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An Algorithmic Approaches of Solving Fuzzy Critical Path Problem using HaarRanking Octagonal Fuzzy Number Method

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Abstract:

In a network analysis, find the critical path of the problem is an important techniques which involves planning and control of the large projects that are very complex in nature. Clear identification of each task will help to implement critical path successfully.But in real life situations the time duration cannot be predicted accurately due to various delay or vagueness while execution of the project. During implementation of the project one may encounter various delay or vagueness while execution of the project.Critical path of the network gives an idea of minimum time one may expect to complete the project. Hence the importance of the critical path play a major role in network analysis. In this paper, the concept of finding fuzzy critical path Haar Octagonal fuzzy number is introduced. New Algebraic arithmetic of Haar Octagonal fuzzy numbers is also discussed. A new method for finding the critical path of the problem is introduced with the help of Floyd - Warshall Algorithm and Haar Octagonal fuzzy numbers. A suitable numerical examples are given to demonstrate the above methods.

Keywords: Fuzzy set, Warshall Algorithm, Haar Algorithm, Octogonal Fuzzy number

1. Introduction

One of the problems in engineering and management sciences is the network problems. The main objective of the network analysis is to analyse the network to examine the total project duration and divides the project into critical and non-critical path. In a network analysis, an activity is said to be critical the longest path is taken from start to finish. It denotes the minimum time necessary to finish the entire project. The main idea in identifying critical path in a network is to determine the main activities involved in the project so that the maximum number of resources can be utilized to finish the work at the earliest time. In the early 19th century Morgan R. Walker and James E. Kelly developed the concept of critical path in network analysis to maximize the utilization of available resources. During the execution of the project in any network the time and cost of the activity is clearly an uncertain. Due to this uncertainty prevailing in the network, the modelling of the problem leads to fuzzy numbers. In a network analysis if the fuzzy numbers are involved one can say that network is a fuzzy network problems. Finding the critical path which involves the fuzzy numbers is called fuzzy critical path problem. Critical Path Method (CPM) is one of the techniques for identifying critical activities that prevails in the network.Due to the presence of uncertainty and vagueness of the various time parameters in the network, which leads us to study of fuzzy CPM for the past one and half decade. The vagueness that occur in the network problem that can be easily overcome by Fuzzy CPM. In [1] authors introduced the concept of PERT using fuzzy number to represent activity duration in the project network. In [2] Mon and cheng, introduced the concept of alpha- cut for fuzzy duration they occured as a linear combination which leads the operation time of each activity and to determine the critical activities and critical paths by using standard PERT technique.

Based on the various alpha values several critical activities with different paths are explored. Later in [3]Colleny et al. directed a straight forword method for applying fuzzy logic to examine uncertainty in prevailing in the critical path analysis. Later in [4] Zielinski et al assumes that the crisp value or fuzzy number to determine the complexity of Criticallity. Huang et al [5], proposed a new method which combines fuzzy set theory along with the combination of PERT techniques to determine the critical degrees of activities and paths, latest and earliest starting time and floats. Eventually the combination of fuzzy with PERT techniques has been emerged with the combination of new algorithms for finding CPM attracted many researchers for last five years. In [6]Ghoseri and Moghadam initiated an algorithm to determine the critical path by the use of fuzzy sets, PERT techniques and Bellmann algorithm to specify the critical path and fuzzy earliest and latest starting time and floats of activities in the continuous fuzzy network. Thus several research articles are published in Fuzzy critical activities using above mentioned methods [7,8,9,10].

Elizabeth et al [11,12] introduced a method for finding fuzzy critical path using ranking method. Although there are several mechanism existed in literature for finding the critical path distance, If the network is complex in nature then it is tedious to get the critical path by the problem in the comparing all the possible paths.The main problem is the comparison between fuzzy number which corresponds to a fuzzy path . It should be taken into account that most of the existing methods gives different values for different alpha -cut method. Moreover the results are not comparable with the previous method. In this paper to overcome the above difficulties we are introduced a new concept based on the wavelet analysis. Wavelet analysis is analogue the Fourier analysis which allows a function over an interval may be expressed as orthonormal basis . Alfred Haar in 1909 constructed an orthonormal basis using the piecewise continuous function . Later this basic function termed as Haar wavelet basis. Using this idea Dhanasekar [13,14] proposed Haar critical path technique, which involes converting triangular fuzzy integers in to Haar tuples using the Haar wavelet principle. Later S. Josh et al.,[15,16]

presented Haar tuples using the Haar ranking system for solving assignment problems which involves transforming Heptagonal fuzzy number into Haar tuples by using Haar wavelet principles. A path through the network is one of the ways in a project network from the star point to the completion point. According to the critical path, the length of a path is equal to the total durations of the activities of the path.

This research paper is organized as follows. In section 2, some fundamental concepts and definition are given for fuzzy sets . In section 3 Haar Ranking method for Octagonal Fuzzy number has been introduced and is section 4 Floyd Warshall Algorithm has been discussed. In section 5 a proposed method for finding a critical path has been discussed. In Section 6 two numerical examples are demonstrated to validate the proposed method. In section 7 conclusion of the problem are discussed.

2. Objectives

This section provides with basic definitions of fuzzy set and its related topics

Definition 2.1

Let Y be a set. A fuzzy set M on Y is defined to be a function $\mu_M \rightarrow [0, 1]$ is a mapping called the membership function value of y in Y in a fuzzy set M.

Definition 2.2

The fuzzy number M is a fuzzy if membership function satisfies i) A fuzzy set of the universe of discourse ii) M is normal if for some y In Y,$\mu_M(Y) = 1$ iii) ,$\mu_M(Y)$ is a piecewise continuous.

Definition 2.3

A fuzzy number M is said to be a normal octagonal fuzzy number denoted byμ which has [m_1, m_2,m_3,m_4,m_5, m_6, m_7, m_8,m_9] with real numbers with membership function as

$$\mu = \begin{cases} 0 & y \leq m_1 \\ \frac{y-m_1}{m_2-m_1} & m_1 \leq y \leq c_2 \\ 1 - 0.5\frac{y-m_2}{m_3-m_2} & , m_2 \leq y \leq m_3 \\ 0.5 + 0.5\frac{y-m_3}{m_4-m_3} & m_3 \leq y \leq m_4 \\ 1 - 0.5\frac{y-m_4}{m_5-m_4} & m_4 \leq y \leq m_5 \\ 1 & m_5 \leq y \leq m_6 \\ 0.5 + 0.5\frac{y-m_6}{m_7-m_6} & m_6 \leq y \leq m_7 \\ 0.5 - 0.5\frac{m_8-y}{m_8-m_7} & , m_7 \leq y \leq m_8 \\ 0 & y \geq m_8 \end{cases} \quad (2.1)$$

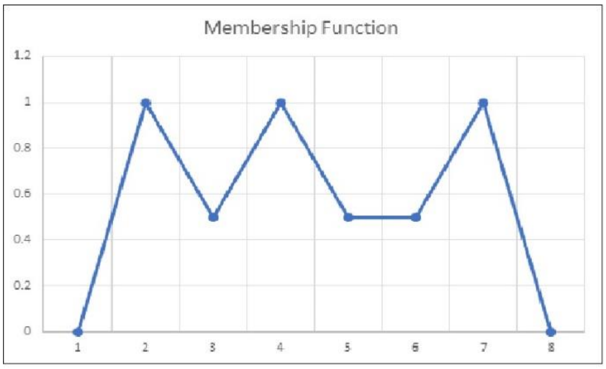


Figure 1: Octagonal Fuzzy number

3. Methods

Haar Ranking method for an Octagonal Fuzzy Number

In this section, we discussed with notion of Haar Octagonal Fuzzy Number (HOFN) and their arithmetic operations.

Let us consider an Octagonal Fuzzy Number (OCFN) as {a, b, c, d, e, f, g, h}.The formula for determining the average and detailed coefficients of the Haar of Octagonal Fuzzy number as follows.

In this sub-section given an OCFN we are converting it into an HOFN using the following steps

Step 1First pair the given OCFN as {(a, b), (c, d), (e,f), (g,h)}

Step 2Compute the Approximation and Detailed coefficients as follows:

2 a Approximation Coefficients is the average of two pair of elements in OCFN which as calculated as $a_1 = (a+b)/2$, $a_2 = (c+d)/2$, $a_3 = (e+f)/(2)$, $a_4 = (g+h)/2$, The calculated average can be represents in a set as $A_1 = (a_1, a_2, a_3, a_4)$

2b] Calculate Detailed coefficients which is the difference average of OFCN which can be calculated as $d_1 = (a-b)/2$, $d_2 = (c-d)/2$, $d_3 = (e-f)/2$, $d_4 = (g-h)/2$, The calculated difference average is represented in the set $D_1 = (d_1, d_2, d_3, d_4)$

Step 3 Consider the elements of A_1 and compute the approximation coefficients and detailed coefficients as in Steps 2a and 2b as follows $a_1^1 = (a_1 + a_2)/2$, $a_2^1 = (a_3 + a_4)/2$,
$d_1^1 = (a_1 - a_2)/2$,$d_2^1 = (a_3 - a_4)/2$ .Represent Approximation Coefficients in $A_2 = (a_1^1, a_2^1)$ and similarly Detailed Coefficients in $D_2 = (d_1^1, d_2^1)$

Step 4 Consider the elements of A_2 and compute the approximation and detailed coefficients as mentioned above $a_1^2 = (a_1^1 + a_2^1)/2$ and detailed coefficients as $d_1^2 = (a_1^1 - a_2^1)/2$

Step 5 Thus the OCFN becomes HOFN as $H =(a_1^2, d_1^2, d_1^1, d_2^1,d_1, d_2, d_3, d_4)$

Step 6 The maximum absolute value in detailed coefficients of HOFN is the Haar Ranking of Octagonal Fuzzy number

Illustration 1:

Conversion of Octagonal Fuzzy number(OCFN) into a Haar Octagonal Fuzzy number(HOFN). Consider a project network whose activity from node 1 to node 2 represented by a octagonal fuzzy number as = (4, 14, 18, 18, 12, 10, 6, 8) By applying the above steps 2 to step 5 as follows:

Step 2 a A_1 = (4+ 14)/2, (18 + 18)/2, (12 + 10)/2, (6+ 8)/2 = (9, 18, 11, 7)

Step 2 b D_1 = (4-14)/2, (18-18)/2),(12-10)/2, (6-8)/2 =(-5, 0, 1, -1)

Step 3 A_2 = (9+18)/2, (11+ 7)/2 = (13.5, 9)D_2 = (9-18)/2, (11-7)/2 = (-4.5, 2)

Step 4 A_3 = (13.5+ 9)/2 = 11.25, D_3 = (13.5 – 9)/2 = 2.25

Step 5 H = (11.25, 2.25, -4.5, 2, -5, 0, 1, -1)

Step 6 Maximum in detailed coefficients gives us Haar Ranking of Octagonal Fuzzy number |H| = 5

Floyd Warshall Algorithm:

Floyd Warshall Algorithm: It is an algorithm is determine the shortest path between any two vertices in a network activity.Let the vertex at which we are starting be called the initial vertex. Let the distance of vertex Y be the distance from the initial vertex to Y.

Step 1 Represent the given network as a rectangular matrix

Step 2 update the rectangular matrix by considering all vertices as an intermediate vertex.

Step 3 pick all vertices and updates all shortest paths which include the picked vertex as an intermediate vertex in the shortest path.

Step 4The picked vertex k as an intermediate vertex, we considered vertices {0, 1, 2, .. k-1} as intermediate vertices.

Step 5For every pair (a, b) of the source and destination vertices respectively, there are two possible cases.

5a If k is not an intermediate vertex in shortest path from a to b. We keep the value of distance[a][b] as it is.

5b k is an intermediate vertex in shortest path from a to b. We update the value of distance[a][b] as distance[a][k] + distance[k][b] if distance[a][n] >distance[a][k] + dist[k][b]

Symbolic Representation

Let $ES_i$and LS_i represents the earliest reaching fuzzy event time, and the latest reaching fuzzy time for event i, respectively. Functions that define the earliest starting times, latest starting times and floats in terms of fuzzy activity duration are always in convex, normal whose membership function are piecewise continuous hence the quantities such as earliest fuzzy event time $ES_i$the latest fuzzy event time LS_i are OCFN fuzzy numbers for an event i respectively.

For basic computations, let us use the following notations:

1.A_{OCFN}(ab) = Total Float time of A_{OCFN}(ab)$

2.E_{OCFN}(ab) = Estimated A_{OCFN}(ab) = Activity of OCFN between event a and event b

3.E_{OCFN}(a) = Earliest occurrence event time a of OCFN

4. L_{OCFN}(b) = Latest occurrence event time b of OCFN

5. ES_{OCFN}(ab) = Earliest starting time from activity a to b of OCFN

6. EF_{OCFN}(ab) = Earliest finishing time from activity a to b of OCFN

7. LS_{OCFN}(ab) = Latest Starting time from activity a to b of OCFN

8. LF_{OCFN}(ab)= Latest Finishing time from activity a to b of OCFN

9. E_{OCFN} (ab) = Estimated completion time.

Procedure for Haar Ranking Octagonal Fuzzy critical path Algorithm

Step 1 In project network, identify the OCFN activities.

Step 2 Establish precedence relationship of all fuzzy activities in terms of OCFN numbers.

Step 3 Draw a project network diagram with OCFN as fuzzy activity times.

Step 4 Find the expected time for the activity a_0-a_1 using the following rule.

Round off the expected time to the nearest largest integer.

Step 4 a) Consider the octagonal fuzzy activity time (4, 14, 18, 18, 12, 10, 6, 8) for the path 1-2.

Step 4 b)] Now convert it into a Haar Octagonal Fuzzy number as explained in the illustration 1

Step 4 c)] Thus for the path 1-2 the Haar Ranking Octagonal Fuzzy number is 5

Step 4 d)] Apply the above three steps for all the path. Thus we got the expected time for each path which was shown in table.

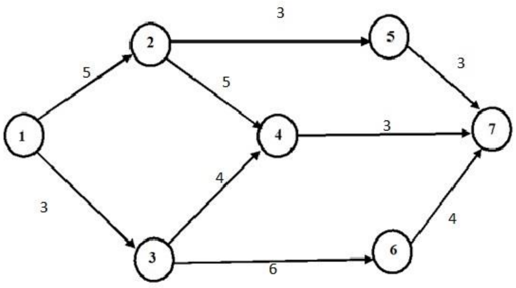
Step 5] At this stage, our project network has an associated integer calculated in Step 3 for any two adjacent nodes.

Step 6] Consider Sand D as the source node and destination nodes in the network.

Apply Floyd- Warshall algorithm to find the shortest path between the source node and the destination node. The path identified by the Floyd -Warshall algorithm is identified as the Haar Ranking Octagonal Fuzzy Critical Path.

Numerical Results

In this section we are going to determine the fuzzy critical path using Floyd Warshall algorithm with the help of Haar octagonal fuzzy number. First consider the following network along with the expected time of each activity.



Example 1

The table 1 represents the fuzzy activities of various nodes in a given network. For finding

Activity	Octagonal Fuzzy Activity Time	Expected time using Haar Ranking
1-2	(4,14,18,18,12, 10, 6, 8)	5
1-3	(2, 4, 5, 7, 8, 9, 10, 12)	3
2-4	(11, 14, 15, 6, 17, 7, 8, 9)	5
3-4	(12, 13, 32, 45, 6, 8, 9, 6)	4
2-5	(5, 6, 8, 9, 12, 12, 4, 9)	3
3-6	(10, 11,12, 14, 16, 6, 2, 6)	5
4-7	(4, 6, 8, 2, 5, 9, 2, 5)	3
5-7	(2, 4, 8, 9, 3, 5, 6, 8)	3
6-7	(5, 7, 8, 11, 14, 17, 5, 7)	4

Table 1: Expected time using Haar Ranking method

critical path first we have to convert the fuzzy activities into a single node using Haar wavelet basis. Consider the activity 1-2along with the octagonal fuzzy activity time (4,14,18,18,12, 10, 6, 8). By applying the Haar ranking method we have to convert the octagonal fuzzy number into a single number as which was illustrated in section 3. Thus the values of column three in table 1 is obtained. After converting all the fuzzy activity time into a single number using Haar ranking method. Now we have to apply the Floyd - Marshall algorithm that was discussed in section 4. In this example, path P2 : 1-3 – 4- 7 is identified as a Haar octagonal Fuzzy critical path. The table below represents the calculations of expected time in each activity of fuzzy critical path Network in the numerical example and identify the critical path.

Path	Project completion time
P1: 1 – 2 – 5 – 7	11
P2: 1 – 2 – 4 – 7	13
P1: 1 – 3 – 4 – 7	10
P1: 1 – 3 – 6 – 7	12

Table 2: Project completion time using Floyd – Warshall Algorithm

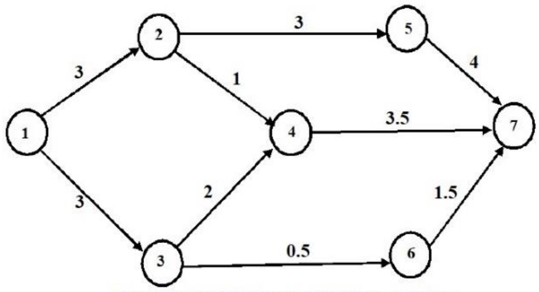
Example 2:

Consider the table 2 which represents fuzzy activity and the expected time using Haar ranking method

Activity	Fuzzy Activity time	Expected time using Haar ranking method
1-2	(13,27, 14, 17,21,27,13, 17)	3
1-3	(23, 15,27, 49, 30,32,23, 52)	3
2-4	(11, 24, 37, 24, 27,32, 17, 19)	1
3-4	(13, 15,27, 39, 40,22, 23, 35)	2
2-5	(15, 27, 20, 33,27, 31, 25, 37)	3
3-6	(17, 19, 11,24, 19, 26, 17, 19)	0.5
4-7	(17, 29, 13, 19, 28, 27,19, 7)	3.5
5-7	(12, 23, 14, 26, 17, 19, 12, 13)	4
6-7	(15, 27, 18, 21, 24, 27, 15, 17)	1.5

Table – 3 Expected time using Haar Ranking method

For the above table the first column represent the activity, second column represent the time duration the done the specified work and third column represents the expected time using Haar ranking method whose network was shown below



Fuzzy Project Network with Expected time

For the above project network various path has been calculated using Ford Warshall Algorithm and by applying the Haar ranking method the minimum path is obtained which was shown in table 4.

Path	Project completion time
P1: 1 – 2 – 5 – 7	10
P2: 1 – 2 – 4 – 7	7.5
P1: 1 – 3 – 4 – 7	8.5
P1: 1 – 3 – 6 – 7	5

Table 4: Project completion time

In the above example among all the paths P4: 1 – 3 – 6 – 7 is identified as fuzzy critical path.

4. Results

In this paper Floyd Warshall Algorithm has been implemented in a fuzzy network environment to determine the critical path using several criteria. Octagonal Fuzzy number have been used as fuzzy activity times, to find critical path with the help of Haar Ranking Octagonal Fuzzy number. A new expected time has been proposed to select critical path using Floyd Warshall Algorithm and Haar

Ranking fuzzy number as activity times. Two numerical examples related to this problem has provided to explain the procedure of the proposed method in determining critical path with different criteria.

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