoud, Wael. , Bataineh, Malik. , Al-Tarawneh, Hassan. , Alahmade, Ayman. , Al-Husban, Abdallah. (2024) On the Geometry of Weak Fuzzy Complex Numbers and Applications to the Classification of Some A-Curves. International Journal of Neutrosophic Science, 23(4), 369-375.

[19] Abualhomos, Mayada. , Mahmoud, Wael. , Bataineh, Malik. , Omar, Mowafaq. , Alahmade, Ayman. , Al-Husban, Abdallah. (2024). An Effective Algorithm for Solving Weak Fuzzy Complex Diophantine Equations in Two Variables. International Journal of Neutrosophic Science, 23(4), 386-394.

[20] Abualhomos, Mayada. , Atallah, Ahmed. , Bataineh, Malik. , Abu-Alkishik, Nabeela. , Al-Shbeil, Isra. , Al-Husban, Abdallah.(2024(. On The Real Inner Products and Orthogonality for Plithogenic Vector Spaces of Orders 4 and 5. International Journal of Neutrosophic Science, 24 (1), 94-103.

[21] Z. Xu, R.R. Yager, Some geometric aggregation operators based on intuitionistic fuzzy sets, Int. J. Gen. Syst. 35, (2006), 417–433.

[22] D.F. Li, Multi-attribute decision making method based on generalized OWA operators with intuitionistic fuzzy sets, Expert Syst. Appl. 37, (2010), 8673–8678.

[23] S. Zeng, W. Sua, Intuitionistic fuzzy ordered weighted distance operator, Knowl. Based Syst. 24, (2011), 1224–1232.

[24] X. Peng, H. Yuan, Fundamental properties of Pythagorean fuzzy aggregation operators, Fundam. Inform. 147, (2016), 415–446.

[25] S. Ashraf, S. Abdullah, T. Mahmood, Spherical fuzzy Dombi aggregation operators and their application in group decision making problems, J. Amb. Intell. Hum. Comput. 11, (2020), 2731–2749.

[26] K. Ullah, H. Garg, T. Mahmood, N. Jan, Z. Ali, Correlation coefficients for T-spherical fuzzy sets and their applications in clustering and multi-attribute decision making, Soft Comput. 24, (2020), 1647–1659.

[27] K. Ullah, T. Mahmood, H. Garg, Evaluation of the performance of search and rescue robots using T-spherical fuzzy hamacher aggregation operators, Int. J. Fuzzy Syst. 22, (2020), 570–582.

[28] Tansu Temel, Salih Berkan Aydemir,Yasar Hoscan, Power Muirhead mean in spherical normal fuzzy environment and its applications to multi-attribute decision-making, Complex and Intelligent Systems, (2022), 1–19.

[29] Abu, Ibraheem. , Al-Husban, Abdallah. , M., Mutaz. , A., Ahmed. , Shatarah, Amani. , Mousa, Norah.(2024) On the Characterization of Some m-Plithogenic Vector Spaces and Their AH-Substructures Under the Condition $6 \leq dimSPV \leq 10$, International Journal of Neutrosophic Science, 24(3), 258-267.

[30] Shihadeh, A., Matarneh, K. A. M., Hatamleh, R., Al-Qadri, M. O., & Al-Husban, A. (2024). On The Two-Fold Fuzzy n-Refined Neutrosophic Rings For 2? 3. Neutrosophic Sets and Systems, 68, 8-25.

[31] Abdallah Shihadeh, Khaled Ahmad Mohammad Matarneh, Raed Hatamleh, Randa Bashir Yousef Hijazeen, Mowafaq Omar Al-Qadri, Abdallah Al-Husban.(2024). An Example of Two-Fold Fuzzy Algebras Based On Neutrosophic Real Numbers, Neutrosophic Sets and Systems, 67,169-178.

[32] A., Ahmad. , Hatamleh, Raed. , Matarneh, Khaled. , Al-Husban, Abdallah. On the Irreversible k-Threshold Conversion Number for Some Graph Products and Neutrosophic Graphs. (2025). International Journal of Neutrosophic Science, 25(2), 183-196.

[33] Abubaker, A. A., Abualhomos, M., Matarneh, K., & Al-Husban, A. (2025). A Numerical Approach For The Algebra of Two-Fold. Neutrosophic Sets and Systems, 75, 181-195.

[34] Abubaker, Ahmad A, Hatamleh, Raed, Matarneh, Khaled, Al-Husban, Abdallah.(2024). On the Numerica Solutions for Some Neutrosophic Singular Boundary Value Problems by Using (LPM) Polynomials, International Journal of Neutrosophic Science,25(2),197-205.

[35] A. Rajalakshmi, Raed Hatamleh, Abdallah Al-Husban, K. Lenin Muthu Kumaran, M. S. Malchijah raj. (2025). Various neutrosophic ideals of an ordered ternary semigroups. 32 (3), PP: 400-417.

[36] Raed Hatamleh, Abdallah Al-Husban, N. Sundarakannan, M. S. Malchijah Raj. (2025). Complex cubic intuitionistic fuzzy set applied to subbisemirings of bisemirings using homomorphism. 32 (3), PP: 418-435.

[37] Hatamleh, R., Zolotarev, V. A. (2016). Triangular Models of Commutative Systems of Linear Operators Close to Unitary Operators. Ukrainian Mathematical Journal, 68(5), 791-811.

[38] Hatamleh, R. (2003). On the Form of Correlation Function for a Class of Nonstationary Field with a Zero Spectrum. Rocky Mountain Journal of Mathematics, 33(1) 159-173.

[39] X. D. Peng and J. Dai, Approaches to single-valued neutrosophic MADM based on MABAC, TOPSIS and new similarity measure with score function, Neural Computing and Applications, 29(10), (2018), 939–954.

[40] X. Lhang and L. Xu, Extension of TOPSIS to multiple criteria decision-making with (α, β)phantine Pythagorean fuzzy sets, International Journal of Intelligent Systems, 29, (2014), 1061–1078.

[41] M Palanikumar, K Arulmozhi, A Iampan, Multi criteria group decision making based on VIKOR and TOPSIS methods for Fermatean fuzzy soft with aggregation operators, ICIC Express Letters 16 (10), (2022), 1129–1138.

[42] M Palanikumar, K Arulmozhi, MCGDM based on TOPSIS and VIKOR using Pythagorean neutrosophic soft with aggregation operators, Neutrosophic Sets and Systems, (2022), 538–555.

[43] M Palanikumar, S Broumi, Square root (α, β) Diophantine neutrosophic normal interval-valued sets and their aggregated operators in application to multiple attribute decision making, International Journal of Neutrosophic Science, 4, (2022).

[44] M Palanikumar, K Arulmozhi, Novel possibility Pythagorean interval valued fuzzy soft set method for a decision making, TWMS J. A and Eng. Math., 13(1), (2023), 327–340.

[45] M Palanikumar, K Arulmozhi, Novel possibility Pythagorean interval valued fuzzy soft set method for a decision making, TWMS J. A and Eng. Math., 13(1), (2023), 327–340.

[46] M Palanikumar, N Kausar, H Garg, A Iampan, S Kadry, M Sharaf, Medical robotic engineering selection based on square root neutrosophic normal interval-valued sets and their aggregated operators, AIMS Mathematics, 8(8), (2023), 17402–17432.

[47] Hatamleh, R.,Zolotarev,V.A. (2015). On Model Representations of Non-Selfadjoint Operators with Infinitely Dimensional Imaginary Component. Journal of Mathematical Physics, Analysis, Geometry,11(2), 174-186.

[48] Hatamleh, R., Zolotarev, V. A. (2014).On Two-Dimensional Model Representations of One Class of Commuting Operators. Ukrainian Mathematical Journal, 66(1), 122-144.

[49] R.Hatamleh,V.A. Zolotarev.(2017).On the abstract inverse scattering problem for traces class pertubations. Journal of Mathematical Physics, Analysis, Geometry 13(1),1-32.

[50] Abdallah Shihadeh, Khaled Ahmad Mohammad Matarneh, Raed Hatamleh, Randa Bashir Yousef Hijazeen, Mowafaq Omar Al-Qadri, Abdallah Al-Husban.(2024). An Example of Two-Fold Fuzzy Algebras Based On Neutrosophic Real Numbers, Neutrosophic Sets and Systems, 67,169-178.

[51] Abdallah Shihadeh, Khaled Ahmad Mohammad Matarneh, Raed Hatamleh, Mowafaq Omar Al-Qadri, Abdallah Al-Husban, On The Two-Fold Fuzzy n-Refined Neutrosophic Rings For $2 \leq n \leq 3$, Neutrosophic Sets and Systems, Vol. 68, 2024, 8-25.