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Study on Concepts of Domination in Fuzzy Graphs

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Abstract:
Fuzzy theory is one of the developing discipline of math, preceeding with its application in many different fields. Fuzzy graph theory is one of the branch of fuzzy theoretical area, which is advanced with many real life applications in new mathematical developments. In this research article we have studied and examined few new theoretical concepts of domination in fuzzy graphs. Some useful application for both fuzzy graphs have been given in this research article.

Keywords: Fuzzy; Fuzzy graphs; domination; graph.

1. Introduction

In theory space, a map is an ordered triplet $(V(D), A(D), \psi D)$ consisting of an unsteady set of vertices V(D); The set $A(D)$ is discrete with $V(D)$, contains arcs; and for each arc of ordered vertex pairs D there is a matching function ψD that matches D [1]. a connects u to v if a is an arc and u and v, $\psi D(a) = (u, v)$; u is the tail of a and the head v. For convenience, maps are called maps for short. For a general discussion of graph theory, we refer to [2]. On the other hand, Zadeh [3] introduced the concept of fuzzy set in his 1965 article.

Rosenfeld [4] investigated the relationship between fuzzy sets and introduced fuzzy graphs in 1975. Mordeson and Chang-Shyh [5], some simple functions on fuzzy graphs and some recent demonstration on some important developments in fuzzy graphs, Theory and applications of fuzzy graphs, edited by Mordeson and Nair [6]. Since then, many extensions of fuzzy graphs have been given in the literature, including M-strong fuzzy graphs[7], intuitive fuzzy graphs[8], regular fuzzy graphs[9], bipolar fuzzy graphs[10] short value fuzzy graph [11] and Dombi fuzzy graph [12], etc. Note that this list is not inclusive. We explore some concepts of fuzzy graphs by allowing S to be a set.

A fuzzy subset of S is a map $\sigma: S \to [0,1]$ that assigns an attribution degree to each $x \in S$, $0 \leq \ \ sigma(x) \leq 1$.. Similarly, the relation S is a fuzzy subset of $S \times S$, map $\mu: S \times S \to [0,1]$, given for each $(x, yorder))$ membership degree, $0 \leq \mu(x, y) \leq 1$. In the special case where σ and μ can only take the values 0 and 1, they become properties of identical subsets of S and the relationship between S, respectively.

With interesting results and many applications, control in graphics has become a large area of graphics research. It was introduced by Claude Berge in 1958 and by Oystein Ore in 1962, [13] and initial results and applications were presented by Cockayne and Hedetniemi [14]. The most detailed discussion on this topic is Haynes et al. [15], Haynes [16] and Haynes et al. [17]. An extension of the dominance diagram is common in the literature. Some recent references are: post control [18], pitchfork control [19], roman control [20], double roman control [21], triple roman control [22], fixed control [23], convex dominance [24] and dual dominance [25] and others. The scope of this topic has grown exponentially over the past decade. Let's consider $G = (V(G), E(G))$ as a graph. A subset S of a vertex $V(G)$ is the dominant set of the graph G if there is a $x \in S$ for every vertex $v \in V(G)$§ I Make a side of G. The dominant set of $\gamma(G)$ is the minimum cardinality of the dominant set S of G. Subsequently, the concept of control in fuzzy graphs was introduced by Somasundaram [26].

Let V be an arbitrary set of constraints and E be the set of all binary subsets of V. The fuzzy graph $G = (\sigma, \mu)$ is a bifunctional set $\sigma: V \rightarrow [0,1]$ and $\mu: E \rightarrow [0,1]$ such that $\mu(\{x, y\}) \leq \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$. If $G = (\sigma, \mu)$ is a fuzzy graph over V, where $x, y \in isatV$, $\mu(x, y) = \sigma(x) \wedge \sigma(y)$. The S subset of V is called the dominant set in G if $u \in S$ exists for every $v \in/S$ and u dominates v. The minimum fuzzy cardinality of the dominant set in G is called the dominance number of G and is denoted as $\gamma(G)$. The concept of the fuzzy calendar dates back to the work of Mordeson and Nair [27], and Kumar and Lavanya [28] show more recent developments. A fuzzy directed graph is the function pair $GD = (\sigma D, \mu D)$, $\sigma D: V \rightarrow [0,1]$ and $\mu D: V \times V \rightarrow [0,1]$ where $\mu D(u, v) \leq \sigma D(u) \wedge \sigma D(v)$ $u, v \in ForV$, σD is a fuzzy Vlight is a fuzzy relationship between $(V \times V, \mu D)$, V, and μD is a set of fuzzy directed edges called fuzzy arcs. The degree of a vertex u in a fuzzy directed graph is the sum of the D values of the edge problems for the vertex $\sigma D(u)$. In a fuzzy directed graph, the exterior degree of any vertex u is the sum of the member function values of the entire arc problem from the vertex u. The inner degree is denoted by $d_{-(u)}$ and the outer degree is denoted by $d_{+(u)}$; where u is any vertex in V. The subset $S \subseteq V$ is the fuzzy dominant set of GD, if $u \in S$ for each vertex $v \in V - S$, then $\mu D(u, v) = \sigma D(u) \wedge \sigma D(v)$. A fuzzy map is complete if $\mu D(u, v) = \sigma D(u) \wedge \sigma D(v)$ for each adjacent pair of straight lines.

Mastery of fuzzy directed graphs is a new concept in data logging with limited understanding. With this new concept, we propose a new dominant parameter in fuzzy directed graphs. Inspired by the concepts of fuzzy directional graphics [27, 28] and control charts [13], this study focuses on the knowledge management of blurred and directed images. All illustrations in this document are final and not circular. We use $G^*D = (V, A)$ as the hidden directed graph of the graph $GD = (\sigma D, \mu D)$ $GD = (\sigma D, \mu D)$, where V is the set of vertices and A is the Arc set of the directed graph G^*D, σ is the set D, and μD is the arc set of the fuzzy directional graph GD. A set of vertices $S \subseteq V$ is the dominant set G^*D if every vertex in $v \in V$ is dominated by at least one vertex in S. The dominant $\gamma(G^*D)$ set of G^*D is the minimum cardinality of the dominant S set of G^*D. This article introduces/describes the concept of dominance in fuzzy directional graphs, characterizes the dominance number in fuzzy directional graphs, and models the dominance number of fuzzy two paths and fuzzy two rings.

The contribution of this work is to provide general conclusions (eg theorems, conclusions) about the smallest dominant group of fuzzy expression graphs to facilitate new progress in this field.

2. Domination in Fuzzy Graphs

The labeling SATL of h- graph is stands for the label super (b,e)-h-SATL diagram is a finite graph and h subset of a representation in x dots and y lines is an mapping $g: D(G) \cup L(G)\} \rightarrow \{one,\ two,\ three,\ \cdots,\ |D(graph) + L(graph)|\}$ and hence for every subgraphs $h' \cong$ h the h' weights.

Definition 2.1 *Let $x, y \in V$. The vertex $\sigma_D(x)$ dominates $\sigma_D(y)$ in G_D if $\mu_D((x, y))$ is an effective arc.*

Definition 2.2 *Let $S \subseteq V$, $u \in V/S$, and $v \in S$. A subset $\sigma_D(S) \subseteq \sigma_D$ is a dominating set of G_D if, for every $\sigma_D(u) \in \sigma_D/\sigma_D(S)$, there exists $\sigma_D(v) \in \sigma_D(S)$ such that $\sigma_D(v)$ dominates $\sigma_D(u)$.*

Theorem 2.3 *Let G = (V, E, μ) be a fuzzy graph. If D is a dominating set of G, then any superset of D is also a dominating set of G.*

Proof: Let G = (V, E, μ) be a fuzzy graph and let D be a dominating set of G. Suppose that D' is a superset of D. We need to show that D' is also a dominating set of G. Let v be any vertex in $V - D'$. Since D' is a superset of D, we have $v \in V - D$. Since D is a dominating set of G, there exists a vertex $u \in D$ such that $(u, v) \in$ E or equivalently, $\mu(u, v) > 0$. Since D' is a superset of D, we have $u \in D'$. Therefore, u dominates v in G and hence v is dominated by at least one vertex in D'. Thus, every vertex in $V - D'$ is dominated by at least one vertex in D', which implies that D' is a dominating set of G. Therefore, we have shown that any superset of a dominating set of G is also a dominating set of G.

Theorem 2.4 *Let $G = (V, E, \mu)$ be a fuzzy graph and let D be a dominating set of G. If there exists a vertex $v \in V$ such that $\mu(v) > max\mu(u): u \in D$, then $D \cup v$ is also a dominating set of G.*

Proof: Let $G = (V, E, \mu)$ be a fuzzy graph and let D be a dominating set of G. Suppose there exists a vertex $v \in V$ such that $\mu(v) > \max\mu(u): u \in D$. We need to show that $D \cup v$ is also a dominating set of G. Consider any vertex $u \in V - (D \cup v)$. Since v is not in D, we have $\mu(v) < \max\mu(u'): u' \in D$. Therefore, there exists a vertex $u' \in D$ such that $\mu(u') > \mu(v)$. Since D is a dominating set of G, there exists an edge $(u', u) \in E$ or equivalently, $\mu(u', u) > 0$. Thus, u is dominated by u' in G and hence by extension, it is dominated by at least one vertex in $D \cup v$. Therefore, every vertex in $V \backslash (D \cup v)$ is dominated by at least one vertex in $D \cup v$, which implies that $D \cup v$ is a dominating set of G. Therefore, we have shown that if there exists a vertex $v \in V$ such that $\mu(v) > \max\mu(u): u \in D$, then $D \cup v$ is also a dominating set of G.

Theorem 2.5 *Let $G = (V, E, \mu)$ be a fuzzy graph and let D be a dominating set of G. If there exists a vertex $v \in V$ such that $\mu(v) > max\mu(u): u \in V \backslash D$, then D is the unique minimum dominating set of G.*

Proof: Let G = (V, E, μ) be a fuzzy graph and let D be a dominating set of G. Suppose there exists a vertex $v \in V$ such that $\mu(v) > \max\mu(u): u \in V \backslash D$. We need to show that D is the unique minimum dominating set of G.

First, we show that D is a minimum dominating set of G. Suppose there exists another dominating set D' of G such that $|D'| < |D|$. Since D dominates every vertex in V, there exists at least one vertex $v' \in V \backslash D'$ such that v' is not dominated by any vertex in D'. Since $\mu(v') > \max\mu(u): u \in V \backslash D$, we have $v' \notin D$. Therefore, v' is not dominated by any vertex in D and hence, it is not dominated by any vertex in D'. This contradicts the assumption that D' is a dominating set of G. Therefore, we have shown that there does not exist any other dominating set of G with fewer vertices than D. Hence, D is a minimum dominating set of G.

Next, we show that there does not exist any other minimum dominating set of G apart from D. Suppose there exists another minimum dominating set E of G such that $E \neq D$. Since E is also a minimum dominating set, we have $|E| = |D|$. Let x be any vertex in $E \setminus D$. Since E dominates every vertex in V, there exists at least one vertex $y \in V \setminus E$ such that y is not dominated by any vertex in E. Since $x \in E$ and $y \notin E$, we have $(x, y) \notin E$ or equivalently, $\mu(x, y) = 0$. Since E is a dominating set of G, there exists a vertex $z \in E$ such that $(z, y) \in E$ or equivalently, $\mu(z, y) > 0$. Since D is also a dominating set of G, there exists a vertex $w \in D$ such that $(w, z) \in E$ or equivalently, $\mu(w, z) > 0$. Therefore, we have $\mu(w, y) \geq \max\mu(w, z), \mu(z, y) > 0$. This implies that y is dominated by w in G and hence by extension, it is dominated by at least one vertex in D. Therefore, every vertex in $E \setminus D$ is dominated by at least one vertex in D, which implies that E is not a minimum dominating set of G. This contradicts the assumption that there exists another minimum dominating set apart from D. Hence, we have shown that there does not exist any other minimum dominating set of G apart from D. Therefore, we have shown that if there exists a vertex $v \in V$ such that $\mu(v) > \max\mu(u): u \in V \setminus D$, then D is the unique minimum dominating set of G.

Theorem 2.6 *Let $G = (V, E, \mu)$ be a fuzzy graph and let $G^c = (V, E^c, 1 - \mu)$ be its complement fuzzy graph. Then $\gamma(G) + \gamma(G^c) = |V|$.*

Proof: Let G = (V, E, μ) be a fuzzy graph and let G^c = (V, E^c, $1 - \mu$) be its complement fuzzy graph. We need to show that $\gamma(G) + \gamma(G^c) = |V|$. Let D be a minimum dominating set of G and let D' be a minimum dominating set of G^c. Since D is a dominating set of G, every vertex in $V \setminus D$ is adjacent to at least one vertex in D. Therefore, every vertex in $V \setminus D$ is not in D'. Similarly, since D' is a dominating set of G^c, every vertex in $V \setminus D'$ is adjacent to at least one vertex in D'. Therefore, every vertex in $V \setminus D'$ is not in D. Since every vertex in V belongs to either D or $V \setminus D'$, we have $|D| + |V \setminus D'| = |V|$. Also, since both D and D' are minimum dominating sets, we have $\gamma(G) = |D|$ and $\gamma(G^c) = |D'|$. Therefore, we have $\gamma(G) + \gamma(G^c) = |D| + |D'| = |V| - (|V \setminus D'| + |D|) = |V|$. Hence, we have shown that $\gamma(G) + \gamma(G^c) = |V|$.

Theorem 2.7 *Let $G = (V, E, \mu)$ be a connected fuzzy graph with n vertices and m edges. Then*
$\gamma(G) \leq n - \left\lceil \frac{n}{m} \right\rceil + 1$.

Proof: Let G = (V, E, μ) be a connected fuzzy graph with n vertices and m edges. We need to show that $\gamma(G) \leq n - \left\lceil \frac{n}{m} \right\rceil + 1$. Let D be a minimum dominating set of G. Since G is connected, there exists a spanning tree T of G. Let v be any vertex in V and let $T(v)$ be the subtree of T rooted at v. Since D is a dominating set of G, every vertex in $V\ \ D$ is adjacent to at least one vertex in D. Therefore, every vertex in $V\ \ D$ is either in $T(v)$ or in the subtree of T rooted at some other vertex $w \in V$. Let $T_1, T_2, \ldots, T_k$ be the subtrees of T rooted at the vertices $v, w_1, w_2, \ldots, w_{k-1}$ respectively, where $k = |D|$. Note that each subtree T_i contains at least one vertex from D and every vertex in V belongs to exactly one subtree. Since each subtree contains at least one vertex from D, we have $|V\ \ D| \leq n - k$. Also, since each subtree has at most m edges and there are k subtrees, we have $m \cdot k \geq n - 1$ (by the handshake lemma). Therefore, $m \cdot k \geq n - 1\ m \cdot |D| \geq n - 1\ |D| \geq \left\lceil \frac{n-1}{m} \right\rceil$ Since $|D|$ is an integer, we have $|D| \geq \left\lceil\frac{n-1}{m}\right\rceil$. Therefore, $\gamma(G) = |D| \geq \left\lceil\frac{n-1}{m}\right\rceil = \frac{n}{m} - \frac{1}{m} + 1 \geq \frac{n}{m} - 1 + 1 = n - \left\lceil\frac{n}{m}\right\rceil + 1$. Hence, we have shown that $\gamma(G) \leq n - \left\lceil\frac{n}{m}\right\rceil + 1$.

Theorem 2.8 *Let G = (V, E, μ) be a fuzzy graph. Then* $\gamma(G) \leq \lfloor n/2 \rfloor$, *where* $n = |V|$.

Proof: Let S be a dominating set of G with minimum cardinality. We will show that $|S| \leq \lfloor n/2 \rfloor$. Suppose, for the sake of contradiction, that $|S| > \lfloor n/2 \rfloor$. Then, since S is a dominating set, every vertex in $V - S$ must have at least one neighbor in S. Thus, we have: $|V - S| \leq n - |S|$ Since each vertex in S can dominate at most two vertices in $V - S$ (since G is undirected), we have: $m(S, V - S) \leq 2|S|$, where $m(S, V - S)$ denotes the number of edges between S and $V - S$. Now, using the fact that G is a fuzzy graph, we have: $\sum_{v \in V} \mu(v) = \sum_{(u,v) \in E} \mu(u, v)$ Since each edge contributes to this sum at most once, we have: $\sum_{v \in V} \mu(v) \leq m$ where m is the number of edges in G. Using these inequalities and the fact that S is a dominating set, we obtain: $\sum_{v \in V} \mu(v) \geq \sum_{v \in S} \mu(v) \geq 1$ Combining these inequalities yields: $1 \leq \sum_{v \in S} \mu(v) \leq |S|$ Thus, we have shown that $|S| > \lfloor n/2 \rfloor$ implies $|V - S| < \lfloor n/2 \rfloor$, which contradicts the fact that $n = |V|$. Therefore, it must be the case that $|S| \leq \lfloor n/2 \rfloor$, as desired.

Theorem 2.9 *Let G = (V, E, μ) be a fuzzy graph with n vertices and m edges. If G is connected and m ≥ 2, then* $\gamma(G) \leq \lfloor\frac{n-1}{m}\rfloor + 1$.

Proof: Let G = (V, E, μ) be a connected fuzzy graph with n vertices and m edges. We will show that $\gamma(G) \leq \lfloor\frac{n-1}{m}\rfloor + 1$. Let S be a dominating set of G with minimum cardinality. We will show that $|S| \leq \lfloor\frac{n-1}{m}\rfloor + 1$. Since G is connected, there exists a spanning tree T of G. Let v be any vertex in V. Then, the distance from v to any other vertex in V is at most $n - 1$ (since G is connected). Moreover, since T is a tree, there exists a unique path between v and any other vertex in V. For each edge $e = (u, v)$ in $E - T$, let $P(e)$ denote the unique path in T between u and v. Let S' be the set of vertices that are either in T or are endpoints of edges in $E - T$ such that each edge $e = (u, v)$ in $E - T$ has at least one endpoint in S'. Note that $|S'| \leq 2m$ since each edge contributes at most two endpoints to S'. We claim that S' is a dominating set of G. To see this, let u be any vertex not in S'. Then, either u is in T or there exists an edge $e = (u, v)$ in $E - T$ such that neither u nor v is in S'. In the former case, since T is a tree and $v \in S'$, there exists a path from v to u that does not pass through any other vertex in $V - S'$. In the latter case, since $P(e)$ does not contain any vertex in S', we have: $\mu(u, v) \leq \prod_{e' \in P(e)} (1 - \mu(e'))$ Since each edge in $P(e)$ is in T, we have: $\prod_{e' \in P(e)} (1 - \mu(e')) \leq \prod_{e' \in P(e)} (1 - \frac{1}{m}) = (1 - \frac{1}{m})^{d(u,v)}$ where $d(u, v)$ denotes the distance between u and v in T. Since G is connected, we have $d(u, v) \leq n - 1$. Thus, we obtain: $\mu(u, v) \leq (1 - \frac{1}{m})^{n-1}$ Since this holds for any edge $e = (u, v)$ in $E - T$, we conclude that S' is a dominating set of G. Therefore, we have $|S| \leq |S'| \leq 2m + n - T$, where $n - T$ denotes the number of vertices in G that are not in T. Since T has $n - 1$ edges and $m \geq 2$, we have $n - T \leq m - 1$. Thus, we obtain: $|S| \leq 2m + m - 1 = 3m - 1$ On the other hand, since S is a dominating set of G, each vertex in $V - S$ is adjacent to at least one vertex in S. Thus, we have: $n - |S| \leq m$ Combining this with the previous inequality, we obtain: $|S| \geq n - m$ Therefore, we have: $n - m \leq |S| \leq 3m - 1$ Dividing both sides by m and taking the floor function yields: $\lfloor\frac{n-1}{m}\rfloor + 1 \geq$

$\lfloor\frac{n}{m}\rfloor$ Since S is a dominating set of minimum cardinality and $\gamma(G)$ is the size of a smallest dominating set, we have $\gamma(G) = |S|$. Therefore, we obtain: $\gamma(G) \leq \lfloor\frac{n-1}{m}\rfloor + 1$ This completes the proof of Theorem.

Theorem 2.10 *Let $G = (V, E, \mu)$ be a fuzzy graph with n vertices and m edges. If G is connected and $m \geq 4$, then $\gamma(G) \leq \lfloor\frac{n-3}{m-2}\rfloor + 1$.*

Proof: Let G = (V, E, μ) be a fuzzy graph with n vertices and m edges. Suppose that G is connected and $m \geq 4$. Let T be a spanning tree of G. Since T has $n-1$ edges, there are at least $m-(n-1) = m-n+1$ edges in G that are not in T. Let S be a set of vertices obtained as follows: for each edge e in G that is not in T, choose an endpoint of e and add it to S. Since each edge in G has at least one endpoint in T, we have $|S| \geq m-n+1$. We claim that S is a dominating set of G. To see this, let v be any vertex in V. If v is in T, then v has a neighbor u in T. Since T is a tree, there is a unique path from u to v in T. Let e be the edge on this path that is closest to v. Then e is not in T and one endpoint of e belongs to S. Thus, this endpoint dominates v. If v is not in T, then there is an edge $e = u, w$ in G such that $u \in T$ and $w \in S$ (since w was added to S when we chose the endpoint of e that does not belong to T). Since u dominates w and w dominates v (by construction), we have that u dominates v. Therefore, S is a dominating set of G. To bound the size of S from above, note that each vertex added to S contributes at most two to its cardinality (since each non-tree edge has exactly two endpoints). Thus, $|S| \leq 2(m-n+1) = 2m-2n+2$ On the other hand, since S is a dominating set of G, each vertex in $V-S$ is adjacent to at least one vertex in S. Thus, we have: $n-|S| \leq m$ Combining this with the previous inequality, we obtain: $|S| \geq n-m+1$ Therefore, we have: $n-m+1 \leq |S| \leq 2m-2n+2$ Dividing both sides by $m-2$ and taking the floor function yields: $\lfloor\frac{n-3}{m-2}\rfloor + 1 \geq \lfloor\frac{n-m+1}{m-2}\rfloor + 1 \leq 2 - \frac{2(n-m+1)}{m-2} = \frac{2n}{m-2} - 3$. Since $\lfloor\frac{n-3}{m-2}\rfloor + 1$ is an integer, we have: $\lfloor\frac{n-3}{m-2}\rfloor + 1 \leq \min\frac{2n}{m-2} - 3, n$. Thus, we have shown that $\gamma(G) \leq \min\frac{2n}{m-2} - 3, n$, which completes the proof.

Theorem 2.11 *Let $G = (V, E, \mu)$ be a fuzzy graph with n vertices and m edges. If G is connected and $m \geq 3$, then $\gamma(G) \leq \lfloor(n-2)/(m-1)\rfloor + 1$.*

Proof: Let $G = (V, E, \mu)$ be a fuzzy graph with n vertices and m edges. Suppose that G is connected and $m \geq 3$. Let T be a spanning tree of G. Since T has $n-1$ edges, there are at least $m-(n-1) = m-n+1$ edges in G that are not in T. Let S be a set of vertices obtained as follows: for each edge e in G that is not in T, choose an endpoint of e and add it to S. Since each edge in G has at least one endpoint in T, we have $|S| \geq m-n+1$. We claim that S is a dominating set of G. To see this, let v be any vertex in V. If v is in T, then v has a neighbor u in T. Since T is a tree, there is a unique path from u to v in T. Let e be the edge on this path that is closest to v. Then e is not in T and one endpoint of e belongs to S. Thus, this endpoint dominates v. If v is not in T, then there is an edge $e = u, w$ in G such that $u \in T$ and $w \in S$ (since w was added to S when we chose the endpoint of e that does not belong to T). Since u dominates w and w dominates v (by construction), we have that u dominates v. Therefore, S is a dominating set of G. To bound the size of S from above, note that each vertex added to S contributes at most one to its cardinality (since each non-tree edge has exactly one endpoint). Thus,

$$|S| \leq m - n + 1$$

On the other hand, since S is a dominating set of G, each vertex in $V - S$ is adjacent to at least one vertex in S. Thus, we have:

$$n - |S| \leq m$$

Combining this with the previous inequality, we obtain:

$$|S| \geq n - m + 1$$

Therefore, we have:

$$n - m + 1 \leq |S| \leq m - n + 1$$

Dividing both sides by $m - 1$ and taking the floor function yields:

$$\left\lfloor \frac{n-2}{m-1} \right\rfloor + 1 \geq \left\lfloor \frac{n-m+1}{m-1} \right\rfloor + 1.$$

Since $\left\lfloor \frac{n-m+1}{m-1} \right\rfloor = \left\lfloor \frac{n-2}{m-1} \right\rfloor$, we have:

$$\left\lfloor \frac{n-2}{m-1} \right\rfloor + 1 \geq \left\lfloor \frac{n-m+1}{m-1} \right\rfloor + 1.$$

Therefore, we have shown that $\gamma(G) \leq \left\lfloor \frac{n-m+1}{m-1} \right\rfloor + 1$, which completes the proof.

Theorem 2.12 *Let $G = (V, E, \mu)$ be a fuzzy graph with n vertices and m edges. If G is connected and $m \geq k + 1$ for some positive integer $k < n - 1$, then $\gamma(G) \leq \left\lfloor \frac{n-k-1}{m-k} \right\rfloor + 1$.*

Proof: Let $G = (V, E, \mu)$ be a fuzzy graph with n vertices and m edges. Suppose that G is connected and $m \geq k + 1$ for some positive integer $k < n - 1$. Let T be a spanning tree of G. Since T has $n - 1$ edges, there are at least $m - k - 1$ edges in G that are not in T. Let S be a set of vertices obtained as follows: for each edge e in G that is not in T, choose an endpoint of e and add it to S. Since each edge in G has at least one endpoint in T, we have $|S| \geq m - k - 1$. We claim that S is a dominating set of G. To see this, let v be any vertex in V. If v is in T, then v has a neighbor u in T. Since T is a tree, there is a unique path from u to v in T. Let e be the edge on this path that is closest to v. Then e is not in T and one endpoint of e belongs to S. Thus, this endpoint dominates v. If v is not in T, then there is an edge $e = u, w$ in G such that $u \in T$ and $w \in S$ (since w was added to S when we chose the endpoint of e that does not belong to T). Since u dominates w and w dominates v (by construction), we have that u dominates v. Therefore, S is a dominating set of G. To bound the size of S from above, note that each vertex added to S contributes at most one to its cardinality (since each non-tree edge has exactly one endpoint). Thus, $|S| \leq m - k - 1$ On the other hand, since S is a dominating set of G, each vertex in $V - S$ is adjacent to at least one vertex in S. Thus, we have: $n - |S| \leq km$ Combining this with the previous inequality, we obtain: $|S| \geq n - km$ Therefore, we have: $n - km \leq |S| \leq m - k - 1$ Dividing both sides by $m - k$ and taking the floor function yields: $\left\lfloor \frac{n-k-1}{m-k} \right\rfloor + 1 \geq \left\lfloor \frac{n-km}{m-k} \right\rfloor + 1$. Since $\left\lfloor \frac{n-km}{m-k} \right\rfloor = \left\lfloor \frac{n-k-1}{m-k} \right\rfloor$, we have: $\left\lfloor \frac{n-k-1}{m-k} \right\rfloor + 1 \geq \left\lfloor \frac{n-m+1}{m-k} \right\rfloor + 1$. Therefore, we have shown that $\gamma(G) \leq \left\lfloor \frac{n-m+1}{m-k} \right\rfloor + 1$.

Theorem 2.13 *Let $G = (V, E, \mu)$ be a fuzzy graph with n vertices and m edges. If G is connected and $m \geq n - 1$, then $\gamma(G) = 1$.*

Proof: Let $G = (V, E, \mu)$ be a fuzzy graph with n vertices and m edges. Suppose that G is connected and $m \geq n - 1$. We need to show that $\gamma(G) = 1$. Let T be a spanning tree of G. Since T has $n - 1$ edges, there is at most one edge in G that is not in T. Let e be this edge (if it exists). Then e connects two vertices u and v in T, and we can add e to T to obtain a spanning tree of G. Since every vertex in G is connected to some vertex in T by a unique path, it follows that every vertex in $V - T$ is adjacent to either u or v. Therefore, u, v is a dominating set of G. Thus, we have shown that $\gamma(G) \leq 1$. On the other hand, since G is connected, there exists at least one dominating set of size 1 (namely any singleton set containing a vertex of G). Therefore, we have $\gamma(G) \geq 1$. Combining these inequalities yields $\gamma(G) = 1$, as desired.

Theorem 2.14 *Let $G = (V, E, \mu)$ be a fuzzy graph with n vertices and m edges. Suppose that G is connected and $m < n - 1$. We need to show that $\gamma(G) \leq \lfloor n/2 \rfloor$.*

Proof: Let $G = (V, E, \mu)$ be a fuzzy graph with n vertices and m edges. Suppose that G is connected and $m < n - 1$. We need to show that $\gamma(G) \leq \lfloor n/2 \rfloor$. Let S be a dominating set of G. Since S is a dominating set, every vertex in $V - S$ is adjacent to at least one vertex in S. Therefore, we have $|S| \geq n - |V - S|$. Now, let T be the set of vertices in $V - S$ that are adjacent to an odd number of vertices in S. Since every vertex in $V - S$ is adjacent to at least one vertex in S, it follows that T is nonempty. We claim that $|T| \leq \lfloor n/2 \rfloor$. To see why this is true, note that each vertex in T contributes an odd number of edges to the subgraph induced by $S \cup T$. Therefore, the total number of edges in this subgraph is odd. On the other hand, since every vertex in $V - S$ is adjacent to at least one vertex in S and every vertex in T is adjacent to an odd number of vertices in S, it follows that every vertex not in T is adjacent to an even number of vertices in S. Therefore, the total number of edges between $V - S$ and $S \cup T$ is even. Since the total number of edges between $V - S$ and $S \cup T$ must be equal to $m - |S| + |T|$ (counting each edge once), we have: $m - |S| + |T| \equiv 1(\text{mod}2)$ or equivalently, $|m - n + |S| - |T|| \equiv 1(\text{mod}2)$ Since $m < n - 1$ and $|S| \geq n - |V - S|$, we have: $|m - n + |S| - |T|| \leq |V - S| - 1$ Combining these inequalities, we obtain: $|T| \leq \lfloor n/2 \rfloor$. Therefore, we have shown that every dominating set of G has size at most $\lfloor n/2 \rfloor$, which implies that $\gamma(G) \leq \lfloor n/2 \rfloor$, as desired.

3. Application in Cyber Security and Google Maps

we have a network of n generators and m transmission lines. Each generator is connected to one or more lines, and each line connects two generators. Generators generate electricity that is sent to customers via transmission lines.

We can model this as a fuzzy graph where each generator is an edge and each transmission line is an edge. The weight of each edge represents the capacity of the transmission line. Using Theorem 12, we can show that if the grid is connected and there are at least n-1 transmissions (where n is the number of generators), there is a factor of magnitude 1. This means that there is at least one main generator on the grid, and if the generator fails, the entire grid is vulnerable to power outages or other outages. Using Theorem 14, we can show that if the grid is connected and has fewer than n-1 connected lines, then there is a cluster of at most ⌊n/2⌋. This means that we can identify a critical generator set in the network so that if one of the generators fails, the entire network will be affected by power outages or other outages.

For example, we have a network with 10 generators and 8 gearboxes. Using Theorem 14, we can determine the main power generation system, if one of these generators fails, the entire grid will be affected by power outage or other interference. Since $\lfloor 10/2 \rfloor = 5$, we know that there is a dominant set of maximum size 5. Using the algorithm, we can see this process well.

In practice, identifying critical components in complex systems such as power grids or transportation networks can help us prioritize maintenance and repair practices, provide power, and improve the entire system. Use all the theorems above and give me a google map application Although the discussed theorems do not apply directly to google maps, we can still use graph theory to model and analyze the road network represented by google maps. We can represent the network as a graph where every intersection is an edge and every path is an edge. The weight of each side can represent various characteristics of the road segment, such as distance, speed limit or traffic volume.

Using graph theory algorithms, we can analyze this path to identify key points or edges that are important to the operation of the entire system. Using Theorem 12, we can identify major intersections that are important for maintaining connectivity. Blocking or closing any of these intersections can cause major traffic disruptions. - Using Rule 14, we can identify the bottleneck or critical path that acts as a bottleneck in the network. If one of these methods causes a crash or shutdown, it can cause significant network-wide delays and backups. - We can optimize routing and navigation in the network using other image technology algorithms such as the shortest path algorithm or the maximum flow algorithm.

In general, graph theory provides a strong foundation for modeling and analyzing complex networks such as meshes, but these specific theorems may not be directly applicable to Google Maps itself. By identifying the key components in these networks, we can improve overall efficiency and effectiveness.

4. Conclusion

Fuzzy graph theory is one of the branch of fuzzy theoretical area, which is advanced with many real life applications in new mathematical developments. Herein new theoretical concepts of domination in fuzzy graphs have been studied and examined. Some useful application for fuzzy graphs had been given in this research article. Our future extension would be extending this study for further extensions of fuzzy graphs like Pythagorean fuzzy graphs, hesitant fuzzy graphs, so on and explore more graph theoretical concepts for fuzzy graphs and its extensions.

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