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Some Common Fixed Point Theorems in Neutrosophic Metric Spaces

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Abstract:
In this article, we construct some fixed point results for pair of self mappings and occasionally weakly compatible mappings on neutrosophic metric spaces. In order to show the strength of these results, some motivating examples are established as well.

Keywords: Fuzzy metric, Neutrosophic metric space, Occasionally weakly compatible, Self mapping.

1. Introduction

The concept of metric spaces and the Banach contraction principle are the backbone of the field of fixed-point theory. Axiomatic interpretation of metric space attracts thousands of researchers towards spaciousness. So far, there have been many generalizations on metric spaces. This tells us of the beauty, attraction and expansion of the concept of metric spaces. Zadeh [12] established the basis for fuzzy mathematics in 1965. Fixed point theory is considered to be the fascination and active area of research and development of nonlinear analysis. Kramosil and Michalek [6] introduced fuzzy metric spaces in a variety of ways in 1975. With the help of continuous t-norm. George and Veeramani [3] present the concept of fuzzy metric spaces in 1994. Atanassov[1] stirred things up by adding the idea of non-membership grade of fuzzy set theory. Smarandache [9] described the concept of neutrosophic logic and neutrosophic sets in 1998. In this study provides a common fixed point theorem for pair of self mappings and occasionally weakly compatible mapping fulfilling various constraints in the neutrosophic metric space

2. Preliminaries

Now, we begin with some basic fundamental aspects, notations and definitions.

Definition 2.1.[6] A binary operation $*: [0,1] \times [0,1] \rightarrow [0,1]$, is named continuous t-norm if it meets the following :

(i) $*$ is associative and commutative,
(ii) $*$ is continuous,
(iii) ŧ $* 1 =$ ŧ for all ŧ $\in [0,1]$,
(iv) ŧ $*$ $\tilde{\varsigma}$ $\leq$ ʒ $*$ ꝺ whenever ŧ $\leq$ ʒ and $\tilde{\varsigma}$ $\leq$ ꝺ.

Definition 2.2. [6] A binary operation $\odot : [0,1] \times [0,1] \to [0,1]$, is named continuous t-conorm if it meets the following:

(i) $\odot$ is associative and commutative,
(ii) $\odot$ is continuous,
(iii) $\mathfrak{k} \odot 0 = \mathfrak{k}$ for all $\mathfrak{k} \in [0,1]$,
(iv) $\mathfrak{k} \odot \tilde{\varsigma} \leq \mathfrak{z} \odot \mathfrak{d}$ whenever $\mathfrak{k} \leq \mathfrak{z}$ and $\tilde{\varsigma} \leq \mathfrak{d}$.

Example 2.3.[2]

(i) $\mathfrak{r} * \mathfrak{s} = \min\{ \mathfrak{r}, \mathfrak{s}\}$ for all $\mathfrak{r}, \mathfrak{s} \in [0,1]$.
(ii) $\mathfrak{r} * \mathfrak{s} = \max\{\mathfrak{r} + \mathfrak{s} - 1, 0\}$ for all $\mathfrak{r}, \mathfrak{s} \in [0,1]$.

Example 2.4.[2]

(i) $\mathfrak{r} \odot \mathfrak{s} = \max\{\mathfrak{r}, \mathfrak{s}\}$ for all $\in [0,1]$.
(ii) $\mathfrak{r} \odot \mathfrak{s} = \min\{\mathfrak{r} + \mathfrak{s}, 1\}$ for all $\mathfrak{r}, \mathfrak{s} \in [0,1]$.

Definition 2.5. The 6-tuple $(\Xi, \mathfrak{N}, \mathfrak{S}, \mathfrak{T} *, \odot)$ is called a Neutrosophic Metric Space [*NMS*] if Ξ is an arbitrary non void set, $*$ is a continuous t-norm, $\odot$ is a continuous t-conorm and $\mathfrak{N}, \mathfrak{S}, \mathfrak{T} : \Xi \times \Xi \times (0, \infty) \to [0,1]$ are fuzzy sets, fulfilling the following assertions:

For all $, \tilde{\varsigma}, \mathfrak{z} \in \Xi$; $\varrho, \rho \in (0, \infty)$.

(1) $\mathfrak{N}(\mathfrak{k}, \tilde{\varsigma}, \varrho) + \mathfrak{S}(\mathfrak{k}, \tilde{\varsigma}, \varrho) + \mathfrak{T}(\mathfrak{k}, \tilde{\varsigma}, \varrho) \leq 3$,

(2) $0 \leq \mathfrak{N}(\mathfrak{k}, \tilde{\varsigma}, \varrho) \leq 1; 0 \leq \mathfrak{S}(\mathfrak{k}, \tilde{\varsigma}, \varrho) \leq 1$ and $0 \leq \mathfrak{T}(\mathfrak{k}, \tilde{\varsigma}, \varrho) \leq 1$,

(3) $\mathfrak{N}(\mathfrak{k}, \tilde{\varsigma}, \varrho) > 0$,

(4) $\mathfrak{N}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = 1$, for all $\varrho \in (0, \infty) \Leftrightarrow \mathfrak{k} = \tilde{\varsigma}$,

(5) $\mathfrak{N}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = \mathfrak{N}(\tilde{\varsigma}, \mathfrak{k}, \varrho)$,

(6) $\mathfrak{N}(\mathfrak{k}, \mathfrak{z}, \varrho + \rho) \geq \mathfrak{N}(\mathfrak{k}, \tilde{\varsigma}, \varrho) * \mathfrak{N}(\tilde{\varsigma}, \mathfrak{z}, \rho)$,

(7) $\mathfrak{N}(\mathfrak{k}, \tilde{\varsigma}, \varrho): (0, \infty) \to [0,1]$ is continuous,

(8) $\mathfrak{S}(\mathfrak{k}, \tilde{\varsigma}, \varrho) < 1$,

(9) $\mathfrak{S}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = 0$, for all $\varrho \in (0, \infty) \Leftrightarrow \mathfrak{k} = \tilde{\varsigma}$,

(10) $\mathfrak{S}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = \mathfrak{S}(\tilde{\varsigma}, \mathfrak{k}, \varrho)$,

(11) $\mathfrak{S}(\mathfrak{k}, \mathfrak{z}, \varrho + \rho) \leq \mathfrak{S}(\mathfrak{k}, \tilde{\varsigma}, \varrho) \odot \mathfrak{S}(\tilde{\varsigma}, \mathfrak{z}, \rho)$,

(12) $\mathfrak{S}(\mathfrak{k}, \tilde{\varsigma}, \varrho): (0, \infty) \to [0,1]$ is continuous,

(13) $\mathfrak{T}(\mathfrak{k}, \tilde{\varsigma}, \varrho) < 1$,

(14) $\mathfrak{T}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = 0$ for all $\varrho \in (0, \infty) \Leftrightarrow \mathfrak{k} = \tilde{\varsigma}$,

(15) $\mathfrak{T}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = \mathfrak{T}(\tilde{\varsigma}, \mathfrak{k}, \varrho)$,

(16) $\mathfrak{T}(\mathfrak{k}, \mathfrak{z}, \varrho + \rho) \leq \mathfrak{T}(\mathfrak{k}, \tilde{\varsigma}, \varrho) \odot \mathfrak{T}(\tilde{\varsigma}, \mathfrak{z}, \rho)$,

(17) $\mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho)$: $(0,\infty) \to [0,1]$ is continuous.

The triplet $(\mathfrak{R},\mathfrak{S},\mathfrak{T})$ is named a *NMS*. The function $\mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho)$, $\mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho)$ and $\mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho)$ indicates the degree of nearness, non-nearness and neutralness between $\mathfrak{k}$ and $\tilde{\varsigma}$ with respect to ϱ.

Example 2.6: Let $\Xi = \mathbb{R}$ and let $\mathfrak{r} * \mathfrak{s} = \min\{\mathfrak{r},\mathfrak{s}\}$ and $\mathfrak{r}\odot\mathfrak{s} = \max\{\mathfrak{r},\mathfrak{s}\}$, for all $\mathfrak{r},\mathfrak{s} \in [0,1]$. For each $\varrho > 0$, $\mathfrak{k},\tilde{\varsigma} \in \Xi$, we define $\mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho) = e^{-\frac{|\mathfrak{k}-\tilde{\varsigma}|}{\varrho}}$, $\mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho) = (e^{\frac{|\mathfrak{k}-\tilde{\varsigma}|}{\varrho}} - 1)e^{-\frac{|\mathfrak{k}-\tilde{\varsigma}|}{\varrho}}$ and

$\mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho) = (e^{\frac{|\mathfrak{k}-\tilde{\varsigma}|}{\varrho}} - 1)$. Then $(\Xi,\mathfrak{R},\mathfrak{S},\mathfrak{T} *,\odot)$ is a NMS.

Definition 2.7: Let $(\Xi,\mathfrak{R},\mathfrak{S},\mathfrak{T} *,\odot)$ be *NMS* and $\mathfrak{L}$ and $\mathfrak{M}$ are self mappings on Ξ. The self mappins $\mathfrak{L}$ and $\mathfrak{M}$ are named to be commuting if $\mathfrak{L}\mathfrak{M}(\mathfrak{k}) = \mathfrak{M}\mathfrak{L}(\mathfrak{k})$, for all $\mathfrak{k} \in \Xi$. The self maps $\mathfrak{L}$ and $\mathfrak{M}$ are named to be compatible if

$\lim_{n\to\infty} |\mathfrak{R}(\mathfrak{L}\mathfrak{M}\mathfrak{k}_n, \mathfrak{M}\mathfrak{L}\mathfrak{k}_n,\varrho)| = 1, \lim_{n\to\infty} |\mathfrak{S}(\mathfrak{L}\mathfrak{M}\mathfrak{k}_n, \mathfrak{M}\mathfrak{L}\mathfrak{k}_n,\varrho)| = 0$ and

$\lim_{n\to\infty} |\mathfrak{T}(\mathfrak{L}\mathfrak{M}\mathfrak{k}_n, \mathfrak{M}\mathfrak{L}\mathfrak{k}_n,\varrho)| = 0, \varrho > 0.$

Whenever $\{\mathfrak{k}_n\}$ is a sequence in Ξ such that $\lim_{n\to\infty} \mathfrak{L}\,\mathfrak{k}_n = \lim_{n\to\infty} \mathfrak{M}\,\mathfrak{k}_n$, for some $\mathfrak{k} \in \Xi$.

Definition 2.8: Let $(\Xi,\mathfrak{R},\mathfrak{S},\mathfrak{T} *,\odot)$ be *NMS* and $\mathfrak{L}$ and $\mathfrak{M}$ are self mappings on Ξ. The self mappings $\mathfrak{L}$ and $\mathfrak{M}$ are named to be Occasionally Weakly Compatible [*OWC*] if and only if there is a coincidence point $\mathfrak{k}$ in Ξ of $\mathfrak{L}$ and $\mathfrak{M}$ commute. i.e., $\mathfrak{L}\mathfrak{M}\mathfrak{k} = \mathfrak{M}\mathfrak{L}\mathfrak{k}$.

Lemma 2.9: Let $(\Xi,\mathfrak{R},\mathfrak{S},\mathfrak{T},*,\odot)$ be a *NMS* with $\lim_{\varrho\to\infty} \mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho) = 1, \lim_{\varrho\to\infty} \mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho) = 0$ and $\lim_{\varrho\to\infty} \mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho) = 0$, for all $\mathfrak{k},\tilde{\varsigma} \in \Xi$. If $\mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\mathfrak{d}\varrho) \geq \mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho), \mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\mathfrak{d}\varrho) \leq \mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho)$ and

$\mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\mathfrak{d}\varrho) \leq \mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho)$ for some $\mathfrak{d} \in (0,1)$, for all $\varrho > 0$, then $\mathfrak{k} = \tilde{\varsigma}$.

Proof: Suppose there exists $\mathfrak{d} \in (0,1)$, such that $\mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\mathfrak{d}\varrho) \geq \mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho)$,$\mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\mathfrak{d}\varrho) \leq \mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho)$

And $\mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\mathfrak{d}\varrho) \leq \mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho)$, for all $\mathfrak{k},\tilde{\varsigma} \in \Xi$ and $\varrho > 0$.

So that $\mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho) \geq \mathfrak{R}\left(\mathfrak{k},\tilde{\varsigma},\frac{\varrho}{\mathfrak{d}}\right)$,$\mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho) \leq \mathfrak{S}\left(\mathfrak{k},\tilde{\varsigma},\frac{\varrho}{\mathfrak{d}}\right)$ and $\mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho) \leq \mathfrak{T}\left(\mathfrak{k},\tilde{\varsigma},\frac{\varrho}{\mathfrak{d}}\right)$.

Repeated application gives,

$\mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho) \geq \mathfrak{R}\left(\mathfrak{k},\tilde{\varsigma},\frac{\varrho}{\mathfrak{d}^n}\right), \mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho) \leq \mathfrak{S}\left(\mathfrak{k},\tilde{\varsigma},\frac{\varrho}{\mathfrak{d}^n}\right)$ and$\mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho) \leq \mathfrak{T}\left(\mathfrak{k},\tilde{\varsigma},\frac{\varrho}{\mathfrak{d}^n}\right)$

for some positive integer n. On taking $n \to \infty$, reduces to

$\mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho) \geq 1$ and $\mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho) \leq 0$ and $\mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho) \leq 0$. Thus, we have $\mathfrak{k} = \tilde{\varsigma}$.

Lemma 2.10: Let $\{\mathfrak{k}_n\}$ be a sequence in a *NMS*, $(\Xi,\mathfrak{R},\mathfrak{S},\mathfrak{T},*,\odot)$ with $\lim_{\varrho\to\infty} \mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho) = 1$,

$\lim_{\varrho\to\infty} \mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho) = 0$ and $\lim_{\varrho\to\infty} \mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho) = 0$, for all $\mathfrak{k},\tilde{\varsigma} \in \Xi$. If there exists $\mathfrak{d} \in (0,1)$ such that

$\mathfrak{R}(\mathfrak{k}_{n+1},\mathfrak{k}_{n+2},\mathfrak{d}\varrho) \geq \mathfrak{R}(\mathfrak{k}_n,\mathfrak{k}_{n+1},\varrho)$,$\mathfrak{S}(\mathfrak{k}_{n+1},\mathfrak{k}_{n+2},\mathfrak{d}\varrho) \leq \mathfrak{S}(\mathfrak{k}_n,\mathfrak{k}_{n+1},\varrho)$ and

$\mathfrak{T}(\mathfrak{k}_{n+1},\mathfrak{k}_{n+2},\mathfrak{d}\varrho) \leq \mathfrak{T}(\mathfrak{k}_n,\mathfrak{k}_{n+1},\varrho)$ for all $\varrho > 0$ and n = 0,1,2.... Then$\{\mathfrak{k}_n\}$ is a Cauchy sequence in Ξ.

Proof: For n = 0, we have

$\mathfrak{R}(\mathfrak{k}_1,\mathfrak{k}_2,\varrho) \geq \mathfrak{R}\left(\mathfrak{k}_0,\mathfrak{k}_1,\frac{\varrho}{\mathfrak{d}}\right)$, $\mathfrak{S}(\mathfrak{k}_1,\mathfrak{k}_2,\varrho) \leq \mathfrak{S}\left(\mathfrak{k}_0,\mathfrak{k}_1,\frac{\varrho}{\mathfrak{d}}\right)$ and $\mathfrak{T}(\mathfrak{k}_1,\mathfrak{k}_2,\varrho) \leq \mathfrak{T}\left(\mathfrak{k}_0,\mathfrak{k}_1,\frac{\varrho}{\mathfrak{d}}\right)$,

for all $\varrho > 0$ and $\mathfrak{d} \in (0,1)$. By induction,

$\mathfrak{R}(\mathfrak{k}_{n+1},\mathfrak{k}_{n+2},\varrho) \geq \mathfrak{R}\left(\mathfrak{k}_0,\mathfrak{k}_1,\frac{\varrho}{\mathfrak{d}^{n+1}}\right)$, $\mathfrak{S}(\mathfrak{k}_{n+1},\mathfrak{k}_{n+2},\varrho) \leq \mathfrak{S}\left(\mathfrak{k}_0,\mathfrak{k}_1,\frac{\varrho}{\mathfrak{d}^{n+1}}\right)$ and

$\mathfrak{T}(\mathfrak{k}_{n+1},\mathfrak{k}_{n+2},\varrho) \leq \mathfrak{T}\left(\mathfrak{k}_0,\mathfrak{k}_1,\frac{\varrho}{\mathfrak{d}^{n+1}}\right)$, for all n.

Thus for any positive integer $\mathfrak{q}$ and using (6), (11) and (16), we have

$$\mathfrak{R}(\mathfrak{k}_n,\mathfrak{k}_{n+\mathfrak{q}},\varrho) \geq \mathfrak{R}\left(\mathfrak{k}_n,\mathfrak{k}_{n+1},\frac{\varrho}{\mathfrak{q}}\right) * \dots * (\mathfrak{q}\ times) * \dots * \mathfrak{R}\left(\mathfrak{k}_{n+\mathfrak{q}-1},\mathfrak{k}_{n+\mathfrak{q}},\frac{\varrho}{\mathfrak{q}}\right)$$

$$\geq \mathfrak{R}\left(\mathfrak{k}_0,\mathfrak{k}_1,\frac{\varrho}{\mathfrak{q}\mathfrak{d}^n}\right) * \dots * (\mathfrak{q}\ times) * \dots * \mathfrak{R}\left(\mathfrak{k}_0,\mathfrak{k}_1,\frac{\varrho}{\mathfrak{q}\mathfrak{d}^{n+\mathfrak{q}-1}}\right).$$

$$\mathfrak{S}(\mathfrak{k}_n,\mathfrak{k}_{n+\mathfrak{q}},\varrho) \leq \mathfrak{S}\left(\mathfrak{k}_n,\mathfrak{k}_{n+1},\frac{\varrho}{\mathfrak{q}}\right) \odot \dots \odot (\mathfrak{q}\ times) \odot \dots \odot \mathfrak{S}\left(\mathfrak{k}_{n+\mathfrak{q}-1},\mathfrak{k}_{n+\mathfrak{q}},\frac{\varrho}{\mathfrak{q}}\right)$$

$$\leq \mathfrak{S}\left(\mathfrak{k}_0,\mathfrak{k}_1,\frac{\varrho}{\mathfrak{q}\mathfrak{d}^n}\right) \odot \dots \odot (\mathfrak{q}\ times) \odot \dots \odot \mathfrak{S}\left(\mathfrak{k}_0,\mathfrak{k}_1,\frac{\varrho}{\mathfrak{q}\mathfrak{d}^{n+\mathfrak{q}-1}}\right).$$

$$\mathfrak{T}(\mathfrak{k}_n,\mathfrak{k}_{n+\mathfrak{q}},\varrho) \leq \mathfrak{T}\left(\mathfrak{k}_n,\mathfrak{k}_{n+1},\frac{\varrho}{\mathfrak{q}}\right) \odot \dots \odot (\mathfrak{q}\ times) \odot \dots \odot \mathfrak{T}\left(\mathfrak{k}_{n+\mathfrak{q}-1},\mathfrak{k}_{n+\mathfrak{q}},\frac{\varrho}{\mathfrak{q}}\right)$$

$$\leq \mathfrak{T}\left(\mathfrak{k}_0,\mathfrak{k}_1,\frac{\varrho}{\mathfrak{q}\mathfrak{d}^n}\right) \odot \dots \odot (\mathfrak{q}\ times) \odot \dots \odot \mathfrak{T}\left(\mathfrak{k}_0,\mathfrak{k}_1,\frac{\varrho}{\mathfrak{q}\mathfrak{d}^{n+\mathfrak{q}-1}}\right).$$

Which on taking $n \to \infty$, reduces to

$\lim_{\varrho\to\infty} \mathfrak{R}(\mathfrak{k}_n,\mathfrak{k}_{n+\mathfrak{q}},\varrho) \geq 1 * 1 * \dots * 1$, $\lim_{\varrho\to\infty} \mathfrak{S}(\mathfrak{k}_n,\mathfrak{k}_{n+\mathfrak{q}},\varrho) \leq 0 \odot \dots \odot 0$ and

$\lim_{\varrho\to\infty} \mathfrak{T}(\mathfrak{k}_n,\mathfrak{k}_{n+\mathfrak{q}},\varrho) \leq 0 \odot \dots \odot 0$.

Since $\mathfrak{d} < 1$, $\lim_{\varrho\to\infty} \mathfrak{R}(\mathfrak{k}_n,\mathfrak{k}_{n+\mathfrak{q}},\varrho) \geq 1$, $\lim_{\varrho\to\infty} \mathfrak{T}(\mathfrak{k}_n,\mathfrak{k}_{n+\mathfrak{q}},\varrho) \leq 0$ and $\lim_{\varrho\to\infty} \mathfrak{S}(\mathfrak{k}_n,\mathfrak{k}_{n+\mathfrak{q}},\varrho) \leq 0$. This necessitates that $\{\mathfrak{k}_n\}$ is a Cauchy sequence in Ξ

3. Main Results

In this section, we present the concept of *NMS* and prove several *FP* results.

Theorem 3.1: Let $(\Xi,\mathfrak{R},\mathfrak{S},\mathfrak{T},*,\odot)$ be a *NMS* with $\lim_{\varrho\to\infty} \mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho) = 1$, $\lim_{\varrho\to\infty} \mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho) = 0$ and $\lim_{\varrho\to\infty} \mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho) = 0$, for all $\mathfrak{k},\tilde{\varsigma} \in \Xi$ and $\varrho > 0$ and let $\mathfrak{L}$ and $\mathfrak{M}$ be self mapping on Ξ. If there exist $\mathfrak{d} \in (0,1)$ such that

$\mathfrak{R}(\mathfrak{L}\mathfrak{k},\mathfrak{M}\tilde{\varsigma},\mathfrak{d}\varrho) \geq \mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho)$, $\mathfrak{S}(\mathfrak{L}\mathfrak{k},\mathfrak{M}\tilde{\varsigma},\mathfrak{d}\varrho) \leq \mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho)$ and

$\mathfrak{T}(\mathfrak{L}\mathfrak{k},\mathfrak{M}\tilde{\varsigma},\mathfrak{d}\varrho) \leq \mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho)$ for all $\mathfrak{k},\tilde{\varsigma},\in \Xi$, and for all $\varrho > 0$ (3.1.1)

Then $\mathfrak{L}$ and $\mathfrak{M}$ have a unique common fixed point in Ξ.

Proof. Let $\mathfrak{k}_0 \in \Xi$ be an arbitrary point and we define the sequence $\{\mathfrak{k}_n\}$ by $\mathfrak{k}_{2n+1} = \mathfrak{L}\mathfrak{k}_{2n}$ and $\mathfrak{k}_{2n+2} = \mathfrak{M}\mathfrak{k}_{2n+1}$; n = 0,1,2,… .

Now, for $\mathfrak{d} \in (0,1)$ and for all $\varrho > 0$, then from (3.1.1) we have

$\mathfrak{R}(\mathfrak{k}_{2n+1}, \mathfrak{k}_{2n+2}, \mathfrak{d}\varrho) = \mathfrak{R}(\mathfrak{L}\mathfrak{k}_{2n}, \mathfrak{M}\mathfrak{k}_{2n+1}, \mathfrak{d}\varrho) \geq \mathfrak{R}(\mathfrak{k}_{2n}, \mathfrak{k}_{2n+1}, \varrho)$,

$\mathfrak{R}(\mathfrak{k}_{2n}, \mathfrak{k}_{2n+1}, \mathfrak{d}\varrho) = \mathfrak{R}(\mathfrak{L}\mathfrak{k}_{2n-1}, \mathfrak{M}\mathfrak{k}_{2n}, \mathfrak{d}\varrho) \geq \mathfrak{R}(\mathfrak{k}_{2n-1}, \mathfrak{k}_{2n}, \varrho)$.

$\mathfrak{S}(\mathfrak{k}_{2n+1}, \mathfrak{k}_{2n+2}, \mathfrak{d}\varrho) = \mathfrak{S}(\mathfrak{L}\mathfrak{k}_{2n}, \mathfrak{M}\mathfrak{k}_{2n+1}, \mathfrak{d}\varrho) \leq \mathfrak{S}(\mathfrak{k}_{2n}, \mathfrak{k}_{2n+1}, \varrho)$,

$\mathfrak{S}(\mathfrak{k}_{2n}, \mathfrak{k}_{2n+1}, \mathfrak{d}\varrho) = \mathfrak{S}(\mathfrak{L}\mathfrak{k}_{2n-1}, \mathfrak{M}\mathfrak{k}_{2n}, \mathfrak{d}\varrho)\mathfrak{S}(\mathfrak{k}_{2n-1}, \mathfrak{k}_{2n}, \varrho)$ and

$\mathfrak{T}(\mathfrak{k}_{2n+1}, \mathfrak{k}_{2n+2}, \mathfrak{d}\varrho) = \mathfrak{T}(\mathfrak{L}\mathfrak{k}_{2n}, \mathfrak{M}\mathfrak{k}_{2n+1}, \mathfrak{d}\varrho) \leq \mathfrak{T}(\mathfrak{k}_{2n}, \mathfrak{k}_{2n+1}, \varrho)$,

$\mathfrak{T}(\mathfrak{k}_{2n}, \mathfrak{k}_{2n+1}, \mathfrak{d}\varrho) = \mathfrak{T}(\mathfrak{L}\mathfrak{k}_{2n-1}, \mathfrak{M}\mathfrak{k}_{2n}, \mathfrak{d}\varrho)\mathfrak{T}(\mathfrak{k}_{2n-1}, \mathfrak{k}_{2n}, \varrho)$.

In general, we have

$\mathfrak{R}(\mathfrak{k}_{n+1}, \mathfrak{k}_{n+2}, \mathfrak{d}\varrho) \geq \mathfrak{R}(\mathfrak{k}_n, \mathfrak{k}_{n+1}, \varrho)$, $\mathfrak{S}(\mathfrak{k}_{n+1}, \mathfrak{k}_{n+2}, \mathfrak{d}\varrho) \leq \mathfrak{S}(\mathfrak{k}_n, \mathfrak{k}_{n+1}, \varrho)$ and

$\mathfrak{T}(\mathfrak{k}_{n+1}, \mathfrak{k}_{n+2}, \mathfrak{d}\varrho) \leq \mathfrak{T}(\mathfrak{k}_n, \mathfrak{k}_{n+1}, \varrho)$ for all $\varrho > 0$ and $\mathfrak{d} \in (0,1)$; n = 0,1,2….

By Lemma (2.10) $\{\mathfrak{k}_n\}$ be a Cauchy sequence in Ξ. Since Ξ is complete then there exists $\vartheta \in \Xi$ such that $\mathfrak{k}_n \to \vartheta$ as $n \to \infty$ and $\{\mathfrak{k}_{2n}\}, \{\mathfrak{k}_{2n+1}\}$ are sub sequences of $\{\mathfrak{k}_{2n}\}$ converge to the same point $\vartheta \in \Xi$, i.e. $\mathfrak{k}_{2n} \to \vartheta, \mathfrak{k}_{2n+1} \to \vartheta$ as $n \to \infty$. Now from equation (3.1.1) we have,

$$\mathfrak{R}(\mathfrak{L}\vartheta, \vartheta, \mathfrak{d}\varrho) = \mathfrak{R}\left(\mathfrak{L}\vartheta, \vartheta, \frac{\mathfrak{d}\varrho}{2} + \frac{\mathfrak{d}\varrho}{2}\right) \geq \mathfrak{R}\left(\mathfrak{L}\vartheta, \mathfrak{k}_{2n+2}, \frac{\mathfrak{d}\varrho}{2}\right) * \mathfrak{R}\left(\mathfrak{k}_{2n+2}, \vartheta, \frac{\mathfrak{d}\varrho}{2}\right)$$

$$= \mathfrak{R}\left(\mathfrak{L}\vartheta, \mathfrak{M}\mathfrak{k}_{2n+1}, \frac{\mathfrak{d}\varrho}{2}\right) * \mathfrak{R}\left(\mathfrak{k}_{2n+2}, \vartheta, \frac{\mathfrak{d}\varrho}{2}\right) \geq \mathfrak{R}\left(\vartheta, \mathfrak{k}_{2n+1}, \frac{\varrho}{2}\right) * \mathfrak{R}\left(\mathfrak{k}_{2n+2}, \vartheta, \frac{\mathfrak{d}\varrho}{2}\right).$$

$$\mathfrak{S}(\mathfrak{L}\vartheta, \vartheta, \mathfrak{d}\varrho) = \mathfrak{S}\left(\mathfrak{L}\vartheta, \vartheta, \frac{\mathfrak{d}\varrho}{2} + \frac{\mathfrak{d}\varrho}{2}\right) \leq \mathfrak{S}\left(\mathfrak{L}\vartheta, \mathfrak{k}_{2n+2}, \frac{\mathfrak{d}\varrho}{2}\right) \odot \mathfrak{S}\left(\mathfrak{k}_{2n+2}, \vartheta, \frac{\mathfrak{d}\varrho}{2}\right)$$

$$= \mathfrak{S}\left(\mathfrak{L}\vartheta, \mathfrak{M}\mathfrak{k}_{2n+1}, \frac{\mathfrak{d}\varrho}{2}\right) \odot \mathfrak{S}\left(\mathfrak{k}_{2n+2}, \vartheta, \frac{\mathfrak{d}\varrho}{2}\right) \leq \mathfrak{S}\left(\vartheta, \mathfrak{k}_{2n+1}, \frac{\varrho}{2}\right) \odot \mathfrak{S}\left(\mathfrak{k}_{2n+2}, \vartheta, \frac{\mathfrak{d}\varrho}{2}\right) \text{ and}$$

$$\mathfrak{T}(\mathfrak{L}\vartheta, \vartheta, \mathfrak{d}\varrho) = \mathfrak{T}\left(\mathfrak{L}\vartheta, \vartheta, \frac{\mathfrak{d}\varrho}{2} + \frac{\mathfrak{d}\varrho}{2}\right) \leq \mathfrak{T}\left(\mathfrak{L}\vartheta, \mathfrak{k}_{2n+2}, \frac{\mathfrak{d}\varrho}{2}\right) \odot \mathfrak{T}\left(\mathfrak{k}_{2n+2}, \vartheta, \frac{\mathfrak{d}\varrho}{2}\right)$$

$$= \mathfrak{T}\left(\mathfrak{L}\vartheta, \mathfrak{M}\mathfrak{k}_{2n+1}, \frac{\mathfrak{d}\varrho}{2}\right) \odot \mathfrak{T}\left(\mathfrak{k}_{2n+2}, \vartheta, \frac{\mathfrak{d}\varrho}{2}\right) \leq \mathfrak{T}\left(\vartheta, \mathfrak{k}_{2n+1}, \frac{\varrho}{2}\right) \odot \mathfrak{T}\left(\mathfrak{k}_{2n+2}, \vartheta, \frac{\mathfrak{d}\varrho}{2}\right).$$

Taking limit $n \to \infty$.

$$\mathfrak{R}(\mathfrak{L}\vartheta, \vartheta, \mathfrak{d}\varrho) \geq 1 * 1 = 1, \mathfrak{S}(\mathfrak{L}\vartheta, \vartheta, \mathfrak{d}\varrho) \leq 0 \odot 0 = 0, \mathfrak{T}(\mathfrak{L}\vartheta, \vartheta, \mathfrak{d}\varrho) \leq 0 \odot 0 = 0.$$

So $\mathfrak{L}\vartheta = \vartheta$; Again,

$$\mathfrak{R}(\vartheta, \mathfrak{M}\vartheta, \mathfrak{d}\varrho) = \mathfrak{R}\left(\vartheta, \mathfrak{W}\vartheta, \frac{\mathfrak{d}\varrho}{2} + \frac{\mathfrak{d}\varrho}{2}\right) \geq \mathfrak{R}\left(\vartheta, \mathfrak{k}_{2n+1}, \frac{\mathfrak{d}\varrho}{2}\right) * \mathfrak{R}\left(\mathfrak{k}_{2n+1}, \mathfrak{W}\vartheta, \frac{\mathfrak{d}\varrho}{2}\right)$$

$$= \mathfrak{R}\left(\vartheta, \mathfrak{k}_{2n+1}, \frac{\mathfrak{d}\varrho}{2}\right) * \mathfrak{R}\left(\mathfrak{L}\mathfrak{k}_{2n}, \mathfrak{W}\vartheta, \frac{\mathfrak{d}\varrho}{2}\right) \geq \mathfrak{R}\left(\vartheta, \mathfrak{k}_{2n+1}, \frac{\varrho}{2}\right) * \mathfrak{R}\left(\mathfrak{k}_{2n}, \vartheta, \frac{\mathfrak{d}\varrho}{2}\right) \text{ and}$$

$$\mathfrak{S}(\vartheta,\mathfrak{M}\vartheta,\mathfrak{d}\varrho)=\mathfrak{S}\left(\vartheta,\mathfrak{W}\vartheta,\frac{\mathfrak{d}\varrho}{2}+\frac{\mathfrak{d}\varrho}{2}\right)\leq\mathfrak{S}\left(\vartheta,\mathfrak{k}_{2n+1},\frac{\mathfrak{d}\varrho}{2}\right)\odot\mathfrak{S}\left(\mathfrak{k}_{2n+1},\mathfrak{W}\vartheta,\frac{\mathfrak{d}\varrho}{2}\right)$$

$$=\mathfrak{S}\left(\vartheta,\mathfrak{k}_{2n+1},\tfrac{\mathfrak{d}\varrho}{2}\right)\odot\mathfrak{S}\left(\mathfrak{L}\mathfrak{k}_{2n},\mathfrak{W}\vartheta,\tfrac{\mathfrak{d}\varrho}{2}\right)\leq\mathfrak{S}\left(\vartheta,\mathfrak{k}_{2n+1},\tfrac{\varrho}{2}\right)\odot\mathfrak{S}\left(\mathfrak{k}_{2n},\vartheta,\tfrac{\mathfrak{d}\varrho}{2}\right).$$

In addition,

$$\mathfrak{T}(\vartheta,\mathfrak{M}\vartheta,\mathfrak{d}\varrho)=\mathfrak{T}\left(\vartheta,\mathfrak{W}\vartheta,\frac{\mathfrak{d}\varrho}{2}+\frac{\mathfrak{d}\varrho}{2}\right)\leq\mathfrak{T}\left(\vartheta,\mathfrak{k}_{2n+1},\frac{\mathfrak{d}\varrho}{2}\right)\odot\mathfrak{T}\left(\mathfrak{k}_{2n+1},\mathfrak{W}\vartheta,\frac{\mathfrak{d}\varrho}{2}\right)$$

$$=\mathfrak{T}\left(\vartheta,\mathfrak{k}_{2n+1},\tfrac{\mathfrak{d}\varrho}{2}\right)\odot\mathfrak{T}\left(\mathfrak{L}\mathfrak{k}_{2n},\mathfrak{W}\vartheta,\tfrac{\mathfrak{d}\varrho}{2}\right)\leq\mathfrak{T}\left(\vartheta,\mathfrak{k}_{2n+1},\tfrac{\varrho}{2}\right)\odot\mathfrak{T}\left(\mathfrak{k}_{2n},\vartheta,\tfrac{\mathfrak{d}\varrho}{2}\right).$$

On taking limit $n\to\infty$.

$\mathfrak{R}(\vartheta,\mathfrak{M}\vartheta,\mathfrak{d}\varrho)\geq 1*1=1$,$\mathfrak{S}(\vartheta,\mathfrak{M}\vartheta,\mathfrak{d}\varrho)\leq 0\odot 0=0$, and $\mathfrak{T}(\vartheta,\mathfrak{M}\vartheta,\mathfrak{d}\varrho)\leq 0\odot 0=0$.

So $\mathfrak{W}\vartheta=\vartheta$, and $\mathfrak{L}\vartheta=\mathfrak{W}\vartheta=\vartheta$. Hence ϑ is a common fixed point of $\mathfrak{L}$ and $\mathfrak{W}$.

For uniqueness, let $\mathfrak{s}$ be any another fixed point of $\mathfrak{L}$ and $\mathfrak{W}$. Now from (3.1.1),

$\mathfrak{R}(\vartheta,\mathfrak{s},\mathfrak{d}\varrho)=\mathfrak{R}(\mathfrak{L}\vartheta,\mathfrak{W}\mathfrak{s},\mathfrak{d}\varrho)\geq\mathfrak{R}(\vartheta,\mathfrak{s},\varrho)$;

$\mathfrak{S}(\vartheta,\mathfrak{s},\mathfrak{d}\varrho)=\mathfrak{S}(\mathfrak{L}\vartheta,\mathfrak{W}\mathfrak{s},\mathfrak{d}\varrho)\leq\mathfrak{S}(\vartheta,\mathfrak{s},\varrho)$and

$\mathfrak{T}(\vartheta,\mathfrak{s},\mathfrak{d}\varrho)=\mathfrak{T}(\mathfrak{L}\vartheta,\mathfrak{W}\mathfrak{s},\mathfrak{d}\varrho)\leq\mathfrak{T}(\vartheta,\mathfrak{s},\varrho)$.

We know that when $(\Xi,\mathfrak{R},\mathfrak{S},*,\odot)$ be *NMS* such that

$\lim_{\varrho\to\infty}\mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho)=1$, $\lim_{\varrho\to\infty}\mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho)=0$ and $\lim_{\varrho\to\infty}\mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho)=0$, for all $\mathfrak{k},\tilde{\varsigma}\in\Xi$.

If $\mathfrak{R}(\mathfrak{L}\mathfrak{k},\mathfrak{L}\tilde{\varsigma},\mathfrak{d}\varrho)\geq\mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho)$, $\mathfrak{S}(\mathfrak{L}\mathfrak{k},\mathfrak{L}\tilde{\varsigma},\mathfrak{d}\varrho)\leq\mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho)$ and

$\mathfrak{T}(\mathfrak{L}\mathfrak{k},\mathfrak{L}\tilde{\varsigma},\mathfrak{d}\varrho)\leq\mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho)$, for some $0<\mathfrak{d}<1$, for all $\mathfrak{k},\tilde{\varsigma},\in\Xi$, $\varrho\in(0,\infty)$, then $\mathfrak{k}=\tilde{\varsigma}$.

Hence $\vartheta=\mathfrak{s}$.

Example: 3.2: Let $\Xi=[0,1]$. Consider the metric $d(\mathfrak{k},\tilde{\varsigma})=|\mathfrak{k}-\tilde{\varsigma}|$ with $\mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho)=\frac{\varrho}{\varrho+d(\mathfrak{k},\tilde{\varsigma})}$,

$\mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho)=\frac{d(\mathfrak{k},\tilde{\varsigma})}{\varrho+d(\mathfrak{k},\tilde{\varsigma})}$ and $\mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho)=\frac{d(\mathfrak{k},\tilde{\varsigma})}{\varrho}$ and the self mappings $\mathfrak{L}$ and $\mathfrak{W}$ on Ξ, defined by

$\mathfrak{L}(\mathfrak{k})=\frac{\mathfrak{k}}{4}$, $\mathfrak{W}(\mathfrak{k})=\frac{\mathfrak{k}}{2}$. The self mappings $\mathfrak{L}$ and $\mathfrak{W}$ satisfies all the conditions that are stated in Theorem (3.1), then $\mathfrak{L}$ and $\mathfrak{W}$ have unique common fixed point at 0.

Corollary 3.3: Let $(\Xi,\mathfrak{R},\mathfrak{S},\mathfrak{T},*,\odot)$ be a *NMS* with $\lim_{\varrho\to\infty}\mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho)=1$, $\lim_{\varrho\to\infty}\mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho)=0$ and $\lim_{\varrho\to\infty}\mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho)=0$, for all $\mathfrak{k},\tilde{\varsigma}\in\Xi$ and $\varrho>0$ and let $\mathfrak{L}$ and $\mathfrak{M}$ be self mapping on Ξ. If there exist $\mathfrak{d}\in(0,1)$ such that

$\mathfrak{R}(\mathfrak{L}\mathfrak{k},\mathfrak{L}\tilde{\varsigma},\mathfrak{d}\varrho)\geq\mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho)$, $\mathfrak{S}(\mathfrak{L}\mathfrak{k},\mathfrak{L}\tilde{\varsigma},\mathfrak{d}\varrho)\leq\mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho)$ and $\mathfrak{T}(\mathfrak{L}\mathfrak{k},\mathfrak{L}\tilde{\varsigma},\mathfrak{d}\varrho)\leq\mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho)$ for all $\mathfrak{k},\tilde{\varsigma},\in\Xi$, and for all $\varrho>0$, then $\mathfrak{L}$ have a unique fixed point in Ξ.

Theorem3.4: Let $(\Xi, \mathfrak{N}, \mathfrak{S}, \mathfrak{T}, *, \odot)$ be a *NMS* with $\lim_{\varrho\to\infty} \mathfrak{N}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = 1$, $\lim_{\varrho\to\infty} \mathfrak{S}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = 0$ and $\lim_{\varrho\to\infty} \mathfrak{T}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = 0$, for all $\mathfrak{k}, \tilde{\varsigma} \in \Xi$ and $\ddot{\mathfrak{A}}, \ddot{\mathfrak{B}}, \mathfrak{L}$ and $\mathfrak{W}$ be self mappings on Ξ. Let the pairs $\{\ddot{\mathfrak{A}}, \mathfrak{L}\}$ and $\{\ddot{\mathfrak{B}}, \mathfrak{W}\}$ be *OWC*. If there exists $\mathfrak{d} \in (0,1)$ such that

$$\mathfrak{N}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \geq \min\{\mathfrak{N}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{N}(\mathfrak{L}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho), \mathfrak{N}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{N}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{N}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{L}\mathfrak{k}, \varrho)\} \quad (3.4.1)$$

$$\mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \leq \max\{\mathfrak{S}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{S}(\mathfrak{L}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho), \mathfrak{S}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{S}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{L}\mathfrak{k}, \varrho)\} \quad (3.4.2)$$

$$\mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \leq \max\{\mathfrak{T}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{T}(\mathfrak{L}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho), \mathfrak{T}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{T}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{L}\mathfrak{k}, \varrho)\} \quad (3.4.3)$$

for all $\mathfrak{k}, \tilde{\varsigma} \in \Xi$ and $\varrho > 0$, then $\ddot{\mathfrak{A}}, \ddot{\mathfrak{B}}, \mathfrak{L}$ and $\mathfrak{W}$ have a unique common fixed point in Ξ.

Proof: Since the pairs $\{\ddot{\mathfrak{A}}, \mathfrak{L}\}$ and $\{\ddot{\mathfrak{B}}, \mathfrak{W}\}$ be *OWC*, so there are point $\mathfrak{k}, \tilde{\varsigma} \in \Xi$ such that

$\ddot{\mathfrak{A}}(\mathfrak{k}) = \mathfrak{L}(\mathfrak{k})$ and $\ddot{\mathfrak{B}}(\tilde{\varsigma}) = \mathfrak{W}(\tilde{\varsigma})$. Now, by the given conditions (3.4.1) and (3.4.2) we get

$$\mathfrak{N}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \geq \min\{\mathfrak{N}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{N}(\mathfrak{L}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho), \mathfrak{N}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{N}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{W}}\tilde{\varsigma}, \varrho), \mathfrak{N}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{L}\mathfrak{k}, \varrho)\}$$

$$= \min\{\mathfrak{N}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{N}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho), \mathfrak{N}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{N}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{N}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho)\}$$

$$= \min\{\mathfrak{N}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), 1, 1, \mathfrak{N}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{N}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho)\} = \mathfrak{N}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho).$$

$$\mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \leq \max\{\mathfrak{S}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{S}(\mathfrak{L}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho), \mathfrak{S}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{W}}\tilde{\varsigma}, \varrho), \mathfrak{S}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{L}\mathfrak{k}, \varrho)\}$$

$$= \max\{\mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho), \mathfrak{S}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{S}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho)\}$$

$$= \max\{\mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), 0, 0, \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{S}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho)\} = \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho).$$

$$\mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \leq \max\{\mathfrak{T}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{T}(\mathfrak{L}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho), \mathfrak{T}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{W}}\tilde{\varsigma}, \varrho), \mathfrak{T}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{L}\mathfrak{k}, \varrho)\}$$

$$= \max\{\mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho), \mathfrak{T}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{T}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho)\}$$

$$= \max\{\mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), 0, 0, \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{T}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho)\} = \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho).$$

In view of Lemma (2.9), we have $\ddot{\mathfrak{A}}\mathfrak{k} = \ddot{\mathfrak{B}}\tilde{\varsigma}$ and therefore $\ddot{\mathfrak{A}}\mathfrak{k} = \mathfrak{L}\mathfrak{k} = \ddot{\mathfrak{B}}\tilde{\varsigma} = \mathfrak{W}\tilde{\varsigma}$. (3.4.4)

Suppose that the pair $\{\ddot{\mathfrak{A}}, \mathfrak{L}\}$ have an another coincidence point $\mathfrak{w} \in \Xi$. i.e., $\ddot{\mathfrak{A}}\mathfrak{w} = \mathfrak{L}\mathfrak{w}$.

Now,

$$\mathfrak{N}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \geq \min\{\mathfrak{N}(\mathfrak{L}\mathfrak{w}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{N}(\mathfrak{L}\mathfrak{w}, \ddot{\mathfrak{A}}\mathfrak{w}, \varrho), \mathfrak{N}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{N}(\ddot{\mathfrak{A}}\mathfrak{w}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{N}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{L}\mathfrak{w}, \varrho)\}$$

$$= \min\{\mathfrak{N}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{N}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{A}}\mathfrak{w}, \varrho), \mathfrak{N}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{N}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{N}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \ddot{\mathfrak{A}}\mathfrak{w}, \varrho)\}$$

$$= \min\{\mathfrak{N}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), 1, 1, \mathfrak{N}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{N}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \ddot{\mathfrak{A}}\mathfrak{w}, \varrho)\} = \mathfrak{N}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho).$$

$$\mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \leq \max\{\mathfrak{S}(\mathfrak{L}\mathfrak{w}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{S}(\mathfrak{L}\mathfrak{w}, \ddot{\mathfrak{A}}\mathfrak{w}, \varrho), \mathfrak{S}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{w}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{S}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{L}\mathfrak{w}, \varrho)\}$$

$$= \max\{\mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{A}}\mathfrak{w}, \varrho), \mathfrak{S}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{S}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \ddot{\mathfrak{A}}\mathfrak{w}, \varrho)\}$$

$$= \max\{\mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), 0, 0, \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{S}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \ddot{\mathfrak{A}}\mathfrak{w}, \varrho)\} = \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho).$$

$$\mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \leq \max\{\mathfrak{T}(\mathfrak{L}\mathfrak{w}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{T}(\mathfrak{L}\mathfrak{w}, \ddot{\mathfrak{A}}\mathfrak{w}, \varrho), \mathfrak{T}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{w}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{T}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{L}\mathfrak{w}, \varrho)\}$$

$$= \max\{\mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{A}}\mathfrak{w}, \varrho), \mathfrak{T}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho)\mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{T}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \ddot{\mathfrak{A}}\mathfrak{w}, \varrho)\}$$

$$= \max\{\mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), 0, 0, \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho), \mathfrak{T}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \ddot{\mathfrak{A}}\mathfrak{w}, \varrho)\} = \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho).$$

Again, in view of Lemma (2.9), we have $\ddot{\mathfrak{A}}\mathfrak{k} = \ddot{\mathfrak{B}}\tilde{\varsigma}$.

Therefore, $\ddot{\mathfrak{A}}\mathfrak{k} = \mathfrak{L}\mathfrak{k} = \ddot{\mathfrak{B}}\tilde{\varsigma} = \mathfrak{W}\tilde{\varsigma}$. (3.4.5)

From (3.4.4) and (3.4.5), $\ddot{\mathfrak{A}}\mathfrak{k} = \ddot{\mathfrak{A}}\mathfrak{w}$ and therefore the pair $\{\ddot{\mathfrak{A}}, \mathfrak{L}\}$ have a unique coincidence point $\zeta = \ddot{\mathfrak{A}}\mathfrak{k} = \mathfrak{L}\mathfrak{k}$. Thus by Lemma (2.9), $\mathfrak{w}$ is the unique common fixed point of the pair $\{\ddot{\mathfrak{A}}, \mathfrak{L}\}$. Similarly, we can show that this pair $\{\ddot{\mathfrak{B}}, \mathfrak{W}\}$ also have a unique common fixed point.

Suppose this is $\eta \in \Xi$. Now,

$$\mathfrak{R}(\zeta, \eta, \mathfrak{d}\varrho) = \mathfrak{R}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \mathfrak{d}\varrho)$$
$$\geq \min\{\mathfrak{R}(\mathfrak{L}\zeta, \mathfrak{W}\eta, \varrho), \mathfrak{R}(\mathfrak{L}\zeta, \ddot{\mathfrak{A}}\zeta, \varrho), \mathfrak{R}(\ddot{\mathfrak{B}}\eta, \mathfrak{W}\eta, \varrho), \mathfrak{R}(\ddot{\mathfrak{A}}\zeta, \mathfrak{W}\eta, \varrho), \mathfrak{R}(\ddot{\mathfrak{B}}\eta, \mathfrak{L}\zeta, \varrho)\}$$
$$= \min\{\mathfrak{R}(\zeta, \eta, \varrho), \mathfrak{R}(\zeta, \zeta, \varrho), \mathfrak{R}(\eta, \eta, \varrho), \mathfrak{R}(\zeta, \eta, \varrho), \mathfrak{R}(\eta, \zeta, \varrho)\}$$
$$= \min\{\mathfrak{R}(\zeta, \eta, \varrho), 1, 1, \mathfrak{R}(\zeta, \eta, \varrho), \mathfrak{R}(\eta, \zeta, \varrho)\} = \mathfrak{R}(\zeta, \eta, \varrho).$$

$$\mathfrak{S}(\zeta, \eta, \mathfrak{d}\varrho) = \mathfrak{S}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \mathfrak{d}\varrho)$$
$$\leq \max\{\mathfrak{S}(\mathfrak{L}\zeta, \mathfrak{W}\eta, \varrho), \mathfrak{S}(\mathfrak{L}\zeta, \ddot{\mathfrak{A}}\zeta, \varrho), \mathfrak{S}(\ddot{\mathfrak{B}}\eta, \mathfrak{W}\eta, \varrho), \mathfrak{S}(\ddot{\mathfrak{A}}\zeta, \mathfrak{W}\eta, \varrho), \mathfrak{S}(\ddot{\mathfrak{B}}\eta, \mathfrak{L}\zeta, \varrho)\}$$
$$= \max\{\mathfrak{S}(\zeta, \eta, \varrho), \mathfrak{S}(\zeta, \zeta, \varrho), \mathfrak{S}(\eta, \eta, \varrho), \mathfrak{S}(\zeta, \eta, \varrho), \mathfrak{S}(\eta, \zeta, \varrho)\}$$
$$= \max\{\mathfrak{S}(\zeta, \eta, \varrho), 0, 0, \mathfrak{S}(\zeta, \eta, \varrho), \mathfrak{S}(\eta, \zeta, \varrho)\} = \mathfrak{S}(\zeta, \eta, \varrho) \text{ and}$$

$$\mathfrak{T}(\zeta, \eta, \mathfrak{d}\varrho) = \mathfrak{T}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \mathfrak{d}\varrho)$$
$$\leq \max\{\mathfrak{T}(\mathfrak{L}\zeta, \mathfrak{W}\eta, \varrho), \mathfrak{T}(\mathfrak{L}\zeta, \ddot{\mathfrak{A}}\zeta, \varrho), \mathfrak{T}(\ddot{\mathfrak{B}}\eta, \mathfrak{W}\eta, \varrho), \mathfrak{T}(\ddot{\mathfrak{A}}\zeta, \mathfrak{W}\eta, \varrho), \mathfrak{T}(\ddot{\mathfrak{B}}\eta, \mathfrak{L}\zeta, \varrho)\}$$
$$= \max\{\mathfrak{T}(\zeta, \eta, \varrho), \mathfrak{T}(\zeta, \zeta, \varrho), \mathfrak{T}(\eta, \eta, \varrho), \mathfrak{T}(\zeta, \eta, \varrho), \mathfrak{T}(\eta, \zeta, \varrho)\}$$
$$= \max\{\mathfrak{T}(\zeta, \eta, \varrho), 0, 0, \mathfrak{T}(\zeta, \eta, \varrho), \mathfrak{T}(\eta, \zeta, \varrho)\} = \mathfrak{T}(\zeta, \eta, \varrho).$$

Therefore using Lemma (2.9), we have $\zeta = \eta$ consequently, ζ is common fixed point of $\ddot{\mathfrak{A}}, \ddot{\mathfrak{B}}, \mathfrak{L}$ and $\mathfrak{W}$. Now,

$$\mathfrak{R}(\zeta, \tau, \mathfrak{d}\varrho) = \mathfrak{R}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\tau, \mathfrak{d}\varrho)$$
$$\geq \min\{\mathfrak{R}(\mathfrak{L}\zeta, \mathfrak{W}\tau, \varrho), \mathfrak{R}(\mathfrak{L}\zeta, \ddot{\mathfrak{A}}\zeta, \varrho), \mathfrak{R}(\ddot{\mathfrak{B}}\tau, \mathfrak{W}\tau, \varrho), \mathfrak{R}(\ddot{\mathfrak{A}}\zeta, \mathfrak{W}\tau, \varrho), \mathfrak{R}(\ddot{\mathfrak{B}}\tau, \mathfrak{L}\zeta, \varrho)\}$$
$$= \min\{\mathfrak{R}(\zeta, \tau, \varrho), \mathfrak{R}(\zeta, \zeta, \varrho), \mathfrak{R}(\tau, \tau, \varrho), \mathfrak{R}(\zeta, \tau, \varrho), \mathfrak{R}(\tau, \zeta, \varrho)\}$$
$$= \min\{\mathfrak{R}(\zeta, \tau, \varrho), 1, 1, \mathfrak{R}(\zeta, \tau, \varrho), \mathfrak{R}(\tau, \zeta, \varrho)\} = \mathfrak{R}(\zeta, \tau, \varrho).$$

$$\mathfrak{S}(\zeta, \tau, \mathfrak{d}\varrho) = \mathfrak{S}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\tau, \mathfrak{d}\varrho)$$
$$\leq \max\{\mathfrak{S}(\mathfrak{L}\zeta, \mathfrak{W}\tau, \varrho), \mathfrak{S}(\mathfrak{L}\zeta, \ddot{\mathfrak{A}}\zeta, \varrho), \mathfrak{S}(\ddot{\mathfrak{B}}\tau, \mathfrak{W}\tau, \varrho), \mathfrak{S}(\ddot{\mathfrak{A}}\zeta, \mathfrak{W}\tau, \varrho), \mathfrak{S}(\ddot{\mathfrak{B}}\tau, \mathfrak{L}\zeta, \varrho)\}$$
$$= \max\{\mathfrak{S}(\zeta, \tau, \varrho), \mathfrak{S}(\zeta, \zeta, \varrho), \mathfrak{S}(\tau, \tau, \varrho), \mathfrak{S}(\zeta, \tau, \varrho), \mathfrak{S}(\tau, \zeta, \varrho)\}$$
$$= \max\{\mathfrak{S}(\zeta, \tau, \varrho), 0, 0, \mathfrak{S}(\zeta, \tau, \varrho), \mathfrak{S}(\tau, \zeta, \varrho)\} = \mathfrak{S}(\zeta, \tau, \varrho) \text{ and}$$

$\mathfrak{T}(\zeta,\tau,\mathfrak{d}\varrho)=\mathfrak{T}(\ddot{\mathfrak{A}}\zeta,\ddot{\mathfrak{B}}\tau,\mathfrak{d}\varrho)$

$\leq \max\{\mathfrak{T}(\mathfrak{L}\zeta,\mathfrak{W}\tau,\varrho),\mathfrak{T}(\mathfrak{L}\zeta,\ddot{\mathfrak{A}}\zeta,\varrho),\mathfrak{T}(\ddot{\mathfrak{B}}\tau,\mathfrak{W}\tau,\varrho),\mathfrak{T}(\ddot{\mathfrak{A}}\zeta,\mathfrak{W}\tau,\varrho),\mathfrak{T}(\ddot{\mathfrak{B}}\tau,\mathfrak{L}\zeta,\varrho)\}$

$=\max\{\mathfrak{T}(\zeta,\tau,\varrho),\mathfrak{T}(\zeta,\zeta,\varrho),\mathfrak{T}(\tau,\tau,\varrho),\mathfrak{T}(\zeta,\tau,\varrho),\mathfrak{T}(\tau,\zeta,\varrho)\}$

$=\max\{\mathfrak{T}(\zeta,\tau,\varrho),0,0,\mathfrak{T}(\zeta,\tau,\varrho),\mathfrak{T}(\tau,\zeta,\varrho)\}=\mathfrak{T}(\zeta,\tau,\varrho).$

By Lemma (2.9), we have $\zeta=\tau$. Hence $\ddot{\mathfrak{A}},\ddot{\mathfrak{B}}$, $\mathfrak{L}$ and $\mathfrak{W}$ have a unique common fixed point.

Example 3.5: Let $\Xi=\mathbb{R}$. Consider the metric $d(\mathfrak{k},\tilde{\varsigma})=|\mathfrak{k}|+|\tilde{\varsigma}|$, for all $\mathfrak{k}\neq\tilde{\varsigma}$ and $d(\mathfrak{k},\tilde{\varsigma})=0$, for $\mathfrak{k}=\tilde{\varsigma}$ on Ξ. Let $\mathfrak{r}*\mathfrak{s}=\min\{\mathfrak{r},\mathfrak{s}\}$ and $\mathfrak{r}\odot\mathfrak{s}=\max\{\mathfrak{r},\mathfrak{s}\}$, for all $\mathfrak{r},\mathfrak{s}\in[0,1]$. For each $\varrho>0$, $\mathfrak{k},\tilde{\varsigma}\in\Xi$, we define $\mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho)=e^{-\frac{|\mathfrak{k}-\tilde{\varsigma}|}{\varrho}}$, $\mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho)=(e^{\frac{|\mathfrak{k}-\tilde{\varsigma}|}{\varrho}}-1)e^{-\frac{|\mathfrak{k}-\tilde{\varsigma}|}{\varrho}}$ and $\mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho)=(e^{\frac{|\mathfrak{k}-\tilde{\varsigma}|}{\varrho}}-1)$. Then $(\Xi,\mathfrak{R},\mathfrak{S},\mathfrak{T}*,\odot)$ is a *NMS* with

$\lim_{\varrho\to\infty}\mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho)=1$, $\lim_{\varrho\to\infty}\mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho)=0$ and $\lim_{\varrho\to\infty}\mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho)=0$, for all $\mathfrak{k},\tilde{\varsigma}\in\Xi$.

Now we define the self maps $\ddot{\mathfrak{A}},\ddot{\mathfrak{B}},\mathfrak{L}$ and $\mathfrak{W}$ on Ξ by $\ddot{\mathfrak{A}}(\mathfrak{k})=\frac{\mathfrak{k}}{9},\ddot{\mathfrak{B}}(\mathfrak{k})=\frac{\mathfrak{k}}{12},\mathfrak{L}(\mathfrak{k})=\frac{\mathfrak{k}}{2},\mathfrak{W}(\mathfrak{k})=\frac{\mathfrak{k}}{4}$.

Let $\mathfrak{d}=\frac{1}{3}$. For $\mathfrak{k}\neq\tilde{\varsigma}$,

$$\mathfrak{R}\left(\ddot{\mathfrak{A}}\mathfrak{k},\ddot{\mathfrak{B}}\tilde{\varsigma},\frac{\varrho}{3}\right)=e^{\frac{-3(|\ddot{\mathfrak{A}}\mathfrak{k}|+|\ddot{\mathfrak{B}}\tilde{\varsigma}|)}{\varrho}}=e^{\frac{-3(|\frac{\mathfrak{k}}{9}|+|\frac{\mathfrak{k}}{12}|)}{\varrho}}=e^{\frac{-(|\frac{\mathfrak{k}}{3}|+|\frac{\mathfrak{k}}{4}|)}{\varrho}}\geq e^{\frac{-(|\frac{\mathfrak{k}}{2}|+|\frac{\mathfrak{k}}{4}|)}{\varrho}}=\mathfrak{R}(\mathfrak{L}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho).$$

$$\mathfrak{S}\left(\ddot{\mathfrak{A}}\mathfrak{k},\ddot{\mathfrak{B}}\tilde{\varsigma},\frac{\varrho}{3}\right)=(e^{\frac{3(|\ddot{\mathfrak{A}}\mathfrak{k}|+|\ddot{\mathfrak{B}}\tilde{\varsigma}|)}{\varrho}}-1)e^{\frac{-3(|\ddot{\mathfrak{A}}\mathfrak{k}|+|\ddot{\mathfrak{B}}\tilde{\varsigma}|)}{\varrho}}=(e^{\frac{3(|\frac{\mathfrak{k}}{9}|+|\frac{\mathfrak{k}}{12}|)}{\varrho}}-1)e^{\frac{-3(|\frac{\mathfrak{k}}{9}|+|\frac{\mathfrak{k}}{12}|)}{\varrho}}=(e^{\frac{(|\frac{\mathfrak{k}}{3}|+|\frac{\mathfrak{k}}{4}|)}{\varrho}}-1)e^{\frac{-(|\frac{\mathfrak{k}}{3}|+|\frac{\mathfrak{k}}{4}|)}{\varrho}}$$

$$\leq(e^{\frac{(|\frac{\mathfrak{k}}{2}|+|\frac{\mathfrak{k}}{4}|)}{\varrho}}-1)e^{\frac{-(|\frac{\mathfrak{k}}{2}|+|\frac{\mathfrak{k}}{4}|)}{\varrho}}=\mathfrak{S}(\mathfrak{L}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho).$$

$$\mathfrak{T}\left(\ddot{\mathfrak{A}}\mathfrak{k},\ddot{\mathfrak{B}}\tilde{\varsigma},\frac{\varrho}{3}\right)=(e^{\frac{3(|\ddot{\mathfrak{A}}\mathfrak{k}|+|\ddot{\mathfrak{B}}\tilde{\varsigma}|)}{\varrho}}-1)=(e^{\frac{3(|\frac{\mathfrak{k}}{9}|+|\frac{\mathfrak{k}}{12}|)}{\varrho}}-1)=(e^{\frac{(|\frac{\mathfrak{k}}{3}|+|\frac{\mathfrak{k}}{4}|)}{\varrho}}-1)\leq(e^{\frac{(|\frac{\mathfrak{k}}{2}|+|\frac{\mathfrak{k}}{4}|)}{\varrho}}-1)$$

$$=\mathfrak{T}(\mathfrak{L}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho).$$

For $\mathfrak{k}=\tilde{\varsigma}$, $\mathfrak{R}\left(\ddot{\mathfrak{A}}\mathfrak{k},\ddot{\mathfrak{B}}\tilde{\varsigma},\frac{\varrho}{3}\right)=1=\mathfrak{R}(\mathfrak{L}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho),\mathfrak{S}\left(\ddot{\mathfrak{A}}\mathfrak{k},\ddot{\mathfrak{B}}\tilde{\varsigma},\frac{\varrho}{3}\right)=0=\mathfrak{S}(\mathfrak{L}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho)$ and

$\mathfrak{T}\left(\ddot{\mathfrak{A}}\mathfrak{k},\ddot{\mathfrak{B}}\tilde{\varsigma},\frac{\varrho}{3}\right)=0=\mathfrak{T}(\mathfrak{L}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho)$. So that for any $\mathfrak{k},\tilde{\varsigma}\in\Xi$,

$\mathfrak{R}\left(\ddot{\mathfrak{A}}\mathfrak{k},\ddot{\mathfrak{B}}\tilde{\varsigma},\frac{\varrho}{3}\right)\geq\mathfrak{R}(\mathfrak{L}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho)$

$=\min\{\mathfrak{R}(\mathfrak{L}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho),\mathfrak{R}(\mathfrak{L}\mathfrak{k},\ddot{\mathfrak{A}}\mathfrak{k},\varrho),\mathfrak{R}(\ddot{\mathfrak{B}}\tilde{\varsigma},\mathfrak{W}\tilde{\varsigma},\varrho),\mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho),\mathfrak{R}(\ddot{\mathfrak{B}}\tilde{\varsigma},\mathfrak{L}\mathfrak{k},\varrho)\}$.

$\mathfrak{S}\left(\ddot{\mathfrak{A}}\mathfrak{k},\ddot{\mathfrak{B}}\tilde{\varsigma},\frac{\varrho}{3}\right)\leq\mathfrak{S}(\mathfrak{L}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho)$

$=\max\{\mathfrak{S}(\mathfrak{L}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho),\mathfrak{S}(\mathfrak{L}\mathfrak{k},\ddot{\mathfrak{A}}\mathfrak{k},\varrho),\mathfrak{S}(\ddot{\mathfrak{B}}\tilde{\varsigma},\mathfrak{W}\tilde{\varsigma},\varrho),\mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho),\mathfrak{S}(\ddot{\mathfrak{B}}\tilde{\varsigma},\mathfrak{L}\mathfrak{k},\varrho)\}$.

$\mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k},\ddot{\mathfrak{B}}\tilde{\varsigma},\mathfrak{d}\varrho)\leq\mathfrak{T}(\mathfrak{L}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho)$

$\mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k},\ddot{\mathfrak{B}}\tilde{\varsigma},\mathfrak{d}\varrho)=\max\{\mathfrak{T}(\mathfrak{L}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho),\mathfrak{T}(\mathfrak{L}\mathfrak{k},\ddot{\mathfrak{A}}\mathfrak{k},\varrho),\mathfrak{T}(\ddot{\mathfrak{B}}\tilde{\varsigma},\mathfrak{W}\tilde{\varsigma},\varrho),\mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho),\mathfrak{T}(\ddot{\mathfrak{B}}\tilde{\varsigma},\mathfrak{L}\mathfrak{k},\varrho)\}$.

Hence, the maps $\ddot{\mathfrak{A}}, \ddot{\mathfrak{B}}, \mathfrak{L}$ and $\mathfrak{W}$ satisfies the condition (3.4.1) of Theorem (3.4) for $\mathfrak{d} = \frac{1}{3}$.

Also, the pairs $\{\ddot{\mathfrak{A}}, \mathfrak{L}\}$ and $\{\ddot{\mathfrak{B}}, \mathfrak{W}\}$ are obviously *OWC*. Thus all the condition of Theorem (3.4) are satisfied at $\mathfrak{k} = 0$ is the unique common fixed point of $\ddot{\mathfrak{A}}, \ddot{\mathfrak{B}}, \mathfrak{L}$ and $\mathfrak{W}$ in Ξ.

Theorem 3.6: Let $(\Xi, \mathfrak{R}, \mathfrak{S}, \mathfrak{T}, *, \odot)$ be a *NMS* with $\lim_{\varrho\to\infty} \mathfrak{R}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = 1$, $\lim_{\varrho\to\infty} \mathfrak{S}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = 0$ and $\lim_{\varrho\to\infty} \mathfrak{T}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = 0$, for all $\mathfrak{k}, \tilde{\varsigma} \in \Xi$ and $\ddot{\mathfrak{A}}, \ddot{\mathfrak{B}}, \mathfrak{L}$ and $\mathfrak{W}$ be self mappings on Ξ. Let the pairs $\{\ddot{\mathfrak{A}}, \mathfrak{L}\}$ and $\{\ddot{\mathfrak{B}}, \mathfrak{W}\}$ be *OWC*. If there exists $\mathfrak{d} \in (0, 1)$ such that

$$\mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \geq \hbar\left(min\{\mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{R}(\mathfrak{L}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho), \mathfrak{R}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{R}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{L}\mathfrak{k}, \varrho)\}\right)$$

$$\mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \leq \hbar(max\{\mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{R}(\mathfrak{L}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho), \mathfrak{R}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{R}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{L}\mathfrak{k}, \varrho)\} \text{ and}$$

$$\mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \leq \hbar(max\{\mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{R}(\mathfrak{L}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho), \mathfrak{R}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{R}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{L}\mathfrak{k}, \varrho)\}$$

for all $\mathfrak{k}, \tilde{\varsigma} \in \Xi$ and $\varrho > 0$, where $\hbar : [0,1] \to [0,1]$ with $\hbar(\mathfrak{k}) > k$ for all $\mathfrak{k} \in [0,1]$.

Then $\ddot{\mathfrak{A}}, \ddot{\mathfrak{B}}, \mathfrak{L}$ and $\mathfrak{W}$ have a unique common fixed point in Ξ.

Proof: The proof follows from Theorem (3.4).

Theorem 3.7. Let $(\Xi, \mathfrak{R}, \mathfrak{S}, \mathfrak{T}, *, \odot)$ be a *NMS* with $\lim_{\varrho\to\infty} \mathfrak{R}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = 1$, $\lim_{\varrho\to\infty} \mathfrak{S}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = 0$ and $\lim_{\varrho\to\infty} \mathfrak{T}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = 0$, for all $\mathfrak{k}, \tilde{\varsigma} \in \Xi$ and $\ddot{\mathfrak{A}}, \ddot{\mathfrak{B}}, \mathfrak{L}$ and $\mathfrak{W}$ be self mappings on Ξ. Let the pairs $\{\ddot{\mathfrak{A}}, \mathfrak{L}\}$ and $\{\ddot{\mathfrak{B}}, \mathfrak{W}\}$ be *OWC*. If there exists $\mathfrak{d} \in (0, 1)$ such that

$$\mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \geq \mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho) * \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{L}\mathfrak{k}, \varrho) * \mathfrak{R}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho) * \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho) \quad (3.7.1)$$

$$\mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \leq \mathfrak{S}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{L}\mathfrak{k}, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho) \quad (3.7.2)$$

$$\mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \leq \mathfrak{T}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{L}\mathfrak{k}, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho) \quad (3.7.3)$$

for all $\mathfrak{k}, \tilde{\varsigma} \in \Xi$ and $\varrho > 0$. Then $\ddot{\mathfrak{A}}, \ddot{\mathfrak{B}}, \mathfrak{L}$ and $\mathfrak{W}$ have a unique common fixed point in Ξ.

Proof: The pairs $\{\ddot{\mathfrak{A}}, \mathfrak{L}\}$ and $\{\ddot{\mathfrak{B}}, \mathfrak{W}\}$ be *OWC*, so there are point $\mathfrak{k}, \tilde{\varsigma} \in \Xi$ such that $\ddot{\mathfrak{A}}(\mathfrak{k}) = \mathfrak{L}(\mathfrak{k})$ and $\ddot{\mathfrak{B}}(\tilde{\varsigma}) = \mathfrak{W}(\tilde{\varsigma})$. Now, by the given conditions (3.7.1), (3.7.2) and (3.7.3), we get

$$\mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \geq \mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho) * \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{L}\mathfrak{k}, \varrho) * \mathfrak{R}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho) * \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho)$$

$$= \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho) * \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho) * \mathfrak{R}(\ddot{\mathfrak{B}}\mathfrak{k}, \ddot{\mathfrak{B}}\mathfrak{k}, \varrho) * \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho)$$

$$= \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho) * 1 * 1 * \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho) = \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho).$$

$$\mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \leq \mathfrak{S}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{L}\mathfrak{k}, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho)$$

$$= \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{B}}\mathfrak{k}, \ddot{\mathfrak{B}}\mathfrak{k}, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho)$$

$$= \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho) \odot 0 \odot 0 \odot \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho) = \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho) \text{ and}$$

$$\mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \leq \mathfrak{T}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{L}\mathfrak{k}, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho)$$

$= \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{B}}\mathfrak{k}, \ddot{\mathfrak{B}}\mathfrak{k}, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho)$

$= \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho) \odot 0 \odot 0 \odot \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho) = \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho)$.

In view of Lemma (2.10), we have $\ddot{\mathfrak{A}}\mathfrak{k} = \ddot{\mathfrak{B}}\tilde{\varsigma}$ and therefore $\ddot{\mathfrak{A}}\mathfrak{k} = \mathfrak{L}\mathfrak{k} = \ddot{\mathfrak{B}}\tilde{\varsigma} = \mathfrak{W}\tilde{\varsigma}$. (3.7.4)

Suppose the pair $\{\ddot{\mathfrak{A}}, \mathfrak{L}\}$ have an another coincidence point $\mathfrak{w} \in \Xi$, i.e.,$\ddot{\mathfrak{A}}\mathfrak{w} = \mathfrak{L}\mathfrak{w}$.

$\mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \geq \mathfrak{R}(\mathfrak{L}\mathfrak{w}, \mathfrak{W}\tilde{\varsigma}, \varrho) * \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{w}, \mathfrak{L}\mathfrak{w}, \varrho) * \mathfrak{R}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho) * \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{w}, \mathfrak{W}\tilde{\varsigma}, \varrho)$

$= \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho) * \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{A}}\mathfrak{w}, \varrho) * \mathfrak{R}(\ddot{\mathfrak{B}}\mathfrak{w}, \ddot{\mathfrak{B}}\mathfrak{w}, \varrho) * \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho)$

$= \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho) * 1 * 1 * \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho) = \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho)$.

$\mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \leq \mathfrak{S}(\mathfrak{L}\mathfrak{w}, \mathfrak{W}\tilde{\varsigma}, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{w}, \mathfrak{L}\mathfrak{w}, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{w}, \mathfrak{W}\tilde{\varsigma}, \varrho)$

$= \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{A}}\mathfrak{w}, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{B}}\mathfrak{w}, \ddot{\mathfrak{B}}\mathfrak{w}, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho)$

$= \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho) \odot 0 \odot 0 \odot \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho) = \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho)$ and

$\mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \leq \mathfrak{T}(\mathfrak{L}\mathfrak{w}, \mathfrak{W}\tilde{\varsigma}, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{w}, \mathfrak{L}\mathfrak{w}, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{w}, \mathfrak{W}\tilde{\varsigma}, \varrho)$

$= \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{A}}\mathfrak{w}, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{B}}\mathfrak{w}, \ddot{\mathfrak{B}}\mathfrak{w}, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho)$

$= \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho) \odot 0 \odot 0 \odot \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho) = \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{w}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \varrho)$.

By lemma (2.10), $\ddot{\mathfrak{A}}\mathfrak{w} = \ddot{\mathfrak{B}}\tilde{\varsigma}$ and consequently $\ddot{\mathfrak{A}}\mathfrak{w} = \mathfrak{L}\mathfrak{w} = \ddot{\mathfrak{B}}\tilde{\varsigma} = \mathfrak{W}\tilde{\varsigma}$. (3.7.5)

From (3.7.4) and (3.7.5) $\ddot{\mathfrak{A}}\mathfrak{k} = \ddot{\mathfrak{A}}\mathfrak{w}$ and therefore the pair $\{\ddot{\mathfrak{A}}, \mathfrak{L}\}$ have a unique point of coincidence $\zeta = \ddot{\mathfrak{A}}\mathfrak{k} = \mathfrak{L}\mathfrak{k}$. ζ is the unique common fixed point of $\{\ddot{\mathfrak{A}}, \mathfrak{L}\}$. Similarly, we can show that there is unique common fixed point $\eta \in \Xi$ of $\{\ddot{\mathfrak{B}}, \mathfrak{W}\}$. Now,

$$\mathfrak{R}(\zeta, \eta, \mathfrak{d}\varrho) = \mathfrak{R}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \mathfrak{d}\varrho) \geq \mathfrak{R}(\mathfrak{L}\zeta, \mathfrak{W}\eta, \varrho) * \mathfrak{R}(\ddot{\mathfrak{A}}\zeta, \mathfrak{L}\zeta, \varrho) * \mathfrak{R}(\ddot{\mathfrak{B}}\eta, \mathfrak{W}\eta, \varrho) * \mathfrak{R}(\ddot{\mathfrak{A}}\zeta, \mathfrak{W}\eta, \varrho)$$

$$= \mathfrak{R}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \varrho) * \mathfrak{R}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{A}}\zeta, \varrho) * \mathfrak{R}(\ddot{\mathfrak{B}}\eta, \ddot{\mathfrak{B}}\eta, \varrho) * \mathfrak{R}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \varrho)$$

$= \mathfrak{R}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \varrho) * 1 * 1 * \mathfrak{R}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \varrho) = \mathfrak{R}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \varrho) = \mathfrak{R}(\zeta, \eta, \varrho)$.

$\mathfrak{S}(\zeta, \eta, \mathfrak{d}\varrho) = \mathfrak{S}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \mathfrak{d}\varrho) \leq \mathfrak{S}(\mathfrak{L}\zeta, \mathfrak{W}\eta, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{A}}\zeta, \mathfrak{L}\zeta, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{B}}\eta, \mathfrak{W}\eta, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{A}}\zeta, \mathfrak{W}\eta, \varrho)$

$= \mathfrak{S}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{A}}\zeta, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{B}}\eta, \ddot{\mathfrak{B}}\eta, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \varrho)$

$= \mathfrak{S}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \varrho) \odot 0 \odot 0 \odot \mathfrak{S}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \varrho) = \mathfrak{S}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \varrho) = \mathfrak{S}(\zeta, \eta, \varrho)$ and

$\mathfrak{T}(\zeta, \eta, \mathfrak{d}\varrho) = \mathfrak{T}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \mathfrak{d}\varrho) \leq \mathfrak{T}(\mathfrak{L}\zeta, \mathfrak{W}\eta, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{A}}\zeta, \mathfrak{L}\zeta, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{B}}\eta, \mathfrak{W}\eta, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{A}}\zeta, \mathfrak{W}\eta, \varrho)$

$= \mathfrak{T}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{A}}\zeta, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{B}}\eta, \ddot{\mathfrak{B}}\eta, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \varrho)$

$= \mathfrak{T}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \varrho) \odot 0 \odot 0 \odot \mathfrak{T}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \varrho) = \mathfrak{T}(\ddot{\mathfrak{A}}\zeta, \ddot{\mathfrak{B}}\eta, \varrho) = \mathfrak{T}(\zeta, \eta, \varrho)$.

By lemma (2.10), we have $\zeta = \eta$ and consequently ζ is common fixed Point of $\ddot{\mathfrak{A}}, \ddot{\mathfrak{B}}$, $\mathfrak{L}$ and $\mathfrak{W}$. For uniqueness, let τ is an another common fixed point of $\ddot{\mathfrak{A}}, \ddot{\mathfrak{B}}$, $\mathfrak{L}$ and $\mathfrak{W}$. Therefore,

$$\mathfrak{R}(\zeta,\tau,\mathfrak{d}\varrho)=\mathfrak{R}(\ddot{\mathfrak{A}}\zeta,\ddot{\mathfrak{B}}\tau,\mathfrak{d}\varrho)\geq\mathfrak{R}(\mathfrak{L}\zeta,\mathfrak{W}\tau,\varrho)*\mathfrak{R}(\ddot{\mathfrak{A}}\zeta,\mathfrak{L}\zeta,\varrho)*\mathfrak{R}(\ddot{\mathfrak{B}}\tau,\mathfrak{W}\tau,\varrho)*\mathfrak{R}(\ddot{\mathfrak{A}}\zeta,\mathfrak{W}\tau,\varrho)$$

$$=\mathfrak{R}(\zeta,\tau,\varrho)*\mathfrak{R}(\zeta,\zeta,\varrho)*\mathfrak{R}(\tau,\tau,\varrho)*\mathfrak{R}(\zeta,\tau,\varrho)$$

$$=\mathfrak{R}(\zeta,\tau,\varrho)*1*1*\mathfrak{R}(\zeta,\tau,\varrho)=\mathfrak{R}(\zeta,\tau,\varrho).$$

$$\mathfrak{S}(\zeta,\tau,\mathfrak{d}\varrho)=\mathfrak{S}(\ddot{\mathfrak{A}}\zeta,\ddot{\mathfrak{B}}\tau,\mathfrak{d}\varrho)\leq\mathfrak{S}(\mathfrak{L}\zeta,\mathfrak{W}\tau,\varrho)\odot\mathfrak{S}(\ddot{\mathfrak{A}}\zeta,\mathfrak{L}\zeta,\varrho)\odot\mathfrak{S}(\ddot{\mathfrak{B}}\tau,\mathfrak{W}\tau,\varrho)\odot\mathfrak{S}(\ddot{\mathfrak{A}}\zeta,\mathfrak{W}\tau,\varrho)$$

$$=\mathfrak{S}(\zeta,\tau,\varrho)\odot\mathfrak{S}(\zeta,\zeta,\varrho)\odot\mathfrak{S}(\tau,\tau,\varrho)\odot\mathfrak{S}(\zeta,\tau,\varrho)$$

$$=\mathfrak{S}(\zeta,\tau,\varrho)\odot 0\odot 0\odot\mathfrak{S}(\zeta,\tau,\varrho)=\mathfrak{S}(\zeta,\tau,\varrho) \text{ and}$$

$$\mathfrak{T}(\zeta,\tau,\mathfrak{d}\varrho)=\mathfrak{T}(\ddot{\mathfrak{A}}\zeta,\ddot{\mathfrak{B}}\tau,\mathfrak{d}\varrho)\leq\mathfrak{T}(\mathfrak{L}\zeta,\mathfrak{W}\tau,\varrho)\odot\mathfrak{T}(\ddot{\mathfrak{A}}\zeta,\mathfrak{L}\zeta,\varrho)\odot\mathfrak{T}(\ddot{\mathfrak{B}}\tau,\mathfrak{W}\tau,\varrho)\odot\mathfrak{T}(\ddot{\mathfrak{A}}\zeta,\mathfrak{W}\tau,\varrho)$$

$$=\mathfrak{T}(\zeta,\tau,\varrho)\odot\mathfrak{T}(\zeta,\zeta,\varrho)\odot\mathfrak{T}(\tau,\tau,\varrho)\odot\mathfrak{T}(\zeta,\tau,\varrho)$$

$$=\mathfrak{T}(\zeta,\tau,\varrho)\odot 0\odot 0\odot\mathfrak{T}(\zeta,\tau,\varrho)=\mathfrak{T}(\zeta,\tau,\varrho).$$

In view of Lemma (2.10), we have $\zeta=\tau$. Hence $\ddot{\mathfrak{A}},\ddot{\mathfrak{B}}$, $\mathfrak{L}$ and $\mathfrak{W}$ have a unique common fixed point.

Example 3.8: Let $\Xi=\mathbb{R}$. Consider the metric $d(\mathfrak{k},\tilde{\varsigma})=|\mathfrak{k}|+|\tilde{\varsigma}|$, for all $\mathfrak{k}\neq\tilde{\varsigma}$ and $d(\mathfrak{k},\tilde{\varsigma})=0$, for $\mathfrak{k}=\tilde{\varsigma}$. Let $\mathfrak{r}*\mathfrak{s}=\min\{\mathfrak{r},\mathfrak{s}\}$ and $\mathfrak{r}\odot\mathfrak{s}=\max\{\mathfrak{r},\mathfrak{s}\}$, for all $\mathfrak{r},\mathfrak{s}\in[0,1]$. For each $\varrho>0$, $\mathfrak{k},\tilde{\varsigma}\in\Xi$, we define $\mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho)=e^{-\frac{|\mathfrak{k}-\tilde{\varsigma}|}{\varrho}}$, $\mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho)=(e^{\frac{|\mathfrak{k}-\tilde{\varsigma}|}{\varrho}}-1)e^{-\frac{|\mathfrak{k}-\tilde{\varsigma}|}{\varrho}}$ and $\mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho)=(e^{\frac{|\mathfrak{k}-\tilde{\varsigma}|}{\varrho}}-1)$.

Then $(\Xi,\mathfrak{R},\mathfrak{S},\mathfrak{T}*,\odot)$ is a NMS with $\lim_{\varrho\to\infty}\mathfrak{R}(\mathfrak{k},\tilde{\varsigma},\varrho)=1$, $\lim_{\varrho\to\infty}\mathfrak{S}(\mathfrak{k},\tilde{\varsigma},\varrho)=0$ and $\lim_{\varrho\to\infty}\mathfrak{T}(\mathfrak{k},\tilde{\varsigma},\varrho)=0$,

for all $\mathfrak{k},\tilde{\varsigma}\in\Xi$.

Now we define the self maps $\ddot{\mathfrak{A}},\ddot{\mathfrak{B}},\mathfrak{L}$ and $\mathfrak{W}$ on Ξ by $\ddot{\mathfrak{A}}(\mathfrak{k})=\frac{\mathfrak{k}}{10},\ddot{\mathfrak{B}}(\mathfrak{k})=\frac{\mathfrak{k}}{15},\mathfrak{L}(\mathfrak{k})=\mathfrak{k},\mathfrak{W}(\mathfrak{k})=\frac{\mathfrak{k}}{3}$.

Let $\mathfrak{d}=\frac{1}{5}$. For $\mathfrak{k}\neq\tilde{\varsigma}$,

$$\mathfrak{R}\left(\ddot{\mathfrak{A}}\mathfrak{k},\ddot{\mathfrak{B}}\tilde{\varsigma},\frac{\varrho}{5}\right)=e^{\frac{-5(|\ddot{\mathfrak{A}}\mathfrak{k}|+|\ddot{\mathfrak{B}}\tilde{\varsigma}|)}{\varrho}}=e^{\frac{-5\left(\left|\frac{\mathfrak{k}}{10}\right|+\left|\frac{\mathfrak{k}}{15}\right|\right)}{\varrho}}=e^{\frac{-\left(\left|\frac{\mathfrak{k}}{2}\right|+\left|\frac{\mathfrak{k}}{3}\right|\right)}{\varrho}}\geq e^{\frac{-\left(|\mathfrak{k}|+\left|\frac{\mathfrak{k}}{3}\right|\right)}{\varrho}}=\mathfrak{R}(\mathfrak{L}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho),$$

$$\mathfrak{S}\left(\ddot{\mathfrak{A}}\mathfrak{k},\ddot{\mathfrak{B}}\tilde{\varsigma},\frac{\varrho}{3}\right)=(e^{\frac{5(|\ddot{\mathfrak{A}}\mathfrak{k}|+|\ddot{\mathfrak{B}}\tilde{\varsigma}|)}{\varrho}}-1)=(e^{\frac{5\left(\left|\frac{\mathfrak{k}}{9}\right|+\left|\frac{\mathfrak{k}}{12}\right|\right)}{\varrho}}-1)e^{\frac{-3\left(\left|\frac{\mathfrak{k}}{9}\right|+\left|\frac{\mathfrak{k}}{12}\right|\right)}{\varrho}}$$

$$=(e^{\frac{\left(\left|\frac{\mathfrak{k}}{2}\right|+\left|\frac{\mathfrak{k}}{3}\right|\right)}{\varrho}}-1)e^{\frac{-\left(\left|\frac{\mathfrak{k}}{2}\right|+\left|\frac{\mathfrak{k}}{3}\right|\right)}{\varrho}}\leq(e^{\frac{\left(|\mathfrak{k}|+\left|\frac{\mathfrak{k}}{3}\right|\right)}{\varrho}}1)e^{\frac{-\left(|\mathfrak{k}|+\left|\frac{\mathfrak{k}}{3}\right|\right)}{\varrho}}=\mathfrak{S}(\mathfrak{L}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho)$$

$$\mathfrak{T}\left(\ddot{\mathfrak{A}}\mathfrak{k},\ddot{\mathfrak{B}}\tilde{\varsigma},\frac{\varrho}{5}\right)=(e^{\frac{5(|\ddot{\mathfrak{A}}\mathfrak{k}|+|\ddot{\mathfrak{B}}\tilde{\varsigma}|)}{\varrho}}-1)=(e^{\frac{5\left(\left|\frac{\mathfrak{k}}{2}\right|+\left|\frac{\mathfrak{k}}{3}\right|\right)}{\varrho}}-1)=(e^{\frac{\left(\left|\frac{\mathfrak{k}}{2}\right|+\left|\frac{\mathfrak{k}}{3}\right|\right)}{\varrho}}-1)\leq(e^{\frac{\left(|\mathfrak{k}|+\left|\frac{\mathfrak{k}}{3}\right|\right)}{\varrho}}-1)$$

$$=\mathfrak{T}(\mathfrak{L}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho).$$

For $\mathfrak{k}=\tilde{\varsigma}$.

$\mathfrak{R}\left(\ddot{\mathfrak{A}}\mathfrak{k},\ddot{\mathfrak{B}}\tilde{\varsigma},\frac{\varrho}{5}\right)=1=\mathfrak{R}(\mathfrak{L}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho)$, $\mathfrak{S}\left(\ddot{\mathfrak{A}}\mathfrak{k},\ddot{\mathfrak{B}}\tilde{\varsigma},\frac{\varrho}{5}\right)=0=\mathfrak{S}(\mathfrak{L}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho)$ and

$\mathfrak{T}\left(\ddot{\mathfrak{A}}\mathfrak{k},\ddot{\mathfrak{B}}\tilde{\varsigma},\frac{\varrho}{5}\right)=0=\mathfrak{T}(\mathfrak{L}\mathfrak{k},\mathfrak{W}\tilde{\varsigma},\varrho)$. So that for any $\mathfrak{k},\tilde{\varsigma}\in\Xi$,

$\mathfrak{R}\left(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \frac{\varrho}{5}\right) \geq \mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho)$

$= \min\{\mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{R}(\mathfrak{L}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho), \mathfrak{R}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{R}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{L}\mathfrak{k}, \varrho)\}$

$\mathfrak{S}\left(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \frac{\varrho}{5}\right) \leq \mathfrak{S}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho)$

$= \max\{\mathfrak{S}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{S}(\mathfrak{L}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho), \mathfrak{S}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{S}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{L}\mathfrak{k}, \varrho)\}$

$\mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{d}\varrho) \leq \mathfrak{T}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho)$

$= \max\{\mathfrak{T}(\mathfrak{L}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{T}(\mathfrak{L}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho), \mathfrak{T}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \widetilde{\mathfrak{W}\varsigma}, \varrho), \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{W}\tilde{\varsigma}, \varrho), \mathfrak{T}(\ddot{\mathfrak{B}}\tilde{\varsigma}, \mathfrak{L}\mathfrak{k}, \varrho)\}.$

Hence, the maps $\ddot{\mathfrak{A}}, \ddot{\mathfrak{B}}, \mathfrak{L}$ and $\mathfrak{W}$ satisfies the condition (3.7.1), (3.7.2) and (3.7.3) of Theorem (3.7) for $\mathfrak{d} = \frac{1}{5}$. Also, the pairs $\{\ddot{\mathfrak{A}}, \mathfrak{L}\}$ and $\{\ddot{\mathfrak{B}}, \mathfrak{W}\}$ are obviously *OWC*.

Thus all the condition of Theorem (3.6) are satisfied at $\mathfrak{k} = 0$ is the unique common fixed point of $\ddot{\mathfrak{A}}, \ddot{\mathfrak{B}}, \mathfrak{L}$ and $\mathfrak{W}$ in Ξ.

Corollary 3.9: Let $(\Xi, \mathfrak{R}, \mathfrak{S}, \mathfrak{T}, *, \odot)$ be a *NMS* with $\lim_{\varrho\to\infty} \mathfrak{R}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = 1$, $\lim_{\varrho\to\infty} \mathfrak{S}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = 0$ and $\lim_{\varrho\to\infty} \mathfrak{T}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = 0$, for all $\mathfrak{k}, \tilde{\varsigma} \in \Xi$ and let $\ddot{\mathfrak{A}}$ and $\mathfrak{L}$ be self mappings on Ξ. Let $\ddot{\mathfrak{A}}, \mathfrak{L}$ be self mappings on Ξ . Let the pair $\{\ddot{\mathfrak{A}}, \mathfrak{L}\}$ be *OWC*. If there exists $\mathfrak{d} \in (0, 1)$ such that

$\mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{A}}\tilde{\varsigma}, \mathfrak{d}\varrho) \geq \mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho) * \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{L}\mathfrak{k}, \varrho) * \mathfrak{R}(\ddot{\mathfrak{A}}\tilde{\varsigma}, \mathfrak{L}\tilde{\varsigma}, \varrho) * \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho),$

$\mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{A}}\tilde{\varsigma}, \mathfrak{d}\varrho) \leq \mathfrak{S}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{L}\mathfrak{k}, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{A}}\tilde{\varsigma}, \mathfrak{L}\tilde{\varsigma}, \varrho) \odot \mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho)$ and

$\mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{A}}\tilde{\varsigma}, \mathfrak{d}\varrho) \leq \mathfrak{T}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{L}\mathfrak{k}, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{A}}\tilde{\varsigma}, \mathfrak{L}\tilde{\varsigma}, \varrho) \odot \mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho)$

for all $\mathfrak{k}, \tilde{\varsigma} \in \Xi$ and $\varrho > 0$. Then $\ddot{\mathfrak{A}}$ and $\mathfrak{L}$ have a unique common fixed point in Ξ.

Theorem 3.10: Let $(\Xi, \mathfrak{R}, \mathfrak{S}, \mathfrak{T}, *, \odot)$ be a *NMS* with $\lim_{\varrho\to\infty} \mathfrak{R}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = 1$, $\lim_{\varrho\to\infty} \mathfrak{S}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = 0$ and $\lim_{\varrho\to\infty} \mathfrak{T}(\mathfrak{k}, \tilde{\varsigma}, \varrho) = 0$, for all $\mathfrak{k}, \tilde{\varsigma} \in \Xi$. Let the pair $\{\ddot{\mathfrak{A}}, \mathfrak{L}\}$ be *OWC*. If there exists $\mathfrak{d} \in (0, 1)$ such that

$\mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \mathfrak{d}\varrho) \geq \tilde{\mathfrak{f}}\mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{A}}\tilde{\varsigma}, \varrho) + \tilde{\mathfrak{g}}\, min\{\mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{A}}\tilde{\varsigma}, \varrho), \mathfrak{R}(\mathfrak{L}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho), \mathfrak{R}(\mathfrak{L}\tilde{\varsigma}, \ddot{\mathfrak{A}}\tilde{\varsigma}, \varrho)\}$

$\mathfrak{S}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \mathfrak{d}\varrho) \leq \tilde{\mathfrak{f}}\mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{A}}\tilde{\varsigma}, \varrho) + \tilde{\mathfrak{g}}\, max\{\mathfrak{S}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{A}}\tilde{\varsigma}, \varrho)\mathfrak{S}(\mathfrak{L}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho), \mathfrak{S}(\mathfrak{L}\tilde{\varsigma}, \ddot{\mathfrak{A}}\tilde{\varsigma}, \varrho)\}$ and

$\mathfrak{T}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \mathfrak{d}\varrho) \leq \tilde{\mathfrak{f}}\mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{A}}\tilde{\varsigma}, \varrho) + \tilde{\mathfrak{g}}\, max\{\mathfrak{T}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{A}}\tilde{\varsigma}, \varrho), \mathfrak{T}(\mathfrak{L}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho), \mathfrak{T}(\mathfrak{L}\tilde{\varsigma}, \ddot{\mathfrak{A}}\tilde{\varsigma}, \varrho)\}$ (3.10.1)

Proof: The pairs are *OWC*, so there exists $\mathfrak{k} \in \Xi$ such that $\ddot{\mathfrak{A}}(\mathfrak{k}) = \mathfrak{L}(\mathfrak{k})$. Suppose that there exists another $\tilde{\varsigma} \in \Xi$ for which $\ddot{\mathfrak{A}}(\tilde{\varsigma}) = \mathfrak{L}(\tilde{\varsigma})$. From the condition (3.10.1),

$\mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \mathfrak{d}\varrho) \geq \tilde{\mathfrak{f}}\, \mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{A}}\tilde{\varsigma}, \varrho) + \tilde{\mathfrak{g}}\, min\{\mathfrak{R}(\ddot{\mathfrak{A}}\mathfrak{k}, \ddot{\mathfrak{A}}\tilde{\varsigma}, \varrho), \mathfrak{R}(\mathfrak{L}\mathfrak{k}, \ddot{\mathfrak{A}}\mathfrak{k}, \varrho), \mathfrak{R}(\mathfrak{L}\tilde{\varsigma}, \ddot{\mathfrak{A}}\tilde{\varsigma}, \varrho)\}$

$=\tilde{\mathfrak{f}}\, \mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho) + \tilde{\mathfrak{g}}\, min\{\mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho), \mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\mathfrak{k}, \varrho), \mathfrak{R}(\mathfrak{L}\tilde{\varsigma}, \mathfrak{L}\tilde{\varsigma}, \varrho)\}$

$= \tilde{\mathfrak{f}}\, \mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho) + \tilde{\mathfrak{g}}\, min\{\mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho), 1,1\}$

$=\tilde{\mathfrak{f}}\, \mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho) + \tilde{\mathfrak{g}}\mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho) = (\tilde{\mathfrak{f}} + \tilde{\mathfrak{g}})\mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho).$

Since $\tilde{\mathfrak{f}} + \tilde{\mathfrak{g}} \geq 1, \mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \eth\varrho) \geq \mathfrak{R}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho)$.

$$\mathfrak{S}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \eth\varrho) \leq \tilde{\mathfrak{f}}\, \mathfrak{S}(\ddot{\mathfrak{U}}\mathfrak{k}, \ddot{\mathfrak{U}}\tilde{\varsigma}, \varrho) + \tilde{\mathfrak{g}}\, max\{\mathfrak{S}(\ddot{\mathfrak{U}}\mathfrak{k}, \ddot{\mathfrak{U}}\tilde{\varsigma}, \varrho), \mathfrak{S}(\mathfrak{L}\mathfrak{k}, \ddot{\mathfrak{U}}\mathfrak{k}, \varrho), \mathfrak{S}(\mathfrak{L}\tilde{\varsigma}, \ddot{\mathfrak{U}}\tilde{\varsigma}, \varrho)\}$$

$$= \tilde{\mathfrak{f}}\, \mathfrak{S}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho) + \tilde{\mathfrak{g}}\, max\{\mathfrak{S}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho), \mathfrak{S}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\mathfrak{k}, \varrho), \mathfrak{S}(\mathfrak{L}\tilde{\varsigma}, \mathfrak{L}\tilde{\varsigma}, \varrho)\}$$

$$= \tilde{\mathfrak{f}}\, \mathfrak{S}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho) + \tilde{\mathfrak{g}}\, max\{\mathfrak{S}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho), 0,0\}$$

$$= \tilde{\mathfrak{f}}\, \mathfrak{S}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho) + \tilde{\mathfrak{g}}\mathfrak{S}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho) = (\tilde{\mathfrak{f}} + \tilde{\mathfrak{g}})\mathfrak{S}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho).$$

Since $\tilde{\mathfrak{f}} + \tilde{\mathfrak{g}} \geq 1, \mathfrak{S}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \eth\varrho) \leq \mathfrak{S}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho)$

$$\mathfrak{T}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \eth\varrho) \leq \tilde{\mathfrak{f}}\, \mathfrak{T}(\ddot{\mathfrak{U}}\mathfrak{k}, \ddot{\mathfrak{U}}\tilde{\varsigma}, \varrho) + \tilde{\mathfrak{g}}\, max\{\mathfrak{T}(\ddot{\mathfrak{U}}\mathfrak{k}, \ddot{\mathfrak{U}}\tilde{\varsigma}, \varrho), \mathfrak{T}(\mathfrak{L}\mathfrak{k}, \ddot{\mathfrak{U}}\mathfrak{k}, \varrho), \mathfrak{T}(\mathfrak{L}\tilde{\varsigma}, \ddot{\mathfrak{U}}\tilde{\varsigma}, \varrho)\}$$

$$= \tilde{\mathfrak{f}}\, \mathfrak{T}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho) + \tilde{\mathfrak{g}}\, max\{\mathfrak{T}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho), \mathfrak{T}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\mathfrak{k}, \varrho), \mathfrak{T}(\mathfrak{L}\tilde{\varsigma}, \mathfrak{L}\tilde{\varsigma}, \varrho)\}$$

$$= \tilde{\mathfrak{f}}\, \mathfrak{T}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho) + \tilde{\mathfrak{g}}\, max\{\mathfrak{T}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho), 0,0\}$$

$$= \tilde{\mathfrak{f}}\, \mathfrak{T}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho) + \tilde{\mathfrak{g}}\mathfrak{T}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho) = (\tilde{\mathfrak{f}} + \tilde{\mathfrak{g}})\mathfrak{T}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho).$$

Since $\tilde{\mathfrak{f}} + \tilde{\mathfrak{g}} \geq 1, \mathfrak{T}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \eth\varrho) \leq \mathfrak{T}(\mathfrak{L}\mathfrak{k}, \mathfrak{L}\tilde{\varsigma}, \varrho)$. In view of Lemma (2.10), we have $\mathfrak{L}\mathfrak{k} = \mathfrak{L}\tilde{\varsigma}$ and consequently $\ddot{\mathfrak{U}}\mathfrak{k} = \ddot{\mathfrak{U}}\tilde{\varsigma}$. Therefore the pair $\{\ddot{\mathfrak{U}}, \mathfrak{L}\}$ have a unique point of coincidence $\zeta = \ddot{\mathfrak{U}}\mathfrak{k} = \mathfrak{L}\tilde{\varsigma}$. Thus, $\ddot{\mathfrak{U}}$ and $\mathfrak{L}$ have a unique common fixed point in Ξ.

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