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Dynamic Effect of Green Building Concept for Sustainable Cities under Climate Change via Variable-Order Fractional Derivatives

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Abstract:

This study introduces an innovative fractional order model that delves into the core principles of sustainable constructions, incorporating Liouville-Caputo, Caputo-Fabrizio, and Atangana-Baleanu derivatives. It examines the increasing adoption of eco-friendly buildings in India as a crucial remedy against climate change. By revealing innovative perspectives on fractional solutions, encompassing their validity and distinctiveness, the research conducts computational analyses to evaluate the impacts of Greenhouse Gas (GHG) elements. Visual representations demonstrate the dynamics of the model, emphasizing the influences of various fractional order parameters across different scenarios.

Keywords: Fractional variable-order model, Liouville-Caputo, Caputo-Fabrizio, Atangana-Baleanu, Numerical simulations.

1. Introduction

In recent years, there has been a notable global increase in the utilization of sustainable construction methods as a key approach to tackling the issues related to climate change. Currently, buildings contribute to 7.85 gigatons (Gt) or 33% of all energy-linked CO_2 emissions worldwide. Forecasts suggest that by 2030, these emissions could potentially rise to between 11 and 15.6 Gt. Specifically, developed nations observe buildings consuming over 41% of total energy usage. In India, the rapid growth in urbanization, population expansion, and the flourishing IT sector and its related industries are major factors contributing to the escalating energy needs.

Green structures are created, constructed, and managed with the aim of minimizing their ecological impact while enhancing efficiency and functionality. This involves strategies like water preservation, reducing energy usage, and employing sustainable resources. In India, there has been a significant rise in the volume of environmentally friendly building space, with the Indian Green Building Council (IGBC) certifying over 7.6 billion square feet of green building space as of August 2023. This reflects a notable increase from the 3.6 billion square feet of certified green building space in 2018.

The expansion of sustainable construction in India is being primarily influenced by several factors, including governmental efforts, rising consumer awareness, and growing demand from multinational corporations. This growth is resulting in numerous favourable outcomes, such as the conservation of resources, decreased greenhouse gas emissions, enhanced indoor air quality, and the creation of employment opportunities. Recently, the Indian government has displayed a stronger dedication to encouraging eco-friendly building practices, while Indian consumers are growing more knowledgeable about the benefits of environmentally conscious constructions. As of October 21, 2023, there are more than 3,088 building projects in India, encompassing a total area of over 1,315 million square feet (122.2×10^6 m^2) of certified space. These initiatives have been approved by the Indian Green Building Council (IGBC), the primary green building assessment system in the nation. It is approximated that less than 10% of buildings in India conform to sustainable standards, a stark contrast to developed countries like the United States, where over 30% of buildings are considered environmentally friendly.

A green building is characterized as one that utilizes less water, enhances energy efficiency, safeguards natural resources, minimizes waste, and provides healthier environments for occupants compared to a traditional building. It is commonly known as a 'high-performance' or sustainable structure [1]. Environmental issues are intertwined with economic and population growth. To achieve sustainability, India needs to effectively waste that contaminates the ecosystem [2]. Through life cycle analysis, eco-friendly buildings will preserve resources and mitigate climate change [3]. The idea of green buildings has developed in a surprising manner to achieve a more realistic sequence of events [4-6]. Sustainable buildings offer reduced energy consumption and minimized negative environmental impacts However, one of the causes of preventable health issues, increased consumption costs, and decreased productivity is the incapacity to adopt a "green" lifestyle [7]. While recent research studies [8, 9] have advised the utilization of innovative technology to enhance the state of buildings constructed from locally sourced materials, this recommendation has yet to be implemented. Concerning thermal comfort, dwellings featuring thatched roofs, bamboo cladding, and mud bricks tend to be more appealing to residents compared to traditional, unsustainable edifices. Enhancing greenery cover to implement nature-based strategies for mitigating heat can significantly reduce energy consumption, as well as the associated risks of mortality and heat-related ailments [10]. The rise in green building construction in urban areas is closely linked to the pressing need for economic development as cities strive to develop intelligent, eco-friendly, and sustainable environment [11-13]. The transition from integer-order differential equations to fractional differential equations represents a broadening of mathematical frameworks. Fractional calculus, widely utilized by scholars and mathematicians, serves as a powerful tool for modeling real-world scenarios [18-22, 33, 34]. The growing significance of variable-order fractional calculus is evident due to its diverse applications across science and engineering fields. These fractional operators with variable orders demonstrate intriguing practical uses [23-30].

In view of the above, this study is driven by the need to comprehend the dynamic effects of the green building concept for sustainable cities under climate change through the application of the variable-order fractional derivative in the Liouville-Caputo, Caputo-Fabrizio, and Atangana-Baleanu senses. In section 2, the necessary preliminary information for the fractional model are provided. Section 3 presents a detailed description of the model in the classical version of the "Green building concept for sustainable cities" and its corresponding fractional order version. The uniqueness and existence of fractional solutions are explained in section 5. Section 6 provides detailed numerical results and simulations for the suggested model on Liouville Caputo, Caputo-Fabrizio, and Atangana-Baleanu derivatives. All obtained results of the work are concluded in the final section.

2. Prelminaries

This section provides some basic definitions of variable order fractional derivatives which are used in subsequent sections.

Definition 2.1:The Liouville–Caputo (LC) fractional derivative with variable-order ν(t) is defined as [32]

$${}^{LC}_{0}D_t^{\nu(t)} f(t) = \frac{1}{\Gamma 1-\nu(t)}\int_0^t (t-u)^{-\nu(t)} f(u)du \text{ , } 0 < \nu(t) \le 1$$

Definition 2.2: The Caputo–Fabrizio (CF) derivative with variable-order $\nu(t)$ in Liouville–Caputo sense is defined as follows

$${}^{CF}_{0}D_t^{\nu(t)} f(t) = \frac{(2-\nu(t))M(\nu(t))}{2(1-\nu(t))}\int_0^t \exp\left[\frac{-\nu(t)}{(1-\nu(t))}(t-u)\right] f'(u)du \text{ , } 0 < \nu(t) < 1$$

where $M(\nu(t)) = \frac{2}{2-\nu(t)}$ is a normalization function.

Definition 2.3: The Atangana–Baleanu (AB) fractional derivative with variable-order $\nu(t)$ in Liouville–Caputo sense is defined as follows [17]

$${}^{AB}_{0}D_t^{\nu(t)} f(t) = \frac{B(\nu(t))}{(1-\nu(t))}\int_0^t E_{\nu(t)}\left[\frac{-\nu(t)}{(1-\nu(t))}(t-u)^{\nu(t)}\right] f'(u)du \text{ , } 0 < \nu(t) \le 1$$

where $B(\nu(t)) = 1-\nu(t) + \frac{\nu(t)}{\Gamma \nu(t)}$ is a normalization function.

Remark: When $\nu(t)$ is a constant, then we retrieve the constant-order fractional derivative in Liouville-Caputo, Caputo-Fabrizio and Atangana-Baleanu sense.

3. Model Formulation in Classical and Fractional Sense

3.1 Classical model of Green Building concept for sustainable cities under climate change

A mathematical model [15] has been developed to examine the effects of global warming excessive greenhouse gas emissions on sustainable urban. This model considers four key variables: G(t) represents the quantity of Green buildings at any given time t, C(t) signifies the level of GHGs at time t, I(t) denotes the energy generated by green buildings at time t, and H(t) reflects the human populations in a specific region at time t.

$$\begin{aligned}
\frac{dG}{dt} &= \Lambda + p_1 G(t)C(t) - p_2 G(t)\\
\frac{dC}{dt} &= \Omega - p_1 G(t)C(t) + p_3 H(t)C(t) - p_4 C(t) \qquad (1)\\
\frac{dI}{dt} &= p_5 G(t)I(t) - p_6 H(t)I(t) - p_7 I(t)\\
\frac{dH}{dt} &= \Psi + p_6 H(t)I(t) - p_8 H(t)
\end{aligned}$$

With initial conditions $G(0) = G_0 \geq 0$; $C(0) = C_0 > 0$; $I(0) = I_0$; $H(0) = H_0 \geq 0$

Where Λ refers the amount of green building , p_1 is the rate at which GHGs are absorbed by green buildings, p_2 is the reduction rate of green building, GHGs are measured by Ω , p_3 is the rate of human originated GHGs, p_4 represents rate of reduction of GHGs, an ingredient production rate by green buildings is measured by p_5, p_6 is the adoption rate of ingredients by human communities, the reduction rate of ingredients is defined as p_7, Ψ is the amount of human community and p_8 is the reduction rate of human communities.

3.2 Fractional version of the classical model

By substituting the classical derivative with the operator $\frac{d^{\nu(t)}f(t)}{dt}$, the fractional model of system (1) is obtained.

$$\frac{d^{\nu(t)}G}{dt} = \Lambda + p_1{}^{\nu(t)}G(t)C(t) - p_2{}^{\nu(t)}G(t)$$

$$\frac{d^{\nu(t)}C}{dt} = \Omega - p_1{}^{\nu(t)}G(t)C(t) + p_3{}^{\nu(t)}H(t)C(t) - p_4{}^{\nu(t)}C(t)$$

$$\frac{d^{\nu(t)}I}{dt} = p_5{}^{\nu(t)}G(t)I(t) - p_6{}^{\nu(t)}H(t)I(t) - p_7{}^{\nu(t)}I(t) \qquad (2)$$

$$\frac{d^{\nu(t)}H}{dt} = \Psi + p_6{}^{\nu(t)}H(t)I(t) - p_8{}^{\nu(t)}H(t)$$

With $G(0) = G_0 \geq 0$; $C(0) = C_0 > 0$; $(0) = I_0$; $H(0) = H_0 \geq 0$

The equilibria of the above fractional-order model can be obtained from $\frac{d^{\nu(t)}G(t)}{dt}$=0 $\frac{d^{\nu(t)}C(t)}{dt}$ = 0, $\frac{d^{\nu(t)}I(t)}{dt}$=0 and $\frac{d^{\nu(t)}H(t)}{dt}$=0.

It was observed that the system (1) has two equilibria, one of them is trivial equilibrium $E_{TE} = (G_0, C_0, I_0, H_0)$ and the other is Global equilibrium $E_{GE} = (G^*, C^*, I^*, H^*)$. Where

$$G^* = \alpha = \frac{\Lambda p_3^{\nu(t)} p_7^{\nu(t)} + \Lambda p_4^{\nu(t)} p_6^{\nu(t)}}{\left[\left(p_2^{\nu(t)} p_3^{\nu(t)} p_7^{\nu(t)} + p_2^{\nu(t)} p_4^{\nu(t)} p_6^{\nu(t)}\right) + \left(\Lambda p_3^{\nu(t)} p_5^{\nu(t)} - \Lambda p_1^{\nu(t)} p_6^{\nu(t)}\right) - p_1^{\nu(t)} \Omega p_6^{\nu(t)}\right]}$$

$$C^* = \frac{\Omega p_6^{\nu(t)}}{\left(p_3^{\nu(t)} p_7^{\nu(t)} + p_4^{\nu(t)} p_6^{\nu(t)}\right) - \alpha\left(p_3^{\nu(t)} p_5^{\nu(t)} + p_1^{\nu(t)} p_6^{\nu(t)}\right)}$$

$$I^* = \frac{p_5^{\nu(t)} p_8^{\nu(t)} \alpha - p_7^{\nu(t)} p_8^{\nu(t)} - \Psi p_6^{\nu(t)}}{p_6^{\nu(t)} (p_5^{\nu(t)} \alpha - p_7^{\nu(t)})}$$

$$H^* = \frac{p_5^{\nu(t)} \alpha - p_7^{\nu(t)}}{p_6^{\nu(t)}}$$

The eigen values are the solutions of the characteristic equation $\det(A_i - \lambda I) = 0$.

Where the matrix A_i and the unit matrix is I with the eigen values calculated at E_{TE} and E_{GE}. For further details of the results can be found in [15]

Since the parameters are dimensionless, the fractional models within LC, CF and AB sense will be the same and it will not be necessary to investigate again.

The Jacobian matrix of the system (2) is as follows

$$J=\begin{bmatrix} p_1{}^{\nu(t)}C - p_2{}^{\nu(t)} & p_1{}^{\nu(t)}G & 0 & 0 \\ -p_1{}^{\nu(t)}C & -p_1{}^{\nu(t)}G + p_3{}^{\nu(t)}H - p_4{}^{\nu(t)} & 0 & p_3{}^{\nu(t)}C \\ p_5{}^{\nu(t)}I & 0 & p_5{}^{\nu(t)}G - p_6{}^{\nu(t)}H - p_7{}^{\nu(t)} & -p_6{}^{\nu(t)}I \\ 0 & 0 & p_6{}^{\nu(t)}H & p_6{}^{\nu(t)}I - p_8{}^{\nu(t)}H \end{bmatrix}$$

4. Existence and Uniqueness of Fractional solutions

4.1 Existence and Uniqueness of Fractional solutions by the Liouville-Caputo model

In this section, we establish the existence and uniqueness of solutions of the Liouville- Caputo model.

Let us construct the system (2) as

$$\begin{aligned}
{}^{LC}_{0}D_t^{\nu(t)}[G(t)] &= F_1(t,G) = \Lambda + p_1{}^{\nu(t)}G(t)C(t) - p_2{}^{\nu(t)}G(t) \\
{}^{LC}_{0}D_t^{\nu(t)}[C(t)] &= F_2(t,C) = \Omega - p_1{}^{\nu(t)}G(t)C(t) + p_3{}^{\nu(t)}H(t)C(t) - p_4{}^{\nu(t)}C(t) \\
{}^{LC}_{0}D_t^{\nu(t)}[I(t)] &= F_3(t,I) = p_5{}^{\nu(t)}G(t)I(t) - p_6{}^{\nu(t)}H(t)I(t) - p_7{}^{\nu(t)}I(t) \qquad (3) \\
{}^{LC}_{0}D_t^{\nu(t)}[H(t)] &= F_4(t,H) = \Psi + p_6{}^{\nu(t)}H(t)I(t) - p_8{}^{\nu(t)}H(t)
\end{aligned}$$

By using Liouville-Caputo fractional integral operator to the above system, we get

$$G(t) - G(0) = \frac{1}{\Gamma\nu(t)}\int_0^t (t-k)^{\nu(t)-1} F_1(K, G(t))dk$$

$$C(t) - C(0) = \frac{1}{\Gamma\nu(t)}\int_0^t (t-k)^{\nu(t)-1} F_2(K, C(t))dk$$

$$I(t) - I(0) = \frac{1}{\Gamma\nu(t)}\int_0^t (t-k)^{\nu(t)-1} F_3(K, I(t))dk$$

$$H(t) - H(0) = \frac{1}{\Gamma\nu(t)}\int_0^t (t-k)^{\nu(t)-1} F_4(K, H(t))dk$$

We will show that the kernel F_i for $i = 1,2,3,4$ follows the Lipschitz condition and contraction.

Theorem 4.1.1

The kernel $F_i(K, G)$ for $i = 1,2,3,4$ satisfies Lipschitz condition and contraction if the following inequality $0 \le r_i < 1$ holds.

Proof

Consider two functions $G \ and \ \bar{G}$

$$\|F_1(t,G) - F_1(t,\bar{G})\|$$

$$\le p_1{}^{\nu(t)}\|C\|\|G-\bar{G}\| - p_2{}^{\nu(t)}\|G-\bar{G}\| \le [p_1{}^{\nu(t)}u_2 - p_2{}^{\nu(t)}]\|G-\bar{G}\|$$

$$\le r_1\|G-\bar{G}\| \qquad (4)$$

where $\|G(t)\| \leq u_1$, $\|C(t)\| \leq u_2$, $\|I(t)\| \leq u_3$, $\|H(t)\| \leq u_4$ and $r_1 = {p_1}^{\nu(t)} u_2 - {p_2}^{\nu(t)}$ are positive constants .As a result, the Lipschitz condition is met for $r_1$and if $0 \leq r_1 < 1$,then $r_1$follows contraction. Similarly, it can be exihibited and demonstrated in the other equations as follows

$$\|F_2(t, C) - F_2(t, \bar{C})\| \leq r_2\|C - \bar{C}\|$$

$$\|F_3(t, I) - F_3(t, \bar{I})\| \leq r_3\|I - \bar{I}\|$$

$$\|F_4(t, H) - F_3(t, \bar{H})\| \leq r_4\|H - \bar{H}\|$$

Therefore F_i satisfies Lipschitz condition. Also, if $0 \leq r_i < 1$, then the kernels follows contractions.

From system (3), the recurrent form can be written as follows

$$\Phi_{1n} = G_n(t) - G_{n-1}(t) = \frac{1}{\Gamma\nu(t)}\int_0^t (t-k)^{\nu(t)-1}\,[F_1(K, G_{n-1}) - F_1(K, G_{n-2})]dk$$

$$\Phi_{2n} = C_n(t) - C_{n-1}(t) = \frac{1}{\Gamma\nu(t)}\int_0^t (t-k)^{\nu(t)-1}\,[F_2(K, C_{n-1}) - F_2(K, C_{n-2})]dk$$

$$\Phi_{3n} = I_n(t) - I_{n-1}(t) = \frac{1}{\Gamma\nu(t)}\int_0^t (t-k)^{\nu(t)-1}\,[F_3(K, I_{n-1}) - F_3(K, I_{n-2})]dk$$

$$\Phi_{4n} = H_n(t) - H_{n-1}(t) = \frac{1}{\Gamma\nu(t)}\int_0^t (t-k)^{\nu(t)-1}\,[F_4(K, H_{n-1}) - F_4(K, H_{n-2})]dk$$

Using the initial conditions

$G(t) = G(0)$,$C(t) = C(0)$, $I(t) = I(0)$, $H(t) = H(0)$ for the above equation and taking norm, we get

$$\|\Phi_{1n}(t)\| = \|G_n(t) - G_{n-1}(t)\| = \left\|\frac{1}{\Gamma\nu(t)}\int_0^t (t-k)^{\nu(t)-1}\,[F_1(K, G_{n-1}) - F_1(K, G_{n-2})]dk\right\|$$

Now using Lipschitz condition in the above equation, we obtain

$$\|\Phi_{1n}(t)\| \leq \frac{r_1}{\Gamma\nu(t)}\int_0^t \left\|\Phi_{1(n-1)}(k)\right\|dk$$

Similarly,

$$\|\Phi_{2n}(t)\| \leq \frac{r_2}{\Gamma\nu(t)}\int_0^t \left\|\Phi_{2(n-1)}(k)\right\|dk$$

$$\|\Phi_{3n}(t)\| \leq \frac{r_3}{\Gamma\nu(t)}\int_0^t \left\|\Phi_{3(n-1)}(k)\right\|dk$$

$$\|\Phi_{4n}(t)\| \leq \frac{r_4}{\Gamma\nu(t)}\int_0^t \left\|\Phi_{4(n-1)}(k)\right\|dk \qquad (5)$$

which implies that it can be written as

$G_n(t) = \sum_{i=1}^{n} \Phi_{1i}(t)$; $C_n(t) = \sum_{i=1}^{n} \Phi_{2i}(t), I_n(t) = \sum_{i=1}^{n} \Phi_{3i}(t), H_n(t) = \sum_{i=1}^{n} \Phi_{4i}(t)$

Theorem 4.1.2

The Liouville -Caputo model (3) has system of solutions if there exists t >1 such that $\frac{r_i t}{\Gamma \nu(t)} \leq 1$ for i=1, 2, 3, 4

Proof

Consider,

$$\|\Phi_{1n}(t)\| \leq \frac{r_1}{\Gamma \nu(t)} \int_0^t \left\|\Phi_{1(n-1)}(k)\right\| dk$$

Replacing n by n-1 in the above inequality

$$\left\|\Phi_{1(n-1)}(t)\right\| \leq \frac{r_1}{\Gamma \nu(t)} \int_0^t \left\|\Phi_{1(n-2)}(k)\right\| dk \quad \leq \left[\frac{r_1}{\Gamma \nu(t)}\right]^2 \int_0^t \left\|\Phi_{1(n-2)}(k)\right\| dk$$

Again, replacing n by n-2 in the given inequality

$$\left\|\Phi_{1(n-2)}(t)\right\| \leq \left[\frac{r_1}{\Gamma \nu(t)}\right]^3 \int_0^t \left\|\Phi_{1(n-3)}(k)\right\| dk$$

On substituting in this way and use the initial condition

We obtain

$$\|\Phi_{1n}(t)\| \leq \|G_n(0)\| \left[\frac{r_1 t}{\Gamma \nu(t)}\right]^n$$

Similarly, we get ν(t)

$$\|\Phi_{2n}(t)\| \leq \|C_n(0)\| \left[\frac{r_2 t}{\Gamma \nu(t)}\right]^n$$

$$\|\Phi_{3n}(t)\| \leq \|I_n(0)\| \left[\frac{r_3 t}{\Gamma \nu(t)}\right]^n$$

$$\|\Phi_{4n}(t)\| \leq \|H_n(0)\| \left[\frac{r_4 t}{\Gamma \nu(t)}\right]^n$$

This result proved the existence and continuity of solutions

To show that G(t) , C(t) , I(t) and H(t) are the solutions of (3) ,we consider the following equations

$$G(t) - G(0) = G_n(t) - R_{1n}(t)$$
$$C(t) - C(0) = C_n(t) - R_{2n}(t)$$
$$I(t) - I(0) = I_n(t) - R_{3n}(t) \qquad (6)$$

$H(t) - H(0) = H_n(t) - R_{4n}(t)$

$$\|R_{1n}(t)\| = \left\|\frac{1}{\Gamma\nu(t)}\int_0^t [F_1(K, G_n) - F_1(K, G_{n-1})]\, dk\right\|$$

$$\leq \frac{1}{\Gamma\nu(t)}\int_0^t \|[F_1(K, G_n) - F_1(K, G_{n-1})]\|\, dk$$

$$\leq \frac{1}{\Gamma\nu(t)} r_1 \|G_n - G_{n-1}\| t$$

Applying the above process recursively,

$$\|R_{1n}(t)\| = \left[\frac{r_1 t}{\Gamma\nu(t)}\right]^{n+1} . M$$

where M is the Lipschitz constant

when $n \to \infty$, $\|R_{1n}(t)\| \to 0$

similarly we prove for

$\|R_{2n}(t)\| \to 0$, $\|R_{3n}(t)\| \to 0$ $and \|R_{4n}(t)\| \to 0$ as $n \to \infty$

Hence the proof.

Theorem 4.1.3

If the condition $\left[1 - \frac{r_i t}{\Gamma\nu(t)}\right] \geq 0$, for $i = 1,2,3,4$ holds then Caputo model have unique solution.

Proof

To establish the uniqueness for a solution of the system (3), consider the different set of solutions for the system (3), say $\bar{G}$, $\bar{C}$ $\bar{I}$ and $\bar{H}$. Then as an outcome of the first equation of (3), we write

$$G(t) - \bar{G}(t) = \frac{1}{\Gamma\nu(t)}\int_0^t [F_1(K, G) - F_1(K, \bar{G})]\, dk$$

Using the norm of above equation

$$\|G(t) - \bar{G}(t)\| = \left\|\frac{1}{\Gamma\nu(t)}\int_0^t [F_1(K, G) - F_1(K, \bar{G})]\, dk\right\|$$

Now by applying Lipschitz condition,

$$\|G(t) - \bar{G}(t)\| = \frac{1}{\Gamma\nu(t)} r_1 t \|G(t) - \bar{G}(t)\|$$

Consequently,

$$\|G(t) - \bar{G}(t)\| - \frac{1}{\Gamma\nu(t)} r_1 t \|G(t) - \bar{G}(t)\| \leq 0$$

$\|G(t)-\bar{G}(t)\|[1-\frac{1}{\Gamma\nu(t)}r_1 t]\leq 0$ (7)

Since $[1-\frac{1}{\Gamma\nu(t)}r_i t]>0$,

equation (7) becomes the form

$\|G(t)-\bar{G}(t)\|=0$

Therefore, $G(t)=\bar{G}(t)$

similarly, we prove

$C(t)=\bar{C}(t)$, $I(t)=\bar{I}(t)$ and $H(t)=\bar{H}(t)$

4.2 Existence and Uniqueness of Fractional solutions by the Caputo-Fabrizio model

Let us construct the system (2) in the sense of Caputo-Fabrizio, we have

$$
\begin{aligned}
{}^{CF}_{0}D_t^{\nu(t)}[G(t)] &= F_1(t,G)=\Lambda+{p_1}^{\nu(t)}G(t)C(t)-{p_2}^{\nu(t)}G(t)\\
{}^{CF}_{0}D_t^{\nu(t)}[C(t)] &= F_2(t,C)=\Omega-{p_1}^{\nu(t)}G(t)C(t)+{p_3}^{\nu(t)}H(t)C(t)-{p_4}^{\nu(t)}C(t) \qquad (8)\\
{}^{CF}_{0}D_t^{\nu(t)}[I(t)] &= F_3(t,I)={p_5}^{\nu(t)}G(t)I(t)-{p_6}^{\nu(t)}H(t)I(t)-{p_7}^{\nu(t)}I(t)\\
{}^{CF}_{0}D_t^{\nu(t)}[H(t)] &= F_4(t,H)=\Psi+{p_6}^{\nu(t)}H(t)I(t)-{p_8}^{\nu(t)}H(t)
\end{aligned}
$$

The Caputo-Fabrizio integral form of the above system is

$$
\begin{aligned}
G(t)-G(0) &= \frac{1-\nu(t)}{M(\nu(t))}F_1(t,G)+\frac{\nu(t)}{M(\nu(t))}\int_0^t F_1(\tau,G)d\tau\\
C(t)-C(0) &= \frac{1-\nu(t)}{M(\nu(t))}F_2(t,X)+\frac{\nu(t)}{M(\nu(t))}\int_0^t F_2(\tau,C)d\tau\\
I(t)-I(0) &= \frac{1-\nu(t)}{M(\nu(t))}F_3(t,I)+\frac{\nu(t)}{M(\nu(t))}\int_0^t F_3(\tau,I)d\tau \qquad (9)\\
H(t)-H(0) &= \frac{1-\nu(t)}{M(\nu(t))}F_3(t,H)+\frac{\nu(t)}{M(\nu(t))}\int_0^t F_4(\tau,H)d\tau
\end{aligned}
$$

Here we have to prove the kernel F_i for $i=1,2,3,4$ follows the Lipschitz condition and a contraction.

Theorem 4.2.1

The kernel $F_i(\tau,G)$, for $i=1,2,3,4$ satisfies the Lipschitz condition and a contraction if the following inequality $0\leq\rho_i<1$ holds.

Proof

This theorem is proved as similar as theorem 4.1.1

The recurrent form of (9) for the first equation is

$\psi_{1n}=G_n(t)-G_{n-1}(t)$

$$=\frac{1-\nu(t)}{M(\nu(t))}[F_1(t,G_{n-1})-F_1(t,G_{n-2})]+\frac{\nu(t)}{M(\nu(t))}\int_0^t[F_1(\tau,G_{n-1})-F_1(\tau,G_{n-2})]d\tau$$

Similarly ψ_{2n} and ψ_{3n} are also be derived

Using the initial condition and taking norm, we get

$$\|\psi_{1n}\|\le\frac{1-\nu(t)}{M(\nu(t))}\|[F_1(t,G_{n-1})-F_1(t,G_{n-2})]\|+\frac{\nu(t)}{M(\nu(t))}\int_0^t\|[F_1(\tau,G_{n-1})-F_1(\tau,G_{n-2})]\|\,d\tau$$

Since $\rho_1$satisfies Lipschitz condition

$$\|\psi_{1n}(t)\|\le\frac{1-\nu(t)}{M(\nu(t))}\rho_1\left\|\left[\psi_{1(n-1)}(t)\right]\right\|+\frac{\nu(t)}{M(\nu(t))}\rho_1\int_0^t\left\|\psi_{1(n-1)}(\tau)\right\|d\tau \quad (10)$$

Similarly $\|\psi_{2n}(t)\|$, $\|\psi_{3n}(t)\|$ and $\|\psi_{4n}(t)\|$can also be obtained. Therefore,

$G_n(t)=\sum_{i=1}^{n}\psi_{1i}(t)$, $C_n(t)=\sum_{i=1}^{n}\psi_{2i}(t)$, $I_n(t)=\sum_{i=1}^{n}\psi_{3i}(t)$, $H_n(t)=\sum_{i=1}^{n}\psi_{4i}(t)$

Theorem 4.2.2

The Caputo Fabrizio fractional derivative model (8) has system of solutions if there exists $v>1$ such that

$\left[\frac{1-\nu(t)}{M(\nu(t))}\rho_i+\frac{\nu(t)}{M(\nu(t))}\rho_i\, v\right]\le 1$, for i=1,2,3,4

Proof

Operating (10) recursively and using the initial conditions, we have

$$\|\psi_{1n}(t)\|\le\|G_n(0)\|\left[\frac{1-\nu(t)}{M(\nu(t))}\rho_1+\frac{\nu(t)}{M(\nu(t))}\rho_1 v\right]^n$$

Similarly, we have for $\|\psi_{2n}(t)\|$, $\|\psi_{3n}(t)\|$ and $\|\psi_{4n}(t)\|$this result proved the existence and continuity of solution.

To show that G(t), $C(t)$, $I(t)$ and $H(t)$ are solutions of (8),

consider, $G(t)-G(0)=G_n(t)-D_{1n}(t)$

$$C(t)-C(0)=C_n(t)-D_{2n}(t)$$

$$I(t)-I(0)=I_n(t)-D_{3n}(t)$$

$$H(t)-H(0)=H_n(t)-D_{3n}(t)$$

Now,

$$\|D_{1n}(t)\|\le\frac{1-\nu(t)}{M(\nu(t))}\|[F_1(t,G_n)-F_1(t,G_{n-1})]\|\quad+\frac{\nu(t)}{M(\nu(t))}\int_0^t\|[F_1(\tau,G_n)-F_1(\tau,G_{n-1})]\|\,d\tau$$

$$\le\frac{1-\nu(t)}{M(\nu(t))}\rho_1\|G_n-G_{n-1}\|+\frac{\nu(t)}{M(\nu(t))}\rho_1\ \|G_n-G_{n-1}\|v$$

$$\le\left[\frac{1-\nu(t)}{M(\nu(t))}\rho_1+\frac{\nu(t)}{M(\nu(t))}\rho_1 v\right]\|G_n-G_{n-1}\|$$

Applying the above process recursively

$$\|D_{1n}(t)\| \le \left[\frac{1-\nu(t)}{M(\nu(t))}\rho_1 + \frac{\nu(t)}{M(\nu(t))}\ \rho_1 v\right]^{n+1} .S$$

where S is the Lipschitz constant, when $n \to \infty$, $\|D_{1n}\| \to 0$

Similarly, we prove for $\|D_{2n}\| \to 0$, $\|D_{3n}\| \to 0$ and $\|D_{4n}\| \to 0$ as $n \to \infty$

Hence the proof

Theorem 4.2.3

If the condition $\left[1 - \left[\frac{1-\nu(t)}{M(\nu(t))}\rho_i + \frac{\nu(t)}{M(\nu(t))}\ \rho_i v\right]\right] \ge 0$, for i=1,2,3,4 holds then the Caputo-Fabrizio fractional derivative model have unique solutions.

Proof

Suppose the system (8) has another solution $\bar{G}, \bar{C}, \bar{I}, and\ \bar{H}$

$$G(t) - \bar{G}(t) = \frac{1-\nu(t)}{M(\nu(t))}[F_1(t,G) - F_1(t,\bar{G})] + \frac{\nu(t)}{M(\nu(t))}\int_0^t [F_1(\tau,G) - F_1(\tau,\bar{G})]\, d\tau$$

Using norm and applying Lipschitz condition

$$\|G(t) - \bar{G}(t)\| \le \left[\frac{1-\nu(t)}{M(\nu(t))}\rho_1 + \frac{\nu(t)}{M(\nu(t))}\ \rho_1 v\right] \|G(t) - \bar{G}(t)\|$$

Consequently, we have

$$\|G(t) - \bar{G}(t)\| \left[1 - \left[\frac{1-\nu(t)}{M(\nu(t))}\rho_1 + \frac{\nu(t)}{M(\nu(t))}\ \rho_1 v\right]\right] \le 0$$

Since $\left[1 - \left[\frac{1-\nu(t)}{M(\nu(t))}\rho_i + \frac{\nu(t)}{M(\nu(t))}\ \rho_i v\right]\right] > 0$,we have

$\|G(t) - \bar{G}(t)\| = 0$

Therefore $G(t) = \bar{G}(t)$

Similarly we prove
$C(t) = \bar{C}(t)$, $I(t) = \bar{I}(t)$ and $H(t) = \bar{H}(t)$

Hence the proof

4.3 Existence and Uniqueness of solutions for the Atangana-Baleanu fractional model

Let us construct (2) in Atangana Baleanu fractional derivative in Caputo sense

$${}^{AB}_{0}D_t^{\nu(t)}[G(t)] = F_1(t,G) = \Lambda + {p_1}^{\nu(t)}G(t)C(t) - {p_2}^{\nu(t)}G(t)$$

$${}^{AB}_{0}D_t^{\nu(t)}[C(t)] = F_2(t,C) = \Omega - {p_1}^{\nu(t)}G(t)C(t) + {p_3}^{\nu(t)}H(t)C(t) - {p_4}^{\nu(t)}C(t)$$

$${}^{AB}_{0}D_t^{\nu(t)}[I(t)] = F_3(t,I) = {p_5}^{\nu(t)}G(t)I(t) - {p_6}^{\nu(t)}H(t)I(t) - {p_7}^{\nu(t)}I(t)$$

$${}^{AB}_{0}D_t^{\nu(t)}[H(t)] = F_4(t,H) = \Psi + {p_6}^{\nu(t)}H(t)I(t) - {p_8}^{\nu(t)}H(t) \quad (11)$$

The Atangana-Baleanu integral form of the above system is

$$G(t) - G(0) = \frac{1-\nu(t)}{AB(\nu(t))} F_1(t,G) + \frac{\nu(t)}{AB(\nu(t))\,\Gamma\nu(t)} \int_0^t (t-\gamma)^{\nu(t)-1} F_1(\gamma,G)\,d\gamma$$

$$C(t) - C(0) = \frac{1-\nu(t)}{AB(\nu(t))} F_2(t,C) + \frac{\nu(t)}{AB(\nu(t))\,\Gamma\nu(t)} \int_0^t (t-\gamma)^{\nu(t)-1} F_2(\gamma,C)\,d\gamma \quad (12)$$

$$I(t) - I(0) = \frac{1-\nu(t)}{AB(\nu(t))} F_3(t,I) + \frac{\nu(t)}{AB(\nu(t))\,\Gamma\nu(t)} \int_0^t (t-\gamma)^{\nu(t)-1} F_3(\gamma,I)\,d\gamma$$

$$H(t) - H(0) = \frac{1-\nu(t)}{AB(\nu(t))} F_4(t,H) + \frac{\nu(t)}{AB(\nu(t))\,\Gamma\nu(t)} \int_0^t (t-\gamma)^{\nu(t)-1} F_4(\gamma,H)\,d\gamma$$

Now to prove the kernel F_i for $i = 1,2,3,4$ follows the Lipschitz condition and contraction.

Theorem 4.3.1

The kernel $F_i(\gamma, G)$, for $i = 1,2,3,4$ satisfies the Lipschitz condition and contraction if $0 \le \delta_i < 1$ holds.

Proof

The proof is similar to the proof of 4.1.1. Now the recurrent form of (12) is $\theta_{1n} = G_n(t) - G_{n-1}(t)$

$$= \frac{1-\nu(t)}{AB(\nu(t))}[F_1(t,G_{n-1}) - F_1(t,G_{n-2})] + \frac{\nu(t)}{AB(\nu(t))\,\Gamma\nu(t)} \int_0^t (t-\gamma)^{\nu(t)-1}[F_1(\gamma,G_{n-1}) - F_1(\gamma,G_{n-2})]\,d\gamma$$

Similarly θ_{2n} , θ_{3n} and $\theta_{4n}$are also be derived. Using the initial condition and taking norm, we get

$$\|\theta_{1n}\| \le \frac{1-\nu(t)}{AB(\nu(t))}\|[F_1(t,G_{n-1}) - F_1(t,G_{n-2})]\| + \frac{\nu(t)}{AB(\nu(t))\,\Gamma\nu(t)} \int_0^t (t-\gamma)^{\nu(t)-1}\|[F_1(\gamma,G_{n-1}) - F_1(\gamma,G_{n-2})]\|\,d\gamma$$

Since δ_1 satisfies Lipschitz condition

$$\|\theta_{1n}\| \le \frac{1-\nu(t)}{AB(\nu(t))}\delta_1\|\theta_{1(n-1)}\| + \frac{\nu(t)}{AB(\nu(t))\,\Gamma\nu(t)}\delta_1 \int_0^t (t-\gamma)^{\nu(t)-1}\|\theta_{1(n-1)}(\gamma)\|\,d\gamma$$

Similarly for$\|\theta_{2n}\|$, $\|\theta_{3n}\|$ and $\|\theta_{4n}\|$

which implies that it can be written as

$$G_n(t) = \sum_{i=1}^{n}\theta_{1i}(t)\,, C_n(t) = \sum_{i=1}^{n}\theta_{2i}(t)\,, I_n(t) = \sum_{i=1}^{n}\theta_{3i}(t)\,, H_n(t) = \sum_{i=1}^{n}\theta_{4i}(t) \quad (13)$$

Theorem 4.3.2

The Atangana- Baleanu derivative model (11) have system of solutions, if there exists μ >1such that

$$\left[\frac{1-\nu(t)}{AB(\nu(t))}\delta_i + \frac{\nu(t)}{AB(\nu(t))\,\Gamma\nu(t)}\delta_i\mu\right] \le 1 \; for \;\; i = 1,2,3,4$$

Proof Consider,

$$\|\theta_{1n}\| \le \|G_n(0)\|\left[\frac{1-\nu(t)}{AB(\nu(t))}\delta_i + \frac{\nu(t)}{AB(\nu(t))\,\Gamma\nu(t)}\delta_i\mu\right]^n$$

Similarly,$\|\theta_{2n}\|$, $\|\theta_{3n}\|$ and $\|\theta_{4n}\|$ can also be obtained

These results proved the existence and continuity of solution.

Now to show that $G(t), C(t), I(t)$ and $H(t)$ are solutions of (11)

Consider

$G(t) - G(0) = G_n(t) - E_{1n}(t)$

$C(t) - C(0) = C_n(t) - E_{2n}(t)$

$I(t) - I(0) = I_n(t) - E_{3n}(t)$

$H(t) - H(0) = H_n(t) - E_{4n}(t)$

Now

$\|E_{1n}(t)\| \leq \frac{1-\nu(t)}{AB(\nu(t))} \|[F_1(t, G_n) - F_1(t, G_{n-1})]\| \qquad + \frac{\nu(t)}{AB(\nu(t))\,\Gamma\nu(t)} \int_0^t (t-\gamma)^{\alpha(t)-1} \|[F_1(\gamma, G_n) - F_1(\gamma, G_{n-1})]\|\, d\gamma$

$\leq \frac{1-\nu(t)}{AB(\nu(t))} \delta_1 \|G_n - G_{n-1}\| + \frac{\nu(t)}{AB(\nu(t))\,\Gamma\nu(t)} \delta_1 \|G_n - G_{n-1}\| \mu$

$\leq \left[\frac{1-\nu(t)}{AB(\nu(t))} \delta_1 + \frac{\nu(t)}{AB(\nu(t))\,\Gamma\nu(t)} \delta_1 \mu\right] \|G_n - G_{n-1}\|$

Applying the above process recursively

$$\|E_{1n}(t)\| \leq \left[\frac{1-\nu(t)}{AB(\nu(t))} \delta_1 + \frac{\nu(t)}{AB(\nu(t))\,\Gamma\nu(t)} \delta_1 \mu\right]^{n+1} . W$$

where W is the Lipschitz constant

when $n \to \infty$, $\|E_{1n}\| \to 0$

Similarly we prove for $\|E_{2n}\| \to 0$, $\|E_{3n}\| \to 0$ and $\|E_{4n}\| \to 0$ as $n \to \infty$

Hence the proof.

Theorem 4.3.3

If the condition $\left[1 - \left[\frac{1-\nu(t)}{AB(\nu(t))} \delta_i + \frac{\nu(t)}{AB(\nu(t))\,\Gamma\nu(t)} \delta_i \mu\right]\right] \geq 0$ For i=1,2,3,4 holds then the Atangana-Baleanu fractional derivative model have unique solutions.

Proof

Suppose the system (16) has another solution $\bar{G}$, $\bar{C}$, $\bar{I}$ and $\bar{H}$ then

$$G(t) - \bar{G}(t) = \frac{1-\nu(t)}{AB(\nu(t))} [F_1(t, G) - F_1(t, \bar{G})]$$

$$+ \frac{\nu(t)}{AB(\nu(t))\,\Gamma\nu(t)} \int_0^t (t-\gamma)^{\alpha(t)-1} [F_1(\gamma, G) - F_1(\gamma, \bar{G})] d\gamma$$

Using norm and apply Lipschitz condition

$\|G(t)-\bar{G}(t)\| \leq (\left[\frac{1-\nu(t)}{AB(\nu(t))}\delta_1 + \frac{\nu(t)}{AB(\nu(t))\,\Gamma\nu(t)}\ \delta_1\mu\right]\|G(t)-\bar{G}(t)\|)$

Consequently we have

$$\left[1-\left[\frac{1-\nu(t)}{AB(\nu(t))}\delta_1 + \frac{\nu(t)}{AB(\nu(t))\,\Gamma\nu(t)}\ \delta_1\mu\right]\right]\|G(t)-\bar{G}(t)\| \leq 0$$

Since$\left[1-\left[\frac{1-\nu(t)}{AB(\nu(t))}\delta_i + \frac{\nu(t)}{AB(\nu(t))\,\Gamma\nu(t)}\ \delta_i\mu\right]\right] > 0$,we have

$\|G(t)-\bar{G}(t)\| = 0$

Therefore, $G(t)=\bar{G}(t)$

Similarly ,we prove
$C(t)=\bar{C}(t)$, $I(t)=\bar{I}(t)$ and $H(t)=H(t)$

Hence the proof

5. Numerical Scheme

In this section Numerical scheme [31] is considered in the sense of Liouville-Caputo, Caputo-Fabrizio and Atangana –Baleanu fractional derivatives.

Let us consider our fractional model as

${}_0^{*}D_t^{\alpha}u(t)$ $=f(t,u(t))$

Where * denotes LC, CF and AB terms and $u(t)=(G(t),C(t),I(t),H(t))$.

Now we use the numerical scheme [31] represented for Liouville-Caputo (14), Caputo-Fabrizio(15) and Atangana-Baleanu(16) fractional derivatives in (2)

$$u_{n+1}(t)=u(0)+\frac{1}{\Gamma\nu(t)}\sum_{m=0}^{n}\begin{pmatrix}\frac{h^{\nu(t)}f(t_m,u_m)}{\nu(t)(\nu(t)+1)} \\ ((n-m+\ 2+\ 2\alpha)-\frac{h^{\nu(t)}f(t_{m-1},u_{m-1})}{\nu(t)(\nu(t)+1)} \\ \begin{pmatrix}(n+1-\ m)^{\nu(t)+1}- \\ (n-m)^{\nu(t)}(n-m+1+\nu(t)\end{pmatrix}\end{pmatrix} \quad (14)$$

$(u_{n+1})=(u_n)+$

$$\left[\frac{(2-\nu(t))(1-\nu(t))}{2}+\frac{3h}{4}\nu(t)(2-\nu(t))\right]f(t_n,u_n)-\begin{bmatrix}\frac{(2-\nu(t))(1-\nu(t))}{2}+ \\ \frac{h}{4}\nu(t)\ (2-\nu(t))\end{bmatrix}f(t_{n-1},u_{n-1})$$

$$u_{n+1}(t)=u(0)+\frac{\Gamma\nu(t)(1-\nu(t))}{\Gamma\nu(t)(1-\nu(t))+\nu(t)}f(t_n,u_n) \quad (15)$$

$$+\frac{1}{(\nu(t)+1)\big((1-\nu(t))\,\Gamma\nu(t)\big)+\nu(t)}\sum_{m=0}^{n}\begin{pmatrix} h^{\nu(t)}f(t_m,u_m) \\ \begin{pmatrix}(n+1-m)^{\nu(t)}\,\big(n-m+2+\nu(t)\big) \\ -\,(n-m)^{\nu(t)}(n-m+2+2\alpha(t)\end{pmatrix} \\ -h^{\nu(t)}f(t_{m-1},u_{m-1}) \\ \begin{pmatrix}(n+1-\,m)^{\nu(t)+1} \\ -\,(n-m)^{\nu(t)}\,\,(n-m+1+\nu(t))\end{pmatrix}\end{pmatrix} \quad (16)$$

6. Results and Discussions

The primary aim of our model is to effectively address the adverse effects of greenhouse gases (GHGs) by integrating them into green building systems using a fractional model. Existence and uniqueness of the fractional operators are established. Numerical or computer simulations are considered the most appropriate method to accurately represent the solution. We employed MATLAB R2023a programming language to conduct numerical simulations of the proposed fractional model. These simulations aim to demonstrate the dynamic behavior of green building dynamics in order to mitigate GHG emissions and gain a comprehensive understanding of the impact on human communities.

Human communities are significant contributors to unwanted GHG emissions. The rise in these emissions due to various human activities poses a threat to the well-being of all organisms. The continuous increase in the human population leads to higher levels of these harmful gases. In cases of rapid emission of these gases by human societies, green constructions utilize their advanced technologies to tackle the issue. Consequently, green buildings offer a range of advantages while effectively addressing the problem of GHGs. By incorporating sustainable practices, energy-efficient designs, and innovative technologies, these buildings help to create a more environmentally friendly and sustainable built environment. These energy-efficient structures also provide benefits to human societies.

The fractional order model is ideal for illustrating the memory effect/history of increasing GHGs on green building systems to enhance their efficiency.

Case 1(Variable-order case): The rate of green building projects varies due to a multitude of factors. These include the progression of eco-friendly building initiatives, government policies, public awareness, and technological advancements. Additionally, economic incentives, market demand, industry standards, access to resources, regulatory environment, collaboration, and public perception play crucial roles. Variable order study of the green building concept allows for a more comprehensive understanding of its dynamics and complexities.

The variable order of $G(t)$ can be represented as $\nu(t)= L/(1+\exp(k(t-t_0))$,where L is the highest percentage value of buildings that are deemed to be green, k is the logistic growth rate and t_0 is the time at which the growth is halfway between its starting and ending values. Various factors, including building materials, energy sources, energy efficiency measures, occupant behavior, and maintenance practices, may have an impact on the function of greenhouse gas emission rates in green buildings.

$C(t)$ takes the variable order as a linear function ν(t)= R_0-λt, where R_0 represents the starting point of greenhouse gas emissions and λ is the decreasing rate of GHGs in green buildings. The usage of ingredients in green buildings $I(t)$ can be represented in variable order as ν(t)=S_0+μt, where S_0 symbolizes the initial adoption of eco-friendly components and μ denotes the speed of growth in usage. These components might include things like renewable energy sources and other elements that enhance a building's green features. When new technologies are created, building codes are updated, and societal priorities change over time, the function may change.

The growth rate of a human community in green buildings is determined by various factors, including population dynamics, urbanization trends, economic conditions, and the availability of green buildings .Here the variable order of $H(t)$ can be represented as ν(t)=ϑ+α exp((ω)(t)(M)),where ϑ denotes the baseline growth factor, α represents a coefficient scaling of the exponential growth, ω indicates the decay rate coefficient of the exponential growth and M represents a coefficient affecting the speed at which the growth function approaches zero as t increases.

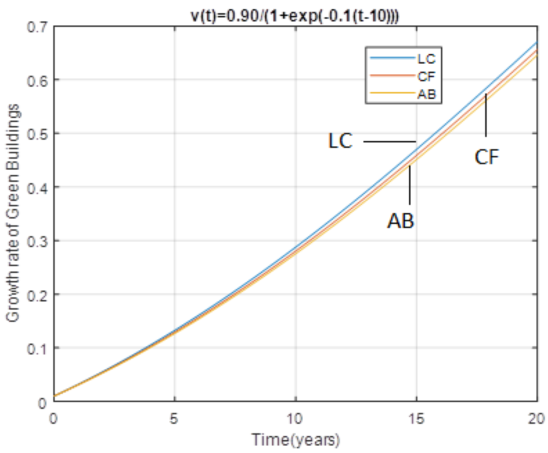


Fig 1: Comparison graph of $G(t)$ via LC,CF and AB

Fig 1 illustrates the growth rate of green buildings with a variable order represented by ν(t)=0.90/(1+exp(0.1(t-10)). Here,0.90 represents buildings with the highest percentage of green features,0.1 represents the rate of logistic growth and 10 indicates the growth reaches a point midway between its initial and final values. It is observed that in all cases of LC,CF and AB the growth rate of green buildings rises accordingly when time increases.

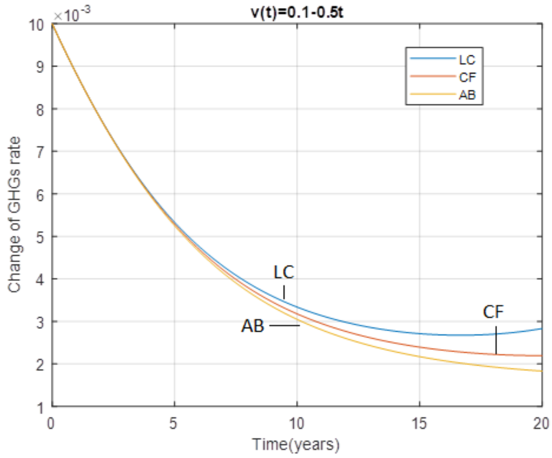


Fig 2: Comparison graph of $C(t)$ via LC, CF and AB

Fig 2 depicts the change of GHGs rate with a variable order ν(t)= 0.1-0.5t.where 0.1 signifies the initial stage of greenhouse gas emissions.0.5 indicates the rate at which greenhouse gas emission decreases. By Operating the above function using LC,CF and AB, AB gives declination of GHGs in Green buildings more appropriately as in [15] than LC and CF. Gradually, as green technologies and practices are adopted and refined, the rate of greenhouse gas emissions declines.

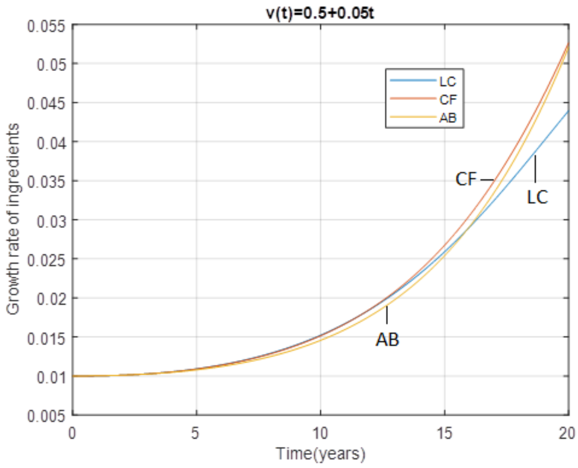


Fig 3: Comparison graph of $I(t)$ via LC, CF and AB

Fig 3 displays the growth rate of ingredients by the variable order $\nu(t) = 0.5 + 0.05t$. Here, 0.5 represents the initial usage of green ingredients and 0.05 represents the rate of increase in usage. Using LC, CF and AB for observing the considered function, the growth rate of ingredients gradually rises over time. Also CF, AB have better memory effect than LC.

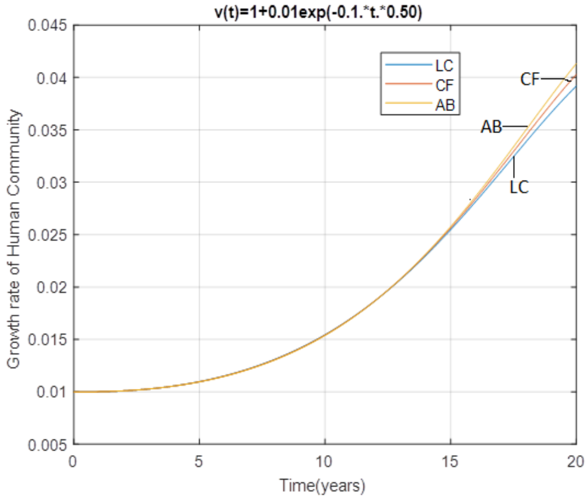
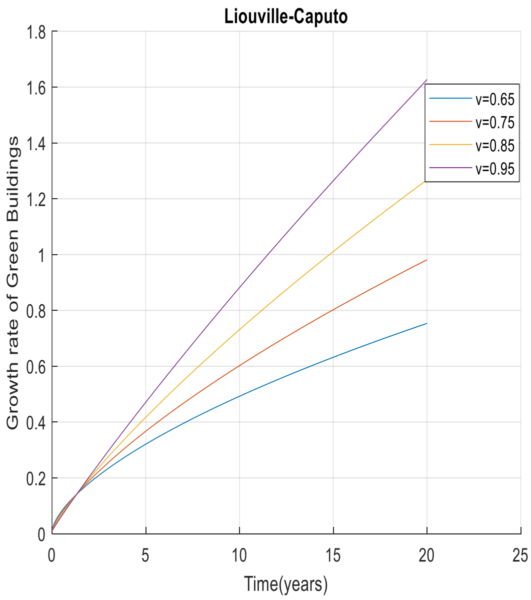
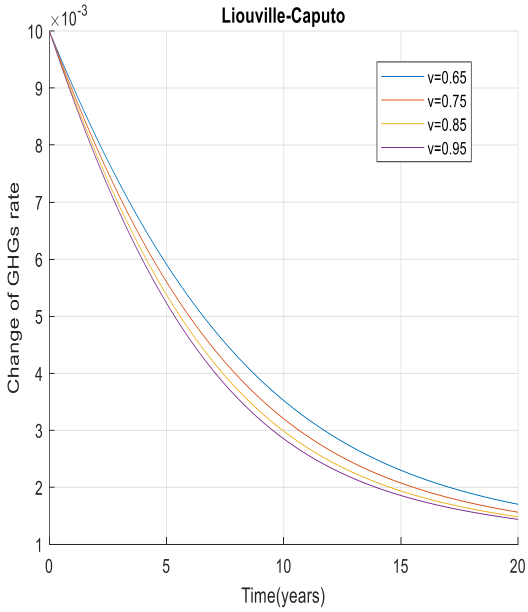


Fig 4: Comparison graph of $H(t)$ via LC,CF and AB

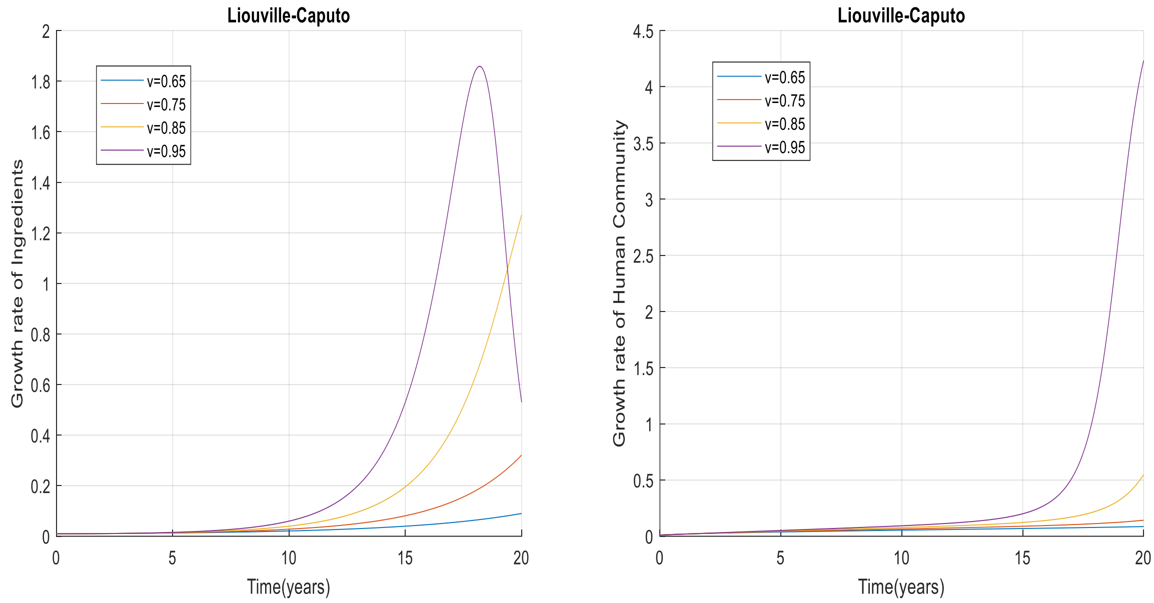
Fig 4 provides a graph of the variable order function ν(t)=1+0.1exp(-(0.1)(t)(0.50)),where 1 represents the fundamental growth element, 0.1 denotes the growth and decay rate coefficient of human community and 0.50 indicates how quickly the growth function approaches zero as time t increases. Here the growth rate of human community in green buildings progressively rises over time in all cases of LC, CF and AB.



(a)

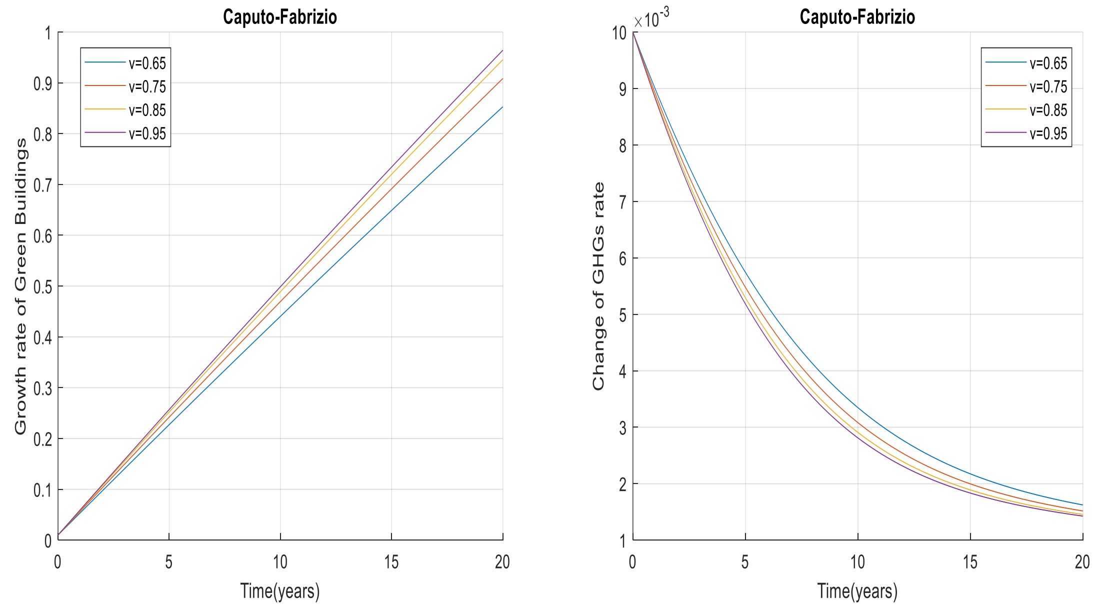


(b)

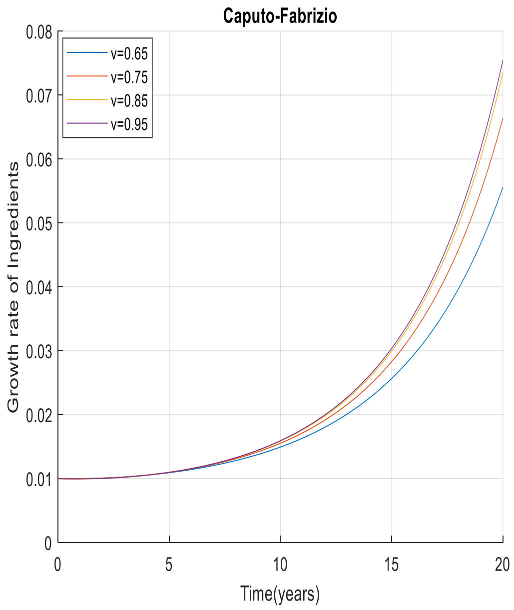


(c) (d)

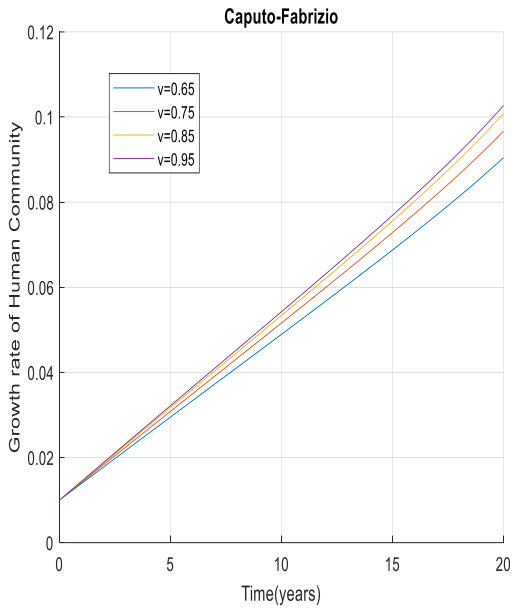
Fig5.Numerical simulation of Green Buildings for various values of ν=0.65,0.75,0.85 and 0.95 in Liouville-Caputo sense



(a) (b)

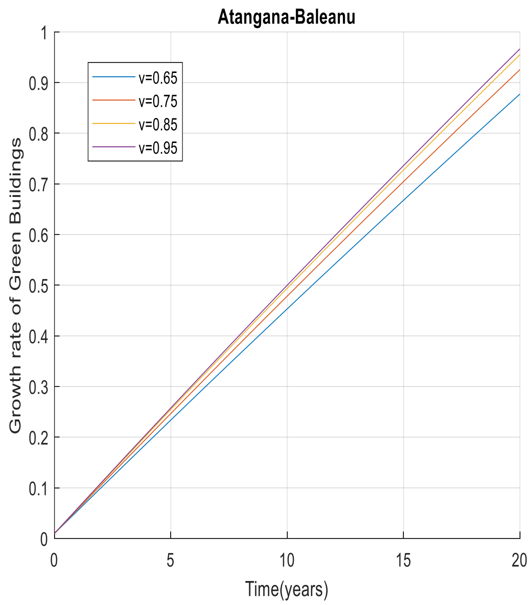


(c)

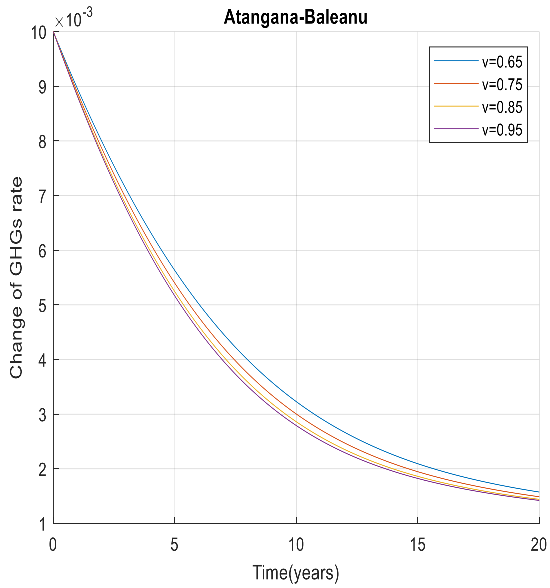


(d)

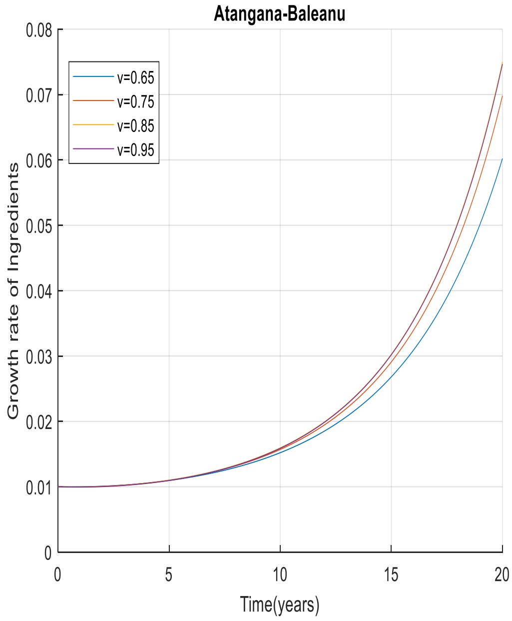
Fig 6.Numerical simulation of Green Buildings for various values of ν=0.65,0.75,0.85 and 0.95 in Caputo-Fabrizio sense.



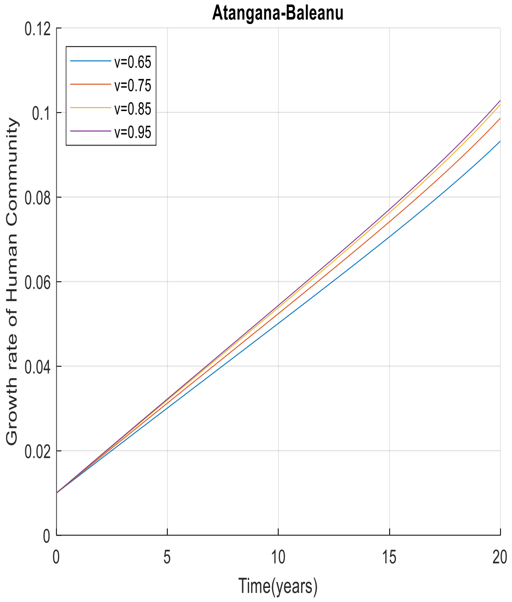
(a)



(b)



(c)



(d)

Fig 7.Numerical simulation of Green Buildings for various values of ν=0.65,0.75,0.85 and 0.95 in Atangana-Baleanu sense.

Case 2(Fractional order case): Fig[5] shows that when ν increases, the growth rate of green buildings absorbing greenhouse gases also increases drastically, but in Fig 6 and 7, it increases more gradually over time, which is more in line with real world circumstances.In Fig[5-7], GHG emissions decrease gradually over time as ν increases. According to Fig 6 and 7, green buildings can produce ingredients proportionally with increasing ν due to their high absorption of greenhouse gases, whereas in Fig 5, as ν close to 1, the production of ingredients reaches a peak and then falls rapidly, this is because of LC's singularity behavior. Fig 6 and 7 show that the human community will benefit from receiving the ingredients, hence it rises proportionally in cases of CF and AB, whereas in Fig 5, we see that for small values of ν , the human community remains stable, which violates the real-time scenario. Hence CF and AB approaches give better results than the LC approach.

Conclusion

This study developed a fractional variable-order model using Liouville-Caputo, Caputo-Fabrizio, and Atangana-Baleanu derivatives to explore sustainable building practices in India, focusing on their impact on greenhouse gas (GHG) emissions. Numerical simulations showed that green buildings can effectively reduce GHGs, with the Caputo-Fabrizio and Atangana-Baleanu approaches providing more realistic and accurate results compared to the Liouville-Caputo method. The findings highlight the potential of green buildings in fostering sustainable communities, where gradual GHG absorption and the growth of eco-friendly components align with real-world scenarios.

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