Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 2s (2025)

Heat Conduction in a Square Plate Involving Multivariable H-Function

Dr.Nitesh Kumar Sahu
Lecturer, Government Polytechnic College, Betul (M.P.). sahunitesh20@gmail.com

Article History:

Received: *14-09-2024*

Revised: *23-10-2024*

Accepted: *02-11-2024*

Abstract:

A rigorous mathematical paradigm incorporating the multivariable H-function is devised to elucidate heat conduction phenomena in square plates. This innovative framework facilitates the derivation of a precise temperature field model, thereby enabling a comprehensive examination of the interplay between plate geometry and heat propagation characteristics. The analytical solution is substantiated through meticulous numerical simulations, underscoring the multivariable H-function's exceptional capability in capturing intricate heat conduction dynamics under disparate boundary conditions. This investigation substantially enhances the theoretical foundations of thermal analysis in square plates, unlocking novel avenues for interdisciplinary applications in engineering and physical sciences.

Keywords: Generalized hypergeometric function,H-function, Multivariable H-function, Heat conduction, Square plate geometry.

1.Introduction :-

Gauss hypergeometric function ${}_2F_1$ [a, b; c; z] has been generalized by the q parameters of the nature of c. This ensuring series

$$ {}_pF_q \left(\begin{matrix} a_1, \ldots, a_p; \\ \\ b_1, \ldots, b_q; \end{matrix} \right) z = \sum_{n=0}^{\infty} \frac{(a_1)n \ldots\ldots (a_p)n \quad z^n}{(b_1)n \ldots\ldots (b_q)n \quad n!} $$

$$ = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p} (a_i)n \, z^n}{\prod_{j=1}^{q} (b_j)n \;\; n!} \qquad (1) $$

is known as the generalized hypergeometric series and the function ${}_pF_q$ is called generalized hypergeometric function of variable z. ${}_pF_q$ is not defined if any denominator parameter b_q is a negative integer or zero. If any numerator parameter a_p is zero or a negative integer, the series terminates.

The class of the hypergeometric series and functions considered above are of single variable. The great success of the theory of hypergeometric series in one variable has stimulated the development of corresponding theory in two and more than two variables.

It was Appell (1880), who for the first time introduced the four series F_1, F_2, F_3, F_4 in two variables. Horn (1931), while giving a general definition for the double power series, constructed ten more hypergeometric functions viz. G_1 to G_3 and H_1 to H_7 and 13 confluent out of these 10 functions. Thus, there are 34 distinct convergent hypergeometric series of two variables as shown by Horn. In 1954, Saran completed Lauricella's series of hypergeometric function of three variables by defining two functions F_E, F_F, F_G, etc.

In recent research work, the double hypergeometric function has been generalized by increasing the number of parameters and the number of variables. Moreover, G and H-function also have been generalized by increasing the number of variables, in terms of contour integral.

The Meijer [2] G-function of one variable was defined in terms of Mellin-Barnes type integrals as follows:

$$G^{m,n}_{p,q}\left[x\Big|^{(a_j,1)_{1,p}}_{(b_j,1)_{1,q}}\right] = (1/2\pi i)\int_L \theta(s)\, x^s\, ds \qquad (2)$$

where $i = \sqrt{(-1)}$,

$$\theta(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j - s)\ \prod_{j=1}^{n}\Gamma(1 - a_j + s)}{\prod_{j=m+1}^{q}\Gamma(1 - b_j + s)\ \prod_{j=n+1}^{p}\Gamma(a_j - s)}$$

an empty product is interpreted as unity; $0 \le m \le q$, $0 \le n \le p$, and the parameters are such that no pole of $\Gamma(b_j - s)$ $(j = 1, \ldots, m)$ coincide with any pole of $\Gamma(1 - a_j + s)$ $(j = 1, \ldots, n)$.

A more general function than the G-function is the H-function introduced by Fox [3] in the form of Mellin-Barnes type integral and symbolically we denote it by

$$H^{m,n}_{p,q}\left[x\Big|^{(a_j,\alpha_j)_{1,p}}_{(b_j,\beta_j)_{1,q}}\right] = (1/2\pi i)\int_L \theta(s)\, x^s\, ds \qquad (3)$$

where $i = \sqrt{(-1)}$,

$$\theta(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j - \beta_j s)\ \prod_{j=1}^{n}\Gamma(1 - a_j + \alpha_j s)}{\prod_{j=m+1}^{q}\Gamma(1 - b_j + \beta_j s)\ \prod_{j=n+1}^{p}\Gamma(a_j - \alpha_j s)}$$

x is not equal to zero and an empty product is interpreted as unity; p, q, m, n are integers satisfying $0 \le m \le q$, $0 \le n \le p$, α_j $(j = 1, \ldots, p)$, β_j $(j = 1, \ldots, q)$ are positive numbers and a_j $(j = 1, \ldots, p)$, b_j $(j = 1, \ldots, q)$ are complex numbers. L is a suitable contour of Barnes type such that poles of $\Gamma(b_j - \beta_j s)$ $(j = 1, \ldots, m)$ lie to the right and poles of $\Gamma(1 - a_j + \alpha_j s)$ $(j = 1, \ldots, n)$ to the left of L. These assumptions for the H-function will be adhered to through out this research work.

According to Braakasma,

$$H_{p,q}^{m,n}\left[x\Big|_{(b_j,\beta_j)_{1,q}}^{(a_j,\alpha_j)_{1,p}}\right] = O(|x|^{\alpha}) \text{ for small } x,$$

where $\sum_{j=1}^{p} \alpha_j - \sum_{j=1}^{q} \beta_j \leq 0$ and $\alpha = \min R(b_h/\beta_h)$ $(h = 1, .., k)$

and

$$H_{p,q}^{m,n}\left[x\Big|_{(b_j,\beta_j)_{1,q}}^{(a_j,\alpha_j)_{1,p}}\right] = O(|x|^{\beta}) \text{ for large } x,$$

where

$$\sum_{j=1}^{n} \alpha_j - \sum_{j=n+1}^{p} \alpha_j + \sum_{j=1}^{m} \beta_j - \sum_{j=m+1}^{q} \beta_j \equiv A > 0,$$

$$\sum_{j=1}^{p} \alpha_j - \sum_{j=1}^{q} \beta_j < 0$$

$|\arg x| < \frac{1}{2} A\pi$ and $\beta = \max R[(a_j - 1)/\alpha_j]$ $(j = 1, .., n)$

Recently Mittal and Gupta [4, p. 117] has given the following notation of the H-function of two variables as:

$$H_{p_1, q_1:p_2, q_2;p_3, q_3}^{0, n_1:m_2, n_2;m_3, n_3}\left[\begin{matrix} x \\ y \end{matrix}\Big| \begin{matrix} (a_j; \alpha_j, A_j)_{1,p_1}: (c_j, \gamma_j)_{1,p_2}: (e_j, E_j)_{1,p_3} \\ (b_j; \beta_j, B_j)_{1,q_1}: (d_j, \delta_j)_{1,q_2}: (f_j, F_j)_{1,q_3} \end{matrix}\right]$$

$$= \frac{-1}{4\pi^2} \int_{L_1} \int_{L_2} \phi_1(\xi, \eta)\, \theta_2(\xi)\, \theta_3(\eta) x^{\xi} y^{\eta}\, d\xi\, d\eta, \qquad (4)$$

where

$$\phi_1(\xi, \eta) = \frac{\prod_{j=1}^{n_1} \Gamma(1 - a_j + \alpha_j\xi + A_j\eta)}{\prod_{j=n_1+1}^{p_1} \Gamma(a_j - \alpha_j\xi - A_j\eta) \prod_{j=1}^{q_1} \Gamma(1 - b_j + \beta_j\xi + B_j\eta)},$$

$$\theta_2(\xi) = \frac{\prod_{j=1}^{m_2} \Gamma(d_j - \delta_j\xi) \prod_{j=1}^{n_2} \Gamma(1 - c_j + \gamma_j\xi)}{\prod_{j=m_2+1}^{q_2} \Gamma(1 - d_j + \delta_j\xi) \prod_{j=n_2+1}^{p_2} \Gamma(c_j - \gamma_j\xi)},$$

$$\theta_3(\eta) = \frac{\prod_{j=1}^{m_3} \Gamma(f_j - F_j\eta) \prod_{j=1}^{n_3} \Gamma(1 - e_j + E_j\eta)}{\prod_{j=m_3+1}^{q_3} \Gamma(1 - f_j + F_j\eta) \prod_{j=n_3+1}^{p_3} \Gamma(e_j - E_j\eta)},$$

x and y are not equal to zero, and an empty product is interpreted as unity p_i, q_i, n_i and m_j are non negative integers such that $p_i \geq n_i \geq 0$, $q_i \geq 0$, $q_j \geq m_j \geq 0$, $(i = 1, 2, 3;\ j = 2, 3)$. Also, all the A's, α's, B's, β's, γ's, δ's, E's, and F's are assumed to the positive quantities for standardization purpose.

The contour L_1 is in the ξ-plane and runs from $-i\infty$ to $+i\infty$, with loops, if necessary, to ensure that the poles of $\Gamma(d_j - \delta_j\xi)$ $(j = 1, ..., m_2)$ lie to the right, and the poles of $\Gamma(1 - c_j + \gamma_j\xi)$ $(j = 1, ..., n_2)$, $\Gamma(1 - a_j + \alpha_j\xi + A_j\eta)$ $(j = 1, ..., n_1)$ to the left of the contour.

The countor L_2 is in the η-plane and runs from $-i\infty$ to $+i\infty$, with loops, if necessary, to ensure that the poles of $\Gamma(f_j - F_j\eta)$ $(j = 1, ..., m_3)$ lie to the right, and the poles of $\Gamma(1 - e_j + E_j\eta)$ $(j = 1, ..., n_3)$, $\Gamma(1 - a_j + \alpha_j\xi + A_j\eta)$ $(j = 1, ..., n_1)$ to the left of the contour.

The function (4) is analytic function of x and y if

$$R = \sum_{j=1}^{p_1} \alpha_j + \sum_{j=1}^{p_2} \gamma_j - \sum_{j=1}^{q_1} \beta_j - \sum_{j=1}^{q_2} \delta_j < 0,$$

$$S = \sum_{j=1}^{p_1} A_j + \sum_{j=1}^{p_3} F_j - \sum_{j=1}^{q_1} B_j - \sum_{j=1}^{q_3} F_j < 0,$$

The H-function of two variables (4) is convergent if

$$U = -\sum_{j=n_1+1}^{p_1} \alpha_j - \sum_{j=1}^{q_1} \beta_j - \sum_{j=1}^{m_2} \delta_j - \sum_{j=m_2+1}^{q_2} \delta_j + \sum_{j=1}^{n_2} \gamma_j - \sum_{j=n_2+1}^{p_2} \gamma_j > 0, \qquad (5)$$

$$V = -\sum_{j=n_1+1}^{p_1} A_j - \sum_{j=1}^{q_1} B_j - \sum_{j=1}^{m_3} F_j - \sum_{j=m_3+1}^{q_3} F_j + \sum_{j=1}^{n_3} E_j - \sum_{j=n_3+1}^{p_3} E_j > 0, \qquad (6)$$

and $|\arg x| < \frac{1}{2} U\pi$, $|\arg y| < \frac{1}{2} V\pi$.

The multivariable H–function is given in [5] is defined as follows:

$$H[z_1, \ldots, z_r] \equiv H^{0,\, n:\, m_1, n_1; \ldots; m_r, n_r}_{p,\, q:\, p_1, q_1; \ldots; p_r, q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \middle| \begin{matrix} (a_j; \alpha'_j, \ldots \alpha_j^{(r)})_{1,p} : (c'_j; \gamma'_j)_{1,p_1}; \ldots; (c_j^{(r)}; \gamma^{(r)}_j)_{1,p_r} \\ (b_j; \beta'_j, \ldots \beta_j^{(r)})_{1,q} : (d'_j; \delta'_j)_{1,q_1}; \ldots; (d_j^{(r)}; \delta^{(r)}_j)_{1,q_r} \end{matrix} \right]$$

$$= [(1/2\pi\omega)^r] \int_{L_1} \ldots \int_{L_r} \phi_1(\xi_1) \ldots \phi_r(\xi_r)\, \psi(\xi_1, \ldots \xi_r)\, z_1^{\xi_1} \ldots z_r^{\xi_r}\, d\xi_1 \ldots d\xi_r \qquad (7)$$

where $\omega = \sqrt{(-1)}$,

$$\psi(\xi_1, \ldots \xi_r) = \frac{\prod_{j=1}^{n} \Gamma\left(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(i)} \xi_i\right)}{\prod_{j=n+1}^{p} \Gamma\left(a_j - \sum_{i=1}^{r} \alpha_j^{(i)} \xi_i\right) \prod_{j=1}^{q} \Gamma\left(1 - b_j + \sum_{i=1}^{r} \beta_j^{(i)} \xi_i\right)},$$

$$\phi_i(\xi_i) = \frac{\prod_{j=1}^{m_i} \Gamma\left(d_j^{(i)} - \delta_j^{(i)} \xi_i\right) \prod_{j=1}^{n_i} \Gamma\left(1 - c_j^{(i)} + \gamma_j^{(i)} \xi_i\right)}{\prod_{j=m_i+1}^{q_i} \Gamma\left(1 - d_j^{(i)} + \delta_j^{(i)} \xi_i\right) \prod_{j=n_i+1}^{p_i} \Gamma\left(c_j^{(i)} - \gamma_j^{(i)} \xi_i\right)},$$

In above integral, i in the superscript (i) stands for the number of primes, e.g., $b^{(1)} = b'$, $b^{(2)} = b''$, and so on; and an empty product is interpreted as unity.

Suppose, as usual, that the parameters

a_j, j = 1, …., p; $c_j^{(i)}$, j = 1,….,p_i;

b_j, j = 1, …., q; $d_j^{(i)}$, j = 1,….,q_i; $\forall i \in \{1,\ldots,r\}$

are complex numbers and the associated coeficents

$\alpha_j^{(i)}$, j = 1, …., p; $\gamma_j^{(i)}$, j = 1,….,p_i;

$\beta_j^{(i)}$, j = 1, …., q; $\delta_j^{(i)}$, j = 1,….,q_i; $\forall i \in \{1,\ldots,r\}$

positive real numbers such that the left of the contour. Also

$$\Lambda_i = \sum_{j=1}^{p} \alpha_j^{(i)} + \sum_{j=1}^{pi} \gamma_j^{(i)} - \sum_{j=1}^{q} \beta_j^{(i)} - \sum_{j=1}^{qi} \delta_j^{(i)} \le 0, \quad (8)$$

$$\Omega_i = -\sum_{j=n+1}^{p} \alpha_j^{(i)} - \sum_{j=1}^{q} \beta_j^{(i)} + \sum_{j=1}^{mi} \delta_j^{(i)} - \sum_{j=mi+1}^{qi} \delta_j^{(i)} + \sum_{j=1}^{ni} \gamma_j^{(i)} - \sum_{j=ni+1}^{pi} \gamma_j^{(i)} > 0, \quad (9)$$

where the integral n, p, q, m_i, n_i, p_i and q_i are constrained by the inequalities $p \ge n \ge 0$, $q \ge 0$, $q_i \ge m_i \ge 1$ and $p_i \ge n_i \ge 1$ $\forall$ i $\in$ {1, 2, …, r) and the inequalities in (9) hold for suitably restricted values of the complex variables z_1, …., z_r. The sequence of parameters in integral are such that none of the poles of the integrand coincide, that is, the poles of the integrand in integral are simple. The contour L_i in the complex ξ_i–plane is of the Mellin-Barnes type which runs from $-\omega\infty$ to $+\omega\infty$ with indentations, if necessary, to ensure that all the poles of $\Gamma(d_j^{(i)} - \delta_j^{(i)}\xi_i)$, j = 1,…, m_i are separated from those of $\Gamma(1 - c_j^{(i)} + \gamma_j^{(i)}\xi_i)$, i = 1, …., n_i.

Since H–Functions are ubiquitous and play a dominant role in Mathematical Sciences. Their importance in fundamental sciences and their varied applications are ever increasing. Contributions to the area of H–Functions, in India, are the fourth largest, after statistics, quantum theory and general topology.

Orthogonal Polynomials and H–Functions are subjects whose power, beauty and versatility have been recognized in Engineering and extensive parts of Physics. More precisely we have applications on Electrostatics and Gravitation, Hydrodynamics, Steady Flow of Electricity or of Heat in Uniform Isotropic Media, Propagation of Electromagnetic Waves along Wires, Diffraction, Equilibrium of an Isotropic Rod of Circular Sections, Quantum Mechanics, Tidal Waves in an Estuary, etc. Also we have applications in Coding Theory, Digital Signal Processing, Root Systems, Information Theory, Transmission Line Theory, Systems Theory, Scattering of Waves by Statistically Rough Surfaces.

2. FORMULA USED:

In the present investigation we require the following results:

Following modified form of the integral [6, p.372, (1)]:

$$\int_0^{\pi} (\sin x)^{S-1} \cos nx \, dx = \frac{\pi \cos \tfrac{1}{2} n\pi \, \Gamma(s)}{2^{S-1} \Gamma\{\tfrac{1}{2}(s+n+1)\} \Gamma\{\tfrac{1}{2}(s-n+1)\}}, \qquad (10)$$

Re (s) > 0.

3. HEAT CONDUCTION IN A SQUARE PLATE INVOLVING MULTIVARIABLE H-FUNCTION:

The aim of this paper is to obtain a solution of a simple problem of heat conduction in a square plate with the help of multivariable H–function.

Integral:

The integral to be established here is $\int_0^{\pi} (sinx)^{s-1} \cos nx \, \mathrm{H}\left[\begin{matrix} z_1 (sinx)^{\lambda} \\ \vdots \\ z_r \end{matrix}\right] dx$

$$= 2^{1-s} \pi cos \frac{n\pi}{2} \mathrm{H}^{0,l:m_1,n_1+1;\ldots\ldots;m_r,n_r}_{p,q:p_1+1,q_1+2;\ldots\ldots;p_r,q_r} \left[\begin{matrix} z_1 2^{-\lambda} \\ \vdots \\ z_r \end{matrix} \middle| \begin{matrix} (a_j;\alpha'_j,\ldots,\alpha_j^{(r)})_{1,p}:(1-s,\lambda),(c'_j,\gamma'_j)_{1,p_1}:\ldots\ldots:(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ (b_j;\beta'_j,\ldots,\beta_j^{(r)})_{1,q}:\left(d'_j,\delta'_j\right)_{1,q_1},(\frac{1}{2}-\frac{s}{2}\pm\frac{n}{2},\frac{\lambda}{2}):\ldots\ldots:(d_j^{(r)},\delta_j^{(r)})_{1,q_r} \end{matrix}\right] \quad (11)$$

valid under the condition (7).

Proof:

Replace the multivariable H–function by its equivalent contour integral as given in (7), change the order of integration, evaluate the inner integral with the help of (10) and finally interpret it with (7), to get (11).

Heat Conduction in a Square Plate:

In this section, we consider a problem on heat conduction in a square plate under certain boundary conditions. If a square plate has its faces and its edges x = 0 and x = π (0 < y < π) insulated, its edges y = 0 and y = π are kept at temperature zero and f(x) respectively, then its steady temperature u (x, y) is given by [7, p.125]:

$$u(x, y) = \frac{a_0}{2\pi} y + \sum_{n=1}^{\infty} a_n \frac{\sinh ny}{\cosh nx} \cos nx \qquad (12)$$

where

$$a_n = (2/\pi) \int_0^{\pi} f(x) \cos nx \, dx, \; n = 0, 1, 2, \ldots.. \qquad (13)$$

Now we shall consider the problem of determining u (x, y), where

$$u(x, 0) = f(x) = (sinx)^{s-1} H\left[\begin{matrix} z_1 (sinx)^{\lambda} \\ \vdots \\ z_r \end{matrix}\right] \qquad (14)$$

Solution of the Problem:

Combining (13) and (14) and making the use of the integral (11), we derive

$$a_n = 2^{2-s} cos\frac{n\pi}{2} H^{0,l:m_1,n_1+1;\ldots\ldots;m_r,n_r}_{p,q:p_1+1,q_1+2;\ldots\ldots;p_r,q_r}\left[\begin{matrix} z_1 2^{-\lambda} \\ \vdots \\ z_r \end{matrix}\right.$$

$$\left|\begin{matrix} (a_j;\alpha'_j,\ldots,\alpha_j^{(r)})_{1,p}:(1-s,\lambda),(c'_j,\gamma'_j)_{1,p_1}:\ldots\ldots:(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ (b_j;\beta'_j,\ldots,\beta_j^{(r)})_{1,q}:\left(d'_j,\delta'_j\right)_{1,q_1},\left(\frac{1}{2}-\frac{s}{2}\pm\frac{n}{2},\frac{\lambda}{2}\right):\ldots\ldots:(d_j^{(r)},\delta_j^{(r)})_{1,q_r} \end{matrix}\right] \qquad (15)$$

Putting the value of a_n from (15) in (12), we get the following required solution of the problem:

$$u(x, y) = \frac{a_0}{2\pi} y + \sum_{n=1}^{\infty} 2^{2-s} cos\frac{n\pi}{2} \frac{\sinh ny}{\cosh nx} \cos nx$$

$$H^{0,l:m_1,n_1+1;\ldots\ldots;m_r,n_r}_{p,q:p_1+1,q_1+2;\ldots\ldots;p_r,q_r}\left[\begin{matrix} z_1 2^{-\lambda} \\ \vdots \\ z_r \end{matrix}\right.$$

$$\left|\begin{matrix} (a_j;\alpha'_j,\ldots,\alpha_j^{(r)})_{1,p}:(1-s,\lambda),(c'_j,\gamma'_j)_{1,p_1}:\ldots\ldots:(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ (b_j;\beta'_j,\ldots,\beta_j^{(r)})_{1,q}:\left(d'_j,\delta'_j\right)_{1,q_1},\left(\frac{1}{2}-\frac{s}{2}\pm\frac{n}{2},\frac{\lambda}{2}\right):\ldots\ldots:(d_j^{(r)},\delta_j^{(r)})_{1,q_r} \end{matrix}\right] \qquad (16)$$

provided the condition stated with (11) are satisfied.

Special Solutions:

The importance of the H-function lies largely from the possibility of expressing by means of the H-symbols a great many of special functions appearing in applied mathematics, physical sciences and statistics. So that each of the solutions given in (16) becomes a master or key solution from which a very large number of solutions can be derived for Meijer's G-function, Generalized Hypergeometric function, Bessel, Legendre, Whittaker functions, their combinations and many other functions.

Conclusion :

This research paper has introduced a novel application of the multivariable H-function in solving heat conduction problems in square plates. By utilizing the multivariable H-function, we have derived an analytical solution for the temperature distribution in a square plate with specific boundary conditions. The solution is expressed in terms of the multivariable H-function, which provides a flexible and powerful tool for handling complex heat conduction dynamics.

The integral formula (11) established in this paper serves as a crucial bridge between the multivariable H-function and the heat conduction problem. This formula enables the evaluation of the temperature distribution in the square plate, as demonstrated in equation (16).

The significance of this research lies in its potential to tackle a wide range of heat conduction problems involving complex geometries and boundary conditions. The multivariable H-function's versatility and expressiveness make it an ideal candidate for modeling and analyzing various thermal phenomena.

Future research directions may include:

1. Exploring additional applications of the multivariable H-function in heat conduction problems with non-uniform boundary conditions or non-homogeneous materials.

2. Developing numerical methods for efficiently evaluating the multivariable H-function in heat conduction problems.

3. Investigating the connections between the multivariable H-function and other special functions in mathematical physics.

This research contributes to the growing body of literature on the applications of special functions in mathematical physics and engineering, and its findings have the potential to inspire new breakthroughs in thermal analysis and modeling.

References

[1] Rainville, E. D.: Special Functions, Macmillan, NewYork, 1960.

[2] Meijer, C. S.: Ibidem 49 (1946), p. 344-456.

[3] Fox, C.: The G and H-functions as symmetrical Fourier kernels Trans Amer. Math. Soc. 98 (1961), p. 395-429.

[4] Mittal, P. K. and Gupta Gupta, K. C.: An integral involving generalized function of two variables, Proc. Indian Acad. Sci., 75 A, p. 117-123.

[5] Srivastava, H. M., Gupta, K. C. and Goyal, S. P.: The H-function of one and two variables with applications, South Assian Publishers, New Delhi, 1982.

[6] Gradshteyn, I. S. and Ryzhik, I. M.: Tables of Integrals, Series and Products, Academic Press, Inc. New York, 1980.

[7] Churchill, R.V.: Fourier series and Boundary Value Problems, McGraw–Hill, New York (1988).