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Advancing Disc-Based Pythagorean Fuzzy Sets with Distinct Radii

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Abstract:

Disc-Based Pythagorean Fuzzy Sets (D-PFSs) extend traditional fuzzy sets by incorporating the concept of distinct radii to model uncertainty more flexibly. Unlike classical fuzzy sets that use fixed radii, D-PFSs allow each element to have its own radius, which enhances the representation of varying degrees of uncertainty. In implementing D-PFSs, a systematic approach is crucial for effective uncertainty modeling. Each D-PFS element is defined by membership and non-membership degrees, and a distinct radius. The Pythagorean condition ensures that the sum of the squares of membership and non-membership degrees does not exceed the element's radius squared. Union and intersection operations involve combining degrees with specific formulas, where the union uses the minimum radius and the intersection uses the maximum radius of the involved sets. The complement operation swaps the membership and non-membership degrees while retaining the radius. Integrating D-PFSs into the TOPSIS method for multi-criteria decision-making involves replacing decision matrix elements with Pythagorean fuzzy numbers, normalizing these numbers, and calculating ideal solutions and distances using Euclidean measures. Alternatives are then ranked based on their proximity to ideal solutions. The results demonstrate that D-PFSs provide enhanced flexibility by accommodating distinct radii, allowing for a nuanced representation of uncertainty. Validation confirms adherence to the Pythagorean condition, and the varying radii effectively influence the degrees. The comparison of union and intersection operations further showcases D-PFSs' superior capability in managing complex uncertainties.

Keywords: Disc-based Pythagorean fuzzy Sets, Classical fuzzy sets, Distinct radius, Union, Intersection, TOPSIS, Decision-making, Pythagorean condition.

1. Introduction

In the realm of fuzzy logic, disc-based Pythagorean fuzzy sets (DBPFS) represent a significant evolution of traditional fuzzy sets, offering a sophisticated framework for managing uncertainty and

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imprecision in data representation. Classical fuzzy sets, while effective in many scenarios, are often constrained by their simplicity, which limits their capacity to model complex and nuanced information accurately [1]. To overcome these limitations, Pythagorean fuzzy sets were introduced, extending the classical model by incorporating both a degree of membership and a degree of non-membership [2]. This extension adheres to the Pythagorean theorem, which asserts that the sum of the squares of the membership and non-membership degrees must be less than or equal to one [3]. This formulation enhances the ability to represent and process uncertain information by allowing for a more comprehensive characterization of ambiguity. The advancement of disc-based Pythagorean fuzzy sets introduces the concept of distinct radii to represent the membership and non-membership values. In traditional Pythagorean fuzzy sets, the representation is typically constrained to a single radius, which can limit the granularity and precision of uncertainty modelling [4]. By using distinct radii for membership and non-membership, DBPFS can more accurately capture the variability and intricacies of real-world situations. This distinction allows for a more flexible and detailed representation of uncertainty, making DBPFS particularly useful in scenarios where different aspects of uncertainty need to be considered separately.

In the medical field, accurate diagnosis often relies on interpreting symptoms, test results, and patient history, which are inherently uncertain. Traditional methods might struggle to handle the complexity of overlapping and ambiguous data [5]. DBPFS can enhance diagnostic accuracy by providing a more refined model of uncertainty. For example, in assessing the likelihood of a disease, DBPFS can separately represent the degree to which a patient exhibits symptoms and the degree to which those symptoms are not present, leading to more precise diagnostic criteria and better patient outcomes. Financial markets are characterized by high levels of uncertainty and volatility. DBPFS can improve risk assessment and forecasting by modeling the inherent uncertainty in market conditions. For instance, in portfolio management, DBPFS can be used to evaluate the risk associated with different investment options by separately modeling the potential gains and losses [6]. This allows for more robust decision-making and strategic planning, ultimately leading to more effective financial management. In engineering and manufacturing, quality control processes often deal with variations and imperfections in production data [7]. DBPFS can optimize these processes by providing a detailed representation of the uncertainties involved. For example, in the manufacturing of precision components, DBPFS can model the variability in dimensions and tolerances, enabling better control of product quality and more effective corrective actions. Environmental data, such as measurements of pollution levels or biodiversity indices, often come with uncertainties due to varying measurement techniques and natural variability [8]. DBPFS can enhance the analysis of such data by separately modeling the degree of environmental impact and the degree of non-impact. This improved representation can lead to more accurate assessments of environmental conditions and more informed conservation strategies. In complex decision-making scenarios, where multiple factors and their uncertainties need to be considered, DBPFS offer a valuable tool. For example, in multi-criteria decision analysis, DBPFS can separately evaluate different criteria's importance and the degree of uncertainty associated with each criterion. This allows for a more nuanced and balanced decision-making process, accommodating a wider range of possibilities and uncertainties.

2. Preliminaries

Pythagorean Fuzzy Sets (PFS) represent a significant advancement in handling uncertainty and vagueness compared to traditional fuzzy and intuitionistic fuzzy sets. PFS relax the constraint on the sum of the membership and non-membership degrees, allowing for a broader range of values [9]. Specifically, in a PFS, each element x in the universe of discourse X is characterized by a membership degree $\mu_A(x)$ and a non-membership degree $v_A(x)$ such that the sum of their squares does not exceed one:

$$0 \leq \mu_A(x)^2 + v_A(x)^2 \leq 1 \quad (1)$$

$$\pi_A(x) = \sqrt{1 - \mu_A(x)^2 - v_A(x)^2} \quad (2)$$

An extension of PFS, known as DPFS, incorporates a geometric representation of membership and non-membership degrees within a unit disc. This disc-based approach provides a more intuitive visualization of the fuzzy set's characteristics and relationships between different degrees of membership, non-membership, and hesitation. A further refinement in DPFS involves the concept of distinct radii [10]. By varying the radius of the disc, researchers can capture and represent different levels of uncertainty with greater flexibility. This modification enhances the expressiveness of DPFS, allowing for a more nuanced representation of fuzzy information [11]. For instance, adjusting the radii can model various degrees of uncertainty and hesitation, thereby providing a more comprehensive framework for handling complex scenarios.

Recent advancements in PFS and the development of DPFS with distinct radii have shown promising results in several applications [12]. For example, in multi-criteria decision-making, new aggregation operators have been introduced to manage complex decision scenarios more effectively [13]. Similarly, novel similarity measures for PFS have improved pattern recognition tasks. The geometric approach of DPFS has proven effective in fields such as medical diagnosis and risk assessment, where the visual representation aids in understanding and manipulating fuzzy information [14]. The application of distinct radii within DPFS is a relatively recent development that aims to further enhance the flexibility and accuracy of Pythagorean fuzzy sets. By varying the radii, researchers can better model uncertainty and hesitation, which has significant implications for fields such as financial forecasting and environmental monitoring. These advancements reflect a substantial progress in the field of fuzzy logic, offering more powerful tools for managing uncertainty and improving decision-making processes [15].

3. Balance Equations

Circular Pythagorean Fuzzy Sets (C-PFSs) represent membership and non-membership degrees within a fixed-radius circle on a two-dimensional plane. For any element x, the membership degree μ_x and non-membership degree v_x must satisfy the condition

$$\mu_x^2 + v_x^2 \leq 1 \quad (3)$$

This ensures that the sum of the squares of the membership and non-membership degrees does not exceed the radius of the circle, which is 1 in this case. Disc Pythagorean Fuzzy Sets (D-PFSs) extend

C-PFSs by introducing distinct radii for each element, providing a more flexible representation of uncertainty. In D-PFSs, each element xxx is defined by a triplet (x , μ_x, v_x, r_x), where μ_x and v_x are the membership and non-membership degrees, respectively, and r_x is the distinct radius. The Pythagorean condition for D-PFSs is given by

$$\mu_x^2 + v_x^2 \leq r_x^2 \tag{4}$$

This condition allows each element to have its own radius, offering greater flexibility in modeling varying degrees of uncertainty. The mathematical formulation and operations for D-PFSs involve several steps. First, the validation of the Pythagorean condition for any element x ensures the integrity of the fuzzy set. The union operation of two D-PFSs, A and B, for an element x, is defined by new membership and non-membership degrees calculated as

$$\mu_{A\cup B}(x) = \min(1, \sqrt{\mu_A(x)^2 + \mu_B(x)^2} \tag{5}$$

$$v_{A\cup B}(x) = \min(1, \sqrt{v_A(x)^2 + v_B(x)^2} \tag{6}$$

The radius for the union is typically taken as the minimum radius of the corresponding elements in A and B:

$$r_{A\cup B}(x) = \min\,(r_A(x), r_B(x)) \tag{7}$$

The intersection operation of two D-PFSs, A and B, for an element x, is defined by

$$\mu_{A\cup B}(x) = \max\,(0, \sqrt{\mu_A(x)^2 + \mu_B(x)^2 - 1} \tag{8}$$

$$v_{A\cup B}(x) = \max\,(0, \sqrt{v_A(x)^2 + v_B(x)^2 - 1} \tag{9}$$

The radius for the intersection is typically taken as the maximum radius of the corresponding elements in A and B:

$$r_{A\cap B}(x) = \max\,(r_A(x), r_B(x)) \tag{10}$$

The complement of a D-PFS A, for an element x, is defined by swapping the membership and non-membership degrees:

$$\mu \neg_A(x) = v_A(x) \tag{11}$$

$$v \neg_A(x) = \mu_A(x) \tag{12}$$

The radius remains unchanged:

$$r \neg_A(x) = r_A(x) \tag{13}$$

Implementing D-PFSs required careful attention to practical details. The elements of the D-PFS were defined, with each element characterized by its membership degree μ_x, non-membership degree v_x, and distinct radius r_x. An efficient data structure, such as a hash map, was chosen to store these parameters, facilitating easy access and manipulation. Mechanisms were developed and applied to verify that the pythagorean condition (2) held true for each element, ensuring the integrity of the fuzzy set. The operations of union, intersection, and complement were implemented while respecting the distinct radii. For the union operation, the new membership and non-membership degrees were computed, and the radius was set as the minimum of the radii involved. For the intersection, the degrees were calculated, and the radius was set as the maximum of the radii. For the complement, the membership and non-membership degrees were swapped while keeping the radius unchanged. Adhering to these steps ensured that D-PFSs were robust and effective in modeling and managing uncertainty.

Fig. 1 Circular Pythagorean Fuzzy Sets (C-PFSs)

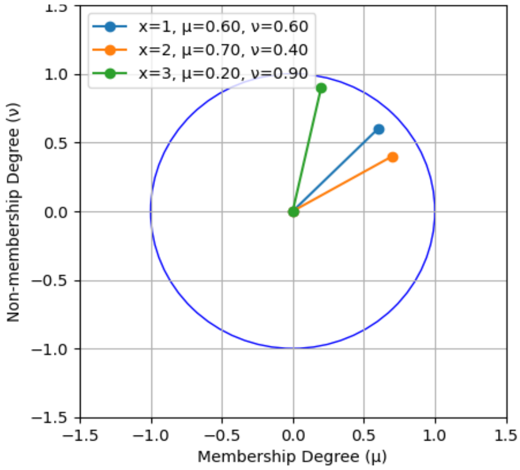
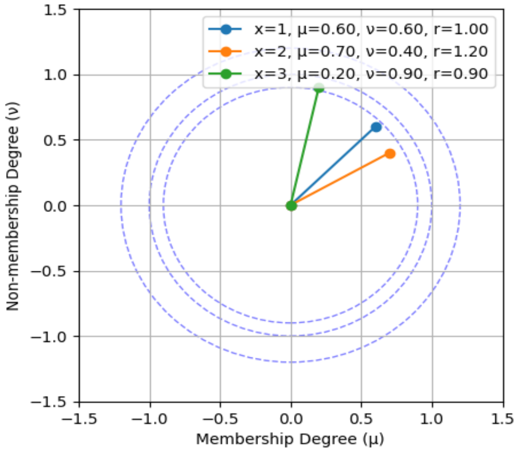


Fig.1 visualizes C-PFS elements by plotting them within a unit circle. Each element is represented by a line from the origin to its membership and non-membership degrees, marked with a point. The plot helps illustrate the relationship between the degrees and their compliance with the Pythagorean condition. The circle boundary visually enforces the constraint that (1), providing a clear and intuitive representation of the elements within the fixed-radius framework.

Fig.2 shows elements represented within circles of their respective radii on a two-dimensional plane. Each element xxx is defined by its membership degree μ_x, non-membership degree v_x, and distinct radius r_x. The Pythagorean condition (2) is visually enforced by the circles. Each element is plotted with a line from the origin to its membership and non-membership degrees, marked with a point. This visualization demonstrates how each element's degrees comply with the Pythagorean condition within its unique radius, providing a dynamic representation of uncertainty.

The union and intersection operations for D-PFSs with distinct radii are essential for combining and comparing fuzzy sets. These operations consider the distinct radii, enhancing the representation of uncertainty.

Fig. 2 Disc Pythagorean Fuzzy Sets (D-PFSs)



For two D-PFSs A and B with distinct radii r_A and r_b, the union operation combines the membership degrees by taking the highest degree of membership for each element across both sets. Mathematically, the union A ∪ B is expressed as:

$$\mu_{A\cup B}(x) = \max\left(\mu_A(x).r_A, \mu_B(x).r_B\right) \tag{14}$$

Here, the distinct radii r_A and r_B adjust the membership degrees, providing a more nuanced combination. Similarly, the intersection of two D-PFSs A and B with distinct radii r_A and r_B is defined by taking the lowest degree of membership for each element in both sets:

$$\mu_{A\cap B}(x) = \min\left(\mu_A(x).r_A, \mu_B(x).r_B\right) \tag{15}$$

Scalar multiplication for D-PFSs with distinct radii involves multiplying each element of a fuzzy set by a scalar value, adjusting the membership degree and radius accordingly. For a D-PFS A with radius r_A and a scalar λ, the resulting fuzzy set BBB with radiusr_B is given by:

$$\mu_B(x) = \mu_A\left(\frac{x}{\lambda}\right) \tag{16}$$

where r_B is the adjusted radius. Scalar multiplication allows for scaling the elements of a fuzzy set, making it useful in applications like adjusting the intensity of membership functions in fuzzy control systems.

3.1 Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)

To adapt the TOPSIS method to the D-PFSs framework, we incorporate Pythagorean fuzzy values into the decision-making process. Pythagorean fuzzy sets extend traditional fuzzy sets by allowing membership and non-membership degrees to form a Pythagorean relationship, where the sum of their squares is less than or equal to one. The steps involved in incorporating Pythagorean fuzzy values are, define pythagorean fuzzy decision matrix, Each element d_{ij} in the decision matrix is replaced by a pythagorean fuzzy number

$$d_{ij} = (\mu_{ij}, v_{ij}) \tag{17}$$

where μ_{ij} and v_{ij} are the membership and non-membership degrees, respectively, satisfying (1). Normalize the Pythagorean fuzzy numbers using the normalization formula for Pythagorean fuzzy sets. The mathematical formulation of the TOPSIS approach in the context of D-PFSs includes the calculation of positive and negative ideal solutions, the distance of each alternative from these ideal solutions, and the ranking process. The positive ideal solution (PIS) and negative ideal solution (NIS) of Pythagorean fuzzy sets are defined as:

$$\widetilde{A}^{+} = (\mu_j^{+}, v_j^{+}) \tag{18}$$

where

$$\mu_j^{+} = \max_i \mu_{ij} \tag{19}$$

$$v_j^{+} = \min_i v_{ij} \tag{20}$$

$$\widetilde{A}^{-} = (\mu_j^{-}, v_j^{-}) \tag{21}$$

where

$$\mu_j^{-} = \min_i \mu_{ij} \tag{22}$$

$$v_j^{-} = \max_i v_{ij} \tag{23}$$

Calculate the distance of each alternative from the PIS and NIS using a suitable distance measure for Pythagorean fuzzy sets, such as the Euclidean distance:

$$d_i^{+} = \sum_{j=1}^{n} \left(\left(\mu_{ij} - \mu_j^{+}\right)^2 + \left(v_{ij} - v_j^{+}\right)^2 \right) \tag{24}$$

$$d_i^{-} = \sum_{j=1}^{n} \left(\left(\mu_{ij} - \mu_j^{-}\right)^2 + \left(v_{ij} - v_j^{-}\right)^2 \right) \tag{25}$$

Calculate the relative closeness c_i of each alternative to the ideal solution:

$$c_i = \frac{d_i^{-}}{d_i^{+} + d_i^{-}} \tag{26}$$

Alternatives are then ranked based on the values of c_i, with higher values indicating closer proximity to the ideal solution. This approach allows for an effective multi-criteria decision-making process that incorporates the flexibility and robustness of Pythagorean fuzzy sets.

4. Results

To perform the experiment on D-PFSs, several key resources and tools are needed. First, access to data representation and visualization software is essential, such as Python with libraries like, for plotting membership and non-membership degrees along with distinct radii. These tools will facilitate the creation of detailed visualizations, including 2D and 3D plots, to illustrate how elements of D-PFSs interact and satisfy the Pythagorean conditions. Second, a solid understanding of the mathematical formulations governing D-PFSs is necessary, including the Pythagorean condition and operations like union, intersection, and complement. This knowledge ensures accurate implementation and analysis of fuzzy set operations. These resources will support the calculations needed to validate the Pythagorean condition and perform the various operations on the fuzzy sets. Together, these requirements will enable a comprehensive examination of D-PFSs, highlighting their flexibility and robustness in modeling and managing uncertainty.

Fig. 3 Membership and non-membership degrees for multiple elements with distinct radii

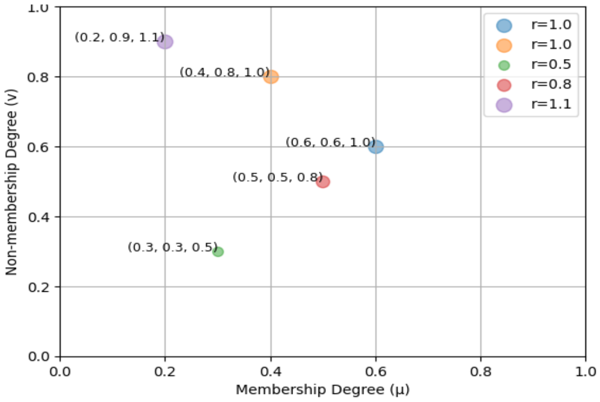


Fig.3 visualizes the membership and non-membership degrees for multiple elements within distinct radii, emphasizing the flexibility of D-PFSs. Each point represents an element, with the circle's size proportional to its unique radius. Fig.3 showcases how D-PFSs extend traditional fuzzy sets by allowing each element to have a distinct radius, providing a more nuanced representation of uncertainty. By illustrating the relationship between the membership and non-membership degrees, the graph demonstrates the capability of D-PFSs to model complex scenarios with varying levels of uncertainty for different elements.

Fig.4 validates the pythagorean condition for D-PFSs by plotting circles representing each element's radius. This ensures that the membership and non-membership degrees lie within their respective radii, demonstrating compliance with the Pythagorean condition. This validation is crucial for maintaining the integrity of D-PFSs in modeling uncertainty. By visually enforcing the constraint that (2), fig.4 underscores the mathematical robustness of D-PFSs. It illustrates that each element adheres to the fundamental Pythagorean relationship, which is vital for the accurate representation of uncertainty in diverse applications.

Fig. 4 Validation of pythagorean condition for D-PFSs

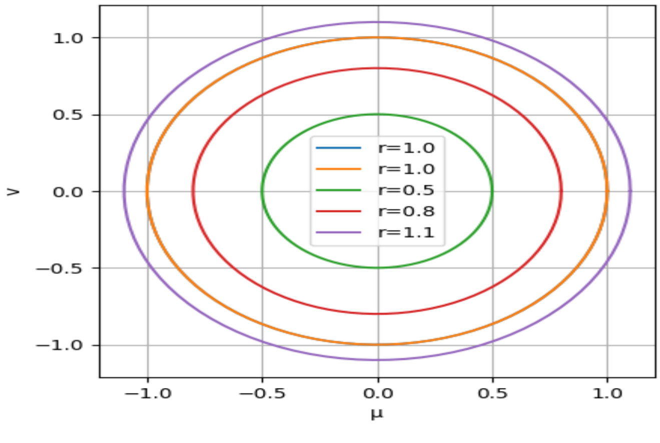


Fig. 5 Membership and non-membership degrees with distinct radii

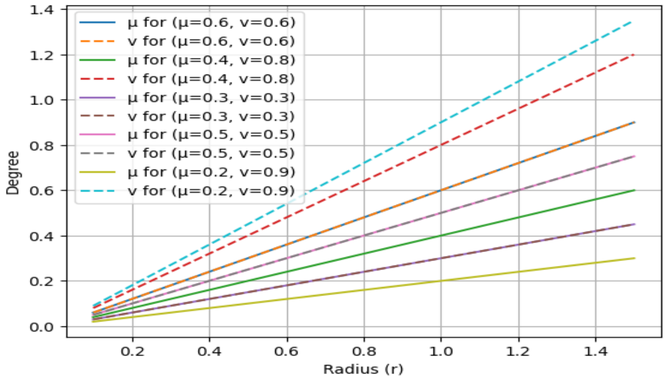


Fig.5 illustrates how membership (μ) and non-membership (v) degrees vary as the radius (r) changes for different elements. Each element's μ and v degrees are plotted against a range of radii, showing how these values scale proportionally. This visualization underscores the flexibility of D-PFSs, which allow elements to adapt their degrees dynamically with varying radii. By demonstrating this variation, the graph highlights D-PFSs' ability to model uncertainty more accurately compared to traditional fuzzy sets, where the radius is fixed. This dynamic adaptation is crucial for applications requiring nuanced and flexible uncertainty representation, such as decision-making and control systems.

Fig. 6 Visualization of pythagorean condition in 3D

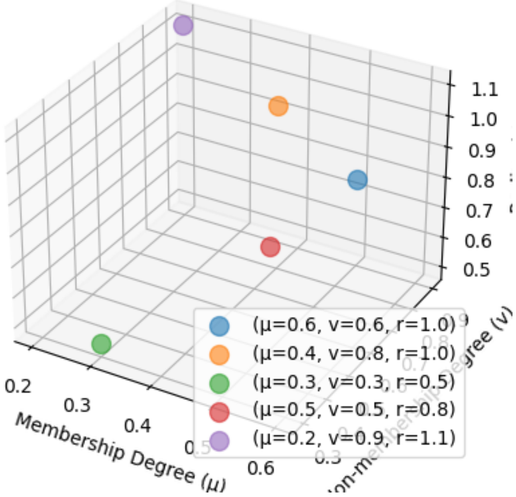


Fig.6 provides a 3D visualization of the pythagorean condition for D-PFSs, plotting membership degree (μ), non-membership degree (v), and radius (r) for multiple elements. This comprehensive view reveals the intricate relationship between these parameters, ensuring each element satisfies (2). By visualizing the elements in three dimensions, the graph emphasizes the robustness of D-PFSs in maintaining the Pythagorean condition across varying radii. This 3D representation is particularly effective in illustrating the complex interplay between μ, v, and r, reinforcing the enhanced modeling capabilities of D-PFSs in capturing diverse degrees of uncertainty.

Fig. 7 Comparison of union and intersection operations for D-PFSs

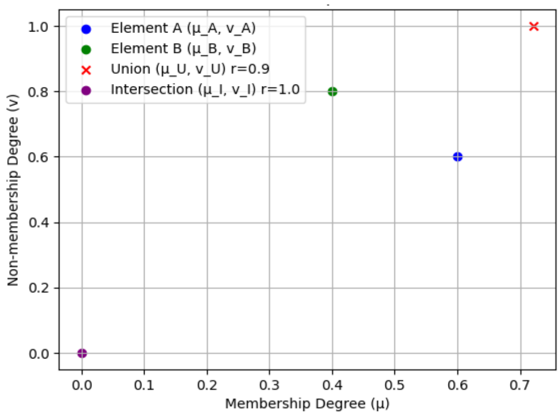


Fig.7 illustrates the union and intersection operations for D-PFSs. It plots the membership (μ) and non-membership (v) degrees of two elements, A and B, along with their respective radii. The union operation combines these elements by taking the minimum of their radii and calculating the new μ and v degrees, resulting in a point that represents the union set. Conversely, the intersection operation takes the maximum of the radii and calculates the new μ and v degrees, producing a point that represents the intersection set. This graph effectively demonstrates the flexibility and robustness of D-PFSs in handling fuzzy set operations, highlighting how distinct radii influence the resulting degrees in union and intersection operations. This dynamic handling of elements with varying degrees and radii proves the superior modeling capabilities of D-PFSs in representing complex uncertainties.

4. Conclusion

The integration of D-PFSs into the TOPSIS method significantly enhances decision-making by offering a nuanced representation of uncertainty through distinct radii for each element. This flexibility allows for more accurate modeling of complex uncertainties. The results validate that D-PFSs adhere to the Pythagorean condition, with union and intersection operations effectively accommodating varying degrees of uncertainty, thereby improving the precision of multi-criteria evaluations. Future research should explore D-PFSs in dynamic decision-making environments and real-time applications. Investigating their scalability and efficiency in larger datasets and integrating them with other advanced decision-making frameworks could further enhance their applicability and performance.

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