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An Introduction to Ternary Γ −Semirings

G. Chandrasekhar[1,2], D.Madhusudana Rao[3],P.Siva Prasad[4]

[1] Research Scholar, Department of Mathematics, Acharya NagarjunaUniversity,Guntur, A. P.,India.

[2]Lecturer in Mathematics, Government College (Autonomous), Rajahmundry, A. P., India.

[3]Professor of Mathematics, Government College For Women(A), Samba Siva Peta Rd, Opp: AC College, Samba Siva Pet, Guntur, Andhra Pradesh, India,mailid:dmrmaths@gmail.com

[4]Associate Professor, Department of Computer Science Engineering School of Computing& Informatics, VFSTR Deemed to University, Vadlamudi,Guntur,A.P,India, mailid:pusapatisivaprasad@gmail.com

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Abstract:

In this research article has to introduce the concept of ternary Γ-semirings.We first consider the congruences and ideals of ternary Γ-semirings then we construct a new ternary Γ-semiring and to be discussed formation of ideals on this ternary Γ-semiring. Also with the help of congruences induced by homomorphism of a ternary Γ-semiring.After that we will establish some of isomorphism theorems and identified the some of commutativity in the diagrams. particularly some fundamental results of ternary Γ-semiring were proved and strengthened.

Keywords: TernaryΓ-semiring,Ideal,Congruence,Quotient ternary Γ-semiring, Homomorphism and Isomorphism

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1.Introduction:

The notion of Γ −semiring was studied by M.K.Rao.as a generalization of Γ −ring as well as of semiring. In the year of 1964 Γ −ring was introduced by N.Nobusawa There have been a few definitions for a Γ −ring.The concepts of ternary Γ −semirings and ternary sub Γ −semiring with left,right,lateral was studied by D.Madhusudana Rao and M.Sajani Lavanya in the year of 2007.T.K Datta and M.L Das were introduced and studied the ideals ,prime ideals semiprime ideals k-idelas and h-ideals of a ternary Γ −semiring ,regular ternary Γ −semiring respectively

2..Priliminaries:

Definition 2.1: Let$(T,+)$ and $(\Gamma,+)$ be two additive commutative semigroups then T is known as ternary Γ −semiring if there exist a mapping from $T\times\Gamma\times T\times\Gamma\times T$ to T which maps $(a,\alpha,b,\beta,c)\to[a\alpha b\beta c]$ satisfying the following conditions

$$i)\big[[\,a\alpha b\beta c]\gamma d\delta e\big]=[a\alpha[b\beta c\gamma d]\delta e]=[a\alpha b\beta[c\gamma d\delta e]]$$

$$ii)[(a+b)\alpha c\beta d]=[a\alpha c\beta d]+[b\alpha c\beta d]$$

$$iii)[a\alpha(b+c)\beta d]=[a\alpha b\beta d]+[\,a\alpha c\beta d]$$

$$iv)[a\alpha b\beta(c+d)]=[a\alpha b\beta c]+[a\alpha b\beta d]\text{for all}$$

$$a,b,c,d\in T\quad and\quad \alpha,\beta,\gamma,\delta\epsilon\Gamma$$

Definition 2.2: A ternaryΓ −semiring T is said to have a zero element provided $0 + x = x = x + 0$ and $[0\alpha a\beta b] = [a\alpha 0\beta b] = [a\alpha b\beta 0] = 0, \forall a, b, x \in T \text{ and } \alpha, \beta \epsilon \Gamma$

Definition 2.3: A ternaryΓ −semiring T is known as commutative ternaryΓ −semiring T provided $[a\alpha b\beta c] = [b\alpha c\beta a] = [c\alpha a\beta b] = [b\alpha a\beta c] = [c\alpha b\beta a] = [a\alpha c\beta b]$ for all$a, b, c \in T$.

Definition 2.4: Let S be a non empty subset of a ternary Γ −semiring T is said to be a ternary sub Γ −semiring of T if and only if $S + S \subseteq S$ and $[S\alpha S\beta S] \subseteq S$ for all $\alpha, \beta \epsilon \Gamma$.

Definition 2.5: Let T be a ternaryΓ −semiring and A be a non empty subset of T is said to be a left ternary Γ-ideal of T if $i)(a + b) \in A$ and $ii) b, c \in T, a \in A, \alpha, \beta \epsilon \Gamma \Longrightarrow [b\alpha c\beta a] \in A$.

Definition 2.6: Let$T$ be a ternaryΓ −semiring and A be a non empty subset of T is said to be a lateral ternary Γ-ideal of T if $i)(a + b) \in A$ and $ii) b, c \in T, a \in A, \alpha, \beta \epsilon \Gamma \Longrightarrow [b\alpha a\beta c] \in A$.

Definition 2.7: Let$T$ be a ternaryΓ −semiring and A be a non empty subset of T is said to be a right ternary Γ-ideal of T if $i)(a + b) \in A$ and $ii) b, c \in T, a \in A, \alpha, \beta \epsilon \Gamma \Longrightarrow [a\alpha b\beta c] \in A$.

Definition 2.8: Let T be a ternaryΓ −semiring and A be a non empty subset of T is said to be a ternary Γ-ideal of T if and only if it is a left ternary Γ-ideal, lateral ternary Γ-ideal, right ternary Γ-ideal of T.

3.Ideals of a ternary Γ −semiring

Entire of this research article T be a ternaryΓ −semiringunless otherwise specified. The below theorems are easily to prove.

Lemma3.1: Let Δ be a non empty index set and $\{I_\lambda\}_{\lambda\in\Delta}$ be a family of ideals of (T,Γ).Then $\bigcap_{\lambda\in\Delta} I_\lambda$ is an ideal of (T,Γ).

Lemma3.2: Let £(T,Γ)be the set of all ideals of (T,Γ).Then $(£(T,\Gamma), \subseteq, \wedge, \vee)$ is a complete lattice ,where $I \wedge J = I \cap J$ and $I \vee J = \langle I \cup J \rangle$ is the unique smallest ideal containing $I \cup J$.

Theorem 3.3: LetT be a ternary Γ −semiring with zero and I be an ideal of (T,Γ).then $\frac{R}{I} = \{\text{p} + I / \text{p} \in R\}$ is a ternary Γ −semiring with the mapping $* : \frac{R}{I} \times \Gamma \times \frac{R}{I} \times \Gamma \times \frac{R}{I} \to \frac{R}{I}$ defined by $(\text{p}+I) * \gamma * (\text{q}+I) * \gamma * (\text{r}+I) = p\gamma q\gamma r + I$ for all $\text{p}, \text{q}, \text{r} \in R \text{ and } \gamma \in \Gamma$

Proof: First we define an operation $\oplus$ on $\frac{R}{I}$ by $(\text{p}+I) \oplus (\text{q}+I) \oplus (\text{r}+I) = p \oplus q \oplus r + I$ for all $p + I$,$q + I$,&$r + I \in \frac{R}{I}$ .It is easy to see that $\oplus$ and * are well defined. Consequently we can verify that ($\frac{R}{I}$,$\oplus$,[ ]) is a commutative semigroup and we have the following equalities.

Left distributive

$$[(p+I)\alpha(q+I)\beta((r+I)\oplus(s+I)) = [(p+I)\alpha(q+I)\beta(r+s+I)]$$
$$= [\mathrm{p}\,\alpha\,\mathrm{q}\,\beta(\mathrm{r+s})]+\mathrm{I}$$
$$= [(\mathrm{p}\,\alpha\,\mathrm{q}\,\beta\,\mathrm{r})\oplus((\mathrm{p}\,\alpha\,\mathrm{q}\,\beta s)]+I$$
$$= [[\mathrm{p}\,\alpha\,\mathrm{q}\,\beta\,\mathrm{r}]+\mathrm{I}]\oplus[[\mathrm{p}\,\alpha\,\mathrm{q}\,\beta s]+I]$$
$$= [(p+I)\alpha(q+I)\beta(r+I)]\oplus[(p+I)\alpha(q+I)\beta(s+I)]$$

Lateral distributive

$$(p+I)\alpha[(r+I)\oplus(s+I)]\beta(\mathrm{q}+I) = [(p+I)\alpha(r+s+I)]\beta(q+I)]$$
$$= [\mathrm{p}\,\alpha(\mathrm{r+s})+\mathrm{I}]\beta(\mathrm{q+I})$$
$$= [p\alpha(r+s)\beta(q+I)]$$
$$= [p\alpha(r+s)\beta\,\mathrm{q}+I]$$
$$= [[p\alpha r\beta q]\oplus[p\alpha s\beta q]+I]$$
$$= [[p\alpha r\beta q]+\mathrm{I}]\oplus[[p\alpha s\beta q]+I]$$
$$= [(p+I)\alpha(r+I)\beta(q+I)]\oplus[(p+I)\alpha(\mathrm{s}+I)\beta(q+I)]$$

Right distributive

$$[(r+I)\oplus(s+I)]\alpha(p+I)\beta(q+I)) = [(r+s)+I]\ \alpha(p+I)\beta(q+I)$$
$$= [(r+s)\alpha p\beta q]+I$$
$$= [(r\alpha p\beta q)\oplus(s\alpha p\beta q)]+I$$
$$= [[r\alpha p\beta q]+\mathrm{I}]\oplus[[s\alpha p\beta q]+\mathrm{I}]$$
$$= [(r+I)\alpha(p+I)\beta(q+I)]\oplus[(s+I)\alpha(p+I)\beta(q+I)]$$

These shows that is a $\frac{R}{I}$ ternary Γ −semiring.

Theorem 3.4(correspondance theorem): Let R be a ternaryΓ −semiring with zero and J an ideal of R such that $I \subseteq J$ then $\frac{J}{I}$ is an ideal of $(\frac{R}{I},\oplus,\ [\ ])$ .Conversely ,if K is an ideal of $(\frac{R}{I},\oplus,\ [\ ])$ then there exist an ideal J of $(R,\Gamma,[\ \ ])$ such that $I \subseteq J$ and $K = \frac{J}{I}$.

Proof: The proof of this theorem is similar to the above.

4.Commutative ternary Γ −semirings and Congruence's on a ternary Γ −semirings

In this entire section R is a commutative ternary Γ −semiring.The following theorems are well known.

Theorem 4.1: The following conditions on an ideal I of a commutative ternary Γ −semiring R with zero are equivalent

(1) $H\oplus(0:\mathrm{I}) = (H\Gamma I:\mathrm{I})$ for all ideals H of $(R,\Gamma,[\ \ ])$

(2) $H\Gamma I = K\Gamma I$ implies that $(0:I)+H=(0:I)+K$ for all ideals H and K of $(R,\Gamma,[\ ])$ where $(I:A)=\{p\in R/\ p\gamma a\gamma b\in I, \forall a,b\in A \ and\ \gamma\in\Gamma\}$ for any $\Phi\neq A\subseteq R.$

Theorem 4.2: Assume that R be a commutative ternary Γ −semiring.If I is an ideal of $(R,\Gamma,[\ ])$, $\Phi\neq A\subseteq R.$ and $\gamma\in\Gamma$ then the following statements are holds good:

(1) $I\subset(I:A)\subseteq(I:A\gamma A\gamma A)\subseteq(I:A\gamma A\gamma A), \forall\gamma\in\Gamma$

(2) $If\ A\subseteq I, then\ (I:A)=R.$

Theorem 4.3: Assume that R be a commutative ternary Γ −semiring.If I is an ideal of $(R,\Gamma,[\ ])$, $\Phi\neq A\subseteq R.$ then $If\ A\subseteq I=\bigcap_{\alpha\in A}(I:a)=(I:A\setminus I)$ An equivalence relation θ on $(R,\Gamma,[\ ])$ is said to be a congruence if for all $p,q,r\in R,\gamma\in\Gamma,$ we have

$p\theta q\theta r \Rightarrow (p+s)\theta(q+s)\theta(r+s)$

$p\theta q \Rightarrow (p\gamma q\gamma s)\theta(q\gamma r\gamma s)\theta(r\gamma p\gamma s)\ and\ (q\gamma p\gamma s)\theta(r\gamma q\gamma s)\theta(p\gamma r\gamma s)$

By R: θ,we mean the set of all equivalence classes of the elements of R with respect to the mapping θ that is, $R:\theta=\{\theta(x)/x\in R\}.$

Lemma 4.4: Let θ be a congruence relation on $(R,\Gamma,[\ ])$.Then
$\theta(x+y)=\theta(\theta(x)+\theta(y))\ and\ \theta(x\gamma y\gamma z)=\theta(\theta(x)\gamma\theta(y)\gamma\theta(z)),\ for\ all\ x,y,z\in R\ and\ \gamma\in\Gamma.$

Proof: First we observe that $\theta(x+y)\subseteq\theta(\theta(x)+\theta(y))$ and $\theta(x\gamma y\gamma z)\subseteq\theta(\theta(x)\gamma\theta(y)\gamma\theta(z))$ .By routine checking, we can easily verify that the above equalities holds.

In the next theorem, we demonstrate how to construct a new ternary Γ −semiring by using the congruence relations.

Theorem 4.5: Let θ be a congruence relation on $(R,\Gamma,[\ ])$. Define $\oplus$ on R :θ by $\theta(x)\oplus\theta(y)=\theta(x+y)\ for\ all\ x,y\in R.$ Then $(R:\theta,\oplus,[\ ])$ is a ternary Γ −semiring with the following mapping $\square:(R:\theta)\times\Gamma\times(R:\theta)\times\Gamma\times(R:\theta)\to(R:\theta),$ defined by
$\theta(x)\square\gamma\square\theta(y)\square\gamma\square\theta(z)=\theta(x\gamma y\gamma z), \text{for all } x,y,z\in R,\gamma\in\Gamma.$

Proof: Let $\theta(x)=\theta(x')\ and\ \theta(y)=\theta(y')\ and\ \theta(z)=\theta(z')$ then by lemma 4.4 we have the following equality

$$\begin{aligned}\theta(x)\oplus\theta(y)=\theta(x+y)&=\theta(\theta(x)+\theta(y))\\&=\theta(\theta(x')+\theta(y'))\\&=\theta(x')\oplus\theta(y')\end{aligned}$$

Also we have an additional euality

$$\theta(x)\square\,\gamma\square\,\theta(y)\square\,\gamma\square\,\theta(\mathrm{z}) = \theta(\mathrm{x}\,\gamma\,\mathrm{y}\,\gamma\,\mathrm{z})$$
$$= \theta(\theta(x)\gamma\theta(\mathrm{y})\gamma\theta(\mathrm{z}))$$
$$= \theta(\theta(\mathrm{x}^{'})\gamma\theta(y^{'})\gamma\theta(\mathrm{z}^{'})$$
$$= \theta(\mathrm{x}^{'})\square\,\gamma\square\,\theta(y^{'})\square\,\gamma\square\,\theta(z^{'})$$

Thus $\oplus$ *and* $\square$ are well defined.

Hence, the author cam verify that $(R:\theta,\oplus,[\ ])$ is a commutative ternary semigroup. Now we deduce that

$$\theta(x)\square\,\gamma\square\,\theta(y)\square\,[(\theta(\mathrm{z})\oplus\theta(t))] = \theta(x)\square\,\gamma\square\,\theta(y)\square\,\theta(z+t)$$
$$= \theta(x\gamma\, y\gamma(\mathrm{z}+t))$$
$$= \theta([x\gamma\, y\gamma \mathrm{z}]+[x\gamma\, y\gamma t])$$
$$= \theta[x\gamma\, y\gamma \mathrm{z}]\oplus\theta[x\gamma\, y y\gamma t]$$
$$= [\theta(x)\square\,\gamma\square\,\theta(\mathrm{y})\square\,\gamma\square\,\theta(\mathrm{z})]\oplus[\theta(x)\square\,\gamma\square\,\theta(\mathrm{y})\square\,\gamma\square\,\theta(\mathrm{t})]$$

In that similar way prove that

$$[\theta(x)\square\,\gamma\square\,[\theta(y)\oplus\theta(\mathrm{z})]\square\,\gamma\square\,\theta(t) = [\theta(x)\square\,\gamma\square\,\theta(y)\square\,\theta(t)]\oplus[\theta(x)\square\,\gamma\square\,\theta(\mathrm{z})\square\,\theta(t)]$$
$$[\theta(x)\oplus\theta(y)]\square\,\gamma\square\,\theta(\mathrm{z})\square\,\gamma\square\,\theta(t) = [\theta(\mathrm{x})\square\,\gamma\square\,\theta(z)\square\,\gamma\square\,\theta(t)]\oplus[\theta(\mathrm{y})\square\,\gamma\square\,\theta(z)\square\,\gamma\square\,\theta(t)]$$

also

$$[[\theta(x)\square\,\alpha\square\,\theta(\mathrm{y})\square\,\beta\square\,\theta(\mathrm{z})]\square\,\gamma\square\,\theta(\mathrm{s})\square\,\delta\square\,\theta(\mathrm{t})]$$
$$= [\theta(x)\square\,\alpha\square\,[\theta(\mathrm{y})\square\,\beta\square\,\theta(z)\square\,\gamma\square\,\theta(\mathrm{s})]\square\,\delta\square\,\theta(\mathrm{t})]$$
$$= [\theta(x)\square\,\alpha\square\,\theta(\mathrm{y})\square\,\beta\square\,[\theta(z)\square\,\gamma\square\,\theta(\mathrm{s})\square\,\delta\square\,\theta(\mathrm{t})]]$$

Therefore R:θ is a ternary Γ −semiring.

Lemma 4.6: If $\prod_R : R\to R:\theta$ is defined by $\prod_R(x)=\theta(x)$ and I_R is the identity function on Γ, then $(\prod_R, I_R):(R,\Gamma)\to(R:\theta,\Gamma)$ is an epi-morphism.

Proof: Let $x,y\in R\, and\, \gamma\in\Gamma$ .Then it is easy to see that

$$\prod_R(x+y)=\theta(x+y)=\theta(x)\oplus\theta(\mathrm{y})$$
$$=\prod_R(x)\oplus\prod_R(\mathrm{y})$$

Also

$$\prod_R(x\gamma\, y\gamma \mathrm{z}) = \theta(x\gamma\, y\gamma \mathrm{z}) = \theta(x)\square\,\gamma\square\,\theta(\mathrm{y})\square\,\gamma\square\,\theta(\mathrm{z})$$
$$=\prod_R(x)\square\,\prod_R(\mathrm{y})\square\,\prod_R(\mathrm{z})$$

Clearly $\prod_R$ is surjective and $(\prod_R, I_R)$ is an epi-morphism.

5 Congruences and products of ternary Γ-semirings

In this section, we show how to use a ternary Γ-ideal and a congruence on a ternary Γ-semiring*R*to construct a new ternary Γ-ideal of *R*and to find the relationship between them.

Theorem5.1. *Let θ be a congruence on* $(R, \Gamma, [\ ])$.*If I is a* ternary Γ *–ideal of* $(R, \Gamma, [\ ])$ *, then* $C_I=\{x \in T|[x\Gamma\theta\Gamma a]\exists a \in I\}$ *is a* ternary Γ*-ideal of* $(R, \Gamma, [\ ])$ *and* $I\subseteq C_I$.

Proof: Clearly $I\subseteq C_I$. Let $x,y\in C_I$. Then$[x\Gamma\theta\Gamma a]$and $[y\Gamma\theta\Gamma b]$for some $a,b\in I$. On the other hand ,$\theta$is a congruence on *R*which implies that $[(x+y)\Gamma\theta\Gamma(a+b)]$ and $x+y \in C_I$. Now, let $x\in C_I$, $r\in R$and $\gamma\in\Gamma$. Then, $[x\Gamma\theta\Gamma a]$for some $a\in I$.In other words, θ is a congruence on T which implies that $[(x\Gamma\gamma\Gamma r)\Gamma\theta\Gamma(a\Gamma\gamma\Gamma r)]$. Thus ,$[x \Gamma \gamma\Gamma r]\in C_I$. Similarly, we can prove that $[r \Gamma \gamma\Gamma x] \in C_I$,also we will prove that $[\gamma \Gamma x \Gamma r]$ C_I,. Therefore, C_I is an ternary Γ *-ideal* of $(R, \Gamma, [\ ])$.

By using the standard arguments , we can prove the following theorem.

Theorem5.2: Let Rbe a ternary Γ -semiring with zero and θ a congruence on (R, Γ,[ ]). Then, θ (0) is an ternary γ-ideal of (R, Γ,[ ]).In the next two theorems, we state the connections between the ideals of $(R, \Gamma, [\ ])$ and $(R{:}\ \theta, \Gamma, [\ ])$.

Theorem5.3.*If I is a* ternary Γ *-ideal of* $(R, \Gamma, [\ ])$,*then* $I{:}\theta$ *is an ideal of* $(R{:}\theta, \Gamma, [\ ])$.

Theorem5.4: *If J is an* ternary Γ*-ideal of* $(R{:}\theta, \Gamma, [\ ])$*, then there exists a* ternary

Γ -ideal I of (R, Γ,[ ]) such that J=I: θ***Proof:*** Define $I=\{x\in R/\theta(x)\in J\}$. Then Wehave

$\theta(x) \in J \Rightarrow x \in I\Rightarrow\theta(x) \in I{:}\theta$,and$\theta(x) \in I{:}\theta\Rightarrow\exists a \in I,\theta(x) = \theta(a) \Rightarrow\theta(x) = \theta(a)\in J$.Thus, $J=I{:}\theta$Now, suppose that $x,y\in I$. Then $\theta(x),\theta(y)\in J$ and by Theorem4.5 , we have $\theta(x+y)=\theta(x) \oplus\theta(y)\in J$. Hence ,$x+y\in I$. Also , assume that $x\in I$, $r\in R$and $\gamma\in\Gamma$ .Then, we have $\theta(x)\in J$ and by Theorem 4.5, we have $\theta([x \Gamma\gamma\Gamma r]) = [\theta(x)\odot \Gamma\odot\gamma\odot \Gamma\odot\theta(r)] \in J$. Hence , $[x \Gamma\gamma \Gamma r] \in I$. Similarly, we can prove that $[r \Gamma\gamma \Gamma x] \in I$ and $[\gamma \Gamma x \Gamma r] \in I$ .Therefore ,*I*is an ideal of $(R ,\Gamma,[\ ])$.

Lemma5.5: *Let* R_i*be a ternary* Γ_i*- semiring*$(1\le i\le n)$. *Then,* $R_1\times\cdots\times R_n$ *is a* Ternary $\Gamma_1\times\cdots\times\Gamma_n$-semiring.The proof is standard and we hence omit the details . It suffices we define$(x_1,\cdots,x_n)+(y_1,\cdots,y_n)= (x_1+y_1,,x_n+y_n)$,and o : $(R_1\times\cdots\times R_n)\times(\Gamma_1\times\cdots\times\Gamma_n)\times(R_1\times\cdots\times R_n)\longrightarrow R_1\times\times R_n$ by $(x_1,\cdots,x_n)$ Γ $_o$ Γ $(\gamma_1,\cdots, \gamma_n)$ Γ $_o$ Γ $(y_1,\cdots,y_n)=[x_1 \Gamma\gamma_1 \Gamma y_1],\ldots\ldots,[x_n \Gamma\gamma_n \Gamma y_n]$,For all $(x_1,\cdots,x_n),(y_1,\cdots,y_n)\in T_1\times\cdots\times T_n$and $(\gamma_1,\cdots,\gamma_n)\in \Gamma_1\times\ldots\times\Gamma_n$.

In the next lemma, we investigate the behavior of congruence on the products of ternary Γ -semirings.

Lemma 5.6: *Let* θ_i *be a congruence on* $(R_i,\Gamma_i,[\ ])$ *for* $1\le i\le n$ *.T hen* θ *is a congruence on* $(T_1\times\cdots\times T_n , \Gamma_1\times\cdots\times\Gamma_n,[\ ])$ *where* $(a_1,\cdots,a_n)\ \Gamma\odot\ \Gamma\theta\ \Gamma\odot\Gamma\ (b_1\ldots\ldots b_n)$ *if*

And only if $[a_i\Gamma_i\theta_i\Gamma_i\, b_i]$*for all* $a_i,b_i\in T_i$*and* $1\le i\le n$.

Proof: If $(x_1,\cdots,x_n)\, {}_o\Gamma_{io}\, \theta\Gamma_{io}(y_1,\cdots,y_n)$, then $[x_i\, {}_o\Gamma_{io}\theta_{io}\Gamma_{io}y_i]$for all $1\le i\le n$. Hence $(x_i+z_i)\, {}_o\Gamma_{io}\theta_i\, {}_o\Gamma_{io}\, (y_i+z_i)$, for all $z_i\in T_i$and $1\le i\le n$. This implies that$(x_1,\cdots,x_n)\mp(z_1,\cdots,z_n)\, {}_o\Gamma_{io}\theta_{io}\Gamma_i(y_1,\cdots,y_n)\mp(z_1,\cdots,z_n)$.Also, $[x_i \Gamma_i\theta_i\Gamma_i y_i]$ for all $1\le i\le n$ implies that $[x_i\, {}_o\Gamma_{io}\gamma_{io}\Gamma_{io}z_i]\theta[y_{io}\Gamma_{io}\gamma_{io}\Gamma_{io}z_i]$ for all $z_i\in T_i$, $\gamma_i\in\gamma_i$and $1 \le i\le n$. Hence $[(x_1,\cdots,x_n)_o\Gamma_{io}(\gamma_1,\cdots,\gamma_n)_o\Gamma_{io}(z_1,\cdots,z_n)\ ]_o\Gamma_{io}\theta\ {}_o\Gamma_{io}\ [(y_1,\cdots,y_n)\ {}_o\Gamma_{io}(\gamma_1,\cdots,\gamma_n)\ {}_o\Gamma_{io}(z_1,\cdots,z_n)]$.Similarly , we can prove that $[(z_1,\cdots,z_n)\ {}_o\Gamma_{io}(\gamma_1,\cdots,\gamma_n)\ {}_o\Gamma_{io}(x_1,\cdots,x_n)]\ {}_{]o}\Gamma_{io}\theta\ {}_o\Gamma_{io}\ [(z_1,\cdots,z_n)\ {}_o\Gamma_{io}(\gamma_1,\cdots,\gamma_n)$

$_{o}\Gamma_{io}(y_1,\cdots,y_n)$]Also $[(y_1,\cdots,y_n)_{o}\Gamma_{io}(\gamma_1,\cdots,\gamma_n)$ $_{o}\Gamma_{io}(x_1,\cdots,x_n)]$ $_{]o}\Gamma_{io}\theta$ $_{o}\Gamma_{io}$ $[(z_1,\cdots,z_n)$ $_{o}\Gamma_{io}(\gamma_1,\cdots,\gamma_n)$ $_{o}\Gamma_{io}(y_1,\cdots,y_n)$]Therefore, θ is a congruence on $(R_1\times\cdots\times R_n,\Gamma_1\times\cdots\times\Gamma_n,[\ ])$.

6 Homomorphism theorems and isomorphism theorems of a ternary Γ –semiring

In the following theorem , we prove an isomorphism theorem of products of ternary Γ-semirings.

Theorem6.1: *Let $\theta_i$be a congruence on $(R_i,\Gamma_i,[\ ])$ for $1\leq i\leq n$ and θ the congruence on $(R_1\times\cdots\times R_n,\Gamma_1\times\cdots\times\Gamma_n,[\ ])$ defined in Lemma 5.6. Then $(R_1:\theta_1)\times\cdots\times(R_n:\theta_n),\Gamma_1\times\cdots\times\Gamma_n \tilde{=} (R_1\times\cdots\times R_n:\theta,\Gamma_1\times\ldots\times\Gamma_n)$.*

Proof: By Theorem 4.5 and Lemmas 5.5 and 5.6 , $(R_1:\theta_1)\times\cdots\times(R_n:\theta_n)$ and $R_1\times\cdots\times R_n:\theta$ are $\Gamma_1\times\cdots\times\Gamma_n$-semirings. Define $\psi:(R_1:\theta_1)\times\cdots\times(R_n:\theta_n)\rightarrow R_1\times\ldots\times R_n:\theta$ by:

$\psi(\theta_1(x_1),\cdots,\theta_n(x_n)=\theta(x_1,...,x_n))$,for all $x_i\in T_i(1\leq i\leq n)$. We can show that $(\psi,1_{\Gamma1\times\cdots\times\Gamma n})$ is an isomorphism between $((R_1:\theta_1)\times\cdots\times(R_n:\theta_n),\Gamma_1\times\cdots\times R_n,[\ ])$ and $(R_1\times\cdots\times R_n:\theta,\Gamma_1\times\cdots\times\Gamma_n,[\ ])$.

We have

$\theta_1(x_1),\cdots,\theta_n(x_n) \Longleftrightarrow \theta_1(y_1),\cdots,\theta_n(y_n)$

$\Longleftrightarrow \theta_i(x_i)=\theta_i(y_i), \quad \forall 1\leq i\leq n$

$\Longleftrightarrow x_i\Gamma_i\theta_i\Gamma_i y_i,\forall 1\leq i\leq n$

$\Longleftrightarrow (x_1\cdots x_n)\Gamma_i\theta\Gamma_i(y_1\cdots y_n)$

$\Longleftrightarrow \theta(x_1\cdots x_n)=\theta(y_1\cdots y_n)$

$\Longleftrightarrow \psi\theta_1(x_1),\cdots,\theta_n(x_n)=\psi\theta_1(y_1),\cdots,\theta_n(y_n)$.

Hence , $(\psi,1_{\Gamma1\times\cdots\times\Gamma n})$ is well-defined and one to one. Clearly ,$(\psi,1_{\Gamma1\times\cdots\times\Gamma n})$is onto. Now, we prove that $(\psi,\Gamma_1\times\cdots\times\Gamma_n)$ is a homomorphism. We have

$$\Psi((\theta_1(x_1),\cdots,\theta_n(x_n))\mp(\theta_1(y_1),\cdots,\theta_n(y_n)))=\Psi(\theta_1(x_1)\oplus\theta_1(y_1),\cdots,\theta_n(x_n)\oplus\theta_n(y_n))=$$
$$\psi(\theta_1(x_1+y_1),\cdots,\theta_n(x_n+y_n))=\theta((x_1+y_1),\cdots,(x_n+y_n))=$$
$$\theta(x_1\cdots,x_n)\oplus\theta(y_1\cdots,y_n)=$$
$$\psi(\theta_1(x_1),\cdots,\theta_n(x_n))\oplus\psi(\theta_1(y_1),\cdots,\theta_n(y_n)).$$

Also, we have
$\Psi((\theta_1(x_1),\cdots,\theta_n(x_n))o(\gamma_1,\cdots,\gamma_n)o(\theta_1(y_1),\cdots,\theta_n(y_n)))=\Psi(\theta_1(x_1)\odot\Gamma\odot\gamma_1\odot\Gamma\odot\theta_1(y_1),\cdots,\theta_n(x_n)\odot\Gamma\odot\gamma_n\odot\Gamma\odot\theta_n(y_n))=\Psi(\theta_1[x_1\Gamma_1 y_1],\cdots,\theta_n[x_n\gamma_n y_n])=\theta[x_1\gamma_1 y_1,\cdots,x_n\gamma_n y_n)=\theta([x_1,\cdots x_n])\odot\Gamma\odot(\gamma_1,\cdots,\gamma_1)\odot\Gamma\odot\theta([y_1,\cdots,y_n])=\psi(\theta_1(x_1),\cdots,\theta_n(x_n))\odot\Gamma\odot 1_{\gamma_1}\times\cdots\times\gamma_n(\gamma_1,\cdots,\gamma_n)\odot\Gamma\odot\psi(\theta_1(y_1),\cdots,\theta_n(y_n))$.

Therefore, $(\psi,1_{\gamma_1}\times\cdots\times\gamma_n)$is an isomorphism.

In the next theorems, we consider the congruence on the ternary γ-semirings$T$ in- duced by the homomorphisms and investigate the corresponding results and properties associated with this congruence on R.

Theorem6.2. *Let* (ϕ,g) :$(R_1,\Gamma_1,[\ ])\longrightarrow(R_2,\Gamma_2,[\ ])$*be a homomorphism . Define the relation* $\theta_{(\phi,g)}$ *on* (R_1,Γ_1) *as follows:*$[x\Gamma_1\theta_{(\phi,g)}\Gamma_1 y]\Longleftrightarrow\phi(x)=\phi(y)$.

Then $\theta_{(\phi,g)}$ *is a congruence on* $(R_1\Gamma_1,[\ ])$.

Proof. Clearly ,$\theta_{(\phi,g)}$ is an equivalence relation. Suppose that $x\theta_{(\phi,g)}y$. We have

$\phi(x)=\phi(y) \Longrightarrow\phi(x)+\phi(z)= \phi(y)+\phi(z) \Longrightarrow\phi(x+z)= \phi(y+z)$ for all $z\in R_1$.Thus $[(x+z)\Gamma_1\theta_{(\phi,g)}\Gamma_1(y+z)]$. Also, we have

$$\phi(x)=\phi(y)\Longrightarrow[\phi(x)\Gamma_1 g(\gamma)\Gamma_1\phi(z)]=[\phi(y)\Gamma_1 g(\gamma)\Gamma_1\phi(z)]\Longrightarrow\phi([x\Gamma_1\gamma\Gamma_1 z])=\phi([y\Gamma_1\gamma\Gamma_1 z])$$

for all $z\in R_1$ and $\gamma\in\Gamma_1$. Therefore ,$\theta_{(\phi,g)}$ is a congruence on$(R_1,\Gamma_1,[\ ])$.

Theorem6.3: *Let* (ϕ,g):$(R_1\Gamma_1,[\ ])\longrightarrow(R_2,\Gamma_2,[\ ])$ *be a homomorphism. Set* $A= \{I\subseteq R_1|\theta_{(\phi,g)}\subseteq I\times I\}$*and* $B= \{J|J\subseteq R_2\}$.*Then, there exists an 1-1 mapping from A to B.*

Proof .Define $\psi :A\rightarrow B$ by $\psi(I)=\phi(I)$. Clearly ,ψ is well-defined.

Suppose that $\psi(I_1)=\psi(I_2)$.Then $\phi(I_1)=\phi(I_2)$.Also we can see that

$x\in I_1 \Longrightarrow \phi(x) \in\phi(I_1)=\phi(I_2) \Longrightarrow \exists y\in I_2,\phi(x)=\phi(y)$

$\Longrightarrow (x,y)\in\theta_{(\phi,g)}\subseteq I_2\times I_2$

$\Longrightarrow x\in I_2$

$\Longrightarrow I_1\subseteq I_2$.Similarly,$I_2\subseteq I_1$andso$I_1=I_2$and hence, ψisone-to-one.

Theorem6.4.*Let* $(R_1,\Gamma_1,[\ ])^{(\phi_1,g_1)}(R_2,\Gamma_2,[\ ])^{(\phi_2,g_2)}(R_3,\Gamma_3,[\ ])$*be a sequence of*

Homomorphisms .Then (ψ,g):$(R_1\times R_2\times R_3,\Gamma_1\times\Gamma_2\times\Gamma_3,[\ ])\rightarrow(R_1\times R_2\times R_3,\Gamma_1\times\Gamma_2\times\Gamma_3,[\ ])$

Defined by $\psi(x,y)=\phi_1(x),\phi_1(y)$ *and* $g(\gamma,\beta)=g_1(\gamma),g_1(\beta)$ *for allx,* $y\in R_1$ *and* $\gamma,\beta\in\Gamma_1$*, is a homomorphism such that* $\psi(\theta_{(\phi_1},g_1))\subseteq\theta_{(\phi_2},g_2)$*. More- over , if* (ψ_1,g_1) *is on to and* (ψ_2,g_2) *is one to one ,then* $\psi(\theta_{(\phi_1},g_1))=\theta_{(\phi_2},g_2)$.

Proof. It is trivial that (ψ,g) is a homomorphism.Hence,we have

$\psi(a,b)\in\psi(\theta_{(\phi_1},g_1)),(a,b)\in\theta_{(\phi_1},g_1) \Longrightarrow \psi_1(a)=\psi_1(b)$

$\Longrightarrow \psi_2(\psi_1(a)=\psi_2(\psi_1(b))$

$\Longrightarrow (\psi_1(a),\psi_1(b))\in\theta(\psi_2,g_2)$

$\Longrightarrow \psi(a,b)\in\theta(\psi_2,g_2)$.

Thus, $\psi(\theta(\psi_1,g_1))\subseteq\theta(\psi_2,g_2)$.Now, if (ψ_1,g_1) is surjective and (ψ_2,g_2) is in-jective, then we have $\psi(\theta(\psi_1,g_1))=\theta(\psi_2,g_2)$It suffices to prove that $\theta(\psi_2,g_2)\subseteq\psi(\theta_{(\phi_1},g_1))$ Hence, we have

$(t,t')\in\theta(\psi_2,g_2)\Longrightarrow$ 22 $\psi_2(t)= \psi_2(t')$

$\Longrightarrow \exists a,b\in R,\psi_1(a)=t,\psi_1(b)=t'$

$\Longrightarrow (t,t')= \psi(a,b)=(\psi_1(a),\psi_1(b))$

$\Longrightarrow (t,t')\in\psi(\theta(\psi_1,g_1))$

This shows that $\theta_{(\phi 2,g_2)} \subseteq \psi\theta_{(\phi 1,g_1)}$, and the proof is completed.

Theorem6.5.. *Let* $(R_1,\Gamma_1,[\ ])^{(\phi_1,g_1)}(R_2,\Gamma_2,[\ ])^{(\phi_2,g_2)}(R_3,\Gamma_3,[\ ])$ *be a sequence of Homomorphism's. Then* $Im\psi_1 \times Im\psi_1 \subseteq \theta_{(\psi_2,g_2)}$ *if and only if* $\psi_{2\,o}\ \psi_1$ *is constant.*

Proof. The proof of the necessary part is routine and we only prove the sufficiency part.

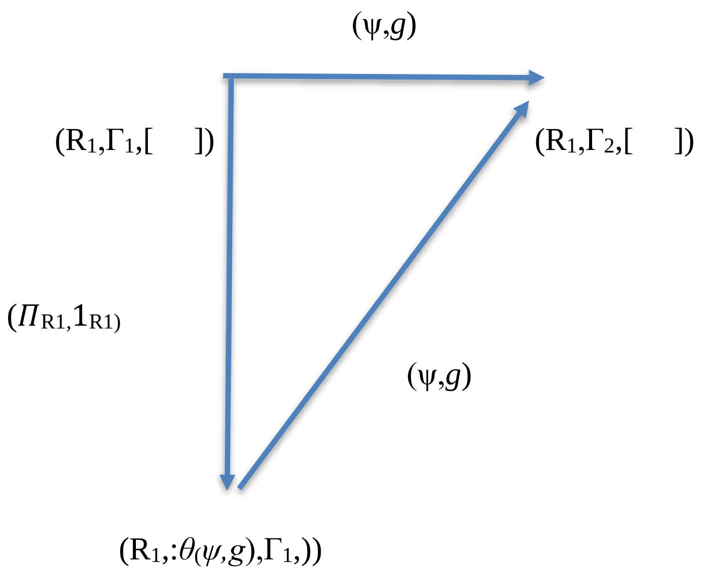
(=⇒):Let $x,y \in R_1$.Then, $(\psi_1(x),\psi_1(y)) \in Im\psi_1 \times Im\psi_1 \subseteq \theta_{(\psi_2,g_2)}$. Hence,

$\psi_2\psi_1(x)=\psi_2\psi_1(y)$.This show that $\psi_2 o \psi_1$is a constant.

Finally, by the congruence on the ternary γ-semiring induced by homomorphism , we are able to establish some isomorphism theorems and investigate the commu- tativity of some diagrams.

Theorem6.6. (IsomorphismTheorem) If $(\psi,g):(R_1, \Gamma_1,[\ ]) \dashrightarrow (R_2,\Gamma_2,[\ ])$ is an epimorphism,then there exists an unique isomorphism

$(\psi,g):(R_1:\theta_{(\phi,g)},\Gamma_1,[\ ]) \dashrightarrow (R_2,\Gamma_2,[\ ])$ Such that the following diagram commutes:



Where $\Pi_{R1},:R_1,\rightarrow R_1,:\ \theta_{(\phi,g)}$ *is defined by* $\Pi_{R1}(x)=\theta_{(\phi,g)}(x)$ *for all* $x \in R_1$,

And 1_{R1} *is identity.*

Proof. Define $\psi:R1:\theta_{(\phi,g)} - R_2$ by $\psi(\theta_{(\phi,g)}(x))=\psi(x)$ for all $x \in R_1$.Then we have $\theta_{(\phi,g)}(x)= \theta_{(\phi,g)}(y) \Longleftrightarrow x\theta_{(\phi,g)}y \Longleftrightarrow \psi(x)=\psi(y)$,and hence ψ is well defined and is a 1-1 mapping.Now,(ψ,g) is a homomorphism. We have

$\psi(\theta_{(\phi,g)}(x) \oplus \theta_{(\phi,g)}(y))=\psi(\theta_{(\phi,g)}(x+y))$

$=\psi(x+y)=\psi(x)+\psi(y)$

$=\psi(\theta_{(\phi,g)}(x))+\psi(\theta_{(\phi,g)}(y))$.

We deduce that $\Psi(\theta_{(\phi,g)}(x)\odot\Gamma\odot\gamma\odot\Gamma\odot\theta_{(\phi,g)}(y)=\psi\theta_{(\phi,g)}(x\odot\Gamma\odot\gamma\odot\Gamma\odot y)$

$=\phi(x\odot\Gamma\odot\gamma\odot\Gamma\odot y)=\phi(x)\odot\Gamma\odot g(\gamma)\odot\Gamma\odot\phi(y)=\psi(\theta_{(\phi,g)}(x))\odot\Gamma\odot g(\gamma)\odot\Gamma\odot\psi(\theta_{(\phi,g)}(y))$.

Therefore,(ψ,g) is a homomorphism . Also $\phi(x) = \psi\theta_{(\phi,g)}(x)= \psi\Pi_{R1}(x)$ and $go1\gamma_1 = g$ which imply that the diagram is commutative . Let

$$(\psi,g):(R_1:\theta_{(\phi,g)}, \Gamma_1,[\ ])\longrightarrow(R_2, \Gamma_2,[\ ])$$

Be such that $\psi_o\Pi_{R1}=\phi$.Then, Wehave

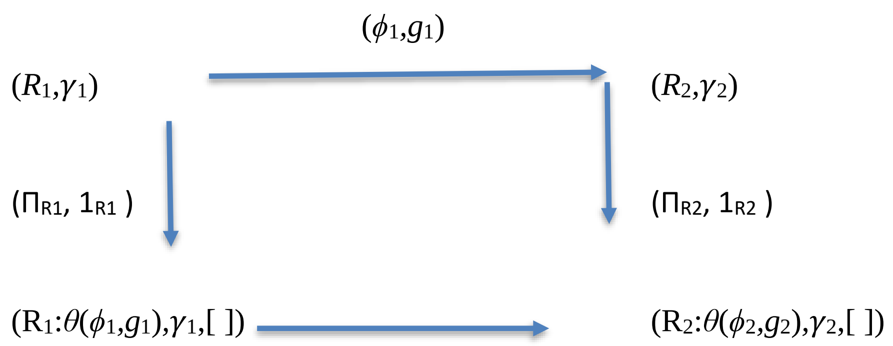
$$\psi\theta_{(\phi,g)}(x)=\psi\Pi_{R1}(x)=\phi(x)=\psi\Pi_{R1}(x)=\psi\theta_{(\phi,g)}(x).$$

Thus ,(ψ,g) is unique and the proof is completed.

Theorem 6.7. *Let* $(R_1, \Gamma_1,[\ ]) \xrightarrow{(\phi_1,g_1)} (R_2,\gamma_2,[\ ]) \xrightarrow{(\phi_2,g_2)} (R_3,\gamma_3,[\ ])$ *be a* Sequence of Homomorphism's .Then ,there exists an unique homomorphism

$$(\psi,g_1):(T_1:\theta_{(\phi_1},g_1),\gamma_1)\rightarrow(T_2:\theta_{(\phi_2},g_2),\gamma_2)$$

Such that the following diagram is commutative:



Moreover ,if(ϕ_1,g_1) *is on to and* (ϕ_2,g_2) *is 1-1,then*(ψ,g_1) *is an isomorphism.*

Proof **.**Define ψ: R_1: $\theta_{(\phi_1},g_1)R_2$: $\theta_{(\phi_2},g_2)$by $\psi(\theta_{(\phi_1},g_1)(x))= \theta_{(\phi_2},g_2)\phi_1(x)$. Then, ψ is well-defined.Finally,we need prove that(ψ_1,g_1) is a homomorphism. We have

$$\Psi(\theta_{(\phi_1},g_1)(x)\oplus\theta_{(\phi_1},g_1)(y))=\psi(\theta_{(\phi_1},g_1)(x+y))=\theta_{(\phi_2},g_2)\phi_1(x+y)=\theta_{(\phi_2},g_2)\phi_1(x)+\phi_1(y)$$

$$=\vartheta_{(\phi_2,g_2)}\phi_1(x)\oplus\vartheta_{(\phi_2,g_2)}\phi_1(y)$$

$$=\varphi(\theta_{(\varphi_1,g_1}(x))\oplus\varphi(\theta_{(\varphi_1,g_1}(y))$$

Also, we have

$$\Psi(\theta_{(\phi_1},g_1)(x)\odot\Gamma\odot\gamma\odot\Gamma\odot\theta_{(\phi_1},g_1)(y))=\psi(\theta_{(\phi_1},g_1)(x\odot\Gamma\odot y)$$

$$=\theta_{(\phi_2},g_2)\phi_1(x\odot\Gamma\odot y)$$

$$=\theta_{(\phi_2},g_2)\phi_1(x)g_1(\gamma)\phi_1(y)$$

$$=\theta(\phi_2,g_2)\phi_1(x)\odot g_1(\gamma)\odot\theta_{(\phi_2},g_2)\phi_1(y)$$

$$= \vartheta_{(\phi_1,g_1)}(x)\odot g_1(\gamma)\odot\psi\vartheta_{(\phi_1,g_1)}(y).$$

Therefore, (ψ,g_1) is a homomorphism .Also ,we have

$\psi\Pi_{R1}(x) = \psi\theta_{(\phi1},g_1)(x) = \theta_{(\phi2},g_2)\phi_1(x) = \Pi_{R2}\phi_1(x)$, and $g_{1o}1_{\gamma1} = 1_{\gamma2}og1$.This shows that the diagram is commutative.

Let$(\bar{\psi}, g_1):(T_1:\theta_{(\phi1},g_1),\gamma_1,[\ ]) \longrightarrow (T_2:\theta_{(\phi2},g_2),\gamma_2,[\ ])$ be a homomorphism which

Makes the diagram commutative. Then, we have

$\bar{\psi}\theta_{(\phi1},g_1)(x) = \bar{\psi}\Pi_{T1}(x) = \Pi_{t2}\phi_1(x) = \theta_{(\phi2},g_2)\phi_1(x) = \psi\theta_{(\phi1},g_1)(x)$.

Thus, (ψ, g_1) is unique and the proof is completed.

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