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# Some Results of Total Domatic Number on Anti Fuzzy Graph

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**Abstract:**

Let $A_G$ = (N, A, σ, μ) be an anti fuzzy graph. A partition DP = {$D_1$, $D_2$, ….., $D_K$} of N($A_G$) is referred to as total domatic partition of $A_G$ if for each $D_i$ is a total dominating set of anti fuzzy graph $A_G$ and N($A_G$)= ⋃$D_i$. The maximum cardinality taken over all maximum number of classes with a minimal total domatic partition of $A_G$ is called the total domatic number of $A_G$ and it is denoted by $d_t(A_G)$. The maximum number of classes with maximum fuzzy cardinality of a partition $D_i$ ($A_G$) is called anti fuzzy total domatic number of anti fuzzy graph $A_G$ and it is denoted by $d_{ft}(A_G)$. In this paper, we gain some preferred results and limits that referring to the full domatic number on anti fuzzy graph.

**Keywords**: Anti fuzzy graph, Dominating set, Total dominating set, Vertex degree.

## 1. Introduction

The notion of an anti-fuzzy structure on a graph was familiar to Muhaamad Akram [1] owing to the fuzzy relation pioneered by Zadeh [11]. E. J. Cockayne, S. T. Hedetniemi[2] delivered the idea of domatic number of a graph. The idea of a graph's anti domatic number was first developed by Bohdan Zelinka [13]. Domatic number and total domatic number of complete uniform hypergraphs and complete bipartite uniform hypergraphs were computed by Dash, S.P. [3]. The generalities of certain different forms of anti-fuzzy graphs were presented by R. Muthuraj and A. Sasireka [6, 7&8] who also determined the domination parameters on anti-fuzzy graphs. Additionally, they invented the concept of the anti-fuzzy graph's total domination number and established boundaries for it. In this paper, we define the definition of total domatic number and partial total domatic number on anti fuzzy graph $A_G$ also extant some general bounds and results that relate the total domatic number of $A_G$.

### Note

The total dominating set D of $A_G$ contained each support node in $A_G$. In both N\D and D, it able to dominate many nodes.

## 2. SOME RESULTS OF TOTAL DOMATIC NUMBER ON ANTI FUZZY GRAPH

### 2.1 Definition

Let $A_G = (N, A, \sigma, \mu)$ be an anti fuzzy graph. A partition DTP = {TD1, TD2, …., $TD_K$} of $N(A_G)$ is called total domatic partition of $A_G$ if for each $TD_i$ is a total dominating set [TDS] of anti fuzzy graph $A_G$ and $N(A_G) = \cup TD_i$.

The maximum cardinality taken over all maximum number of classes with a minimal total domatic partition of $A_G$ is called the total domatic number [TDTN] of $A_G$ and it is denoted by $d_t(A_G)$.

The maximum number of classes with maximum fuzzy cardinality of a partition $TD_i$ ($A_G$) is called anti fuzzy total domatic number of anti fuzzy graph and it is denoted by $d_{ft}(A_G)$.

### 2.2 Example

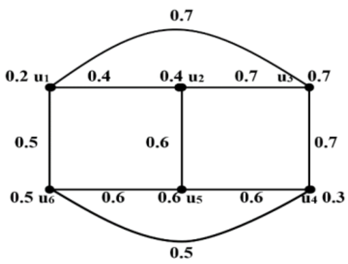


**Figure. 1. Anti Fuzzy Graph $A_G$**

From figure 1, the total dominating sets are

$TD_1 = \{u_2, u_5\} = \{0.4, 0.6\} = 1$

$TD_2 = \{u_3, u_4\} = \{0.7, 0.3\} = 1$

$TD_3 = \{u_1, u_6\} = \{0.2, 0.5\} = 0.7$

$TDP = \{TD_1, TD_2, TD_3\}$

TDT number of anti fuzzy graph $A_G$, $d_t(G_A) = 3$

Anti fuzzy total domatic number of anti fuzzy graph $A_G$, $d_{ft}(A_G) = \max\{1, 1, 0.7\} = 1$

### 2.3 Definition

Let $A_G = (N, A, \sigma, \mu)$ be an anti fuzzy graph. A partition TDP = {$TD_1$, $TD_2$, …., $TD_K$} of $N(A_G)$ is called partial total domatic partition of $A_G$ if for every $TD_i$ is a total dominating set of anti fuzzy graph $A_G$ and at the minimum of single node does not in any one of $TD_i$ and all $TD_i$'s are minimal total dominating sets.

The maximum fuzzy cardinality taken over all maximum number of classes with minimal partial total domatic partition of $A_G$ is called the partial total domatic number [PTDTN] of $A_G$ and it is denoted by $d_{pt}(A_G)$.

The maximum number of classes with maximum fuzzy cardinality of a partition $TD_i$ ($A_G$) is called the anti fuzzy partial total domatic number of $A_G$ and it is denoted by $d_{fpt}(A_G)$.

**2.4 Example**

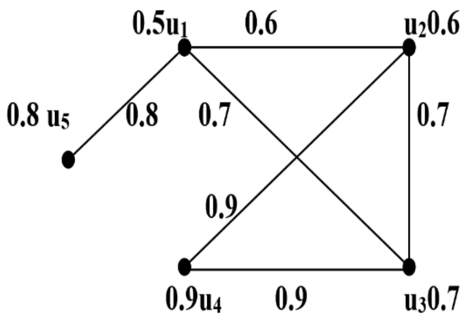


**Figure. 2. Anti Fuzzy Graph $A_G$**

From figure 2, the total dominating set is

$TD_1$ = {$u_1$, $u_3$} = {0.5, 0.7}

$d_{fpt}$ ($A_G$) = {0.5, 0.7}   = 1.2

For finding $TD_2$, $u_5$ is isolated node. So, it dominates itself and <$TD_2$> is not a total dominating set. Since, $TD_2$ does not exist.

Therefore TDP = {$TD_1$}

Partial total domatic number of $A_G$ domatic number of $A_G$, $d_{pt}(A_G) = 1$

Anti fuzzy partial total domatic number of $A_G$ domatic number of $A_G$, $d_{fpt}$ = 1.2

**2.5 Theorem**

Let $A_G$ be a finite undirected AFG with n nodes of order ρ, and τ($A_G$) be the minimum degrees of nodes of $A_G$. Then $d_{ft}(A_G) \geq$ [ρ/ (ρ- $\tau_f(A_G)$ + 1)] and each total dominating set consists n - τ($A_G$) + 1 nodes of $A_G$.

**Proof**

Let $A_G$ be an AFG and $\overline{A_G}$ is complement of $A_G$. TD is a total dominating set of $A_G$ which is a subset of the node set N($A_G$). For every k ϵ N($A_G$) there exists a node $l$ which is not adjacent to k in $\overline{A_G}$. Suppose the node has degree r in $A_G$, then its degree is n-r-1 in $\overline{A_G}$.

Therefore, the maximum degrees of $\overline{A_G}$ is $\rho - \tau_f(A_G) - 1$. Let TD be a subset of N($A_G$) having at the minimum of n - τ($A_G$) + 1 nodes. Then every node k ϵ N($A_G$) can be contiguous to at most n - τ($A_G$) + 1 nodes of TD in $\overline{A_G}$; although k ϵ TD, then ∃a node $l$ ϵ TD which is not contiguous to k in $\overline{A_G}$ and thus is contiguous to k in $\overline{A_G}$. Which mean it every subset of N($A_G$) with at the minimum of n - τ ($A_G$) + 1 nodes is a total dominating set in $A_G$. Consider a partition of N($A_G$) into a class which having n - τ ($A_G$) + 1 nodes each, with the exception of at most one which would have more nodes. Evidently ∃ such a partition having [n/ (n - τ($A_G$) + 1)] classes with $d_{ft}(A_G) \geq$ [ρ/(ρ- $\tau_f$($A_G$) + 1)] this is a total domatic partition.

Hence $d_{ft}(A_G) \geq$ [ρ/ (ρ- $\tau_f$($A_G$) + 1)].

**2.6 Theorem**

$A_G$ is an AFG with n nodes, 3≤n≤7 for which τ($A_G$) = n – 3 and

$$d_{ft}(A_G) = \begin{cases} \leq \left[\frac{\rho}{2}\right] ; for\ n = 4,7 \\ \geq \left[\frac{\rho}{2}\right] ; for\ n = 6 \\ does\ not\ exist\ ;\ for\ n = 3, 5 \end{cases}.$$

**Proof**

For n = 3, $A_G$ is an AFG with three isolated nodes. Since τ($A_G$) = n – 3.

Therefore, total dominating set does not exist. Hence $d_{ft}(A_G) = 0$.

For n = 4, $A_G$ is a disconnected anti fuzzy graph with two components with two nodes each.

Therefore, there exist one partition of total dominating set. Therefore, $d_{ft}(A_G) = \left[\frac{\rho}{2}\right]$.

For n = 5, $A_G$ is an anti fuzzy cycle. We know that, for any anti fuzzy cycle total domatic partition does not exist.

For n = 6, $A_G$ is an anti fuzzy wheel with two TDS of at most $\frac{n}{2}$ nodes which has at the minimum of $\frac{\rho}{2}$. Therefore, $d_{ft}(A_G) \geq \left[\frac{\rho}{2}\right]$.

For n = 7, $A_G$ is an AFG with $\Delta(A_G) = n - 2$. It forms three total dominating sets with at most $\frac{n}{2}$ nodes which has at most $\left[\frac{\rho}{2}\right]$ each. Therefore, $d_{ft}(A_G) \leq \left[\frac{\rho}{2}\right]$.

**2.7 Theorem**

If $A_G$ is an AFG (n=5) with τ($A_G$) = n – 3 then $d_{fpt}(A_G) = \left[\frac{\rho}{2}\right]$.

**Proof**

If $A_G$ is an AFG with n nodes and τ($A_G$) = n – 3 then $A_G$ has an anti fuzzy cycle. Therefore, there exist one partial total dominating set exist with at most $\left[\frac{\rho}{2}\right]$.

Hence, $d_{fpt}(A_G) = \left[\frac{\rho}{2}\right]$.

**2.8 Theorem**

Let $A_G$ be a complete bipartite AFG with 'n' nodes. $N_1$ and $N_2$ are node partition of N($A_G$). Then $d_t(A_G) = \left[\frac{n}{2}\right]$.

**Proof**

Let $A_G$ be any complete bipartite AFG with disjoint node partitioned set $N_1$, $N_2$ and $|N_1(A_G)| = n$, $|N_2(A_G)| = m$ consider n < m (= n+1) There is no edge between the nodes in $N_1$ and also in $N_2$. Let $k_1 \epsilon N_1(A_G)$ which dominates all the nodes in $N_2$. Let $k_1 \epsilon N_2(A_G)$ which dominates all the nodes in $N_1$. Since there exist an edge between $k_1$ and $l_1$. Therefore $\{k_1, l_1\}$ forms a minimal TDS of $A_G$. Similarly, $\{k_2, l_2\}$, $\{k_3, l_3\}$, ..., $\{k_{n-1}, l_{n-1}\}$ forms a minimal TDS of $A_G$. forms a minimal TDS of $A_G$ and $\{k_n, l_n, l_{n+1}\}$ forms a TDS of $A_G$. Therefore, $d_t(A_G) = \left\lfloor \frac{n}{2} \right\rfloor$. Consider if n = m then $\{k_1, l_1\}$, $\{k_2, l_2\}$, $\{k_3, l_3\}$, …., $\{k_n, l_n\}$ are classes of total domatic partition of $A_G$. Therefore, $d_t(A_G) = \left\lfloor \frac{n}{2} \right\rfloor$.

**2.9 Proposition**

For AFG $A_G$, $d_{ft}(A_G) \leq \frac{2\rho}{3}$ where $A_G$ is a not an anti fuzzy cycle.

**Proof**

Let $A_G$ is an AFG and consider that $A_G$ is not an anti fuzzy cycle with n nodes and its order ρ. Since every node in $A_G$ has adjacent to at the minimum of two nodes and does not have any pendent node. So, each node of $A_G$ dominates at the minimum of two. Therefore, it frames at most three minimal total dominating sets of TDTP of $A_G$. Hence $d_{ft}(A_G) \leq \frac{2\rho}{3}$.

**2.10 Theorem**

Let $A_G$ be a simple connected AFG and $\overline{A_G}$ be an anti-complement of $A_G$ then
$d_{ft}(A_G) + d_{ft}(\overline{A_G}) \leq \frac{5\rho}{3}$.

**Proof**

$A_G$ is a simple connected AFG without isolated nodes then $\overline{A_G}$ does not have any isolated nodes then $d_{ft}(A_G) \leq \frac{2\rho}{3}$ & $d_{ft}(\overline{A_G}) \leq \rho$.

$$d_{ft}(A_G) + d_{ft}(\overline{A_G}) \leq \frac{2\rho}{3} + \rho$$

$$\leq \frac{5\rho}{3}.$$

**2.11 Theorem**

For any complete uninodal AFG $A_G$ with n nodes, $d_{ft}(A_G) = \begin{cases} 2\sigma(u_1); & if\ n\ is\ even \\ 3\sigma(u_1); & ifn\ is\ odd \end{cases}$ for all $k_1 \in N(A_G)$.

**Proof**

Consider $A_G$ is a complete uninodal AFG and TD is a total domatic partition of $A_G$ which has $TD_1, TD_2, \ldots,$ are its classes. Which yields the classes $TD_1, TD_2, \ldots, TD_{n/2}$ are total dominating sets with same cardinality. Let $k_1 \epsilon TD_1$ and has adjacent to n-1 nodes with degree (n-1) $k_1$. $l_1 \epsilon N(A_G)$ and $k_1, l_1 \epsilon TD_1$. Since, $k_1, l_1$ are also adjacent and dominates all other nodes in $A_G$. If n is an even number, we get $TD_1, TD_2, \ldots, TD_{n/2}$ classes in total domatic partition of $A_G$. Hence, $d_{ft}(A_G) = 2\sigma(k_1)$.

If n is odd then $A_G$ has $TD_1, TD_2, \ldots, TD_{\frac{n}{2}-1}$ classes have equal number of nodes which forms a TDS classes in total domatic partition of $G_A$. But $k_n$ does not belongs to any other classes of total domatic partitions of $A_G$. Therefore, the node $k_n$ adding into the total dominating set $\left|TD_{\frac{n}{2}}\right| = 2\sigma(k_1) + \sigma(k_n)$

$= 2\sigma(k_1) + \sigma(k_1)$ {since $A_G$ is uninodal anti fuzzy graph}

$= 3\sigma(k_1)$

Hence $d_{ft}(A_G) = 3\sigma(k_1)$.

**2.12 Theorem**

If $A_G$ is an AF path, then $d_{ft}(A_G) \leq \rho - \tau$, where τ is minimum degree of $A_G$.

**Proof**

Consider $A_G$ is an AF path with order 'ρ' and has minimum degree τ. Let TD be minimal TDS of $A_G$. Let k and *l* are the initial and end node of an anti fuzzy path $A_G$. Since, it has the degree as one and the remaining nodes of $A_G$ has degree two. Therefore, alternative pair of nodes consist in TDS. Hence, $d_{ft}(A_G) \leq \rho - \tau$

**2.13 Theorem**

For any two anti fuzzy graphs $A_G$ and $A_H$ without an isolated node, then the following conditions holds.

(i) $d_{ft}(A_G \times A_H) \geq d(A_G) \vee d(A_H)$

(ii) $d(A_G \times A_H) \geq d_t(A_G \times A_H)$.

**Proof**

(i) Consider $A_G$ and $A_H$ are anti fuzzy graphs with order $\rho_1$ and $\rho_2$ respectively. Let $\rho_1 \geq \rho_2$ with $n_1 \geq n_2$ where $n_1$ and $n_2$ are number of nodes of $A_G$ and $A_H$. Let TD be a total domatic partition of $A_G \times A_H$ which having at most $n_2$ classes. Let $\rho_1$, $\rho_2$ be the domatic numbers of $A_G$ and $A_H$ respectively. If $\rho_1 \geq \rho_2$ then $TD_1, TD_2, \ldots, TDn_1$ be the domatic partition of $N(G_A)$ for $1 \leq i \leq n_1$, any node $u_1 \in TD_i$ and $v_1 \in N(H_A)$ then the node $(u_1, v_1) \in A_G \times A_H$ dominates at most four nodes in N $(A_G \times A_H)$. Since $H_A$ does not have any isolated node then TD is a total domatic partition of $A_G \times A_H$ with $\rho_1$,

Hence, $d_{ft}(A_G \times A_H) \geq d(A_G) \vee d(A_H)$.

(ii) Since DP is a domatic partition of $A_G$ which have at most $n_1$ classes. Since, a single node can dominate all other nodes in $A_G \times A_H$. But to form a total dominating set we need at the minimum of two nodes in each domatic partition of $A_G \times A_H$. Therefore, $d(A_G \times A_H) \geq d_t(A_G \times A_H)$.

## 3. CONCLUSION

Total and partial total domatic number on an anti fuzzy graph $A_G$, and they are applied to different types of anti-fuzzy graphs to produce bounds. The bounds on them were established by applying the total domatic number concept to the anti-cartesian product of anti-fuzzy graphs such as path, anti-fuzzy cycle, and full anti-fuzzy graph. A few theorems and propositions are produced for the results once they have been analysed.

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