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Application of the Inverse Weibull Distribution to Agricultural Data Based on Intuitionistic Fuzzy Sets

S. Sujatha[1*], A. Dinesh Kumar[2], R. Sivaraman[3] & M. Vasuki[4]

[1*] Research Scholar, Department of Mathematics, Khadir Mohideen College (Affiliated to Bharathidasan University), Adhirampattinam, Tamil Nadu, India. Email: suja01rakshi@gmail.com

[2] Assistant Professor, Department of Mathematics, Khadir Mohideen College (Affiliated to Bharathidasan University), Adhirampattinam, Tamil Nadu, India. Email: dradineshkumar@gmail.com

[3] Associate Professor, Department of Mathematics, Dwaraka Doss Goverdhan Doss Vaishnav College, Chennai, Tamil Nadu, India. Email: rsivaraman1729@yahoo.co.in

[4] Assistant Professor, Department of Mathematics, Srinivasan College of Arts and Science (Affiliated to Bharathidasan University), Perambalur, Tamil Nadu, India. Email: vasuki.scas@gmail.com

*Corresponding Author: suja01rakshi@gmail.com

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Abstract:

The inverse Weibull distribution (IWD) is a frequently used model in dependability analysis that finds widespread use in a variety of scientific domains. This work examines the intuitionistic fuzzy lifespan data-based dependability estimation of the IWD. Prior to deriving the ideas of intuitionistic fuzzy conditional expectation, intuitionistic fuzzy probability function, and intuitionistic fuzzy conditional density, the associated concepts of fuzzy set theory are examined. In conventional estimations, the maximum likelihood estimators for reliability and parameters are obtained. The maximum likelihood estimates are obtained using the EM algorithm because of the nonlinearity. The gamma prior is chosen in the Bayesian estimation process, and the symmetric entropy and scale square error loss functions, respectively, are used to estimate the parameters and reliability. The Lindley approximation is used to approximate the Bayesian estimates due to the complexity of the integrals. According to the simulation results, the maximum likelihood estimate is not as appropriate for reliability estimation as the Bayesian estimation. Ultimately, the efficacy of these suggested techniques is demonstrated using a collection of agriculture production data. These techniques yield an accurate evaluation of the intuitive fuzzy life data's reliability, serving as a crucial point of reference for reliability analysis in the scientific community. Using intuitionistic fuzzy values for real-time data, the present work investigated the Reliability Estimation, cumulative density function, and probability density function of the inverse Weibull distribution. We examined Andra agricultural output in 2019 in this analysis. The comparison thus showed that the estimation of fuzziness values is better than the real-time data.

Keywords: Agriculture data, Inverse Weibull distribution, intuitionistic fuzzy set, Reliability Estimation.

Mathematical Subject Classification (2020): 03E72, 62A86.

1. Introduction

Real-world models can be more accurate when fuzzy sets of numbers are used, especially when it comes to uncertainty models like statistical fluctuations in observed lifetime. Voids are another type of uncertainty that arises when an observation is not a precise number but rather a fuzzy one. Zadeh used the term "fuzzy variable" in 1965 [1] to refer to incorrect linguistic idiom and vernacular. Fuzzy set theory got its start with this. A fuzzy set is made up of elements with different membership levels.

The Rayleigh lifetime distribution to study the dependability properties of systems, where the lifespan parameter is taken to be a generalized intuitionistic fuzzy number, When the systems follow a generalised intuitionistic fuzzy Rayleigh lifetime distribution, generalised intuitionistic fuzzy dependability, generalised intuitionistic fuzzy hazard function, and generalised intuitionistic fuzzy mean time to failure are explored along with their cut sets. With this method, the danger and dependability curves for each unique cut set resemble a band with upper and lower bounds. A numerical example is provided to demonstrate the suggested methodology. Additional reliability study is conducted for both the parallel and series systems [2].

An intuitionistic fuzzy Weibull lifetime distribution to examine the fuzzy dependability of a few systems, Because of data imperfection and uncertainty, fuzzy lifetime parameters are postulated. When systems follow intuitionistic fuzzy Weibull lifetime distribution, expressions for fuzzy dependability, fuzzy mean time to failure, fuzzy hazard function, and their α-cut have been addressed. To demonstrate the process for calculating the fuzzy reliability characteristics of systems, a numerical example is also provided by [3].

An intuitionistic fuzzy random variable with exact parameters was created and used to assess the reliability functions of a k-out-of-n system. Some reliability evaluation criteria were explored and interpreted. Numerical assessments were also presented to demonstrate the determination of system dependability criteria in the form of intuitionistic fuzzy numbers [4]. Finally, many potential engineering applications for the suggested technique were discussed.

Generalized intuitionistic fuzzy numbers are used to assess the reliability of various systems. The reliability features of systems using the Pareto lifespan distribution are explored, under the assumption that the lifetime scale parameter is a generalized intuitionistic fuzzy number. In general, the cut sets for the generalized intuitionistic fuzzy reliability function, generalised intuitionistic fuzzy conditional reliability function, generalized intuitionistic fuzzy hazard function, and generalized intuitionistic fuzzy mean time to failure are examined. The reliability functions listed above are discussed for generalized intuitionistic fuzzy Pareto lifespan systems. Furthermore, reliability analysis of series and parallel systems is performed, and a numerical example is provided [5].

The intuitionistic fuzzy sets (IFS) idea, along with the triangle fuzzy number (TFN) and Weibull lifetime distribution, is used to determine the same system's fuzzy dependability. In addition, an averaging operator with equal weights is applied to a set of three triangular intuitionistic fuzzy numbers. A numerical example is solved as an illustration [6].

The simulation results demonstrated [7] that the fuzzy reliability at the estimation stage of the Maximum Likelihood Method and the Mixed Thompson Method outperform the other methods in terms of Mean Squared Error (MSE), indicating that the use of this type of estimation is recommended. The fuzzy values also outperform the real values for all sample sizes.

A new technique for analysing fuzzy system reliability based on intuitionistic trapezoidal fuzzy set theory is described, in which the reliabilities of system components are represented by intuitionistic trapezoidal fuzzy numbers to simulate uncertainty and vagueness in real-world circumstances [8]. The suggested method may describe and analyse fuzzy system reliability in a more adaptable and intelligent way. The suggested method also compares different complex systems using the score and accuracy functions. The score function can assist the decision maker in making his decision more efficiently in a decision-making dilemma.

This method is then applied to a wireless communication system to evaluate its intuitionistic fuzzy reliability and availability, with each performance state and probability represented by a TIFN. The acquired results are also shown graphically for better understanding. This work contributes to a better understanding of FMSS, making them more dependable [9].

The foundation of reliability engineering is reliability theory. Reliability analysis is vital for developing several system improvement alternatives during the design, configuration, and tuning stages, all of which are required for a complex system to operate efficiently. This work focuses on estimating the reliability of a degradable system with imperfect coverage and unknown information about its components[10]. The Weibull intuitionistic fuzzy set (WIFS) idea was utilised to address data uncertainty. The trapezoidal intuitionistic fuzzy number (TrIFN) and its arithmetic operations are discussed. Trapezoidal intuitionistic fuzzy numbers (TrIFN) are used to represent the failure rate of the system.

Simulation results indicate that Bayesian estimation outperforms maximum likelihood estimation for estimating dependability. The proposed approaches are tested against real data to demonstrate their usefulness. These approaches reliably evaluate the reliability of intuitive fuzzy life data, serving as a valuable reference for reliability analysis in science [11].

The current work used a fuzzy intuitionistic fuzzy set to analyze the inverse Weibull distribution's probability density function, cumulative density function, and Reliability Estimation with real-time data. This study focused on Andra's agriculture in 2019.

2. Fuzzy Mathematical Approach.

The probability density function (pdf), cumulative distribution function (cdf), and reliability function of IWD are defined as follows:

$$y(t;\lambda,\eta) = \lambda\eta t^{-\eta-1}\exp(-\lambda t^{-\eta}),\, t > 0$$
$$Y(t;\lambda,\eta) = \exp(-\lambda t^{-\eta}),\, t > 0$$
$$R(t) = 1 - \exp(-\lambda t^{-\eta}),\, t > 0$$

where $\lambda > 0$ is the scale parameter and $\eta > 0$ is the shape parameter, A fuzzy set expresses the concept of fuzziness. Similarly to the definition of the characteristic function of a classical set, the definition of a fuzzy set can be achieved by expanding its domain.

Definition 2.1

Let T represent a non-empty universal set. The fuzzy set $\tilde{A}$ is defined as $\tilde{A} = \{< t, \mu_{\tilde{A}}(t)\rangle \mid t \in \mathbb{T}\}$, where $\mu_{\tilde{A}}: \mathbb{T} \rightarrow [0,1]]$ represents the degree of membership of t in $\tilde{A}$. .

Definition 2.2

Let T represent a non-empty universal set. IFS A˜ is defined as $\tilde{A} = \{\langle t, \mu_{\tilde{A}}(t), v_{\tilde{A}}(t) > | t \in \mathbb{T}\}$, where $\mu_{\tilde{A}}: \mathbb{T} \rightarrow [0,1]$ is the degree of membership of t in $\tilde{A}$ and $v_{\tilde{A}}: \mathbb{T} \rightarrow [0,1]$ is the degree of non-membership of t in $\tilde{A}$. They satisfy $0 \leq \mu_{\tilde{A}}(t) + v_{\tilde{A}}(t) \leq 1$ for each t. When T has only one element, $\tilde{A} =< \mu_{\tilde{A}}, v_{\tilde{A}} >$ is frequently referred to as intuitionistic fuzzy number.

The membership and non-membership functions are:

$$\mu_{\tilde{A}}(t) = \begin{cases} \alpha \dfrac{t-a}{b-a} & t \in [a,b] \\ \alpha & t \in (b,c) \\ \alpha \dfrac{d-t}{d-c} & t \in [c,d] \\ 0 & \text{else} \end{cases}$$

$$v_{\tilde{A}}(t) = \begin{cases} \dfrac{b-t}{b-a} + \beta \dfrac{t-a}{b-a} & t \in [a,b] \\ \beta & t\text{this } \in (b,c) \\ \dfrac{t-c}{d-c} + \beta \dfrac{d-t}{d-c} & t \in [c,d] \\ 1 & \text{else} \end{cases}$$

where α is the maximum membership degree and β is the minimum membership degree.

In paper, we assume T be a set of real numbers, which is T=R. Additionally, we assumed that the IFSs discussed in this paper were TraIFNs. To better investigate the estimation problem on the basis of intuitionistic fuzzy data, some concepts in the probability theory were extended to intuitionistic fuzzy random variables.

Definition 2.3

Consider a probability space $(\mathbb{R}^n, \mathfrak{A}, \mathcal{P})$, The probability of an intuitionistic fuzzy observation $\tilde{x}$ in $\mathbb{R}^n$ is defined as

$$P(\tilde{x}) = \int_{\mathbb{R}^n} \frac{1 - v_{\tilde{x}}(t) + \mu_{\tilde{x}}(t)}{2} d\mathcal{P}$$

The continuous random variable $T = (T_1, T_2, \ldots, T_n)$ follows the IW (λ, η), and its intuitionistic fuzzy observations are denoted by $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n)$. The conditional density of random variables in probability theory is introduced, and the intuitionistic fuzzy conditional density is given as follows:

$$y(t \mid \tilde{x}) = \frac{s(t) y(t; \lambda, \eta)}{\int_{\mathbb{R}} s(t) y(t; \lambda, \eta) dt}$$

where $s(t) = \frac{1 - v_{\tilde{x}}(t) + \mu_{\tilde{x}}(t)}{2}$. In this case, the intuitionistic fuzzy likelihood function of IW(λ, η) is:

$$h(\lambda, \eta \mid \tilde{x}) = \prod_{i=1}^{n} P(\tilde{x}_i \mid \lambda, \eta) = \prod_{i=1}^{n} \int_{\mathbb{R}} s_i(t) y(t; \lambda, \eta) dt$$

where $s_i(t) = \frac{1 - v_{\tilde{x}_i}(t) + \mu_{\tilde{x}_i}(t)}{2}$.

Finally, intuitionistic fuzzy conditional expectation is defined. Using the intuitionistic fuzzy conditional density and observation $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n)$, the intuitionistic fuzzy conditional expectation of a random variable $T = (T_1, T_2, \ldots, T_n)$ is:

$$\begin{aligned} E(T \mid \tilde{x}) &= \int_{\mathbb{R}} t y(t \mid \tilde{x}) dt \\ &= \int_{\mathbb{R}} t \frac{s(t) y(t; \lambda, \eta)}{h(\lambda, \eta \mid \tilde{x})} dt \end{aligned}$$

2.4 Maximum Likelihood Estimation

The above Equation shows the intuitionistic fuzzy likelihood function of IW (λ, η) Thus, the intuitionistic fuzzy log-likelihood function is given as follows:

$$H(\lambda, \eta \mid \tilde{x}) = \ln h(\lambda, \eta \mid \tilde{x}) = \sum_{i=1}^{n} \ln \left[\int_{\mathbb{R}} s_i(t) y(t; \lambda, \eta) dt \right]$$

The MLEs $\hat{\lambda}_{ML}$ and $\hat{\eta}_{ML}$ are obtained by the below equations:

$$\begin{cases} \frac{\partial H(\lambda, \eta \mid \tilde{x})}{\partial \lambda} = 0 \\ \frac{\partial H(\lambda, \eta \mid \tilde{x})}{\partial \eta} = 0 \end{cases}$$

where $\frac{\partial H(\lambda, \eta \mid \tilde{x})}{\partial \lambda}$ and $\frac{\partial H(\lambda, \eta \mid \tilde{x})}{\partial \eta}$ are shown in below

$$\frac{\partial H(\lambda, \eta \mid \tilde{x})}{\partial \lambda} = \sum_{i=1}^{n} \frac{1}{h(\lambda, \eta \mid \tilde{x}_i)} \int_{\mathbb{R}} s_i(t) \frac{\partial y(t; \lambda, \eta)}{\partial \lambda} dt$$

$$\frac{\partial H(\lambda, \eta \mid \tilde{x})}{\partial \eta} = \sum_{i=1}^{n} \frac{1}{h(\lambda, \eta \mid \tilde{x}_i)} \int_{\mathbb{R}} s_i(t) \frac{\partial y(t; \lambda, \eta)}{\partial \eta} dt$$

Here, $h(\lambda, \eta \mid \tilde{x}_i) = \int_0^{+\infty} s_i(t) y(t; \lambda, \eta) dt$, $\frac{\partial y(t; \lambda, \eta)}{\partial \lambda}$ and $\frac{\partial y(t; \lambda, \eta)}{\partial \eta}$ are shown in below:

$$\frac{\partial y(t;\lambda,\eta)}{\partial\lambda}=\frac{1}{\lambda}y(t;\lambda,\eta)-t^{-\eta}y(t;\lambda,\eta)$$
$$\frac{\partial y(t;\lambda,\eta)}{\partial\eta}=\frac{1}{\eta}y(t;\lambda,\eta)-y(t;\lambda,\eta)\ln t+\lambda t^{-\eta}y(t;\lambda,\eta)\ln t$$

It is evident that the equations above are nonlinear and difficult to solve. Then we considered the EM algorithm.

The EM technique is equally applicable to intuitionistic fuzzy data because the observed intuitionistic fuzzy data can be viewed as imperfect characterizations of the completed data. To better illustrate the iterative process of the EM method,

From above equation we get,

$$\begin{aligned}\frac{\partial H(\lambda,\eta\mid\tilde{x})}{\partial\lambda}&=\sum_{i=1}^{n}\frac{1}{\overline{\overline{h(\lambda,\eta\mid\tilde{x}_i)}}}\int_0^{+\infty}s_i(t)\frac{\partial y(t;\lambda,\eta)}{\partial\lambda}dt\\&=\sum_{i=1}^{n}\frac{1}{\bar{h}(\lambda,\eta\mid\tilde{x}_i)}\int_0^{+\infty}s_i(t)\left[\frac{1}{\lambda}y(t;\lambda,\eta)-t^{-\eta}y(t;\lambda,\eta)\right]dt\\&=\sum_{i=1}^{n}\int_0^{+\infty}\frac{1}{\lambda}\frac{s_i(t)y(t,\lambda,\eta)}{h(\lambda,\eta\mid\tilde{x}_i)}dt-\sum_{i=1}^{n}\int_0^{+\infty}t^{-\eta}\frac{s_i(t)y(t;\lambda,\eta)}{h(\lambda,\eta\mid\tilde{x}_i)}dt\\&=n\frac{1}{\lambda}-\sum_{i=1}^{n}E_{1i}.\end{aligned}$$

Let $\frac{\partial H(\lambda,\eta\mid\tilde{x})}{\partial\lambda}=0$,

where

$$E_{1i}=E(T^{-\eta}\mid\tilde{x}_i)=\int_0^{+\infty}t^{-\eta}\frac{s_i(t)y(t;\lambda,\eta)}{h(\lambda,\eta\mid\tilde{x}_i)}dt$$

$$\begin{aligned}\frac{\partial H(\lambda,\eta\mid\tilde{x})}{\partial\eta}&=\sum_{i=1}^{n}\frac{1}{\overline{\overline{h(\lambda,\eta\mid\tilde{x}_i)}}}\int_0^{+\infty}s_i(t)\frac{\partial y(t;\lambda,\eta)}{\partial\eta}dt\\&=n\frac{1}{\eta}-\sum_{i=1}^{n}E_{2i}+\lambda\sum_{i=1}^{n}E_{3i}\end{aligned}$$

Let $\frac{\partial H(\lambda,\eta\mid\tilde{x})}{\partial\eta}=0$,

where

$$E_{2i}=E(\ln\ T\mid\tilde{x}_i)=\int_0^{+\infty}\frac{s_i(t)y(t;\lambda,\eta)}{h(\lambda,\eta\mid\tilde{x}_i)}(\ln\ t)dt$$

and

$$E_{3i}=E(T^{-\eta}\ln\ T\mid\tilde{x}_i)=\int_0^{+\infty}\frac{s_i(t)y(t;\lambda,\eta)}{h(\lambda,\eta\mid\tilde{x}_i)}(t^{-\eta}\ln\ t)dt$$

The iterative processes for obtaining MLEs with the EM method are as follows:

Step 1: Let the initial value be $\theta^{(0)}=\left(\lambda^{(0)},\eta^{(0)}\right)$, and set $j=0$. Give the accuracy $\varepsilon>0$. Step 2: At the $(j+1)$th iteration, compute the intuitionistic fuzzy conditional expectations shown below.

$$E_{1i}=\int_0^{+\infty}t^{-\eta}\frac{s_i(t)y(t;\lambda,\eta)}{h(\lambda,\eta\mid\tilde{x}_i)}\Bigg|_{\theta^{(j+1)}=\theta^{(j)}}dt,$$
$$E_{2i}=\int_0^{+\infty}\frac{s_i(t)y(t;\lambda,\eta)}{h(\lambda,\eta\mid\tilde{x}_i)(\ln t)}\Bigg|_{\theta^{(j+1)}=\theta^{(j)}}dt,$$
$$E_{3i}=\int_0^{+\infty}\frac{s_i(t)y(t;\lambda,\eta)}{h(\lambda,\eta\mid\tilde{x}_i)}(t^{-\eta}\ln t)\Bigg|_{\theta(j+1)\&=\theta^{(j)}}dt.$$

Step 3.

$$\lambda^{(j+1)}=n\left(\sum_{i=1}^{n}E_{1i}\right)^{-1}$$

From above equations we get,

$$\eta^{(j+1)}=n\left(\sum_{i=1}^{n}E_{2i}-\lambda^{(j)}\sum_{i=1}^{n}E_{3i}\right)^{-1}$$

Step 4: If $\left|\theta^{(j+1)}-\theta^{(j)}\right|<\varepsilon$, the MLEs are produced by $\hat{\lambda}_{ML}=\lambda^{(j)}$ and $\hat{\eta}_{ML}=\eta^{(j)}$. If not, set $j=j+1$ and go back to step 2.

The MLE MLE $\hat{R}_{ML}(t)$can be calculated using maximum likelihood estimation invariance as follows:

$$\hat{R}_{ML}(t)=1-\exp\left(-\hat{\lambda}_{ML}t^{-\hat{\eta}_{ML}}\right)$$

2.5 Bayesian Estimation

In Bayesian statistical inference, the prior distribution is important. It indicates our prior knowledge or assumption about the parameters and can be used to better precisely predict the posterior distribution .Choosing an appropriate prior distribution is critical since it influences the final inference results.

The gamma distribution is a versatile continuous probability distribution with numerous desirable qualities, making it a popular candidate for the prior distribution of parameters in Bayesian statistics.

The parameters of the gamma distribution can be changed to fit various prior beliefs. Furthermore, the gamma distribution is conjugate, which means that when employed as a prior distribution, the product with the likelihood function remains a gamma distribution; simplifying posterior distribution computations .The pdf for the gamma distribution is [11]

$$\pi(\omega)=\frac{b^a}{\Gamma(a)}\omega^{a-1}e^{-b\omega},\omega>0,a,b>0$$

In this section, we assume that λ and η are independent random variables. λ follows Gamma (c_1,d_1) and η follows Gamma (c_2,d_2). That means:

$$\pi_1(\lambda)\propto\lambda^{d_1-1}e^{-c_1\lambda}\lambda>0,c_1>0,d_1>0$$
$$\pi_2(\eta)\propto\eta^{d_2-1}e^{-c_2\eta}\eta>0,c_2>0,d_2>0.$$

Thus, the joint prior distribution of λ and η is:

$$\pi(\lambda,\eta)=\pi_1(\lambda)\times\pi_2(\eta)\propto\lambda^{d_1-1}\eta^{d_2-1}e^{-c_1\lambda-c_2\eta}$$

With reference to the Bayesian formulation, the posterior distribution of λ and η is

$$\pi(\lambda,\eta\mid\tilde{x})\propto h(\lambda,\eta\mid\tilde{x})\times\pi(\lambda,\eta)$$

The posterior expectation of the function $g(\lambda,\eta)$ of λ and η is:

$$E[g(\lambda,\eta)\mid\tilde{x}]=\int_0^{+\infty}\int_0^{+\infty}g(\lambda,\eta)\frac{\pi(\lambda,\eta\mid\tilde{x})}{\int_0^{+\infty}\int_0^{+\infty}\pi(\lambda,\eta\mid\tilde{x})d\lambda d\eta}d\lambda d\eta$$

$$E[g(\lambda,\eta)\mid\tilde{x}]=\frac{\int g(\lambda,\eta)e^{H(\lambda,\eta|\tilde{x})+G(\lambda,\eta)}d(\lambda,\eta)}{\int e^{H(\lambda,\eta|\tilde{x})+G(\lambda,\eta)}d(\lambda,\eta)}$$

where $G(\lambda,\eta)=\ln\ \pi(\lambda,\eta)$.

$$E[\,g(\lambda,\eta)\mid\tilde{x}\,]=g(\lambda\hat{}_{ML},\eta\hat{}_{ML})+\frac{1}{2(A+B+C+D)}$$
$$A=(g\hat{}_{\lambda\lambda}+2g\hat{}_{\lambda}G\hat{}_{\lambda})\phi\hat{}_{\lambda\lambda}+\left(g\hat{}_{\eta\lambda}+2g\hat{}_{\eta}G\hat{}_{\lambda}\right)\phi\hat{}_{\eta\lambda}$$
$$B=\left(g\hat{}_{\lambda\eta}+2g\hat{}_{\lambda}G\hat{}_{\eta}\right)\phi\hat{}_{\lambda\eta}+\left(g\hat{}_{\eta\eta}+2g\hat{}_{\eta}G\hat{}_{\eta}\right)\phi\hat{}_{\eta\eta}$$
$$C=\left(g\hat{}_{\lambda}\phi\hat{}_{\lambda\lambda}+g\hat{}_{\eta}\phi\hat{}_{\lambda\eta}\right)\left(H\hat{}_{\lambda\lambda\lambda}\phi\hat{}_{\lambda\lambda}+H\hat{}_{\eta\lambda\lambda}\phi\hat{}_{\eta\lambda}+H\hat{}_{\lambda\eta\lambda}\phi\hat{}_{\lambda\eta}+H\hat{}_{\eta\eta\lambda}\phi\hat{}_{\eta\eta}\right)$$
$$D=\left(g\hat{}_{\lambda}\phi\hat{}_{\eta\lambda}+g\hat{}_{\eta}\phi\hat{}_{\eta\eta}\right)\left(H\hat{}_{\lambda\lambda\eta}\phi\hat{}_{\lambda\lambda}+H\hat{}_{\eta\lambda\eta}\phi\hat{}_{\eta\lambda}+H\hat{}_{\lambda\eta\eta}\phi\hat{}_{\lambda\eta}+H\hat{}_{\eta\eta\eta}\phi\hat{}_{\eta\eta}\right)$$

The element $\phi_{ij}(i,j=\lambda,\eta)$represents the inverse matrix of $-H_{ij}$.. The symbol $\hat{g}_{\lambda\lambda}$ signifies taking the second derivative of $g(\lambda,\eta)$ with regard to λ and putting $\hat{\lambda}_{ML}$ into it. Similarly, the rest can be illustrated as follows:

$$H_{\lambda\lambda\lambda}=\sum_{i=1}^{n}\left[2h^{-3}(\lambda,\eta\mid\tilde{x}_i)\left(\int_0^{+\infty}s_i(t)\frac{\partial y(t;\lambda,\eta)}{\partial\lambda}dt\right)^3 - h^{-1}(\lambda,\eta\mid\tilde{x}_i)\int_0^{+\infty}s_i(t)\frac{\partial^3 y(t;\lambda,\eta)}{\partial\lambda^3}dt\right]$$

$$H_{\lambda\lambda\eta}=H_{\lambda\eta\lambda}=H_{\eta\lambda\lambda}$$

$$= \sum_{i=1}^{n} \left[2h^{-3}(\lambda, \eta \mid \tilde{x}_i) \int_0^{+\infty} s_i(t) \frac{\partial y(t; \lambda, \eta)}{\partial \eta} dt \left(\int_0^{+\infty} s_i(t) \frac{\partial y(t; \lambda, \eta)}{\partial \lambda} dt \right)^2 \right]$$

$$- \sum_{i=1}^{n} \left[h^{-2}(\lambda, \eta \mid \tilde{x}_i) \int_0^{+\infty} s_i(t) \frac{\partial y(t; \lambda, \eta)}{\partial \eta} dt \int_0^{+\infty} s_i(t) \frac{\partial^2 y(t; \lambda, \eta)}{\partial \lambda^2} dt \right]$$

$$+ \sum_{i=1}^{n} h^{-1}(\lambda, \eta \mid \tilde{x}_i) \int_0^{+\infty} s_i(t) \frac{\partial^3 y(t; \lambda, \eta)}{\partial \lambda^2 \, \partial \eta} dt$$

$$H_{\eta\eta\lambda} = H_{\eta\lambda\eta} = H_{\lambda\eta\eta}$$

$$= \sum_{i=1}^{n} \left[2h^{-3}(\lambda, \eta \mid \tilde{x}_i) \int_0^{+\infty} s_i(t) \frac{\partial y(t; \lambda, \eta)}{\partial \lambda} dt \left(\int_0^{+\infty} s_i(t) \frac{\partial y(t; \lambda, \eta)}{\partial \eta} dt \right)^2 \right]$$

$$- \sum_{i=1}^{n} \left[h^{-2}(\lambda, \eta \mid \tilde{x}_i) \int_0^{+\infty} s_i(t) \frac{\partial y(t; \lambda, \eta)}{\partial \lambda} dt \int_0^{+\infty} s_i(t) \frac{\partial^2 y(t; \lambda, \eta)}{\partial \eta^2} dt \right]$$

$$+ \sum_{i=1}^{n} \left[h^{-1}(\lambda, \eta \mid \tilde{x}_i) \int_0^{+\infty} s_i(t) \frac{\partial^3 y(t; \lambda, \eta)}{\partial \eta^2 \, \partial \lambda} dt \right]$$

$$H_{\eta\eta\eta} = \sum_{i=1}^{n} \left[2h^{-3}(\lambda, \eta \mid \tilde{x}_i) \left(\int_0^{+\infty} s_i(t) \frac{\partial y(t; \lambda, \eta)}{\partial \eta} dt \right)^3 + h^{-1}(\lambda, \eta \mid \tilde{x}_i) \int_0^{+\infty} s_i(t) \frac{\partial^3 y(t, \lambda, \eta)}{\partial \eta^3} dt \right]$$

The loss function plays an important role in Bayesian statistical inference since it evaluates the difference between model predictions and actual outcomes. In the Bayesian framework, we used the posterior distribution to represent uncertainty and the loss function to select the best decision or prediction. Different loss functions produce different conclusions or predictions; thus, choosing an appropriate loss function is critical to the precision and reliability of Bayesian inference. Then, we investigate Bayesian estimate of unknown parameters using the SE and SSE loss functions.

2.6 Bayesian Estimation under the SE Loss Function

The SE loss function is defined as follows,

$$L_1(\theta, \hat{\theta}) = \frac{\hat{\theta}}{\theta} + \frac{\theta}{\hat{\theta}} - 2$$

where $\hat{\theta}$ is the estimator of unknown parameter θ.

3. Real Data Analysis

In this application, we used real-time agriculture data from Kaggle.com [12] to analyze the Andra agriculture data set in 2019. We evaluated the data using an inverse Weibull distribution with intuitionistic fuzzy data, as shown in Tables 1 and 2. Mean Square Error for each crop 1.0551, 0.0419, 0.2632, 28.5692, 0.0126, 0.0507, 0.0915, 0.0125, 1.0093, 0.0397 and Bayesian Estimates and Reliability (MSE) for each crop 0.0172, 0.0173, 0.0129, 0.0241, 0.0075, 0.0212, 0.0139, 0.0151,

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0.0372, and 0.0073 respectively, Estimated fuzzy parameters for the λ are 2.5554, 2.8394 and 3.1233 for the η are 3.6139, 4.0155 and 4.4170.

Crop	Season	Area(ha)	(MT)	(kg/ha)	Value(₹)	(₹/kg)	Crop
Arecanut	Kharif	1096	10418	899.2	188249	405.52	7.253333
Arhar/Tur	Kharif	237647	114451	899.2	40818249	87929.39	0.44
Arhar/Tur	Rabi	5940	3747	899.2	1020254	2197.8	0.43
Bajra	Kharif	20484	47045	899.2	3518332	7579.08	2.004545
Bajra	Rabi	4592	11322	899.2	788721.9	1699.04	2.48
Banana	Whole Year	97695	5861700	899.2	16780093	36147.15	60
Black pepper	Whole Year	17645	17645	899.2	3030705	6528.65	1
Cashewnut	Kharif	115785	115785	899.2	19887232	42840.45	1
Castor seed	Kharif	36827	25270	899.2	6325406	13625.99	0.7625
Castor seed	Rabi	1056	637	899.2	181378.6	390.72	0.684

Table.1. Agriculture data of Andra in the year of 2019.

Crop	Season	Membership Degree (μ\muμ)	Non-Membership Degree (ν\nuν)	Degree of Uncertainty (π\piπ)
Arecanut	Kharif	0.70	0.20	0.10
Arhar/Tur	Kharif	0.80	0.10	0.10
Arhar/Tur	Rabi	0.60	0.30	0.10
Bajra	Kharif	0.65	0.25	0.10
Bajra	Rabi	0.55	0.35	0.10
Banana	Whole Year	0.90	0.05	0.05
Black Pepper	Whole Year	0.85	0.10	0.05
Cashewnut	Kharif	0.75	0.20	0.05
Castor Seed	Kharif	0.60	0.30	0.10
Castor Seed	Rabi	0.50	0.40	0.10

Table.2. intuitionistic fuzzy data values of Andra in the year of 2019 .

Maximum Likelihood Estimator (MLE) Table $\lambda = 4.3956$ and $\eta = 7.3543$.

S.No	Observed Yield(kg/ha)	Predicted Yield(kg/ha)
1	5.00	5.00
2	4.67	4.67
3	4.00	4.00
4	3.60	3.60
5	3.33	3.33

Mathematical Results of the inverse Weibull distribution.

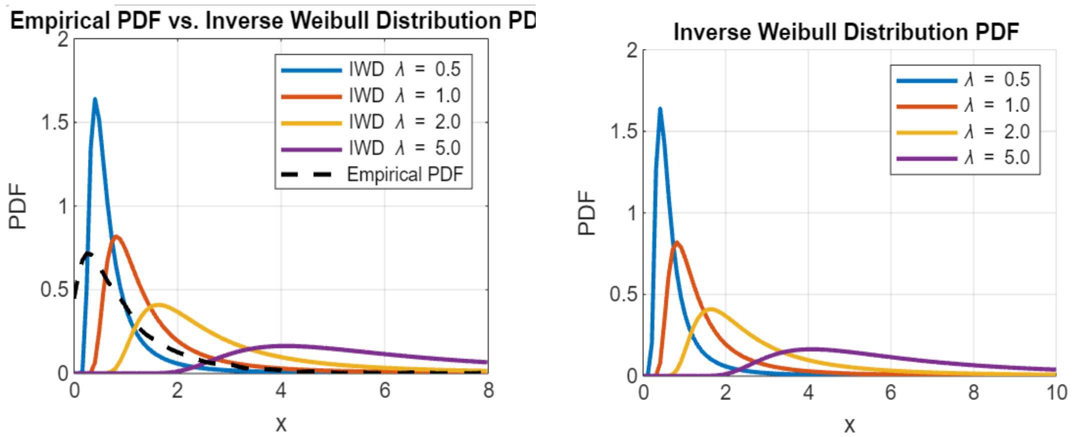

Fig. 1: PDF and Empirical PDF of inverse Weibull distribution using intuitionistic fuzzy set.

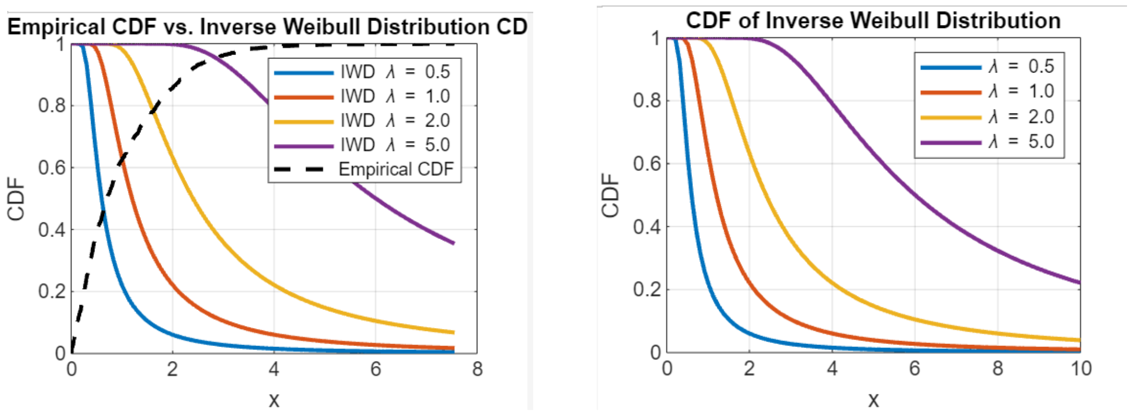

Fig. 2: CDF and Empirical CDF of inverse Weibull distribution using intuitionistic fuzzy set.

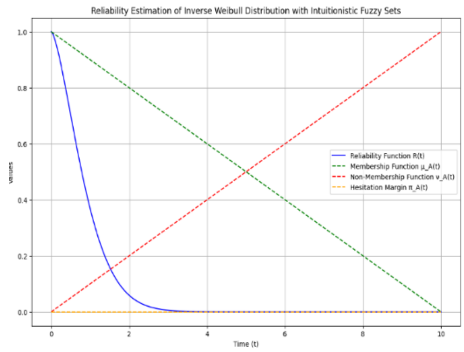

Fig.3 : Reliability Estimation of inverse Weibull distribution with intuitionistic fuzzy set.

4. Results and Discussion

Fuzzy lifetime data can be used to estimate the reliability of IWD in real-life scenarios due to uncontrollable factors. However, fuzzy sets only have one membership degree parameter, resulting in a less precise description of the objective world. In contrast, intuitionistic fuzzy sets can better express uncertainty and fuzziness when dealing with fuzzy information.

The EM algorithm was used to obtain MLEs, followed by the Lindley approximation to obtain BEs under SE and SSE loss functions. Results showed that changing the true values of multiple sets of parameters resulted in a significantly smaller mean square error under Bayesian estimation compared to the maximum likelihood estimator.

Kaggle.com supports the current research findings [11]. We discovered results for the Probability density function as well as PDF and Empirical PDF (Fig.1), Cumulative density function as well as CDF and Empirical CDF (Fig.2), and Reliability Estimation (Fig.3) in the Inverse Weibull distribution intuitionistic fuzzy set to the agriculture problem of Andra in the year of 2019 with fuzzy parameters and supporting MATLAB tool, which shows how well the suggested technique performs in predicting crop production for all seasons.

5. Conclusion

In this study, the Inverse Weibull distribution model is applied to real-time data in a fuzzy environment. This value is generated by combining the inverse Weibull distribution with the intuitionistic fuzzy set, and the resulting curve is used to assess the performance of the given data. The calculated Mean Squared Error (MSE), Maximum Likelihood Estimator (MLE) and Fuzzy operating characteristic curves for several functions are examined. This includes the probability density function, empirical PDF, cumulative density function, empirical CDF, and reliability estimation for the inverse Weibull distribution. This study determined that the proposed approach can be implemented if the product's quality is uncertain. The study discovered that the strategy is quite efficient in unpredictable agriculture and situations.

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