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On Arithmetical Traits of Doubt Fuzzy T-Ideals beneath the Normalization is a T-Algebra

CT. Nagaraj[(a)] , M. Premkumar , Y. Immanuel[(b)], Abdul Salam[(c)] , M. S Franklin Thamil Selvi[(d)] , M. I. Mary Metilda[(e)] and J. Juliet Jeyapackiam[(f)]

[(a)]Department of Mathematics, Sree Sevugan Annamalai College, Devakottai-630303, India[*(1a)][0000-0002-8637-063X]

[(*1a, b, d & e)] Department of Mathematics, Sathyabama Institute of Science and Technology (Deemed To Be University) Chennai-600119, Tamilnadu, India. [(B)][0000-0003-0719-375X]

[(c)]Gulf Asian English School, Sharjah, United Arab Emirates.

[(f)]Department of Mathematics, Jayaraj Annapackiam CSI College of Engineering Nazareth, Tuticorin-628617, India.

[(a)]mathsnagaraj.ct@gmail.com, [(*1a)] mprem.maths3033@gmail.com, [(b)]y_immanuel@yahoo.com, [(c)]abdulsalam.maths@gmail.com, [(d)]thamizanand@gmail.com, [(e)]metilda81@gmail.com

[(f)] jeyasjjjeyas@gmail.com, Corresponding Author Email Id: [(*1a)] mprem.maths3033@gmail.com[0000-0003-4656-3370]

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Abstract:

The normal doubt fuzzy T-ideal and poset under the set of inclusion principle in T-algebra are defined in this article, along with several algebraic properties and instances that are covered in detail.

Keywords: Doubt Fuzzy Set (DFS), Doubt Fuzzy Subset (DFSb),T-Algebra, T-Ideal, Doubt Fuzzy T-ideal (DFTI), Normal Doubt Fuzzy T-ideal (NDFTI).

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I. Introduction

Abu Ayub Ansari[1] introduced the novel idea of T-FβSA of β-algebras in 2014. Prasanna, A., et al. [2&3]. outlined the new FBI Normalization notation in B-Algebra and presented the idea of FBGI Normalization in BG-Algebra in 2018. Priya's FPSIs and FPSSAs for PS-algebras were standardized in 2015[4]. In 2015, Rajam[5] presented the idea of L-FTI in β-algebras. In 2016, Sithar Selvam[6] learned about the FPMSA Normalization study. Tamil created FSA and FTI in TM-Algebras in 2011[7]. Zadeh[8] introduced fuzzy sets for the first time in 1965.

This work describes the normal fuzzy T-ideal and poset under the set of inclusion principle over T-algebra and explores some algebraic characteristics.

II. Preliminaries

Basic Reference: 2.1 [8]

Let X be a non-empty set . A FSb of the set X is a mapping μ : X→ [0, 1].

Basic Reference: 2.2[7]

A FS μ in a BP-algebra X is called a FTI of X if it satisfies the following conditions:

(i) $\mu(0) \geq \mu(x)$

(ii) $\mu(x * z) \geq min\{\mu((x * y) * z), \mu(y)\}, \forall\, x, y \in X.$

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Definition: 3.1

Let $DFTI$ $'\Omega$ of $\tilde{A}$ is called to be $NDFTI$ if $\exists\ \theta \in \tilde{A}$ s.t $'\Omega(0) = 1.$

Example: 3.1.1

Let $\tilde{A} = \{0, a, b, c, d\}$ be a T-Algebra

*	0	*a*	*b*	*C*	*d*
0	0	*a*	*b*	*c*	*d*
a	0	0	0	0	*a*
b	0	*c*	0	*c*	*d*
c	0	*a*	*b*	0	*a*
d	0	0	0	0	0

Then $(\tilde{A}, *, 0)$ is a T-Algebra. Define DFS $'\Omega$ in $\tilde{A}$ by $'\Omega(0) = 0.9,$

$'\Omega(a) = 0.7, '\Omega(b) = 0.8, '\Omega(c) = 0.6\ and\ '\Omega(d) = 0.5.$

$\Rightarrow$Then $'\Omega$ is a $NDFTI$ of $\tilde{A}$.

Remark: 3.2

Let $NDFTI$ $'\Omega$ of $\tilde{A}$ if and only if $'\Omega(0) = 1.$

Theorem: 3.3

Let any $DFTI$ $'\Omega$ of $\tilde{A}$, we can generate the $NDFTI$ of $\tilde{A} \subset {'\Omega}$.

Proof:

Let $'\Omega$ be a $DFTI$ of $\tilde{A}$.

Define a DFS $'\Omega^{n}$ of $\tilde{A}$ as $'\Omega^{n}(\tilde{a}) = {'\Omega}(\tilde{a}) + {'\Omega^{c}}(0), \forall \tilde{a} \in \tilde{A}.$

Let $\tilde{a}, ɓ \in \tilde{A}$

(i) $'\Omega^{n}(0) = {'\Omega}(0) + {'\Omega^{c}}(0) \leq {'\Omega}(\tilde{a}) + {'\Omega^{c}}(0) = {'\Omega^{n}}(\tilde{a})$

$$\Rightarrow {'\Omega^{n}}(0) \leq {'\Omega^{n}}(\tilde{a})$$

(ii) $'\Omega^{n}(\tilde{a} * \hat{c}) = {'\Omega}((\tilde{a} * ɓ) * \hat{c}) + {'\Omega^{c}}(0) \leq max\{{'\Omega}((\tilde{a} * ɓ) * \hat{c}), {'\Omega}(ɓ)\} + {'\Omega^{c}}(0)$
$= max\{[{'\Omega}((\tilde{a} * ɓ) * \hat{c}) + {'\Omega^{c}}(0)], [{'\Omega}(ɓ) + {'\Omega^{c}}(0)]\}$
$= max\{{'\Omega^{n}}((\tilde{a} * ɓ) * \hat{c}), {'\Omega^{n}}(ɓ)\}$

$$\Rightarrow {'\Omega^{n}}(\tilde{a} * \hat{c}) \leq max\{{'\Omega^{n}}((\tilde{a} * ɓ) * \hat{c}), {'\Omega^{n}}(ɓ)\}$$

Also $'\Omega^{n}(0) = {'\Omega}(0) + {'\Omega^{c}}(0) = {'\Omega}(0) + 1 - {'\Omega}(0) = 1.$

$$\therefore {'\Omega^{n}} \text{ is a } NDFTI \text{ of } \tilde{A}.$$

Lemma: 3.4

Let $'\Omega^n$ be an DFS in Ᾰ defined by $'\Omega^n(\tilde{a}) = '\Omega(\theta) + '\Omega^c(0),\ \forall\ \tilde{a} \in$ Ᾰ. If ∃ element $\tilde{a} \in$ Ᾰ in s.t $'\Omega^n(\tilde{a}) = 0,$ then $'\Omega(\tilde{a}) = 0$.

Lemma: 3.5

Let $'\Omega$ be $DFTI$ of Ᾰ. Then,

(i) if $'\Omega$ itself is $DFTI$ then $'\Omega(\tilde{a}) = '\Omega^n(\tilde{a})$.

(ii) if $'\Omega$ is a $DFTI$ of X then $\left('\Omega^n(\tilde{a})\right)^n = '\Omega^n(\tilde{a})$.

Proposition: 3.6

Let $'\Omega$ $DFTI$ Ᾰ. If $'\Omega$ contains the $NDFTI$ of Ᾰ, generated by any other $DFTI$ of Ᾰ then $'\Omega$ is normal .

Proof:

Let δ be a $DFTI$ of Ᾰ.

by the. 3.3, let δ^n is a $DFTI$ of Ᾰ

$\therefore \delta^n(0) = 1($ lem. 3.4)

Let $'\Omega$ be a $DFTI$ of Ᾰ s.t $\delta^n \subset\ '\Omega$.

$\Rightarrow '\Omega(\tilde{a}) \le \delta^n(\tilde{a}), \forall \tilde{a} \in$ Ᾰ

Put $\tilde{a} = 0 \Rightarrow '\Omega(0) \le \delta^n(0) = 1$

$\Rightarrow '\Omega(0) \le 1 \therefore '\Omega$ is normal

Theorem: 3.7

A set $N_\Omega = \{\tilde{a} \in X/'\Omega(\tilde{a}) = '\Omega(0)\}$. Let $'\Omega$ and δ be $NDFTI$ of Ᾰ. If $'\Omega \subset \delta$ then $N_\Omega \subset N_\delta$.

Proof:

Let $\tilde{a} \in N_\Omega$

Since $'\Omega \subset \delta, \delta(\tilde{a}) \le '\Omega(\tilde{a}) = '\Omega(0) = 1 = \delta(0)$

$\Rightarrow \tilde{a} \in N_\delta$

$\therefore N_\Omega \subset N_\delta$

Theorem: 3.8

Let $'\Omega$ be the $DFTI$ of Ᾰ. Let $f: [0, '\Omega(0)] \to [0,1]$ be an increasing function. Let's define a DFS $'\Omega_f$: Ᾰ $\to [0,1]$ by $'\Omega_f(\tilde{a}) = f\left('\Omega(\tilde{a})\right), \forall\ \tilde{a} \in$ Ᾰ. Therefore

(i) If $'\Omega_f$ is a $DFTI$ of Ᾰ

(ii) If $f\left('\Omega(0)\right) = 1$, then $'\Omega_f$ is normal

(iii) If $f(t) \le t,\ \forall\ t \in [0, '\Omega(0)]$ then $'\Omega \subset '\Omega_f$.

Proof:

Let Ω be $DFTI$ of $\tilde{A}$.

Let $f: [0, \Omega(0)] \to [0,1]$ be an increasing function.

Define a DFS $\Omega_f: \tilde{A} \to [0,1]$ by $\Omega_f(\tilde{a}) = f(\Omega(\tilde{a})), \forall\, \tilde{a} \in \tilde{A}$.

(i) (a) $\Omega_f(0) = f(\Omega\,(0)) \le f(\Omega(\tilde{a})) \;= \Omega_f(\tilde{a})$

$$\Rightarrow \Omega_f(0) \le \Omega_f(\tilde{a})$$

(b) $\Omega_f(\tilde{a} * \hat{c}) = f(\Omega\,(\tilde{a} * \hat{c})) \le f\left\{max\{\Omega((\tilde{a} * b) * \hat{c}), \Omega(b)\}\right\}$

$$= max\{f(\Omega(\tilde{a} * b) * \hat{c}), f(\Omega(b))\}$$

$$= max\{\Omega_f((\tilde{a} * b) * \hat{c}), \Omega_f(b)\}$$

$\Rightarrow \Omega_f$ is a FTI.

(ii) If $f(\Omega\,(0)) = 1 \quad \Rightarrow \Omega_f(0) = 1$
$\Rightarrow \Omega_f$ is normal
(iii) Let $f(t) \le t, \forall\; t \in [0, \Omega(0)]$
Then $\Omega_f(\tilde{a}) = f(\Omega(\tilde{a}) \le \Omega(\tilde{a})), \forall\, \tilde{a} \in \tilde{A} \;\; \therefore \Omega \subseteq \Omega_g$.

Definition: 3.9

Let $\vartheta = \left(\frac{\tau}{\tau}\text{is the } NDFTI \text{ of } \tilde{A}\right)$ then the ϑ is called a Poset according to the principle of inclusion.

Definition: 3.10

Let $s > 0$ be a real number. If $\beta \in [0,1]$, β^s be the positive root in case $s < 1$. We define $\Omega^s: K \to [0,1]$ by $\Omega^s(\tilde{a}) = (\Omega(\tilde{a}))^s, \forall \tilde{a} \in \tilde{A}$.

Theorem: 3.11

Let, $\Omega \in \vartheta$ be a constant s.t it is a maximum element of $(\vartheta, \subseteq)$. Then, Ω only accept the values of 0 & 1.

Proof: Let, $\Omega \in \vartheta$. Then $\Omega(0) = 1$,

Let, $\tilde{a} \in \tilde{A}$ s.t $\Omega(\tilde{a}) \ne 1$.

We claim that $\Omega(0) = 0$. If not, then $\exists\; b \in X$ s.t $0 < \Omega(b) < 1$.

We now define a DFS, $\pi: \tilde{A} \to [0,1]$ by $\pi(\tilde{a}) = \frac{1}{2}\{\Omega(\tilde{a}) + \Omega(b)\}, \forall\, \tilde{a} \in \tilde{A}$.

Then Ω obviously is well defined

Now ,(i) $\pi(0) = \frac{1}{2}\{\Omega(0) + \Omega(b)\}$

$$\le \frac{1}{2}\{\Omega(\tilde{a}) + \Omega(b)\} = \pi(\tilde{a})$$

$$\Rightarrow \pi(0) \le \pi(\tilde{a})$$

(ii) $\pi(ã * ĉ) = \frac{1}{2}\{'\Omega(ã * ĉ) + '\Omega(b)\}$

$$\leq \frac{1}{2}\{max\{'\Omega((ã * ĉ) * ĉ), '\Omega(ɓ)\} + '\Omega(b)\}$$

$$= \frac{1}{2}\{max(\{'\Omega((ã * ĉ) * ĉ) + '\Omega(b)\}, \{'\Omega(ɓ) + '\Omega(b)\})\}$$

$$= max\left\{\frac{1}{2}\{'\Omega((ã * ĉ) * ĉ) + '\Omega(b)\}, \frac{1}{2}\{'\Omega(ɓ) + '\Omega(b)\}\right\}$$

$$= max\{\pi((ã * ĉ) * ĉ), \pi(ɓ)\}$$

$$\Rightarrow \pi(ã * ĉ) \leq max\{\pi((ã * ĉ) * ĉ), \pi(ɓ)\}.$$

$\Rightarrow \pi$ is a FTI.

$\Rightarrow \pi^n$ is a $NFTI$.

$$\pi^n(ã) = \pi(ã) + \pi^c(0)$$

$$= \pi(ã) + (1 - \pi(0))$$

$$= \frac{1}{2}\{\pi(ã) + \pi(b)\} + \left(1 - \frac{1}{2}\{\pi(0) + \pi(b)\}\right)$$

$$= \frac{1}{2}'\Omega(ã) + 1 - \frac{1}{2}(1)$$

$$= \frac{1}{2}'\Omega(ã) + \frac{1}{2}$$

$$= \frac{1}{2}('\Omega(ã) + 1) \leq '\Omega(ã), \forall ã \in X$$

$\therefore \pi^n(0) = \frac{1}{2}('\Omega(0) + 1) = 1$

$\therefore \pi^n$ is normal $\Rightarrow \pi^n \in \vartheta$

Also $\pi^n(ã) < '\Omega(ã), \forall ã \in$ Ḁ̃.

The contradiction the fact that of $'\Omega$ is normal.

$\Rightarrow '\Omega(ã) = 0, \forall ã \in$ Ḁ̃.

Theorem: 3.12

Let $'\Omega$ is a $DFTI$ of X, then so is $'\Omega^s$ and ${N_\Omega}^s = N_\Omega$.

Proof:

Let ã, ɓ ∈ Ḁ̃.

Now, (i) $'\Omega^s(0) = ('\Omega(0))^s$

$$\leq ('\Omega(ã))^s$$

$$= '\Omega^s(ã).$$

$$\Rightarrow \quad \acute{\Omega}^{s}(0) \geq \acute{\Omega}^{s}(\tilde{a})$$

$$\text{(ii) } \acute{\Omega}^{s}(\tilde{a} * \hat{c}) = \left(\acute{\Omega}(\tilde{a} * \hat{c})\right)^{s}$$
$$\leq \left(max\left\{\acute{\Omega}\left((\tilde{a} * \text{ɓ}) * \hat{c}\right), \acute{\Omega}(\text{ɓ})\right\}\right)^{s}$$
$$= max\left\{\left(\acute{\Omega}\left((\tilde{a} * \text{ɓ}) * \hat{c}\right)\right)^{s}, (\acute{\Omega}(\text{ɓ}))^{s}\right\}$$
$$= max\left\{\acute{\Omega}^{s}\left((\tilde{a} * \text{ɓ}) * \hat{c}\right), \acute{\Omega}^{s}(\text{ɓ})\right\}$$
$$\Rightarrow \acute{\Omega}^{s}(\tilde{a} * \hat{c}) \leq max\left\{\acute{\Omega}^{s}\left((\tilde{a} * \text{ɓ}) * \hat{c}\right), \acute{\Omega}^{s}(\text{ɓ})\right\}$$

$\therefore \acute{\Omega}^{s}$ is a $DFTI$.

$$N_{\Omega}{}^{s} = \{\tilde{a} \in J / \acute{\Omega}^{s}(\tilde{a}) = \acute{\Omega}^{s}(0)\}$$
$$= \{\tilde{a} \in J / \acute{\Omega}(\tilde{a}) = \acute{\Omega}(0)\}$$

$\Rightarrow N_{\Omega}{}^{s} = N_{\Omega}$.

III. Conclusion

Hence we have to this paper discussed about the $NDFTI$ and Poset under the set of inclusion principle over T-Algebra. This idea can be further extended to normalization of intuitionistic FS, normalization of interval valued FS, and normalization of bipolar FSs for new findings in future studies.

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