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Optimizing Phasor Measurement Units Placement in Power Networks Using Betweenness Centrality

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Abstract:

Introduction: Centrality concepts play a crucial role in determining the significance of a vertex based on its necessity. A network always prefers the shortest path, also known as the geodesic path, for information transmission between nodes. Betweenness centrality, which reduces geodesic distance, makes it easier to strategically position phasor measurement units (PMUs) in a power network. We present a research paper that describes an algorithm specifically designed for PMU placement in power networks. The algorithm's performance is assessed by running it on the widely used IEEE 14 and IEEE 30 buses, allowing for comparison with results obtained using other methodologies.

Keywords: Power network, Power domination, centrality, betweenness, centrality

1. Introduction

The importance of graph theory in power network optimization is that it allows the effective control of the complicated systems that result in transmitting electric power from producers to consumers. The networks can be described as graphs, with nodes representing different entities such as generators and substations and edges representing connections such as transmission lines. A few key areas have been outlined here: optimal power flow, fault detection and restoration, distribution network optimization, and smart grid communication and control. Graph theory plays an important role in helping engineers deal with these issues.

In electric power systems, phasor measurement units (PMUs) are specialized tools for monitoring and analysing the dynamic behaviours of a system in real time. They measure the voltage and current phasors at specific points on the grid, providing synchronized time-stamped information essential for efficient operation, control and protection. Key components and functions of a Phasor Measurement Unit include Phasor Measurement, Wide Area Monitoring, Time Stamping, Power System Stability, and Control.

Haynes et al. [8] first introduced the dominance problem in electrical networks. Kirchhoff's law and Ohm's law are both used to derive this algorithm. Within a graph, dominance refers to a subset *S* of a vertex set V such that every vertex not in *S* is adjacent to at least one vertex in *S*. The same can be defined for power domination in power networks. A PMU-observed vertex is one whose voltage and phase angle have been measured by PMU. The paper uses concepts of centrality to determine the best positions for placing PMUs on graphs representing power networks.

Centrality, as a metric in graph theory, is crucial in measuring the importance or influence of nodes within a graph. This metric provides invaluable information that helps us understand real world systems such as social networks, biological networks, and transportation networks. Centrality concepts used in the analysis of graphs include Eigen vector centrality, Closeness centrality, Degree centrality, Betweenness centrality, and Katz centrality. Networks often employ the shortest path for information transmission through nodes, with high centrality regulating data communication among other nodes. The inefficiency of degree centrality that uses the node's degrees as a basis for placing PMUs on the power network graphs was shown by Baldwin et al. [3]. They recommended using concepts related to spanning trees as an alternative method instead. In contrast to that notion of betweenness centrality is computed based on geodesic paths and identifies such nodes that serve as bridges or intermediaries between different parts of a graph. Highly centralized betweenness facilitates interactions and communication across diverse groups or clusters.

The position of a PMU at a power node makes it interact with all other vertices in the surrounding area. The optimal placement of PMUs within the power network can be effectively determined through minimizing the geodesic distance, which is based on the betweenness centrality concept. Vertices are said to be neighbours when their geodesic distance is one. Gago et.al [7] provides a mathematical basis, interconnections and restrictions of betweenness centrality in their paper. It is possible to optimize many scenarios using this method, such as dynamic water distribution networks [12], vast sparse networks [13], extensive social networks [9], and identification of protein complexes [1]. In 2019, the discourse on cost optimization focused on the use of dominance centrality over betweenness centrality for PMU placement [5].

In this paper, a methodology is introduced that represents an electric network as a power network graph. To tackle the challenges of PMU placement in the power network, the paper utilizes betweenness centrality. The study is summarized in Section 1, while Section 2 provides an explanation of graph theory and the technical terms used in the paper. Section 3 presents an algorithm for PMU placement within a power network. The algorithm's performance is evaluated on standard IEEE 6, IEEE 14, and IEEE 30 buses in Section 4. Section 5 compares the outcomes of the proposed method with those of other techniques. Finally, Section 6 concludes the study and discusses its broader implications.

2. Graph theory basic definitions

In a graph G=(V,E), V is a non-empty set of vertices and E is a set of edges contained in V X V. The path consists of n vertices. A cycle is defined as having the same starting and ending vertices. The graph's order corresponds to the number of vertices, whereas its size represents the number of edges. A graph is said to be connected when every pair of vertices in it has a path connecting them. If there is no such path, the graph is considered disconnected. The minimum length of a path between two vertices is referred to as the distance. The geodesic path denotes the shortest path between any two nodes, and its length is termed the geodesic number. All the terms mentioned are defined according to [4].

2.1. Power domination integrity

The graph that corresponds to a power network is defined as a power network. The power domination of a power network is defined as a subset S of the vertex set V such that every vertex and every edge in that power network is observed by S. The minimum cardinality of a power dominating set is known as the power dominance number γ_P. In [8], the rules to find the observed vertices are provided as follows:

1. Any vertex that is incident to an observed edge is observed.
2. Any edge joining two observed vertices is observed.
3. If a vertex is incident to a total of k >1 edges and if k –1 of these edges are observed, then all k of these edges is observed.

2.2 Centralities

In social networks, the centrality concepts are employed to assess the importance of a vertex. The degree centrality, introduced by Freeman [6], relies on the degree of a vertex in a graph. Closeness centrality [6] of a vertex *u* is determined by the reciprocal of the sum of the shortest paths from *x* to all other vertices. Furthermore, Freeman [6] defined betweenness centrality as the amount of information that passes through a vertex..

$$\sum_{l\neq m\neq n} \frac{G_{lnm}}{G_{lm}}$$

where G_{lnm} denotes the number of geodesic paths from vertex i to vertex m via vertex n and G_{lj} denotes the number of geodesic paths from the vertex l to vertex m.

3. Betweeness centrality in PMU placing

An observed vertex is a node that can be measured by a PMU. An observed network is one where all nodes are observed. A power dominating set P is a collection of PMU placed nodes where each node is either in P or observes at least one vertex in P. The power domination number of a power graph is the minimum number of power dominating sets required.

To place a PMU in a power network the following algorithm is used:

1. Find the betweenness centrality of each nodes and arrange it in descending order by centrality.
2. Place the PMU in the node which has highest centrality.
3. Apply rule 1, 2 and 3 and check all the vertices are observed.
4. If all the vertices are observed, then the set of PMUs placed nodes are required power dominating set. Otherwise, repeat step 2.

4. Implementation of proposed method in IEEE buses

In this section, we implement the above algorithms in IEEE buses.

4.1. **IEEE 6 bus**

The line diagram of IEEE bus 6 is given in Fig. 1 and The power network diagram of IEEE 6 bus is given in the Fig 2.

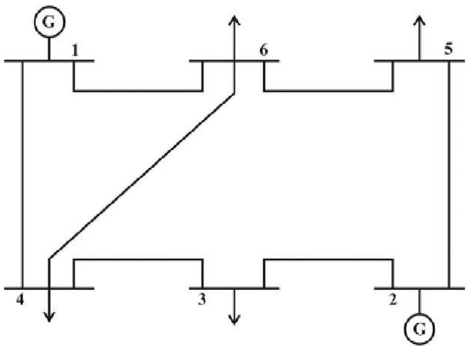

Figure 1: Line Diagram of IEEE 6 bus system

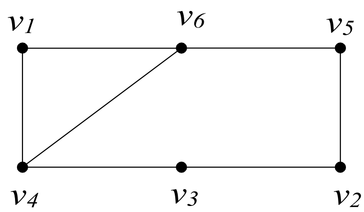

Figure 2: IEEE 6 bus - Power network graph

The table 1 shows the betweenness centrality (BC) and degree of all vertices in the IEEE 6 power network graph. The vertices v_4 and v_6 exhibit the highest betweenness centrality among the given vertices. Therefore, it is recommended to position the first PMU at either node v_4 or v_6. Since all the vertices are already observed, placing one PMU at either v_4 or v_6 would suffice.

Table 1: Calculation of edge integrity of a fuzzy graph

Vertex	**BetweenessCentrality**	**Placement**
v_1	0	-
v_2	1	-
v_3	1.5	-
v_4	2.5	1
v_5	1.5	-
v_6	2.5	-

4.2 IEEE 14 bus

Given in Fig.3 is the line diagram for the IEEE 14 bus system and the power network graph of IEEE 14 bus is given in Fig.4.

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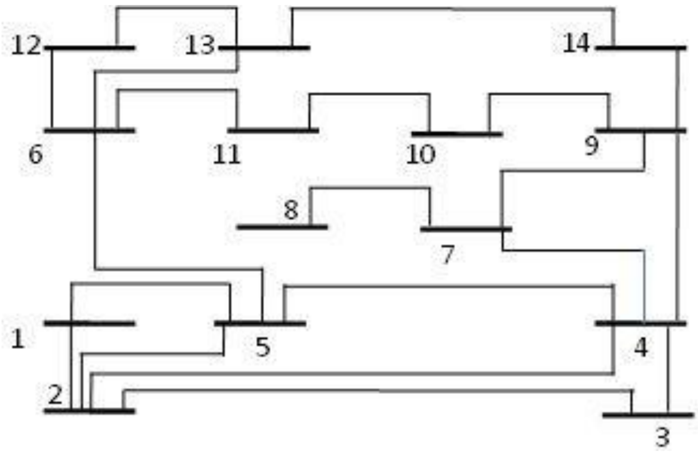

. Figure 3: Line diagram of IEEE 14 bus

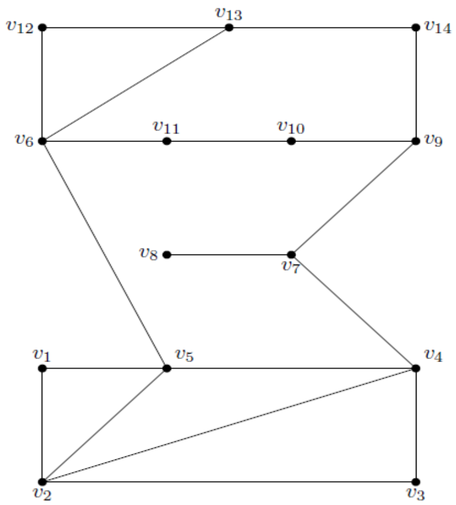

Figure 4: IEEE 14 bus - Power Network Graph

Table 2: IEEE 14 bus BC values

Vertex	Betweeness Centrality	Placement	Vertex	Betweeness Centrality	Placement
v_1	0	-	v_8	0	-
v_2	5.833	-	v_9	21	*2*
v_3	0	-	v_{10}	4.667	-
v_4	24.5	-	v_{11}	3.667	-
v_5	21	*1*	v_{12}	0	-
v_6	20	*3*	v_{13}	5.667	-
v_7	12	-	v_{14}	6.667	-

According to the data provided in table 2, vertex v_4 has the highest betweenness centrality. Therefore, the first PMU should be placed in vertex v_4. This PMU will observe the vertices v_2, v_3, v_5, v_7 and v_9, as all of these vertices are connected to vertex v_4. However, since there are still some vertices that are not observed, the next PMUs should be placed in the vertices with the next highest betweeness centrality values. Hence, the next PMUs should be placed in vertices v_5

and v_9. However, it is important to note that these two vertices are already observed by vertex v_4 and the distance from the newly placed PMU to these vertices is not v_2. Therefore, the PMU at vertex v_4 should be removed. At this point, the PMUs should only be placed in vertices v_5 and v_9. However, vertices v_{12} and v_{13} are still not observed. Therefore, the next placement should be in vertices v_6, which has the next highest betweenness centrality value. As a result, the PMU placed in vertex v_5 will observe the vertices v_1, v_2, v_4, and v_5. The PMU placed in vertex v_9 will observe the vertices v_7, v_9, v_{10}, and v_{14}. Lastly, the PMU placed in vertex v_6 will observe the vertices v_6, v_{11}, v_{12}, and v_{13}. By following rules 2 and 3, the remaining vertices v_3 and v_8 are observed. Consequently, all the vertices in the power network graph are observed. By placing 3 PMUs in vertices v_5, v_6, and v_9, the network is fully observed.

4.3 IEEE 30 bus

The line diagram of IEEE 30 is given in Fig. 5.

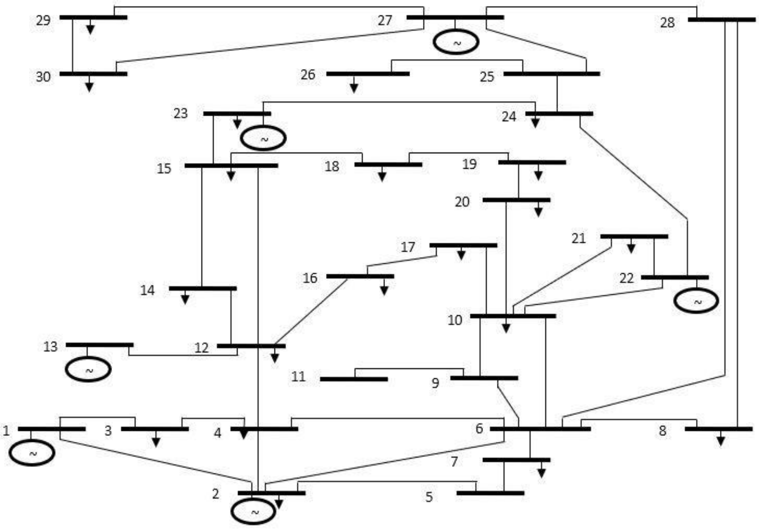

Figure 5: Line diagram of IEEE 30 bus

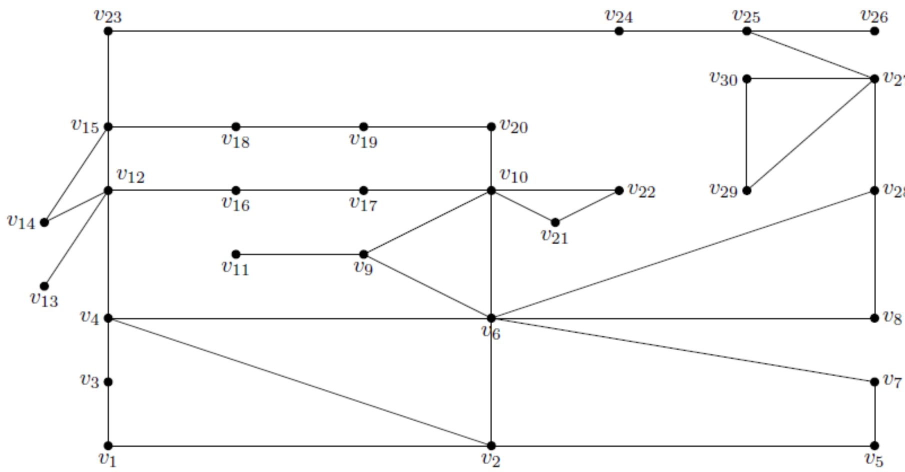

Figure 6: IEEE 30 bus - Power network graph

Table 3: IEEE 30 bus B C values

Vertex	Betweeness Centrality	Placement	Vertex	Betweeness Centrality	Placement
v_1	1	-	v_{16}	10.417	-
v_2	40.5	-	v_{17}	15.917	-
v_3	4	-	v_{18}	11.417	-
v_4	89.75	3	v_{19}	11.417	-
v_5	1	-	v_{20}	26.25	-
v_6	176.583	1	v_{21}	0	-
v_7	8.5	-	v_{22}	34.917	-
v_8	0	-	v_{23}	31.25	-
v_9	28	-	v_{24}	56.417	6
v_{10}	115.667	2	v_{25}	48.833	-
v_{11}	0	-	v_{26}	0	-
v_{12}	87.5	4	v_{27}	76.833	5
v_{13}	0	-	v_{28}	72.833	-
v_{14}	0	-	v_{29}	0	-
v_{15}	54	-	v_{30}	0	-

According to the information provided in table 3, we will begin by placing the first PMU in vertex v_6, which will observe the vertices v_2, v_4, v_6, v_7, v_8, v_9, v_{10}, and v_{28}. Additionally, based on rule 1 and 2, vertex v_{11} will also be observed. Next, we will place the second and third PMUs in vertices v_{10} and v_4, respectively, in order to observe the vertices v_3, v_4, v_{10}, v_{12}, v_{17}, v_{20}, v_{21}, and v_{22}. Following rule 1 and 2, vertices v_1 and v_5 will also be observed. For the fourth and fifth PMUs, we will place them in vertex v_{12} and v_{27} to measure the values in vertices v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{25}, v_{27}, v_{28}, v_{29} and v_{30}. At this stage, vertex v_{26} will also be observed based on rule 1 and 2. Since all the vertices adjacent to vertex v_{28} have already been observed, placing any additional PMU in this location will not be effective. Moving on to the next maximum betweenness value, we will place a PMU in vertex v_{24} and another in vertex v_5 to observe the vertices v_{18}, v_{23}, and v_{24}. Following rule 1 and 2, vertex v_{19} will also be observed. Therefore, with a total of 7 PMUs, we will be able to observe all the vertices in the power network.

5. Results and Discussions

To tackle the issue of high costs, it is essential to reduce the number of PMUs and implement measures originating from the vertices. There are several well-known techniques for calculating the number of PMUs and determining their suitable placements. By comparing the outcomes of the proposed method with other popular methods like power domination integrity [11], topology transformation [10] and Immunity genetic algorithm [2] the results can be organized in a tabulated manner.

Table 4: Number of PMUs in various methods

Method	IEEE 14	IEEE 30
Betweenness Centrality method	3	7
Power domination method [11]	3	7
Topology transformation method [10]	3	7
Immunity Genetic algorithm [2]	3	7

6. Conclusion

Saravanan et al. [11] proposed a methodology involving two distinct steps to determine the minimum number of PMUs and their respective placements. However, a more efficient approach can be achieved by utilizing Betweenness centrality. By calculating the betweenness centrality for all nodes and arranging the PMU placement from highest to lowest, this task can be effectively completed. The results obtained through this technique are consistent with those obtained through other methods, as indicated in Table 4.

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