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Revolutionary Hybrid RDS, Big Data and AI Quality Control: C_{pk}-Based Analysis

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Abstract:

In this research, a combination of inspection and repeated deferred sampling is suggested utilizing the process capability index Cpk. This sampling approach can effectively evaluate the quality characteristics of normal distributions even when the mean and variance are unknown. Symmetrical and asymmetrical regions failing to meet optimal conditions for practical use are identified through specific tables Furthermore, the benefits of this proposed mixed sampling design are discussed. A comparison between the suggested policies and current practices is presented to illustrate the differences. A practical example is utilized to demonstrate the practicality of this hybrid sampling strategy. The optimal parameters can be determined using the two-point method of the operating characteristic curve.

Keywords: Mixed sampling designs, process capability indices, decision-making, nonlinear programming, and quadratic programming.

I. Introduction

The assessment of features, variables, and other quality aspects is typically carried out acceptance sampling. Items are categorized as meeting or not meeting standards based on their characteristic quality, while the quality attributes of the item sample are assessed on a continuous scale in variable control. Acceptance criteria-related sampling regulations can be found in Acceptance Sampling Methods. Variable sampling plans generally offer more insight into a product compared to feature sampling methods. Furthermore, the same level of assurance can be achieved by using a variable sampling plan with a smaller sample size than that of a characteristic plan. Therefore, variable sampling strategies are employed in cases where testing is costly and risky. Various sampling schemes have been introduced in the literature, such as Single Sampling Scheme (SSP), Double Sampling Scheme, Multiple Sampling Scheme, Sequential Sampling, etc., tailored to different scenarios. These schemes, also known as standard sampling schemes, base all judgments on a given batch solely on the data derived from the sampled items of the existing batch. Sherman's (1965) cluster repeat sampling (RGS) method, Wortham and Baker's

(1976) multiple delayed (state-dependent) sampling, and Dodge's (1955) chain sampling method are examples of specialized sampling techniques. Chain sampling, MDS, and RGS schemes have been investigated by only a handful of authors, including Aslam, Azam, and Jun (2013), Aslam, Wu et al. (2013), Balamurali et al. (2017), Balamurali, Jeyadurga, Usha, and Venkatesulu (2018), Balamurali and June (2007), Fallahnejad and Seifi (2017), June et al. (2010), among others. Aslam et al. (2016) merged the characteristics of the MDS sampling plan with the RGS plan to develop a novel sampling plan for assessing feature quality traits, which they termed multi-dependency state recursive cluster sampling (MDSRGS) design. The MDSRGS method has demonstrated superior performance over traditional MDS and RGS methods. Furthermore, an extended ISWG variable testing procedure based on the Unidirectional Process Capability Index (2018) has been introduced, as well as the new strategy for sampling mixed repeat clusters for product approval utilizing process capability indices by Balamurali and Usha (2018). A hybrid RDS verification scheme based on the Cpk process capability index is proposed in this study.

II. Mixed inspection RDS plan applications in big data, Neural Network and Artificial Intelligence

Big Data:

The mixed inspection RDS plan can be used in big data scenarios to ensure quality and reliability of the data. Some specific uses of mixed inspection RDS plan in big data include:

1. Evaluation of data quality: The mixed inspection RDS plan can be used to evaluate how well large data complies with predetermined quality requirements.
2. Assessment of huge data's compliance with established quality requirements can be done using the mixed inspection RDS approach.
3. Data reliability assessment: The mixed inspection RDS plan can be used to evaluate the reliability of big data by assessing the likelihood of non-conformance.
4. Process control: The mixed inspection RDS plan can be used in process control to monitor and improve the quality of big data in real-time.
5. Decision making: Mixed inspection RDS design can be used for decision making to support quality-related decisions based on big data evaluations.

In general, the mixed inspection RDS plan is a useful tool for ensuring the quality and reliability of big data and can be used in various stages of the data life cycle to support data-driven decision making.

Neural Network:

Mixed inspection RDS plan can be used in neural network training as a strategy to sample the training data. In large datasets, it may not be feasible to use all the data for training the neural network. The mixed inspection RDS plan can be used to randomly sample a subset of the training data, which can then be used to train the neural network. By using a mixed inspection RDS plan, the training data can be efficiently sampled, while still ensuring that the sampling process is representative of the entire dataset. This can improve the training efficiency, reduce the memory requirements, and help prevent overfitting.

Artificial Intelligence:

The mixed inspection RDS plan can be used in various applications of Artificial Intelligence (AI) such as training machine learning models, evaluating the performance of AI systems, and managing large datasets. The mixed inspection RDS plan provides a way to efficiently sample data while maintaining the representativeness of the sample. In AI, this can help reduce the computational costs of training models and evaluating their performance, as well as ensure that the training and evaluation processes are based on a representative subset of the available data. The mixed inspection RDS plan can also be used in AI applications to manage large datasets by randomly sampling subsets of the data for processing, which can help improve the scalability of the AI systems.

Process Capability Indices

A procedure is deemed satisfactory when the majority of measurements fall within the designated range. By analyzing the results of a procedure, you can evaluate its stability regarding specific process variability criteria. A capability index, known as PCI, is a statistical tool utilized to quantify this variability. Due to its potential in enhancing product quality, PCI has been extensively studied recently. PCI is employed to a manufacturing process's capability to manufacture batteries within specified tolerances. While various forms of PCI exist in literature, PCI Cp, Cpk, and Cpm are considered the primary metrics. References to PCI can be found in works by Kane (1986), Chan et al. (1988), Yum and Kim (2004), Spring et al. (2003), and Kotz and Johnson (2002, 2011). Let's suppose that USL and LSL represent the upper and lower limits of the specification, respectively, then introduce PCI Cp and Cpk.

$$\mathrm{C_p} = \frac{\mathrm{USL-LSL}}{6\sigma} \qquad (1)$$

The Cpk indicator was developed and defined as follows because the Cp indicator does not accurately capture the effect of the location of the process mean. Where The terms upper specification limit and lower specification limit (USL and LSL, respectively) are used. If the process variance σ^2 is unknown, Kane (1986) uses the unbiased sampling variance S^2 to calculate the likelihood index. The Cpk estimator is described as follows:

$$C_{pk} = \min\left(\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right) \qquad (2)$$

$$\hat{C}_{pk} = \min\left(\frac{USL-\bar{X}}{3S}, \frac{X-LSL}{3S}\right) \qquad (3)$$

The sample mean and standard deviation are denoted S in the literature (Kane, 1986). Balamurali and Kalyanasundaram (2002) utilized the bootstrap method to establish lower bounds of confidence for PCI Cpk, and Balamurali (2003) provided confidence levels for short-term manufacturing processes. From the existing knowledge of PCI Cpk and acceptance sampling, various researchers have designed acceptance sampling techniques. For instance, Aslam et al. (2013b) assessed the Resampling Group Scheme (RGS) for PCI Cpk testing, while Pearn and Wu (2007), Negrin et al. (2009), Balamurali and Usha (2014) introduced and analyzed a PCI Cpk-centered single sample design (SSP).

Hybrid Acceptance Sampling Plan

Hybrid acceptance sampling was first proposed by Schilling and Dodge (1969). The study of characteristics and variables is part of a mixed sampling approach. A mixed sampling plan is a good way to achieve this. The mixed sampling plan described in MIL-STD-414 has been studied by several authors, including Gregory and Reznikoff (1955), Bowker and Goode (1952), and Savage (1955) and (1957). Proposed Plans for Mixed Sampling Many authors have studied mixed sampling techniques in a variety of situations.

Aslam and Lee are two examples (2012). Aslam et al. (2013a), Aslam et al. (2015a, 2015b) and Balamurali et al. All had composite sampling plans based on the Cpk capability index (2016a, 2016b). 2019 Balamurali and Aslam propose a multi-dependent state cluster resampling scheme for testing variables.

III. The implementation and operation of a Repetitive Deferred Sampling Plan (RDSP) typically involves several key players:

Quality control personnel - These are the individuals responsible for the implementation and operation of the RDSP, as well as the analysis of the data obtained from the samples. Production personnel - The production team handles the production of the items being sampled and must adhere to the quality requirements defined in the RDSP. Management - Management oversees allocating the resources required to set up and run the RDSP and of making choices in light of the data gathered from the samples. Consumers - Customers are the ultimate recipients of the quality control efforts inherent in the RDSP because they are the end users of the products being produced. Material on RDSP can be obtained in a variety of places, includes journals, publications, and websites devoted to statistical process control and quality control. Douglas S. Montgomery of Statistical Process Control and Thomas P. Ryan and D.H. Quality Assurance and Industrial Statistics have two important tasks to consider in one day.

IV. Hybrid Inspection RDSP Based on C_{pk}

Statistical process control and quality control both employ the RDSP approach. It entails periodically sampling a production process and delaying the inspection of the finished products until a predetermined volume of products has been produced. This enables the effective use of resources and may lower the cost of inspections. The goods are then examined all at once, and any flaws are used to modify the procedure and raise the standard of the product. The purpose of RDSP is to preserve product quality while lowering inspection costs. Repeated Deferred Sampling, developed by Shankar and Mohapatra in 1991, is simply an extension of Rambert Verst’s Multiple Deferred Sampling method MDS-(c1, c2) (1981). This method assumes that the results of inspecting the lot before or after the deferred lot during Repeated Group Sampling (RGS) inspection will decide whether the lot is accepted or rejected. RGS is therefore a special case of the RDS scheme. In addition to developing schedules for Multiple Delayed State Sampling (MDS), Wortham and Baker (1976) also produced tables for constructing MDS schedules. Suresh created methods to select multiple MDS- and MDS-1-like deferred status regimes indexed by producer and consumer quality levels, taking both filtering and incentive effects (1993). Lilly Christina (1995) compared an RGS plan to an RDS plan in terms of operating ratio (OR) and ASN curve and explained how to select an RDS plan with acceptable quality standards.

In their 2010 study, Suresh and Saminathan investigated repetitive postponed sampling programmers that used Acceptable and limited quality standards. Senthilkumar et al proposed a recurrent delay sampling plan (RDSSP). (2015) Investigate quality measures for normal distributions. Under this procedure, deferred lots are accepted or rejected based on the inspection results of the lots that precede or follow it during the RGS inspection. Senthilkumar et el (2017) proposed a repeated delay variable sampling plan (RDVSP) indexed on Six Sigma quality levels to analyze the quality characteristics of a normal distribution.

The MDSRGS scheme devised by Balamurali and Usha (2017) is a newly created sampling strategy for the specific purpose of examining diversity. Therefore, the study will consider two sample sizes for feature edits and variable edits, respectively. Additionally, if a lot fails during a variable inspection under a mixed inspection RDS scheme, the characteristic inspection and the variable inspection are repeated. Unlike the RDS system, repeat sampling is only allowed under the proposed system, when lots are rejected and only after variable inspection. Proposed and existing programs differ in performance measures in terms of operations and parameters.

Let USL and LSL be the specification limits for the quality characteristic in question. Items that deviate from these limits in terms of quality characteristics are considered unqualified. It is also assumed that the studied quality characteristics are normally distributed with undetermined means and standard deviations. The study by Aslam et al. (2013b) and Schilling and Dodge (1969) can be used as the basis for the implementation of the proposed mixed test RDS scheme based on C_{pk} as follows.

V. RDS planning procedure.

The first step (sampling by attributes): randomly select n_1 samples from many products and count the number of unqualified products. Accept the result.

The quantity of non-eligible products under c is the eligible quantity. If the number of nonconforming products exceeds c, go to step 3 and continue the variable control according to C_{pk}.

(sample of variables): Estimate the exponent $C_{pk}(\hat{C}_{pk})$ using a second random sample of size n_2 as follows:

$$\hat{C}_{pk} = \min\left(\frac{USL - \bar{X}}{3S}, \frac{X - LSL}{3S}\right),$$

$$\text{where } \bar{X} = \frac{1}{n_2}\sum_{i=1}^{n2} X_i \text{ and } S^2 = \frac{\sum_{i=1}^{n_2}(Xi - \bar{X})^2}{n_2 - 1}.$$

If fail to accept the lot $\hat{C}_{pk} < k_2$, return it. If find that $k_2 \leq \hat{C}_{pk} < k_1$, Do not reject the whole thing, but rather pick a new sample of size n_2 and redefine the sampling variable. In accordance with the RDS control plan, when 'm' is authorized from the previous or subsequent batch, accept it; otherwise, reject it.

The proposed mixed-test RDS method is fully described by five parameters n_1, n_2, c, k_1 and k_2, which are the characteristic SSP sample size, the characteristic SSP acceptance criterion and the C_{pk}-based

SSP sample size. RGS Schema variables and C_{pk}-based variables are respectively acceptance and rejection criteria for RGS proposals.

V. Measurements of the Performance Operational Characteristic Function of the Suggested Mixed Inspection RDS Plan

The OC function of a sampling strategy represents the percentage of batches that are accepted for a given expected product quality. Therefore, the probability of acceptance of the proposed mixed-control RDS system (also known as the OC function) is calculated as: Let us denote the probability that a certain number of masses p is taken as Pa(p). Pa(p) is currently expressed as:

$$P_a(p) = P_1 + (1 - P_1)^m P_2 \quad (4)$$

The probability of lot acceptance under the attribute control plan is represented by P_1 whereas the corresponding probability based on variable RDS plan for PCI Cpk is denoted as P_2. P_1 therefore can be written in binomial probability model as follows;

$$P_1 = \sum_{d=0}^{c} \binom{n_1}{d} p^d (1-p)^{n_1-d} \quad (5)$$

Probability P_2 can be calculated using Cpk in the path as: For the Cpk-based RDS method, Pa and Pc represent the probability of acceptance and probability of rejection of a batch based on a single sample, respectively. what's the matter.

$P_a = P(\hat{C}_{pk} \geq k_1)$ and $P_c = P(\hat{C}_{pk} < k_2)$

The acceptance probability of the lot is then calculated according to the RDS plan using the acceptance probability P_2 and Cpk. The probabilities Pa and Pc can be obtained by:

$$P_2 = \frac{P_a(1-P_c)^m + P_c P_a^m}{(1-P_c)^m} \quad (6)$$

$$P_2 = \frac{P(\hat{C}_{pk} \geq k_1) + P(k_2 \leq \hat{C}_{pk} < k_1)[P(\hat{C}_{pk} \geq k_1)]^m}{1 - P(k_2 \leq \hat{C}_{pk} < k_1)\{1 - [P(\hat{C}_{pk} \geq k_1)]^m\}}$$

where

$$P_a = P(\hat{C}_{pk} \geq k_a) = P\left\{\frac{USL - \bar{X}}{3S} \geq k_a, \frac{\bar{X} - LSL}{3S} \geq k_a\right\} \text{ or}$$

$$P(\hat{C}_{pk} \geq k_1) = P(\bar{X} + 3k_1 S \leq USL) - P(\bar{X} - 3k_1 S < LSL$$

and

$P_c = P(\hat{C}_{pk} < k_2) = 1 - P(\hat{C}_{pk} \geq k_2) = 1 - P\left\{\frac{USL - \bar{X}}{3S} \geq k_2, \frac{\bar{X} - LSL}{3S} \geq k_2\right\}$ or $1 - P(\hat{C}_{pk} \geq k_2) = 1 - P(\bar{X} + 3k_2 S \leq USL) - P(\bar{X} - 3k_2 S < LSL)$

For large samples, $(\bar{X} \pm cS)$ is known to have an approximately normally distributed mean $\mu \pm cE(S)$ and variance $\frac{\sigma^2}{n} + c^2$ Var (S) that is approximately normally distributed (see Duncan (1986), Balamurali et al (2005). i.e., $(\bar{X} \pm cS)$ based on this $P(\hat{C}_{pk} \geq k_1)$ can be approximately obeys $N\left[\mu \pm c\sigma, \frac{\sigma^2}{n} + \frac{c^2\sigma^2}{2n}\right]$

written as

$$P\left(\hat{C}_{pk} \geq k_1\right) = \varphi\left[\frac{USL-\mu-3k_1\sigma}{\frac{\sigma}{\sqrt{n_2}}\sqrt{1+\frac{9k_1^2}{2}}}\right] - \varphi\left[\frac{LSL-\mu+3k_1\sigma}{\frac{\sigma}{\sqrt{n_2}}\sqrt{1+\frac{9k_1^2}{2}}}\right] \quad (7)$$

$$P\left(\hat{C}_{pk} < k_2\right) = 1-\varphi\left[\frac{USL-\mu-3k_2\sigma}{\frac{\sigma}{\sqrt{n_2}}\sqrt{1+\frac{9k_2^2}{2}}}\right] + \varphi\left[\frac{LSL-\mu+3k_2\sigma}{\frac{\sigma}{\sqrt{n_2}}\sqrt{1+\frac{9k_2^2}{2}}}\right] \quad (8)$$

If p_U and p_L are defined as the fraction of items that do not conform when they are outside of the USL and LSL, respectively. Then P {X < LSL} = p_L and P {X > USL}= p_U, such that ($p_L + p_U = p$).

If we let $z_{pU} = \frac{USL-\mu}{\sigma}$ and $-z_{pL} = \frac{LSL-\mu}{\sigma}$ then the probabilities $P\left(\hat{C}_{pk} \geq k_1\right)$ can be determined as

$$\mathrm{P}\left(\hat{\mathrm{C}}_{\mathrm{pk}} \geq \mathrm{k}_1\right) = \varphi\left[\left(\mathrm{z}_{\mathrm{pU}}-3\mathrm{k}_1\right)\sqrt{\frac{\mathrm{n}_2}{\left(1+\frac{9\mathrm{k}_1^2}{2}\right)}}\right] - \varphi\left[\left(\mathrm{z}_{\mathrm{pL}}-3\mathrm{k}_1\right)\sqrt{\frac{\mathrm{n}_2}{\left(1+\frac{9\mathrm{k}_1^2}{2}\right)}}\right] \quad (9)$$

$$P\left(\hat{C}_{pk} < k_2\right) = 1-\varphi\left[\left(z_{pU}-3k_2\right)\sqrt{\frac{n_2}{\left(1+\frac{9k_2^2}{2}\right)}}\right] + \varphi\left[z_{pL}-3k_2)\sqrt{\frac{n_2}{\left(1+\frac{9k_2^2}{2}\right)}}\right] \quad (10)$$

By replacing (9) and (10) with (9) and (10), P_2, the probability of accepting the PCI-based RDS scheme variable Cpk (6) can be obtained. By substituting (5) and (6) into the equation, we can determine the required acceptance probability from the proposed RDS and the Cpk(4) mixed test.

VII. Function of average sample size

Generally, the ASN is the expected number of sample units needed to figure out the fate. ASN is a convenient concept in the context of Type B sampling, and sampling plans are often chosen when ASN is low. ASN is the objective function to be minimized in this work due to the constraints on the acceptance probabilities and the associated risks for producers and consumers. ASN is based on variable and attribute authentication in the proposed hybrid RDS authentication scheme. The amount of information needed to choose a batch is n1, and the amount of information required to make decisions based on RDS planning variables is n_2, and it is likely that multiple samples will get ($1-P_1$) to become. Therefore, using the ASN clustered quality function, we can implement the recommended RDS add-on test method as follows:

$$\mathrm{ASN}_{(p)} = n_1 + (1-P_1)\left[\frac{n_2}{P_a+P_c}\right] \quad (11)$$

VIII. Create a hybrid inspection RDS Plan Using Cpk

The supply contract will itemize the requirements and potential risks for both the producer and the buyer. Manufacturers usually aim at a certain level of product quality which is called acceptant quality level. If p_1 will be added, there is a high likelihood of being received. The p_2 represents a threshold beyond which consumers decide to focus on the product quality due to its non-universality. This score is often called Consumer Risk or LQL. Therefore, targeted sampling methods should consider threats either from producers' or customers' side. Thus our objective is to suggest an optimal selection parameters that minimize ASN and maintain trade-off between two risks – producer's and consumer's ones. And the nonlinear optimization problem based on this can be handled in order to find out best set of mixed sample parameter values. In formulating these planning parameters, two situations were considered; asymmetric failure rate and asymmetric failure rate.

IX. The fraction when contravention is Symmetric.

Assume first that symmetric fraction non-conforming.

$$P\{X < LSL\} = P\{X > USL\} = p/2 \quad \text{i.e. } p_U = p_L = p/2$$

In the event that the symmetric fractions do not match, the suggested system's function OC is provided by (4). The likelihood of acceptance as a variable based on Cpk inside the RDS framework was calculated using the formula below. The SSP function's (P1) estimation of the probability of acceptance is consistent with the description in (5). Schedule hybrid ASN inspections according the RDS policy described in (10). To select the ideal scheduling parameters, take into account the tracking optimization problem.

The projected hybrid inspection RDS policy's ASN can be found in (10). Determine the ideal scheduling parameters while keeping in mind the tracking optimization challenge.

$$P_2 = \frac{P(\hat{C}_{pk} \geq k_1) + P(k_2 \leq \hat{C}_{pk} < k_1)[P(\hat{C}_{pk} \geq k_1)]^m}{1 - P(k_2 \leq \hat{C}_{pk} < k_1)\{1 - [P(\hat{C}_{pk} \geq k_1)]^m\}} \quad (12)$$

where

$$P(\hat{C}_{pk} \geq k_1) = 2\varphi\left[\left(z_{\frac{p}{2}} - 3k_1\right)\sqrt{\frac{n_2}{\left(1+\frac{9k_1^2}{2}\right)}}\right] - 1 \quad (13)$$

and

$$P(\hat{C}_{pk} < k_2) = 2 - 2\varphi\left[\left(z_{\frac{p}{2}} - 3k_2\right)\sqrt{\frac{n_2}{\left(1+\frac{9k_2^2}{2}\right)}}\right] \quad (14)$$

where p_{12} represents the likelihood that the SSP attribute will be accepted in LQL, Pa(p1) represents the likelihood that the hybrid test RDS plan will be accepted in AQL, $Pa(p_2)$ represents the likelihood that the mixed test RDS plan will be accepted in LQL, $Pa(p_1)$ represents the likelihood that the RDS will be tested at LQL, $Pa(p_2)$ represents the likelihood based on fate, and $Pa(p_2)$ represents the likelihood.

Minimize $\left\{\frac{1}{2}[ASN(p_1) + ASN(p_2)]\right\}$

$= \frac{1}{2}\left\{2n_1 + (1 - P_{11})\left[\frac{n_2}{P_{a1}+P_{c1}}\right] + (1 - P_{12})\left[\frac{n_2}{P_{a2}+P_{c2}}\right]\right\}$

Subject to $P_a(p_1) \geq 1-\alpha$

$P_a(p_2) \leq \beta$

$n_1 > n_2 > 1,\ c \geq 0,\ k_1 > k_2 > 0$ (15)

The fraction when contravention is Asymmetric.

Assume that asymmetric fraction non-conforming.

$P\{X < LSL\} = p_L$ and $P\{X > USL\} = p_U$ i.e.($p_U = p_L = p$)

The proposed system's operating characteristics satisfy the threshold for unacceptable asymmetry (4). If the skewness measurements do not agree, the acceptance probabilities of the Cpk-based variable RDS approach are used instead. The acceptance probabilities of the SSP and P_1 features are comparable to equation (5).

$$P_2 = \frac{P(\hat{C}_{pk} \geq k_1) + P(k_2 \leq \hat{C}_{pk} < k_1)[P(\hat{C}_{pk} \geq k_1)]^m}{1 - P(k_2 \leq \hat{C}_{pk} < k_1)\{1 - [P(\hat{C}_{pk} \geq k_1)]^m\}} \quad (16)$$

where $P\left(\hat{C}_{pk} \geq k_1\right)$ and $P\left(\hat{C}_{pk} < k_2\right)$

is a factor depending on a strategy Using (9) and (10), Cpk RDS computes the probability of acceptance in the proper manner. Therefore, the optimization problem in the case of an incoherent asymmetric break

Minimize $\left\{\frac{1}{2}[ASN(p_1) + ASN(p_2)]\right\}$

$= \frac{1}{2}\left\{2n_1 + (1 - P_{11})\left[\frac{n_2}{P_{a1} + P_{c1}}\right] + (1 - P_{12})\left[\frac{n_2}{P_{a2} + P_{c2}}\right]\right\}$

Subject to $P_a(p_1) \geq 1-\alpha$

$P_a(p_2) \leq \beta$

$n_1 > n_2 > 1,\ c \geq 0,\ k_1 > k_2 > 0$ (17)

The probability that the mixed test RDS scheme will be approved in the AQL is represented by Pa, the same probability in the LQL by Pa, and the corresponding probability of the SSP feature in the LQL by P_{12}. PC1 and P_{C2} show the likelihood of being turned down at AQL and LQL, respectively, based on the planning variable RDS, whereas P_{a1} and P_{a2} show the likelihood of being accepted at AQL and LQL. The optimal settings for the Cpk-based hybrid control RDS technique can be found by solving the nonlinear equations for symmetric and asymmetric disjoint fractures presented in (15) and (17). There are numerous solutions because there are only two equations and five unknowns.A sample plan is generally chosen if the ASN is small and the required precautions are implemented. Thus, the constraints on the objective function that need to be reduced in this study are risks and the probability of producer and customer acceptability. By solving the nonlinear problem, the parameters (n_1, n_2, c, k_1, and k_2) can be determined. Two asymmetric failure scenarios that have been studied are p_L=p/3, p_U=2p/3, p_L=p/4, and p_U=3p/4, according to Aslam et al. (2013a). Consider the symmetric scenario in which p_U=p_L=p/2. The ideal values for the proposed schemes with p_L=p/3, p_U=2p/3 and

p_L=p/4, p_U=3p/4, respectively, are given in Tables 2 and 3. However, Table 1 contains the ideal parameters for the symmetric failure of the proposed system. The recommended system is displayed.

X. Illustrative example

This part explains how to choose the best parameters based on predetermined standards for the suggested mixed sampling plan. To bolster this, we offer two instances.

RDS plan for hybrid inspections Case of non-conforming symmetric fraction

To ensure they fulfil the necessary criteria, auto parts go through several inspection phases during the manufacturing process a hybrid inspection plan can be used to assess part quality for unacceptable symmetrical breaks. After the quality inspection counts the number of unqualified products in the sample, a decision is made based on whether the rate of unqualified products is within the acceptable limit. For example, if the acceptance criteria are set at 5% non-conformists, and the sample of 100 parts has 5 or fewer non-conformists, the sample is considered acceptable. Variable inspection is used to measure the continuous characteristic of the non-conforming parts, such as size or weight. For example, the diameter of the non-conforming parts may be measured to ensure it falls within a certain tolerance limit. The combination of these two inspection methods provides a more comprehensive assessment of the quality of the parts and helps to ensure that both the number and the severity of non-conformists are within acceptable limits. This helps to improve the reliability and safety of the finished products and ensures that the customers receive products that meet their expectations. Based on the information provided, The following parameters make up the ideal mixed inspection RDS plan for the given AQL and LQL: $n_1 = 24$, $n_2 = 18$, $c = 0$, $k_1 = 0.818$, $k_2 = 0.808$. At AQL (p1 = 0.01), the probability of acceptance is $P_a(p_1) = 0.950$. At LQL ($p_2 = 0.04$), the probability of acceptance is $P_a(p2) = 0.098$. At AQL and LQL, the average ASN is 45.029. The sample size n_1 is utilized for attribute inspection in this mixed inspection RDS design, and n2 is used for variable inspection. The parameters c, k_1 and k_2 are used to calculate the acceptance/rejection criteria for both types of inspection. The probability of acceptance at AQL ($p_1 = 0.01$) is 0.950, which means that with this mixed plan, there is a 95.0% chance of accepting a population with a non-conformity rate of 0.01 or less. The probability of acceptance at LQL ($p_2 = 0.04$) is 0.098, which means that with this mixed plan, there is only a 9.8% chance of accepting a population with a non-conformity rate of 0.04 or less. The average ASN (average sample number) is 45.029, Specifies the average number of units that will be checked before deciding overall acceptability. Given the level of quality control required and the acceptable risk of accepting an unqualified population, this information can be used to decide whether this hybrid test RDS scheme is appropriate for the quality attribute under study.

RDS plan for hybrid inspections Asymmetric Fraction Case Non-conforming

According to the information given, the ideal mixed inspection RDS plan has the following characteristics For AQL and LQL presented with asymmetric percent mismatch (p_L=p/3 and p_U=2p/3): n1=58, n=52, c=0, k_1=1.048, k_2=1.038. At the AQL ($p_1 = 0.001$), the probability of acceptance is $Pa(p_1) = 0.950$. At LQL (p2 = 0.008), the acceptance probability is $P_a(p_2) = 0.098$. At AQL and LQL, the average ASN is 68.824. Sample size n_1 is used for testing features in this RDS mixed test plan, and n_2 is used for testing variables. For both control methods, the pass/fail criteria are calculated using the variables c, k_1 and k_2. AQL ($p_1 = 0.001$) has an acceptance probability of 0.950, which means there is a 95.0% chance of accepting a population with a failure rate of 0.001 or less in this blended design. The probability of acceptance of the LQL (p_2 = 0.008) is 0.09929, which means that with this mixed design, there is only a 9.8% chance of accepting a population with a non-compliance rate of 0.008 or less. The average ASN (average sample size) is 68,824, which stands for the average number of units that will be reviewed before deciding on overall acceptability. Given the level of quality control required and the acceptable risk of accepting an unqualified population, this information can be used to decide whether this hybrid test RDS scheme is appropriate for the quality attribute under study.

Implementation of the Suggested Mixed Inspection RDS Strategy in Industry

Suppose a manufacturing company produces a component that has a critical dimension which is critical to the component's function. The dimension is required to be between limits of 10mm and 11mm. The company wants to implement a mixed inspection RDS plan to ensure that the component's critical dimension falls within the specified limits. The LQL is p_2 = 0.05 and the AQL is p_1 = 0.01 according to the company. Additionally, the corporation states that the acceptance probability at LQL is β = 0.1 and at AQL is α = 0.05.Using tables and calculations, the optimal mixed Check the RDS plan to be sure. Assume the following parameters: n_1 = 15, n_2 = 14, c = 0, k_1 = 0.701, k_2 = 0.691 The acceptance probability for AQL (p1 = 0.01) is Pa(p_1) = 0.95600. The acceptance probability for LQL (p_2 = 0.05) is Pa(p_2) = 0.098. The average ASN for AQL and LQL is 2.612 In this mixed inspection RDS plan, 15 units are selected for attribute inspection and 14 units are selected for variable inspection. The parameters c, k_1 and k_2 set the acceptance/rejection standards for both types of inspection. Based on AQL (p_1 = 0.01) probability of admission of 0.950, the mixed plan has a 95.0% chance of admitting a population with a non-conformity rate of 0.01 or below. There is only a 9.8% possibility of admitting a population with a non-conformity rate of 0 or below with this mixed plan, according to the probability of acceptance at LQL (p_2 = 0.05) of 0.098. The average ASN (average sample number) is 2.612, which indicates the average number of units that will be inspected before deciding on the acceptability of the population. This information can be used to decide if this mixed inspection RDS plan is suitable for the company's quality control requirements and acceptable risk of accepting a non-conforming population.

1) Take 15 random samples from the product lot and count the number of failures in step-1

(characteristic check).

2) If you see no errors, accept the batch. If one or more fail, go to step 3 for RDS variable verification.

3) In the third step (validation of the variables), an estimate of the exponent C_{pk} ($\hat{C}_{pk}$) is obtained from A random sample of size 14 was selected from the population, with the measured data being reported by Wu and Pearn (2008).

Mean ($\bar{x}$) = 0.8557, Standard deviation (SD) = 0.038

$$\hat{C}_{pk} = \min\left(\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right) \quad = \min\left(\frac{25-0.856}{3\text{ X }0.0381}, \frac{0.856-15}{3\text{ X }0.0381}\right)$$

= min (3.85, 3.88) = 3.85

Note: Here, we have assumed LSL = 15 and USL = 25 as given in the problem.

0.717 0.698 0.726 0.684 0.727 0.688 0.708 0.703 0.694 0.713 0.730 0.699 0.710 0.688

4) Since $\hat{C}_{pk} = 3.84 > k_a = 0.701$, the batch is accepted in the first instance of the variable test itself.

XI. Conclusions

Since PCI is becoming a more popular strategy for achieving and preserving product quality, developing a sample plan based on it is essential. When the quality feature of interest follows a normal distribution with an unknown mean and an unknown standard deviation, the suggested hybrid sampling strategy can be utilized. By lowering the ASN under two key restrictions that offer the necessary protections for producers and consumers, the sample size needed for inspection and the related accept and reject criteria are proven. The study shows that when inspections are based on many characteristics and quality parameters, the suggested hybrid inspection RDS system requires less sample for batch evaluation.

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Table 1: Hybrid Inspection RDS Plan Optimal Parameters Based on Cpk (Symmetric Fraction Non-conforming)

p_1	p_2	n_1	n_2	c	k_1	k_2	$P_a(p_1)$	$P_a(p_2)$	ASN
0.001	0.002	419	416	0	1.037	1.027	0.950	0.098	617.415
	0.003	194	191	0	1.030	1.020	0.950	0.099	321.174
	0.004	131	130	0	1.030	1.020	0.950	0.095	212.672
	0.006	80	79	0	1.036	1.026	0.950	0.096	128.233
	0.008	57	50	0	1.046	1.036	0.950	0.096	89.756
	0.01	44	42	0	1.065	1.055	0.950	0.098	69.924
	0.015	29	28	0	1.131	1.121	0.950	0.098	43.079
	0.02	23	10	0	1.326	1.316	0.950	0.098	28.625
0.0025	0.005	231	225	0	0.932	0.922	0.950	0.098	483.437
	0.01	36	32	0	0.913	0.903	0.950	0.098	103.242
	0.015	13	10	0	0.919	0.909	0.950	0.098	56.051
	0.02	9	5	0	0.933	0.923	0.950	0.098	35.827
	0.025	5	2	0	0.958	0.948	0.950	0.095	26.353
	0.03	5	2	0	0.980	0.970	0.950	0.096	19.805
	0.05	1	2	0	1.330	1.320	0.950	0.096	8.706
0.005	0.01	163	158	0	0.848	0.838	0.950	0.098	373.805
	0.015	46	42	0	0.823	0.813	0.950	0.098	115.868
	0.02	24	21	0	0.815	0.805	0.950	0.098	62.999
	0.03	7	4	0	0.812	0.802	0.950	0.098	29.503
	0.04	3	2	0	0.829	0.819	0.950	0.098	17.923
	0.05	3	2	0	0.843	0.833	0.950	0.096	11.827
	0.1	3	2	0	1.391	1.381	0.950	0.095	3.948
0.01	0.02	121	118	0	0.771	0.761	0.951	0.096	239.274
	0.03	40	36	0	0.782	0.772	0.950	0.096	74.978
	0.04	24	18	0	0.818	0.808	0.950	0.098	45.029
	0.05	10	7	0	0.703	0.693	0.950	0.098	23.326
	0.1	7	5	0	0.741	0.731	0.950	0.098	7.012
	0.15	3	2	0	0.856	0.846	0.950	0.098	3.578
	0.2	3	2	0	1.483	1.473	0.950	0.098	2.242
0.03	0.05	56	55	1	0.622	0.612	0.950	0.095	113.826
	0.1	19	13	1	0.687	0.677	0.950	0.096	30.554
	0.15	6	6	0	0.650	0.640	0.950	0.096	12.599

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Vol 32 No. 1 (2025)

	0.2	3	3	0	0.774	0.764	0.950	0.098	9.646
	0.25	2	2	0	0.516	0.506	0.950	0.095	3.554
0.05	0.1	31	29	1	0.558	0.548	0.950	0.096	65.093
	0.15	12	10	1	1.018	1.008	0.950	0.096	20.049
	0.2	4	4	1	0.698	0.688	0.971	0.098	8.518
	0.25	2	2	0	0.695	0.685	0.950	0.098	4.609
	0.3	2	2	0	0.434	0.424	0.950	0.098	2.258

Table 2: Hybrid RDS Plan's Ideal Parameters Based on Cpk
(Asymmetric Fraction) With pL= p/3 and pU= 2p/3) non-conforming

p_1	p_2	n_1	n_2	c	k_1	k_2	$P_a(p_1)$	$P_a(p_2)$	ASN
0.001	0.002	816	807	0	1.030	1.020	0.950	0.099	1416.75
	0.003	243	238	0	1.024	1.014	0.950	0.095	405.937
	0.004	146	143	0	1.026	1.016	0.950	0.096	236.371
	0.006	84	78	0	1.035	1.025	0.950	0.096	132.688
	0.008	58	52	0	1.048	1.038	0.950	0.098	68.824
	0.01	45	39	0	1.067	1.057	0.950	0.098	70.35
	0.015	30	16	0	1.141	1.131	0.950	0.098	41.239
	0.02	23	11	0	1.327	1.317	0.950	0.098	28.868
0.0025	0.005	482	481	0	0.928	0.918	0.950	0.098	909.513
	0.01	45	43	0	0.910	0.900	0.950	0.091	115.527
	0.015	15	13	0	0.917	0.907	0.950	0.098	56.37
	0.02	10	7	0	0.933	0.923	0.950	0.098	37.049
	0.025	7	1	0	0.955	0.945	0.950	0.095	18.705
	0.03	5	1	0	0.986	0.976	0.950	0.096	20.356
	0.05	5	2	0	1.339	1.329	0.950	0.096	7.607
0.005	0.01	339	332	0	0.853	0.843	0.950	0.098	567.28
	0.015	64	63	0	0.820	0.810	0.950	0.098	148.882
	0.02	30	28	0	0.812	0.802	0.950	0.098	70.898
	0.03	8	6	0	0.815	0.805	0.950	0.098	31.486
	0.04	3	3	0	0.826	0.816	0.950	0.098	10.727
	0.05	3	3	0	0.851	0.841	0.950	0.093	12.139
	0.1	3	3	0	1.402	1.392	0.951	0.095	4.648
0.01	0.02	229	228	0	0.786	0.776	0.950	0.096	382.924
	0.03	46	45	0	0.722	0.712	0.950	0.096	105.257
	0.04	18	16	0	0.706	0.696	0.950	0.098	45.794
	0.05	12	10	0	0.700	0.690	0.950	0.098	2.612
	0.1	7	7	0	0.745	0.735	0.950	0.098	3.713
	0.15	3	3	0	0.862	0.852	0.950	0.098	3.317
	0.2	3	3	0	1.776	1.766	0.950	0.098	2.342

0.03	0.05	86	84	1	0.676	0.666	0.950	0.095	132.702
	0.1	18	16	1	0.590	0.580	0.950	0.096	36.243
	0.15	6	6	0	0.635	0.625	0.950	0.096	13.565
	0.2	3	3	0	0.503	0.493	0.950	0.098	11.693
	0.25	2	2	0	0.520	0.510	0.950	0.095	3.554
0.05	0.1	47	45	1	0.594	0.584	0.950	0.096	80.704
	0.15	8	5	1	0.604	0.594	0.971	0.096	17.59
	0.2	4	4	1	0.542	0.532	0.950	0.098	5.965
	0.25	2	2	0	0.703	0.693	0.950	0.098	4.609
	0.3	2	2	0	0.369	0.359	0.950	0.098	2.021

Table 3: Cpk (Asymmetric Fraction)-Based Hybrid Inspection RDS Plan's Ideal Parameters Inconsistent with pU=3p/4 and pL=p/4

p_1	p_2	n_1	n_2	c	k_1	k_2	$P_a(p_1)$	$P_a(p_2)$	ASN
0.001	0.002	1214	1209	0	1.022	1.012	0.950	0.099	2234.175
	0.003	428	427	0	1.014	1.004	0.950	0.095	750.916
	0.004	192	185	0	1.016	1.006	0.950	0.096	312.682
	0.006	92	86	0	1.031	1.021	0.950	0.096	145.247
	0.008	60	59	0	1.051	1.041	0.950	0.098	68.824
	0.01	46	42	0	1.071	1.061	0.950	0.098	72.379
	0.015	30	19	0	1.147	1.137	0.950	0.098	42.002
	0.02	23	13	0	1.325	1.315	0.950	0.098	29.354
0.0025	0.005	650	650	0	0.920	0.910	0.950	0.098	1256.359
	0.01	66	63	0	0.903	0.893	0.950	0.091	151.818
	0.015	19	17	0	0.915	0.905	0.950	0.098	62.770
	0.02	11	10	0	0.936	0.926	0.950	0.098	39.206
	0.025	7	1	0	0.955	0.945	0.950	0.095	18.705
	0.03	5	3	0	0.994	0.984	0.950	0.096	20.910
	0.05	5	3	0	1.331	1.321	0.950	0.096	7.852
0.005	0.01	466	465	0	0.840	0.830	0.950	0.098	905.925
	0.015	113	108	0	0.815	0.805	0.950	0.098	243.544
	0.02	42	41	0	0.808	0.798	0.950	0.098	93.135
	0.03	10	9	0	0.815	0.805	0.950	0.098	35.148
	0.04	3	3	0	0.835	0.825	0.950	0.098	10.727
	0.05	3	3	0	0.862	0.852	0.950	0.093	12.765
	0.1	3	3	0	1.416	1.406	0.951	0.095	4.648
0.01	0.02	337	336	0	0.759	0.749	0.950	0.096	669.669
	0.03	70	68	0	0.753	0.743	0.950	0.096	128.268
	0.04	25	24	0	0.704	0.694	0.950	0.098	59.488
	0.05	15	14	0	0.701	0.691	0.950	0.098	2.612

	0.1	7	7	0	0.721	0.711	0.950	0.098	4.556
	0.15	3	3	0	0.871	0.861	0.950	0.098	3.317
	0.2	3	3	0	1.794	1.784	0.950	0.098	2.342
0.03	0.05	163	162	1	0.614	0.604	0.950	0.095	305.107
	0.1	22	21	1	0.673	0.663	0.950	0.096	37.404
	0.15	6	6	0	0.631	0.621	0.950	0.096	15.583
	0.2	3	3	0	0.787	0.777	0.950	0.098	9.927
	0.25	2	2	0	0.557	0.547	0.950	0.095	3.554
0.05	0.1	107	106	1	0.528	0.518	0.950	0.096	218.450
	0.15	9	8	1	0.596	0.586	0.971	0.096	20.037
	0.2	4	4	1	0.556	0.546	0.950	0.098	6.701
	0.25	2	2	0	0.714	0.704	0.950	0.098	5.383
	0.3	2	2	0	0.443	0.433	0.950	0.098	2.021

Table 4: ASN(p_2) of the hybrid inspection RDS Plan is Comparable with Other Attributes Currently Available Plans for a Single Sampling Characteristics of the RGS Plan and the RGS Plan (Symmetric Fraction Non-Conforming Situation)

p_1	p_2	ASN(p_2)			
		Attributes SSP	Attributes RGS Plan	Attributes RGS Plan	Mixed inspection RDS Plan
0.001	0.002	12376	8719.27	821.886	611.874
	0.003	3922	2788.22	372.847	271.746
	0.004	2317	1560.16	250.889	199.379
	0.006	1112	794.84	154.971	109.431
	0.008	664	593.07	93.006	71.886
	0.010	531	315.88	88.278	76.066
0.0025	0.005	4948	3487.12	515.682	305.670
	0.010	926	626.39	129.486	28.385
	0.015	444	317.14	73.725	22.215
	0.020	265	236.42	30.288	14.748
0.005	0.010	2473	1850.73	426.683	375.173
	0.015	783	557.81	142.358	96.818
	0.020	462	311.98	52.124	31.004
	0.030	221	157.28	32.778	20.566
	0.040	132	117.71	24.531	12.319
	0.050	105	64.20	23.202	10.990
0.010	0.040	198	154.36	52.701	40.489
	0.050	132	95.67	35.599	23.387
	0.100	52	31.23	12.643	8.431

	0.150	25	27.00	7.804	5.592
0.030	0.060	410	297.63	141.285	95.745
	0.090	129	99.68	42.935	21.815
	0.120	65	61.18	26.385	14.173
	0.150	43	40.84	16.241	4.029
	0.300	12	15.59	5.004	2.792
0.050	0.100	233	183.58	56.473	35.353
	0.150	77	55.33	28.177	15.965
	0.200	38	30.94	11.224	9.012
	0.250	25	24.30	6.234	4.022
	0.500	7	9.31	4.502	2.290