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Characterization of Bipolar Valued Vague Ideals of a Semiring

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Abstract:

Bipolar valued vague ideal of a semiring (BVVI) is described and examined in the present paper. Some characterization theorems are introduced in this paper and intersection, product and strongest bipolar valued vague relation of bipolar valued vague ideals (BVVI) of a semirings are introduced.

Keywords: Fuzzy subset, vague subset, bipolar valued fuzzy subset, bipolar valued vague subset, bipolar valued vague ideals.

1 Introduction

Zadeh's [17] research from 1965 was the first to create the idea of a fuzzy subset of a set, and the mathematical construct known as a fuzzy set is useful for expressing a group of things with ambiguous borders. There have been many generalisations of this core idea since it has developed into a lively area of research in other fields, including intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets, soft sets, etc. Grattan-Guiness Fuzzy membership mapped onto interval and multiple valued quantities was discussed in [9]. A fuzzy set extension known as a vague set is a special instance of a fuzzy set that depends on the context. D.J.Buehrer and W.L. Gau [8] introduced the vague set. Lee presented the idea of bipolar valued fuzzy sets in his article from [8]. Fuzzy sets with membership degree ranges that vary from [0, 1] to [-1, 1] are considered extensions of fuzzy sets. In a bipolar valued fuzzy set, elements with a membership degree of 0 are unrelated to the associated property, those with a membership degree of (0, 1] are somewhat in agreement with the property, and those with a membership degree of [-1, 0) are somewhat in agreement with the implicit counter property. Both intuitionistic and bipolar valued fuzzy sets have a similar appearance. They differ from one another, but, [10,11]. Azriel Rosenfeld[5] introduced the fuzzy subgroup. [5]. The vague groups were introduced by Ranjit Biswas ([13]). A new class of generalised bipolar ambiguous sets has been presented by S. Cicily Flora and I. Arockiarani in [7]. Described as bipolar valued fuzzy subgroups of a group by Anitha.M.S., et. al.[1] and The bipolar interval valued fuzzy subgroups of a group were defined by A. Balasubramanian. [6] . K. Murugalingam and K. Arjunan talked about the interval-valued fuzzy subsemiring of a semiring in their discussion in [12]. and Yasodara.B and KE.Sathappan [14] developed bipolar valued multi fuzzy semiring subsemirings. Bipolar valued vague subsemirings of a semiring were defined by Anitha.K., et al [2,3,4].This article makes use of the idea of bipolar valued vague ideals (BVVI) of a semiring.

2 Preliminaries

In this step, we recollect a few key standards and definitions that are likely to be significant for this work.

Definition 2.1 [12] A mapping $\mathfrak{Y}: \mathfrak{G} \longrightarrow [0, 1]$ *is called fuzzy subset of the Universal set* $\mathfrak{G}$.

Definition 2.2 [5] *The ordered structure* $\mathfrak{N} = \{(\mathfrak{y}, \mathcal{V}_{\mathfrak{N}}(\mathfrak{y})): \mathfrak{y} \in \mathbb{T}\}$ *is called* a vague set *of the set* $\mathbb{T}$, *where* $\mathcal{V}_{\mathfrak{N}}(\mathfrak{y}) = [\mathcal{T}_{\mathfrak{N}}(\mathfrak{y}), 1 - \mathcal{F}_{\mathfrak{N}}(\mathfrak{y})]$, $\mathcal{T}_{\mathfrak{N}}: \mathbb{T} \to [0,1]$ *is a truth membership* map and $\mathcal{F}_{\mathfrak{N}}: \mathbb{T} \to [0,1]$ is a false membership map.

Example 2.3 $\mathfrak{N}$ = { ($\mathfrak{y}$, [0.4, 0.7]), ($\mathfrak{v}$, [0.5, 0.8]), ($\mathscr{k}$, [0.6, 0.9])} is a vague subset of the Universal set $\mathbb{T}$ = {$\mathfrak{y}$, $\mathfrak{v}$, $\mathscr{k}$}.

Definition 2.4 [9] *The ordered structure* $\mathfrak{X} = \{(\mathfrak{v}, \mathfrak{X}^{+}(\mathfrak{v}), \mathfrak{X}^{-}(\mathfrak{v})): \mathfrak{v} \in \mathbb{T}\}$ *is called* a bipolar *valued fuzzy subset of* $\mathbb{T}$, *where* $\mathfrak{X}^{+}: \mathbb{T} \to [0,1]$ *is a positive membership* map and $\mathfrak{X}^{-}: \mathbb{T} \to [-1,0]$ is a negative membership map.

Example 2.5 $\mathfrak{N}$ = { ($\mathfrak{y}$, 0.05, –0.003), ($\mathfrak{v}$, 0.04, –0.6), ($\mathscr{k}$, 0.004, –0.07)} is a bipolar valued fuzzy subset of the set $\mathbb{T}$ = { $\mathfrak{y}$, $\mathfrak{v}$, $\mathscr{k}$ }.

Definition 2.6 [7] *The ordered structure* $\mathfrak{E} = \{(\mathfrak{v}, \mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v}), \mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v})): \mathfrak{v} \in \mathbb{T}\}$ *is called* a bipolar *valued vague subset* ($\mathbb{BVVSS}$) *of* $\mathbb{T}$, *where* $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v}) = [\mathcal{T}_{\mathfrak{E}}^{+}(\mathfrak{v}), 1 - \mathcal{F}_{\mathfrak{E}}^{+}(\mathfrak{v})]$ and

$\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v}) = [-1 - \mathcal{F}_{\mathfrak{E}}^{-}(\mathfrak{v}), \mathcal{T}_{\mathfrak{E}}^{-}(\mathfrak{v})]$, $\mathcal{T}_{\mathfrak{E}}^{+}: \mathbb{T} \to [0, 1]$, $\mathcal{F}_{\mathfrak{E}}^{+}: \mathbb{T} \to [0, 1]$, $\mathcal{T}_{\mathfrak{E}}^{-}: \mathbb{T} \to [-1, 0]$ and

$\mathcal{F}_{\mathfrak{E}}^{-}: \mathbb{T} \to [-1, 0]$ *such that* $\mathcal{T}_{\mathfrak{E}}^{+}(\mathfrak{v}) + \mathcal{F}_{\mathfrak{E}}^{+}(\mathfrak{v}) \leq 1$ *and* $-1 \leq \mathcal{F}_{\mathfrak{E}}^{-}(\mathfrak{v}) + \mathcal{T}_{\mathfrak{E}}^{-}(\mathfrak{v})$.

Example 2.7 $\mathfrak{E}$ = { ($\mathfrak{y}$, [0.03, 0,6], [–0.6, –0.02]), ($\mathfrak{v}$, [0.002, 0.04], [–0.05, –0.004]), ($\mathscr{k}$, [0.02, 0.7], [–0.05, –0.005])} is a $\mathbb{BVVSS}$ of $\mathbb{T}$ = {$\mathfrak{y}$, $\mathfrak{v}$, $\mathscr{k}$}.

Definition 2.8. [4] Let A = $\langle V_A^{+}, V_A^{-} \rangle$ and B = $\langle V_B^{+}, V_B^{-} \rangle$ be two $\mathbb{BVVSS}$s of a set X. We define the following relations and operations:

(i) [A] $\subset$ [B] if and only if V_A^{+}(u) $\leq$ V_B^{+}(u) and V_A^{-}(u) $\geq$ V_B^{-}(u), $\forall$ u$\in$X.

(ii) [A] = [B] if and only if V_A^{+}(u) = V_B^{+}(u) and V_A^{-}(u) = V_B^{-}(u), $\forall$ u$\in$X.

(iii) [A]$\cap$[B] = { $\langle$ u, rmin (V_A^{+}(u), V_B^{+}(u)), rmax (V_A^{-}(u), V_B^{-}(u)) $\rangle$ / u$\in$X }.

(iv) [A]$\cup$[B] = { $\langle$ u, rmax(V_A^{+}(u), V_B^{+}(u)), rmin (V_A^{-}(u), V_B^{-}(u))$\rangle$ / u$\in$X}. Here rmin (V_A^{+}(u), V_B^{+}(u)) = [min { $t_A^{+}(x)$, $t_B^{+}(x)$ }, min {$1-f_A^{+}(x)$, $1-f_B^{+}(x)$}], rmax (V_A^{+}(u), V_B^{+}(u)) = [max { $t_A^{+}(x)$, $t_B^{+}(x)$ }, max {$1-f_A^{+}(x)$, $1-f_B^{+}(x)$ }], rmin (V_A^{-}(u), V_B^{-}(u)) = [min {$-1-f_A^{-}(x)$, $-1-f_B^{-}(x)$}, min { $t_A^{-}(x)$, $t_B^{-}(x)$ }], rmax (V_A^{-}(u), V_B^{-}(u)) = [max {$-1-f_A^{-}(x)$, $-1-f_B^{-}(x)$ }, max { $t_A^{-}(x)$, $t_B^{-}(x)$ }].

Definition 2.9. A $\mathbb{BVVSS}$ $\mathfrak{E} = \langle \mathcal{V}_{\mathfrak{E}}^{+}, \mathcal{V}_{\mathfrak{E}}^{-} \rangle$ *of a semiring* $\mathbb{R}$ is said to be a bipolar valued vague subsemiring ($\mathbb{BVVSSR}$) *of* $\mathbb{R}$ if

(i) $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v} + \mathfrak{h}) \geq$ rmin{ $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v})$, $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{h})$},

(ii) $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v}\mathfrak{h}) \geq$ rmin{ $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v})$, $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{h})$},

(iii) $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v}+\mathfrak{h}) \leq$ rmax{ $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v})$, $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{h})$},

(iv) $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v}\mathfrak{h}) \leq$ rmax{ $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v})$, $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{h})$}, $for\ all\ \mathfrak{v}, \mathfrak{h} \in \mathbb{R}$.

Example 2.10. Let R = Z_3 = {0, 1, 2} be a semiring in terms of standard addition and multiplication. Then A = { <0, [0.5, 0.7], [– 0.8, – 0.5]>, <1, [0.4, 0.6], [–0.7, –0.4]>, <2, [0.4, 0.6], [–0.7, –0.4] >} is a BVVSSR of R.

3 BIPOLAR VALUED VAGUE IDEALS:

This section introduced bipolar valued vague ideals (BVVIs) and looked at their characteristics.

Definition 3.1. A $\mathbb{BVVSS}$ $\mathfrak{E} = \langle \mathcal{V}_{\mathfrak{E}}^{+}, \mathcal{V}_{\mathfrak{E}}^{-} \rangle$ $of\ a\ semiring$ $\mathbb{R}$ is said to be a bipolar valued vague ideal ($\mathbb{BVVI}$) of $\mathbb{R}$ if

(i) $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v}+\mathfrak{h}) \geq$ rmin{ $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v})$, $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{h})$},

(ii) $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v}\mathfrak{h}) \geq$ rmax{ $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v})$, $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{h})$},

(iii) $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v}+\mathfrak{h}) \leq$ rmax{ $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v})$, $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{h})$},

(iv) $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v}\mathfrak{h}) \leq$ rmin{ $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v})$, $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{h})$}, $for\ all\ \mathfrak{v}, \mathfrak{h} \in \mathbb{R}$.

Theorem 3.2. Let $\mathfrak{E} = \langle \mathcal{V}_{\mathfrak{E}}^{+}, \mathcal{V}_{\mathfrak{E}}^{-} \rangle$ be a $\mathbb{BVVI}$ of a semiring $\mathbb{R}$.

(i) If $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v}+\mathfrak{h})$ = [0] then either $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v})$= [0] or $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{h})$ = [0] for $\mathfrak{v}, \mathfrak{h} \in \mathbb{R}$

(ii) If $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v}\mathfrak{h})$ = [0] then either $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v})$ = [0] or $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{h})$= [0] for $\mathfrak{v}, \mathfrak{h} \in \mathbb{R}$

(iii) If $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v}+\mathfrak{h})$= [0] then either $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v})$ = [0] or $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{h})$ = [0] for $\mathfrak{v}, \mathfrak{h} \in \mathbb{R}$

(iv) If $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v}\mathfrak{h})$ = [0] then either $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v})$ = [0] or $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{h})$ = [0] for $\mathfrak{v}, \mathfrak{h} \in \mathbb{R}$

Proof. Let $\mathfrak{v}, \mathfrak{h} \in \mathbb{R}$. (i) By the definition $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v}+\mathfrak{h}) \geq$ rmin { $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v})$, $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{h})$ } which implies that [0] ≥ rmin {$\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v})$, $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{h})$}. Therefore either $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v})$ = [0] or $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{h})$ = [0]. (ii) By the definition $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v}\mathfrak{h}) \geq$ rmax { $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v})$, $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{h})$} which implies that [0] ≥ rmax { $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v})$, $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{h})$}. Therefore either $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{v})$ = [0] or $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{h})$= [0]. (iii) By the definition $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v}+\mathfrak{h}) \leq$ rmax {$\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v})$, $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{h})$} which implies that [0] ≤ rmax {$\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v})$, $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{h})$}. Therefore either $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v})$ = [0] or $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{h})$ = [0] . (iv) By the definition $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v}\mathfrak{h}) \leq$ rmin {$\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v})$, $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{h})$} which implies that [0] ≤ rmin {$\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v})$, $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{h})$}. Therefore either $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{v})$ = [0] or $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{h})$ = [0].

Theorem 3.3. If $\mathfrak{E} = \langle \mathcal{V}_{\mathfrak{E}}^{+}, \mathcal{V}_{\mathfrak{E}}^{-} \rangle$ $is\ a$ $\mathbb{BVVI}$ $of\ a\ semiring$ $\mathfrak{R}$, $then$ $\mathcal{H}$ = {$\mathfrak{o} \in \mathfrak{R}$ / $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{o})$ = [1], $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{o})$ = [–1] } is either a subideal or empty of $\mathfrak{R}$.

Proof. There is no $\mathfrak{o} \in \mathfrak{R}$ $such\ that$ $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{o})$ = [1] and $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{o})$ = [–1], then $\mathcal{H}$ is empty. If $\mathfrak{o}$ and $\mathfrak{s}$ in $\mathcal{H}$, then $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{o}+\mathfrak{s}) \geq$ rmin{ $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{o})$, $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{s})$} = rmin{[1], [1]} = [1]. Thus $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{o}+\mathfrak{s})$ =[1]. And $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{o}\mathfrak{s}) \geq$ rmax{ $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{o})$, $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{s})$} = rmax{[1], [1]}= [1]. So, $\mathcal{V}_{\mathfrak{E}}^{+}(\mathfrak{o}\mathfrak{s})$ =[1]. Also $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{o}+\mathfrak{s}) \leq$ rmax{ $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{o})$, $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{s})$}= rmax{[–1],[–1]} = [–1]. That is, $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{o}+\mathfrak{s})$ = [–1]. And $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{o}\mathfrak{s}) \leq$ rmin{ $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{o})$, $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{s})$} = rmin{[–1], [–1]} = [–1]. So, $\mathcal{V}_{\mathfrak{E}}^{-}(\mathfrak{o}\mathfrak{s})$ = [–1]. That is $\mathfrak{o}+\mathfrak{s}$, $\mathfrak{o}\mathfrak{s} \in \mathcal{H}$. Hence $\mathcal{H}$ $is\ a\ suideal\ of$ $\mathfrak{R}$.

Theorem 3.4. If $\mathfrak{E} = \langle \mathcal{V}^{+}_{\mathfrak{E}}, \mathcal{V}^{-}_{\mathfrak{E}} \rangle$ and $\mathfrak{H} = \langle \mathcal{V}^{+}_{\mathfrak{H}}, \mathcal{V}^{-}_{\mathfrak{H}} \rangle$ are two 𝔹𝕍𝕍𝕀s of a ring R, then their intersection $\mathfrak{E} \cap \mathfrak{H}$ is a 𝔹𝕍𝕍𝕀 of ℝ .

Proof. Let Ç = $\mathfrak{E} \cap \mathfrak{H}$ and let $\mathfrak{v}, \mathfrak{h} \in \mathbb{R}$. Now $\mathcal{V}^{+}_{Ç}(\mathfrak{v}+\mathfrak{h})$ = rmin { $\mathcal{V}^{+}_{\mathfrak{E}}(\mathfrak{v}+\mathfrak{h}), \mathcal{V}^{+}_{\mathfrak{H}}(\mathfrak{v}+\mathfrak{h})$ } ≥ rmin{rmin{ $\mathcal{V}^{+}_{\mathfrak{E}}(\mathfrak{v})$, $\mathcal{V}^{+}_{\mathfrak{E}}(\mathfrak{h})$}, rmin{ $\mathcal{V}^{+}_{\mathfrak{H}}(\mathfrak{v})$, $\mathcal{V}^{+}_{\mathfrak{H}}(\mathfrak{h})$ }} ≥ rmin{rmin{ $\mathcal{V}^{+}_{\mathfrak{E}}(\mathfrak{v})$, $\mathcal{V}^{+}_{\mathfrak{H}}(\mathfrak{v})$}, rmin{ $\mathcal{V}^{+}_{\mathfrak{E}}(\mathfrak{h})$, $\mathcal{V}^{+}_{\mathfrak{H}}(\mathfrak{h})$}} = rmin{ $\mathcal{V}^{+}_{Ç}(\mathfrak{v})$, $\mathcal{V}^{+}_{Ç}(\mathfrak{h})$}. Therefore $\mathcal{V}^{+}_{Ç}(\mathfrak{v}+\mathfrak{h})$ ≥ rmin{ $\mathcal{V}^{+}_{Ç}(\mathfrak{v})$, $\mathcal{V}^{+}_{Ç}(\mathfrak{h})$, for all $\mathfrak{v}, \mathfrak{h} \in \mathbb{R}$. And $\mathcal{V}^{+}_{Ç}(\mathfrak{v}\mathfrak{h})$= rmin{ $\mathcal{V}^{+}_{\mathfrak{E}}(\mathfrak{v}\mathfrak{h}), \mathcal{V}^{+}_{\mathfrak{H}}(\mathfrak{v}\mathfrak{h})$} ≥ rmin{rmax{ $\mathcal{V}^{+}_{\mathfrak{E}}(\mathfrak{v}), \mathcal{V}^{+}_{\mathfrak{E}}(\mathfrak{h})$}, rmax{ $\mathcal{V}^{+}_{\mathfrak{H}}(\mathfrak{v}), \mathcal{V}^{+}_{\mathfrak{H}}(\mathfrak{h})$}} ≥ rmax{rmin{ $\mathcal{V}^{+}_{\mathfrak{H}}(\mathfrak{v})$, $\mathcal{V}^{+}_{\mathfrak{H}}(\mathfrak{h})$}, rmin{ $\mathcal{V}^{+}_{\mathfrak{H}}(\mathfrak{v})$, $\mathcal{V}^{+}_{\mathfrak{H}}(\mathfrak{h})$}} = rmax{ $\mathcal{V}^{+}_{Ç}(\mathfrak{v})$, $\mathcal{V}^{+}_{Ç}(\mathfrak{h})$}. Therefore $\mathcal{V}^{+}_{Ç}(\mathfrak{v}\mathfrak{h})$ ≥ rmax{ $\mathcal{V}^{+}_{Ç}(\mathfrak{v})$, $\mathcal{V}^{+}_{Ç}(\mathfrak{h})$}, for all $\mathcal{V}^{+}_{Ç}(\mathfrak{v}+\mathfrak{h})$. Also $\mathcal{V}^{-}_{Ç}(\mathfrak{v}+\mathfrak{h})$ = rmax{ $\mathcal{V}^{-}_{\mathfrak{E}}(\mathfrak{v}+\mathfrak{h}), \mathcal{V}^{-}_{\mathfrak{H}}(\mathfrak{v}+\mathfrak{h})$} ≤ rmax{rmax{ $\mathcal{V}^{-}_{\mathfrak{E}}(\mathfrak{v})$, $\mathcal{V}^{-}_{\mathfrak{E}}(\mathfrak{h})$}, rmax{ $\mathcal{V}^{-}_{\mathfrak{H}}(\mathfrak{v})$, $\mathcal{V}^{-}_{\mathfrak{H}}(\mathfrak{h})$}}≤ rmax{rmax{ $\mathcal{V}^{-}_{\mathfrak{E}}(\mathfrak{v})$, $\mathcal{V}^{-}_{\mathfrak{H}}(\mathfrak{v})$, rmax{ $\mathcal{V}^{-}_{\mathfrak{E}}(\mathfrak{h})$, $\mathcal{V}^{-}_{\mathfrak{H}}(\mathfrak{h})$}} = rmax{ $\mathcal{V}^{-}_{Ç}(\mathfrak{v})$, $\mathcal{V}^{-}_{Ç}(\mathfrak{h})$}. Therefore $\mathcal{V}^{-}_{Ç}(\mathfrak{v}+\mathfrak{h})$ ≤ rmax{ $\mathcal{V}^{-}_{Ç}(\mathfrak{v})$, $\mathcal{V}^{-}_{Ç}(\mathfrak{h})$}, for all$\mathfrak{v}, \mathfrak{h} \in \mathbb{R}$. And $\mathcal{V}^{-}_{Ç}(\mathfrak{v}\mathfrak{h})$ = rmax{ $\mathcal{V}^{-}_{\mathfrak{E}}(\mathfrak{v}\mathfrak{h}), \mathcal{V}^{-}_{\mathfrak{H}}(\mathfrak{v}\mathfrak{h})$} ≤ rmax{rmin{ $\mathcal{V}^{-}_{\mathfrak{E}}(\mathfrak{v})$, $\mathcal{V}^{-}_{\mathfrak{E}}(\mathfrak{h})$}, rmin { $\mathcal{V}^{-}_{\mathfrak{H}}(\mathfrak{v})$, $\mathcal{V}^{-}_{\mathfrak{H}}(\mathfrak{h})$}} ≤ rmin{rmax{ $\mathcal{V}^{-}_{\mathfrak{E}}(\mathfrak{v})$, $\mathcal{V}^{-}_{\mathfrak{H}}(\mathfrak{v})$ }, rmax{ $\mathcal{V}^{-}_{\mathfrak{E}}(\mathfrak{h})$, $\mathcal{V}^{-}_{\mathfrak{H}}(\mathfrak{h})$}}= rmin{ $\mathcal{V}^{-}_{Ç}(\mathfrak{v})$, $\mathcal{V}^{-}_{Ç}(\mathfrak{h})$}. Therefore $\mathcal{V}^{-}_{Ç}(\mathfrak{v}\mathfrak{h})$ ≤ rmin{ $\mathcal{V}^{-}_{Ç}(\mathfrak{v})$, $\mathcal{V}^{-}_{Ç}(\mathfrak{h})$}, for all $\mathfrak{v}, \mathfrak{h} \in \mathbb{R}$. Hence $\mathfrak{E} \cap \mathfrak{H}$is a 𝔹𝕍𝕍𝕀 of Ç = $\mathfrak{E} \cap \mathfrak{H}$.

Theorem 3.5. The intersection of a family of 𝔹𝕍𝕍𝕀s of a semiring ℝ is a 𝔹𝕍𝕍𝕀 of ℝ.

Proof. The proof follows from the Theorem 3.4.

Theorem 3.6. If $\mathfrak{E} = \langle \mathcal{V}^{+}_{\mathfrak{E}}, \mathcal{V}^{-}_{\mathfrak{E}} \rangle$ and $\mathfrak{H} = \langle \mathcal{V}^{+}_{\mathfrak{H}}, \mathcal{V}^{-}_{\mathfrak{H}} \rangle$ are two 𝔹𝕍𝕍𝕀s of a semiring ℝ, then their union $\mathfrak{E} \cup \mathfrak{H}$ need not be a 𝔹𝕍𝕍𝕀 of ℝ.

Proof. It can be easily proved.

Remark 3.7. If one is contained other, then the union is a 𝔹𝕍𝕍𝕀s of a semiring ℝ.

Definition 3.8. Let $\mathfrak{E} = \langle \mathcal{V}^{+}_{\mathfrak{E}}, \mathcal{V}^{-}_{\mathfrak{E}} \rangle$ and $\mathfrak{P} = \langle \mathcal{V}^{+}_{\mathfrak{P}}, \mathcal{V}^{-}_{\mathfrak{P}} \rangle$ be any two 𝔹𝕍𝕍𝕊𝕊s of sets $\mathfrak{R}_1$ and $\mathfrak{R}_2$. The product of $\mathfrak{E}$ and $\mathfrak{P}$, denoted by $\mathfrak{E} \times \mathfrak{P}$, is defined as $\mathfrak{E} \times \mathfrak{P}$ = {⟨($\mathfrak{v}, \mathfrak{h}$), $\mathcal{V}^{+}_{\mathfrak{E}\times\mathfrak{P}}(\mathfrak{v}, \mathfrak{h})$, $\mathcal{V}^{-}_{\mathfrak{E}\times\mathfrak{P}}(\mathfrak{v}, \mathfrak{h})$⟩ / for all $\mathfrak{v} \in \mathfrak{R}_1$ and $\mathfrak{h} \in \mathfrak{R}_2$ }, where $\mathcal{V}^{+}_{\mathfrak{E}\times\mathfrak{P}}(\mathfrak{v}, \mathfrak{h})$ = rmin{ $\mathcal{V}^{+}_{\mathfrak{E}}(\mathfrak{v})$, $\mathcal{V}^{+}_{\mathfrak{P}}(\mathfrak{h})$} and $\mathcal{V}^{-}_{\mathfrak{E}\times\mathfrak{P}}(\mathfrak{v}, \mathfrak{h})$ = rmax{ $\mathcal{V}^{-}_{\mathfrak{E}}(\mathfrak{v})$, $\mathcal{V}^{-}_{\mathfrak{P}}(\mathfrak{h})$} for all $\mathfrak{v} \in \mathfrak{R}_1$ and $\mathfrak{h} \in \mathfrak{R}_2$.

Theorem 3.9. If A = ⟨V^{+}_{A} , V^{-}_{A} ⟩ and B = ⟨V^{+}_{B} , V^{-}_{B} ⟩ are any two 𝔹𝕍𝕍𝕀s of the semirings R_1 and R_2 respectively, then A×B = ⟨ $V^{+}_{A\times B}$, $V^{-}_{A\times B}$ ⟩ is a 𝔹𝕍𝕍𝕀 of $R_1 \times R_2$.

Proof. Let x_1, x_2 be in R_1, y_1 and y_2 be in R_2. Then (x_1, y_1) and (x_2, y_2) are in $R_1 \times R_2$. Now, $V^{+}_{A\times B}$ [(x_1, y_1)+(x_2, y_2)] = $V^{+}_{A\times B}$ (x_1+x_2, y_1+y_2) = rmin{V^{+}_{A} (x_1+x_2), V^{+}_{B} (y_1+y_2)} ≥ rmin{rmin {V^{+}_{A} (x_1), V^{+}_{A} (x_2)}, rmin{V^{+}_{B} (y_1), V^{+}_{B} (y_2)}} = rmin{rmin{V^{+}_{A} (x_1), V^{+}_{B} (y_1)}, rmin{V^{+}_{A} (x_2), V^{+}_{B} (y_2)}} = rmin{$V^{+}_{A\times B}$ (x_1, y_1), $V^{+}_{A\times B}$ (x_2, y_2)}. Therefore $V^{+}_{A\times B}$ [(x_1, y_1)+(x_2, y_2)] ≥ rmin {$V^{+}_{A\times B}$ (x_1, y_1), $V^{+}_{A\times B}$ (x_2, y_2)}. And $V^{+}_{A\times B}$ [(x_1, y_1)(x_2, y_2)] = $V^{+}_{A\times B}$ (x_1x_2, y_1y_2) = rmin{V^{+}_{A} (x_1x_2), V^{+}_{B} (y_1y_2)} ≥ rmin{rmax{V^{+}_{A} (x_1), V^{+}_{A} (x_2)}, rmax{V^{+}_{B} (y_1), V^{+}_{B} (y_2)}} = rmax{rmin{V^{+}_{A} (x_1), V^{+}_{B} (y_1)}, rmin{V^{+}_{A} (x_2), V^{+}_{B} (y_2)}} = rmax{ $V^{+}_{A\times B}$ (x_1, y_1), $V^{+}_{A\times B}$ (x_2, y_2)}.Therefore $V^{+}_{A\times B}$ [(x_1, y_1) (x_2, y_2)] ≥ rmax{$V^{+}_{A\times B}$ (x_1, y_1), $V^{+}_{A\times B}$ (x_2, y_2)}. Also $V^{-}_{A\times B}$ [(x_1, y_1)+(x_2, y_2)] = $V^{-}_{A\times B}$ (x_1+x_2, y_1+y_2) = rmax{V^{-}_{A} (x_1+x_2), V^{-}_{B} (y_1+y_2)} ≤ rmax{rmax{

V_A^- (x1), V_A^- (x2)}, rmax{V_B^- (y1), (y2)}} = rmax{rmax{V_A^- (x1), V_B^- (y1)}, rmax{V_A^- (x2), V_B^- (y2)}} = rmax{$V_{A\times B}^-$ (x1, y1), $V_{A\times B}^-$ (x2, y2)}. Therefore $V_{A\times B}^-$ [(x1, y1)+(x2, y2)] ≤ rmax{$V_{A\times B}^-$ (x1, y1), $V_{A\times B}^-$ (x2, y2)}. And $V_{A\times B}^-$ [(x1, y1)(x2, y2)] = $V_{A\times B}^-$ (x1x2, y1y2) = rmax{V_A^- (x1x2), V_B^- (y1y2)} ≤ rmax{rmin{V_A^- (x1), V_A^- (x2)}, rmin{V_B^- (y1), V_B^- (y2)}} = rmin{rmax{V_A^- (x1), V_B^- (y1)}, rmax{V_A^- (x2), V_B^- (y2)}} = rmin{$V_{A\times B}^-$ (x1, y1), $V_{A\times B}^-$ (x2, y2)}. Therefore $V_{A\times B}^-$ [(x1, y1)(x2, y2)] ≤ rmin{$V_{A\times B}^-$ (x1, y1), $V_{A\times B}^-$ (x2, y2) }. Hence A×B is a 𝔹𝕍𝕍𝕀 of $R_1\times R_2$.

Theorem 3.10. A product of 𝔹𝕍𝕍𝕀s of the semirings is also a 𝔹𝕍𝕍𝕀 in .

Proof. From the Theorem 3.9, the proof follows.

Definition 3.11. Let A = ⟨V_A^+ , V_A^- ⟩ be a 𝔹𝕍𝕍𝕊𝕊 in a set S, the strongest 𝔹𝕍𝕍 relation on S, that is a 𝔹𝕍𝕍 relation on A is V = {⟨(x, y), V_V^+ (x, y), V_V^- (x, y)⟩ / x, y∈S} given by V_V^+ (x, y) = rmin{ V_A^+ (x), V_A^+ (y) } and V_V^- (x, y) = rmax{V_A^- (x), V_A^- (y)}, ∀ x, y∈S.

Theorem 3.12. Let A = ⟨V_A^+ , V_A^- ⟩ be a 𝔹𝕍𝕍𝕊𝕊 of a semiring R and V = ⟨V_V^+ , V_V^- ⟩ be the strongest 𝔹𝕍𝕍 relation of R. Then A is a 𝔹𝕍𝕍𝕀 of R ⇔ V is a 𝔹𝕍𝕍𝕀 of R×R.

Proof. Suppose that A is a 𝔹𝕍𝕍𝕀 of R. Then for any x = (x1, x2), y = (y1, y2) are in R×R. Now V_V^+ (x+y) = V_V^+ [(x1, x2)+(y1, y2)] = V_V^+ (x1+y1, x2+y2) = rmin{V_A^+ (x1+y1), V_A^+ (x2+y2)} ≥ rmin{rmin{V_A^+ (x1), V_A^+ (y1)}, rmin{V_A^+ (x2), V_A^+ (y2)}} = rmin{rmin{V_A^+ (x1), V_A^+ (x2)}, rmin {V_A^+ (y1), V_A^+ (y2)}} = rmin{V_V^+ (x1, x2), V_V^+ (y1, y2)} = rmin{V_V^+ (x), V_V^+ (y)}. Therefore V_V^+ (x+y) ≥ rmin{ V_V^+ (x), V_V^+ (y)}, ∀ x, y∈R×R. And V_V^+ (xy) = V_V^+ [(x1, x2)(y1, y2)] = V_V^+ (x1y1, x2y2) = rmin{ V_A^+ (x1y1), V_A^+ (x2y2)} ≥ rmin{rmax{V_A^+ (x1), V_A^+ (y1)}, rmax{V_A^+ (x2), V_A^+ (y2)}} ≥ rmax{rmin{V_A^+ (x1), V_A^+ (x2)}, rmin{V_A^+ (y1), V_A^+ (y2)}} = rmax{V_V^+ (x1, x2), V_V^+ (y1, y2)}= rmax{V_V^+ (x), V_V^+ (y)}. Therefore V_V^+ (xy) ≥ rmax{V_V^+ (x), V_V^+ (y)}, ∀ x, y∈R×R. Also we have V_V^- (x+y) = V_V^- [(x1, x2)+(y1, y2)] = V_V^- (x1+y1, x2+y2) = rmax{V_A^- (x1+y1), V_A^- (x2+y2)} ≤ rmax{rmax{V_A^- (x1), V_A^- (y1)}, rmax{V_A^- (x2), V_A^- (y2)}} = rmax{rmax {V_A^- (x1), V_A^- (x2)}, rmax{V_A^- (y1), V_A^- (y2)}}= rmax{V_V^- (x1, x2), V_V^- (y1, y2)} = rmax{V_V^- (x), V_V^- (y)}. Therefore V_V^- (x+y) ≤ rmax{V_V^- (x), V_V^- (y)}, ∀ x, y∈R×R. And V_V^- (xy) = V_V^- [(x1, x2)(y1, y2)] = V_V^- (x1y1, x2y2) = rmax{V_A^- (x1y1), V_A^- (x2y2)} ≤ rmax{rmin{V_A^- (x1), V_A^- (y1)}, rmin{V_A^- (x2), V_A^- (y2)}} = rmin{rmax{V_A^- (x1), V_A^- (x2)}, rmax{V_A^- (y1), V_A^- (y2)}}= rmin{V_V^- (x1, x2), V_V^- (y1, y2) } = rmin{V_V^- (x), V_V^- (y)}. Therefore V_V^- (xy) ≤ rmin{V_V^- (x), V_V^- (y)}, ∀ x, y∈R×R. This proves that V is a 𝔹𝕍𝕍𝕀 of R×R. Conversely assume that V is a 𝔹𝕍𝕍𝕀 of R×R, then for any x = (x1, x2) and y = (y1, y2) are in R×R, we have rmin{V_A^+ (x1+y1), V_A^+ (x2+y2)} = V_V^+ (x1+y1, x2+y2) = V_V^+ [(x1, x2)+(y1, y2)] = V_V^+ (x+y) ≥ rmin{V_V^+ (x), V_V^+ (y)} = rmin { V_V^+ (x1, x2), V_V^+ (y1, y2)} = rmin{rmin{V_A^+ (x1), V_A^+ (x2)}, rmin{V_A^+ (y1), V_A^+ (y2)}}. If V_A^+

(x1+y1) ≤ V_A^+ (x2+y2), we get, V_A^+ (x1+y1) ≥ rmin{ V_A^+ (x1), V_A^+ (y1)}, ∀ x1, y1∈R. And rmin { V_A^+ (x1y1), V_A^+ (x2y2)} = V_V^+ (x1y1, x2y2) = V_V^+ [(x1, x2)(y1, y2)] = V_V^+ (xy) ≥ rmax{ V_V^+ (x), V_V^+ (y)} = rmax{ V_V^+ (x1, x2), V_V^+ (y1, y2)} = rmax{rmin{ V_A^+ (x1), V_A^+ (x2)}, rmin{ V_A^+ (y1), V_A^+ (y2)}}. If V_A^+ (x1y1) ≤ V_A^+ (x2y2), we get V_A^+ (x1y1) ≥ rmax{ V_A^+ (x1), V_A^+ (y1)}, ∀ x1, y1∈R. Also we have rmax{ V_A^- (x1+y1), V_A^- (x2+y2)} = V_V^- (x1+y1, x2+y2) = V_V^- [(x1, x2)+(y1, y2)] = V_V^- (x+y) ≤ rmax{ V_V^- (x), V_V^- (y)} = rmax{ V_V^- (x1, x2), V_V^- (y1, y2)}= rmax{rmax{ V_A^- (x1), V_A^- (x2)}, rmax{ V_A^- (y1), V_A^- (y2)}}. If V_A^+ (x1+y1) ≥ V_A^+ (x2+y2), we get V_A^- (x1+y1) ≤ rmax { V_A^- (x1), V_A^- (y1)}, ∀ x1, y1∈R. And rmax{ V_A^- (x1y1), V_A^- (x2y2) } = V_V^- (x1y1, x2y2) = V_V^- [(x1, x2)(y1, y2)] = V_V^- (xy) ≤ rmin{ V_V^- (x), V_V^- (y)} = rmin{ V_V^- (x1, x2), V_V^- (y1, y2) }= rmin{rmax { V_A^- (x1), V_A^- (x2)}, rmax{ V_A^- (y1), V_A^- (y2)}}. If V_A^+ (x1y1) ≥ V_A^+ (x2y2), we get V_A^- (x1y1) ≤ rmin{ V_A^- (x1), V_A^- (y1)},∀ x1, y1∈R. Hence A is a 𝔹𝕍𝕍𝕀 of R.

4 CONCLUSION

The concept of characterization of bipolar valued vague ideal a semiring is discussed in this section and Bipolar valued vague ideal a semiring properties have been introduced. These ideas are applied to further research in the creation of bipolar valued vague semiring subsemirings. As a result, our upcoming research will examine some of the qualities based on the idea of bipolar valued vague ideals with translations.

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