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Accurate Domination in Fuzzy Digraphs using Strong Arc

[1]S.Sureka, [2]Dr.G.Suganthi, [3]Vallabh Shinde, [4]S. Ramya, [5]A.marydayana, [6]R.Poornavalli, [7]Dr.S.Vasantha Gowri

[1]Research scholar, Department of Mathematics,SRM Institute of Science and Technology, Trichy campus, Trichy - 621105. [1]Email id : sureka24122001@gmail.com

[2]Assistant Professor,Department of Mathematics,Sona college of Technology, Salem -636005. [2]Email id: suganthig@sonatech.ac.in

[3]Assistant Professor, Nutan Maharashtra Institute of Engineering and Technology, (NMIET),Talegaon.* [3]Email id: vallabh.shinde@nmiet.edu.in

[4]Associate Professor,Department of Mathematics Division,Saveetha Institue of Medical and Technical sciences,Thandalam, Tamilnadu. [4]Email id: ramyadheeru10@gmail.com

[5]Assistant Professor, Department of Mathematics, Erode Sengunthar Engineering College ,Perundurai, Erode-638057. [5]Email Id:Dayanalilly@gmail.com

[6]Assistant professor, Department of Mathematics,SNS college of Technology,Coimbatore [6] Email ID: poorna.r.math@snsct.org

[7]Assistant Professor,Department of Mathematics,SRM Institute of Science and Technology, Trichy campus, Trichy - 621105. [7]Corresponding Author Email id : vasanthagowri.s@ist.srmtrichy.edu.in

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Abstract:
In this paper, we introduce the concept of accurate dominating set in fuzzy digraphs using strong arc and also an accurate domination number of a fuzzy digraph. In a fuzzy digraph, a subset D of V is an accurate fuzzy dominating set of a fuzzy digraph if every node σD(v) Є V-D is not dominated by atleast one node σD(u) Є D with same cardinality |D|. The accurate domination number of a fuzzy digraph is the minimum cardinality of an accurate dominating set which is denoted by γfa(GD).The upper domination number Γfa(GD) is the maximum cardinality of an accurate dominating set. We prove some results on accurate dominating set of a fuzzy digraph. Also we have modified folyd warshall’s Algorithm to fuzzy domination digraphs for finding an accurate dominating set.

Keywords: Fuzzy digraph, Strong arc, Dominating set, Accurate dominating set, Accurate domination number.

1. INTRODUCTION

Graph theory was introduced by the Swiss Mathematician Leonhard Euler in 1736[1]. Euler is often credited as the founder of graph theory due to his work on the seven bridges of Konigsberg problem. Graph theory is the study of graphs which are mathematical structures that represent a function by connecting vertices. The concept of domination in graph theory was introduced in the year 1950 by American Mathematician Claude Berge and Ore[2,3]. Berge is widely regarded for his work in graph theory, Combinatorics and published extensively on these subjects. The specific idea of domination where a set of vertices in a graph has the property that every other vertex is adjacent to atleast one vertex in the set. Domination in graph theory has expanded into many variations such as independent domination, total domination, each adding specific conditions or constraints to the original concept.

The concept of independent domination in graph theory was introduced by Cockayne and Hedetniemi to explore a set of vertices that is both an independent set and a dominating set[4]. Research into total domination was significantly advanced in the 1970’s and 1980’s with contributions from E.J. Cockayne, RM.Dawes and S.T.Hedetniemi who formalized and studied various properties and applications of these concepts in graphs. The concept of an accurate domination in graphs was presented by Kulli and Kattimani[5]. Accurate domination is a relatively recent addition to the study of domination in graphs and is a variation on traditional domination concepts. Zadeh [8] introduced the concept of fuzzy set theory in 1965. The concept of fuzzy graph was introduced by A.Rosenfeld in 1975 [8]. Rosenfeld extended classical graph theory by incorporating fuzzy set theory which was introduced by zadeh. Fuzzy graphs apply the principles of fuzzy sets to graphs allowing the edges and nodes to have degree of membership between 0 and 1. This approach allows for modelling uncertain or imprecise relationships. A.Somasundaram and N.Somasundaram [9] originated the concept of domination in fuzzy graphs using effective arcs. Nagoor Gani and Chandrasekaran [10] discussed domination in fuzzy graphs using strong arcs. Domination, Independent domination and Irredundance in fuzzy graphs using strong arc was discussed by Nagoor Gani and Vadivel [11]. C.Y.Ponnayan and A.Selvam presented the concept of accurate domination in fuzzy graphs using strong arc[12]. The concept of domination in fuzzy digraphs was discussed by G.Nirmala and M.Sheela [13]. The Folyd Warshall Algorithm was published by Robert Folyd in 1962 [14]. It is essentially the same as algorithm previously by Bernard Roy in 1959 and also by Stephen warshall in 1962 for find the transitive closure of a graph.

2.PRELIMINARIES

Definition 2.1

A graph G = (V,E) consists of a set V of vertices (also called nodes) and a set E of edges (also called arcs).

Definition 2.2

If an edge connects to a vertex we say the edge is incident to the vertex and say the vertex is an end point of the edge.

Definition 2.3

A simple graph is a graph with no loop edges or multiple edges. Edges in a simple graph may be specified by a set $\{v_i,v_j\}$ of the two vertices that the edge makes adjacent. A graph with more than one edge between a pair of vertices is called a multigraph with loop edges is called a pseudograph.

Definition 2.4

A directed graph is a graph in which the edges may only be traverses in one direction. Edges in a simple directed graph may be specified by an ordered pair (V_i,V_j) of the two vertices that the edge connects.

We say that V_i is adjacent to V_j and V_j is adjacent from V_i.

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Definition 2.5

A graph H is a subgraph of a graph G if all vertices and edges in H are also in G.

Definition 2.6

A fuzzy graph is G: (V,σ,μ), V is the vertex set, σ: V→[0,1] and μ: V×V→[0,1] where for all p,q Є V and we have μ(p,q) ≤ σ(p) Λ σ(q).

Definition 2.7

A fuzzy graph H = (τ, ρ) is called a fuzzy subgraph of G if τ (v_i) ≤ σ (v_i) for all v_i Є V and ρ (v_i,v_j) ≤ μ (v_i,v_j) for all v_i,v_j Є V.

Definition 2.8

The underlying crisp graph of a fuzzy graph G = (σ,μ) is denoted by G* = (σ*,μ*) where σ* -= {v_i Є V / σ (v_i) > 0 } and μ* = {(v_i,v_j) Є V×V / μ (v_i,v_j) is a strong arc}.

Definition 2.9

Let G be a fuzzy graph. Let u and v be two distinct nodes of G. We say that u dominates v if (u,v) is a strong arc.

Definition 2.10

A subset D of V is called a dominating set of G if for every v Є V-D, there exists u Є D such that u dominates v. The domination number γ(G) of a fuzzy graph G is the minimum cardinality of a fuzzy dominating set in G.

Definition 2.11

An arc (u,v) is said to be a strong arc if $\mu(u,v) \geq \mu^{\infty}(u,v)$. If μ (u,v) = 0 for every

v Є V, then u is called as an isolated node.

Definition 2.12

A fuzzy digraph $G_D = (\sigma_D, \mu_D)$ is a pair of two functions $\sigma_D : V \rightarrow [0,1]$ and $\mu_D : A \rightarrow [0,1]$ such that $\mu_D(x,y) \leq \sigma_D(x) \wedge \sigma_D(y)$ for all x,y Є V.

Definition 2.13

An arc (x,y) of a fuzzy digraph is called an effective arc if $\mu_D(x,y) = \sigma_D(x) \wedge \sigma_D(y)$.

Definition 2.14

An arc (u,v) is said to be a strong arc or strong edge of a fuzzy digraph if

$\mu_D(u,v) \geq \mu_D{}^{\infty}(u,v)$ and node v is said to be a strong neighbour of u.

Definition 2.15

Let x,y Є V. The vertex $\sigma_D(x)$ dominates $\sigma_D(y)$ in a fuzzy digraph G_D if $\mu_D(x,y)$ is a strong arc.

Definition 2.16

A subset D of V is a fuzzy dominating set of G_D if every node σ_D (v) Є V-D is dominated by atleast one node σ_D(u) Є D.

Definition 2.17

The fuzzy domination number γ_f (G_D) of a fuzzy digraph is the minimum cardinality of a fuzzy dominating set in G_D.

Definition 2.18

In fuzzy digraph, an isolated vertex refers to a vertex that has no incoming and outgoing edges, meaning that its membership degree in any relation or connection to other vertices is zero.

2. ACCURATE DOMINATION IN FUZZY DIGRAPHS

In this section we define accurate dominating set and accurate domination number in fuzzy digraphs using strong arc with suitable examples. We also discuss some results on accurate dominating set of a fuzzy digraph.

Definition 3.1

A subset D of V is an accurate fuzzy dominating set of a fuzzy digraph G_D if every node σ_D(v) ЄV-D is not dominated by atleast one node σ_D(u)ЄD with same cardinality |D|. An accurate fuzzy domination number γ_{fa} (G_D) of a fuzzy digraph is the minimum cardinality of an accurate fuzzy dominating set in G_D. The upper domination number Γ_{fa} (G_D) is the maximum cardinality of an accurate dominating set.

Example 3.2

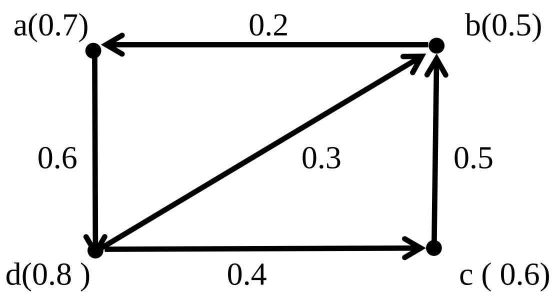

Figure 1

D = {a,c,d} is an accurate dominating set, since V-D is not dominated by atleast one node σ_D (u) Є D with cardinality |D|.

Theorem 3.3

Every accurate dominating set of a fuzzy digraph is a dominating set of a fuzzy digraph.

Proof :

Let D be an accurate dominating set of a fuzzy digraph.

By definition, V-D is not dominated by atleast one node in D with cardinality |D|.

Therefore V-D is adjacent to atleast one vertex in D.

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This implies V-D is dominated by atleast one node in D.

Hence D is a dominating set of fuzzy digraph.

Theorem 3.4

For any fuzzy digraph $\gamma_f (G_D) \leq \gamma_{fa}(G_D)$

Proof :

Domination number of a fuzzy digraph is a minimum cardinality of a fuzzy dominating set.For any fuzzy digraph, an accurate dominating set is a dominating set of a fuzzy digraph.

This implies a minimum accurate dominating set is also a dominating set of a fuzzy digraph. Thus $\gamma_f (G_D) \leq \gamma_{fa}(G_D)$.

Theorem 3.5

In a fuzzy digraph, if there exists an isolated vertex then a minimal dominating set of a fuzzy digraph is an accurate dominating set of a fuzzy digraph.

Proof :

Assume that D is a dominating set of a fuzzy digraph.

Let u Є V be an isolated vertex of a fuzzy digraph which has no incoming and outgoing edges.

This means it does not dominate or dominated by any other vertices.

Therefore V-D is not dominated by u with cardinality |D|.

This implies D is an accurate dominating set of a fuzzy digraph.

Hence the result.

Theorem 3.6

For any fuzzy digraph, if D be an accurate dominating set of a fuzzy digraph then V-D is need not be accurate dominating set of a fuzzy digraph.

Proof :

Let G_D be a fuzzy digraph. Assume that u Є D ia an accurate dominating set of a fuzzy digraph. Then V-D is not dominated by atleast one vertex in u Є D with cardinality |D|.

Case (i):

If V-D has no strong neighbour then V-D cannot be a dominating set of a fuzzy digraph. So it cannot be an accurate dominating set of a fuzzy digraph.

Case (ii):

Suppose it has atleast one strong neighbour which dominates u, then V-D is a

dominating set of a fuzzy digraph and we have $|V\text{-}D| \neq |D|$. Hence it can be an accurate dominating set of a fuzzy digraph.

4. FOLYD WARSHALL'S ALGORITHM FOR FUZZY DOMINATION

This section presents modified Folyd Warshall's algorithm for domination in fuzzy digraphs. The Folyd Warshall's algorithm for fuzzy domination is a graph analysis algorithm that can be used to find an accurate dominating set of a fuzzy digraph. We discuss this algorithm with suitable example.

4.1 ALGORITHM

Step 1: Let us fix the initial matrix W_0 through arc weight.

Step 2: We can produce W_k from W_{k-1}.

Step 3: List the locations $v_1,v_2,\ldots$.. in column k of W_{k-1} where the entry is non zero and the locations $v_1,v_2,\ldots$in row k of W_{k-1} where the entry is non zero.

Step 4: Frame the possible combinations of non zero elements of column k with non zero elements of row k.

Step 5: If (v_i,v_j) is a strong arc and i = j put 1 otherwise 0.

Step 6: Proceed till W_n.

Step 7: We reach the row having maximum number of 1's which is a dominating set of a fuzzy digraph.

Step 8: Finally from the dominating set of a fuzzy digraph, a vertex u Є V-D is not dominated by atleast one vertex in D with same cardinality |D| is called as an accurate dominating set of a fuzzy digraph.

Example 4.2

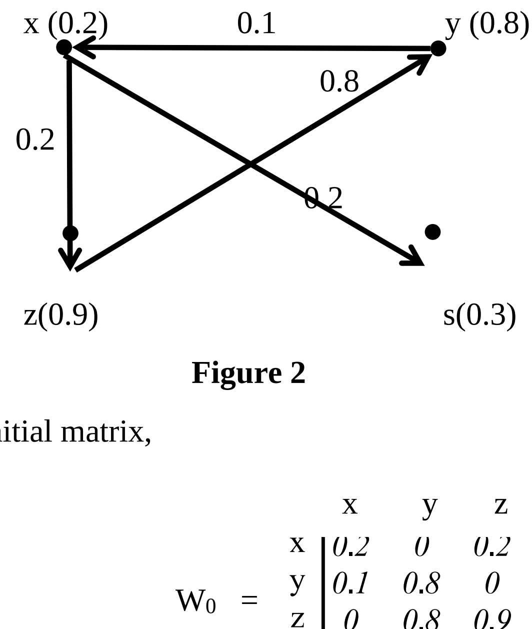

Figure 2

Consider the initial matrix,

$$W_0 = \begin{array}{c} \\ x \\ y \\ z \\ s \end{array} \begin{array}{c} \begin{array}{cccc} x & y & z & s \end{array} \\ \begin{bmatrix} 0.2 & 0 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0 & 0 \\ 0 & 0.8 & 0.9 & 0 \\ 0 & 0 & 0 & 0.3 \end{bmatrix} \end{array}$$

R(1) = { (x,x), (x,z), (x,s), (y,x), (y,z), (y,s) }

$$W_1 = \begin{array}{c} \\ x \\ y \\ z \\ s \end{array} \begin{array}{c} \begin{array}{cccc} x & y & z & s \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0.8 & 0.9 & 0 \\ 0 & 0 & 0 & 0.3 \end{bmatrix} \end{array}$$

R(2) = {(y,y), (z,y)}

$$W_2 = \begin{array}{c} \\ x \\ y \\ z \\ s \end{array} \begin{array}{c} \begin{array}{cccc} x & y & z & s \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0.9 & 0 \\ 0 & 0 & 0 & 0.3 \end{bmatrix} \end{array}$$

R(3) = { (x,y), (x,z), (z,y), (z,z) }

$$W_3 = \begin{array}{c} \\ y \\ z \\ s \end{array} \begin{array}{c} \begin{array}{cccc} x & y & z & s \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0.3 \end{bmatrix} \end{array}$$

R(4) = {(s,s)}

$$W_4 = \begin{array}{c} \\ y \\ z \\ s \end{array} \begin{array}{c} \begin{array}{cccc} x & y & z & s \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

D = {x,z} is a dominating set of a fuzzy digraph and it also a accurate dominating set of a fuzzy digraph.

5.CONCLUSION

In this work, we have introduced the concept of accurate domination and accurate domination number in fuzzy digraphs using strong arc. Some results on accurate dominating set of a fuzzy digraph are discussed. Also, We found modified warshall's algorithm for fuzzy domination to find the accurate dominating set of a fuzzy digraphs with suitable example.

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