

On Neutrosophic Ideal of KK-algebras

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Abstract:

In this paper, we introduce the notion of a neutrosophic ideal KK-algebra. Also, we investigate some properties of the neutrosophic ideal with suitable illustrations. Further, we describe how to deal with the homomorphism of the image and the inverse image of the neutrosophic ideal of the KK-algebra.

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1. Introduction:

The concept of KK-algebra was first developed by Asawasamrit and A. Sudprasert [3]. Asawasamrit and A. Sudprasert [4-6] established the concepts of ideals, subalgebras of KK-algebras, then they examined the relationships between them by using the idea of KK-algebra homomorphism and looked into a few associated properties. In 1965, Zadeh [11] introduced the concept of a fuzzy set. U. Leerawat and C. Prabpayak [8] introduced the idea of KU-algebras and examined certain associated properties and provided the homomorphism of KU-algebras. Fuzzy ideals of KK-algebras have been introduced by Huda Ali Faleh, Dr. Ahmed Hamzah Abed, and Dr. Areej Tawfeeq Hameed [2]. Samarandache [10] in 1998 first developed the concept of neutrosophic sets. The authors [7] looked into some properties and presented the notation of a neutrosophic ideal of BN-algebra.

In this work, we first introduce the notion of the neutrosophic ideal of KK-algebra's and then look into a number of fundamental properties that are connected to it. For the neutrosophic ideal, We explain how to handle the image and inverse image homomorphism.

2. Preliminaries

In order to better understand the primary findings, we go over the basic definitions of KK-algebra, ideals, and ideal characteristics in this section. We also go over the concepts of neutrosophic sets and neutrosophic ideals of KK-algebra.

Definition 2.1[2]: An algebra $(X, *, 0)$ of type $(2, 0)$ is called a KK-algebra, if it satisfied the following axioms for all $\lambda, \mu, \nu \in X$,

$$(i) \quad (\lambda * y) * ((y * v) * (x * v)) = 0$$

$$(ii) \quad 0 * \lambda = \lambda$$

$$(iii) \quad \lambda * y = 0 \text{ and } y * \lambda = 0 \Leftrightarrow \lambda = \mu.$$

Definition 2.2[2]: A binary relation $' \leq '$ on KK-algebra $(X, *, 0)$, such that $\lambda \leq \mu$ if and only if $\mu * \lambda = 0$ for all $\lambda, \mu \in X$.

Proposition 2.3[2]: A any KK-algebra $(X, *, 0)$, the following axioms are hold for all $\lambda, \mu, v \in X$,

$$(i) \quad \lambda * ((\lambda * \mu) * \mu) = 0$$

$$(ii) \quad \lambda * \lambda = 0$$

$$(iii) \quad \lambda * (\mu * v) = \mu * (\lambda * v)$$

$$(iv) \quad ((\lambda * \mu) * \mu) * \mu = \lambda * \mu$$

$$(v) \quad (\lambda * \mu) * 0 = (\lambda * 0) * (\mu * 0)$$

$$(vi) \quad (\lambda * \mu) * ((v * \lambda) * (v * \mu)) = 0$$

$$(vii) \quad \lambda \leq \mu \text{ implies } y * v \leq \lambda * v$$

$$(viii) \quad \lambda \leq \mu \text{ implies } v * \lambda \leq v * \mu.$$

Definition 2.4[2]: Let $(X, *, 0)$ be a KK-algebra, and let S be a nonempty subset of X , S is called subalgebra of X if $\lambda * \mu \in S$ for all $\lambda, \mu \in S$.

Definition 2.5[2]: An ideal of X is a non-empty subset I of a KK-algebra $(X, *, 0)$ that satisfies the following for all $\lambda, \mu \in X$,

$$(i) \quad 0 \in I$$

$$(ii) \quad \lambda * \mu \in I \text{ and } \lambda \in I \text{ imply } \mu \in I.$$

Proposition 2.6[2]: Each ideal in the KK-algebra $(X, *, 0)$ is a subalgebra of X .

Proposition 2.7[2]: Let a family of ideals of KK-algebra X be denoted by $\{I_i / i \in \Delta\}$. The intersection of any set of ideals of KK-algebra X is also is an ideal.

Definition 2.8[2]: Let $(X, *, 0)$ and $(Y, *, 0')$ be an any two KK-algebras. Then, the mapping $f: X \rightarrow Y$ is called homomorphism, if it satisfied $f(\lambda * \mu) = f(\lambda) *' f(\mu)$ for all $\lambda, \mu \in X$.

Definition 2.9 [10]: Let U be the discourse universe. A neutrosophic set N of U is characterized by a truth membership function T_N , an indeterminacy membership function I_N , and a falsity membership function F_N , where T_N , I_N , and F_N are real standard elements of $[0, 1]$. It can be written as

$$N = \{(\lambda, T_N(\lambda), I_N(\lambda), F_N(\lambda)) / \lambda \in U\} \text{ Where } T_N, I_N, F_N \in]0, 1[.$$

There is no restriction on the sum of $T_N(\lambda)$, $I_N(\lambda)$ and $F_N(\lambda)$, and so $0^- \leq T_N(\lambda) + I_N(\lambda) + F_N(\lambda) \leq 3^+$.

Definition 2.10 [7]: A neutrosophic set $N = \{T_N, I_N, F_N\}$ of BN-algebra $(A, *, 0)$, if N is called an neutrosophic ideal of BN-algebra A , then it satisfies the following for all $\lambda, \mu \in A$,

- (i) $T_N(0) \geq T_N(\lambda), I_N(0) \geq I_N(\lambda), F_N(0) \leq F_N(\lambda)$
(ii) $T_N(\mu) \geq \min\{T_N(\lambda * \mu), T_N(\mu)\}; I_N(\mu) \geq \min\{I_N(\lambda * \mu), I_N(\mu)\}$
 $F_N(\mu) \leq \max\{F_N(\lambda * \mu), F_N(\mu)\}.$

Definition 2.11[7]: Let A be any neutrosophic set. If for any $\rho \in [0, 3]$, then an neutrosophic level set of ρ is defined by $L(A, \rho) = \{\alpha \in U : T_N(\alpha) \geq \rho, I_N(\alpha) \geq \rho, F_N(\alpha) \leq \rho\}.$

3. Main results

This part presents the key findings of the study, starting with an idea of neutrosophic ideal of KK algebra, and an explanation of neutrosophic ideal. Furthermore, various properties of the neutrosophic ideal in KK-algebra are investigated.

Definition 3.1: A non empty subset N of a KK-algebra X is called neutrosophic ideal of KK-algebra, if it satisfies the following for all $\lambda, \mu \in N$

- (i) $T_N(0) \geq T_N(\lambda); I_N(0) \geq I_N(\lambda); F_N(0) \leq F_N(\lambda),$
(ii) $T_N(\mu) \geq \min\{T_N(\lambda * \mu), T_N(\lambda)\};$

$$I_N(\mu) \geq \min\{I_N(\lambda * \mu), I_N(\lambda)\};$$

$$F_N(\mu) \leq \max\{F_N(\lambda * \mu), F_N(\lambda)\}.$$

Example 3.2: Consider a set $N = \{0, \lambda, \mu, \nu, 1\}$. Define a binary operation $' * '$ on N given by the following Table 3.1, and neutrosophic set by the Table 3.2 as shown below:

$*$	0	λ	μ	ν	1
0	0	λ	μ	ν	1
λ	λ	0	ν	μ	1
μ	μ	ν	0	λ	1
ν	ν	μ	λ	0	1
1	1	1	1	1	0

Table 3.1: $' * '$ Operation

$A \backslash N$	0	λ	μ	ν	1
T_N	0.7	0.5	0.5	0.5	0.5
I_N	0.6	0.4	0.4	0.4	0.4
F_N	0.2	0.1	0.1	0.1	0.1

Table 3.2: Neutrosophic set

It is easily verified that N is a neutrosophic ideal of X , and that it satisfies the conditions of definition 3.1.

Definition 3.3: Let $(X, *, 0)$ be a KK-algebra, and let be N a nonempty neutrosophic subset N of X , N is called neutrosophic sub algebra of X , if it satisfies the following axioms for all $\lambda, \mu \in X$.

- (i) $T_N(\lambda * \mu) \geq \min\{T_N(\lambda), T_N(\mu)\};$
(ii) $I_N(\lambda * \mu) \geq \min\{I_N(\lambda), I_N(\mu)\};$
(iii) $F_N(\lambda * \mu) \leq \max\{F_N(\lambda), F_N(\mu)\}.$

Example 3.4: Consider a set $N = \{0, \lambda, 1\}$ be a neutrosophic subset of X . Define a binary operation $' * '$ on N given by the following Table 3.3, and neutrosophic set by the Table 3.4 as shown below:

$*$	0	λ	1
0	0	λ	1
λ	λ	0	1
1	1	1	0

Table 3.3: $' * '$ Operation

$\begin{matrix} A \\ N \end{matrix}$	0	λ	1
T_N	0.7	0.5	0.5
I_N	0.6	0.4	0.4
F_N	0.2	0.1	0.1

Table 3.4: Neutrosophic set

It is easily verified that N is neutrosophic sub algebra of X , and that it satisfies the conditions of definition 3.3.

Proposition 3.5: Let N be a neutrosophic ideal in KK-algebra $(X, *, 0)$ and if $\lambda \leq \mu$, then $T_N(\lambda) \geq T_N(\mu)$ for all $\lambda, \mu \in X$.

Proof: Let N be a neutrosophic ideal of KK-algebra X and $\lambda \leq \mu$. Then from the definition 2.2, we have $\mu * \lambda = 0$ for all $\lambda, \mu \in X$.

From (ii) of definition 3.1 we have, $T_N(0 * \lambda) = T_N(\lambda)$

$$\begin{aligned} &\geq \min\{T_N(0 * \mu), T_N(\mu * \lambda)\} \\ &= \min\{T_N(\mu), T_N(0)\} \\ &= T_N(\mu). \end{aligned}$$

Then, we get $T_N(\lambda) \geq T_N(\mu)$.

Similarly, we can prove for $I_N(\mu)$.

Next, from (ii) of the definition 3.1, we have $F_N(0 * \lambda) = F_N(\mu)$

$$\begin{aligned} &\leq \max\{F_N(0 * \lambda), F_N(\mu * \lambda)\} \\ &= \max\{F_N(\lambda), F_N(0)\} \\ &= F_N(\lambda). \end{aligned}$$

Hence, we get $F_N(\lambda) \geq F_N(\mu)$. ■

Definition 3.6: Let $(X, *, 0)$ be a nonempty set and N be a neutrosophic subset of X , for $t \in [0, 1]$ the set $N_t = \{\lambda \in X / T_N(\lambda) \geq t\}$ is called a level subset of N .

Theorem 3.7: Let N be a neutrosophic subset of KK-algebra X . If N is a neutrosophic sub algebra of X if and only if the level set N_t is a subalgebra of X for every $t \in [0, 1]$.

Proof: Let N be neutrosophic subset of KK-algebra X and also neutrosophic sub algebra of

KK-algebra X . Let $\lambda, \mu \in X$ be such that $\lambda \in N_t$ and $\mu \in N_t$, then $T_N(\lambda) \geq t$ and $T_N(\mu) \geq t$. Since N is neutrosophic sub algebra. It follows that $T_N(\lambda * \mu) \geq \min\{T_N(\lambda), T_N(\mu)\} \geq t$,

and that $\lambda * \mu \in N_t$. Hence N_t is a sub algebra of X .

Similarly, we can prove for I_N and F_N .

Conversely, If $T_N(\lambda * \mu) \geq \min\{T_N(\lambda), T_N(\mu)\}$ is not true. Then there exists $\lambda', \mu' \in X$ such that $T_N(\lambda' * \mu') < \min\{T_N(\lambda'), T_N(\mu')\}$

Putting $t' = T_N(\lambda' * \mu') + \min\{T_N(\lambda'), T_N(\mu')\}/2$ then $T_N(\lambda') < t'$ and

$0 \leq t' < \min\{T_N(\lambda'), T_N(\mu')\} \leq 1$. Hence, $T_N(\lambda') > t'$ and $T_N(\mu') > t'$ which imply that $\lambda' \in N_{t'}$ and $\mu' \in N_{t'}$, since $N_{t'}$ is a subalgebra it follows that $\lambda' * \mu' \in N_{t'}$, and that

$T_N(\lambda' * \mu') \geq t'$. This is contradiction.

Therefore, N is neutrosophic sub algebra of X

Similarly, we can prove for I_N and F_N . ■

Theorem 3.8: Let N be a neutrosophic ideal of KK-algebra $(X, *, 0)$, N be a neutrosophic ideal of X if and only if for every $t \in [0, 1]$, N_t is an ideal of X

Proof: Let N be a neutrosophic ideal of KK-algebra X . Then, from the definition of 3.1 we have $T_N(0) \geq T_N(\lambda)$ for all $\lambda \in X$, Therefore, $T_N(0) \geq T_N(\lambda) \geq t$ for $\lambda \in N_t$ and so $0 \in N_t$,

Let $\lambda, \mu \in X$ be such that $\lambda * \mu \in N_t$ and $\mu \in N_t$ then $T_N(\lambda * \mu) \geq t$ and $T_N(\lambda) \geq t$.

Since N is a neutrosophic ideal, then from (ii) of the definition 3.1, it follows that

$T_N(\mu) \geq \min\{T_N(\lambda * \mu), T_N(\lambda)\} \geq t$ and, we have that $\mu \in N_t$.

Hence, N_t is an ideal of X .

Similarly, we can prove for I_N and F_N .

Conversely, we need to show that (i) and (ii) of definition 3.1 are true.

If (i) of definition 3.1 is not satisfied, then there exists $\lambda' \in X$ such that $T_N(0) < T_N(\lambda')$.

If we take $t' = \{T_N(\lambda') + T_N(0)\}/2$ then $T_N(0) < t'$ and $0 \leq t' < T_N(\lambda') \leq 1$

then $\lambda' \in N$ and $N \neq \varphi$ as $N_{t'}$ is an ideal of X we have $0 \in N_{t'}$ and so $T_N(0) \geq t'$.

This is a contradiction. Therefore we have $T_N(0) \geq T_N(\lambda')$.

If (ii) is not true then there exists $\lambda', \mu' \in X$ such that $T_N(\mu') < \min\{(\lambda' * \mu'), T_N(\lambda')\}$

Putting $t' = T_N(\mu') + \min\{T_N(\lambda' * \mu'), T_N(\lambda')\}/2$

then $T_N(\mu') < t'$ and $0 \leq t' < \min\{T_N(\lambda' * \mu'), T_N(\lambda')\} \leq 1$,

Hence, $T_N(\lambda' * \mu') > t'$ and $T_N(\lambda') > t'$ which imply that $\lambda' * \mu' \in N_{t'}$, and $\lambda' \in N_{t'}$.

Since $N_{t'}$ is an ideal. It follows that $\mu' \in N_{t'}$ and $T_N(\mu') \geq t'$.

This is a contradiction. Thus, N is neutrosophic ideal of X .

Similarly, we can prove for I_N and F_N . ■

Proposition 3.9: Every neutrosophic ideal of KK-algebra $(X, *, 0)$ is neutrosophic sub algebra of X .

Proof: Assume that N be an neutrosophic ideal of a KK-algebra X . Next based on theorem 3.8 every $t \in [0, 1]$, N_t is an ideal of X . According to proposition 2.6 for every $t \in [0, 1]$, N_t is a subalgebra of X . Thus, from the theorem 3.7, N is neutrosophic subalgebra of KK-algebra X . ■

Theorem 3.10: Let I be an ideal of KK-algebra of X . Then for any fixed number t in an open interval $(0, 1)$, there exists neutrosophic ideal N of X such that $N_t = I$

Proof: Define $N: X \rightarrow [0, 1]$ by $T_N(\lambda) = \begin{cases} t & \text{if } \lambda \in I \\ 0 & \text{otherwise} \end{cases}$

Where ' t ' is a fixed number in $(0, 1)$ clearly $T_N(0) \geq T_N(\lambda)$ for all $\lambda \in X$.

Let $\lambda, \mu \in X$, if $\lambda \notin I$, then $T_N(\lambda) = 0$ and so $T_N(\mu) \geq 0 = \min\{T_N(\lambda * \mu), T_N(\lambda)\}$,

If $\lambda * \mu \in I$, Then clearly $T_N(\mu) \geq \min\{T_N(\lambda * \mu), T_N(\lambda)\}$

If $\mu \notin I$ and $\lambda \in I$ then $\lambda * \mu \notin I$, since I is an ideal, then $T_N(\mu) = 0 = \min\{T_N(\lambda * \mu), T_N(\lambda)\}$. Hence, N is neutrosophic ideal of X . It is clear that $N_t = I$.

Similarly, we can prove for I_N and F_N . ■

Theorem 3.11: Let N be neutrosophic ideal of a KK-algebra X and assume that N_{t_1}, N_{t_2} be level ideals of N , as well as $t_1 < t_2$ then the following are equivalent

- (i) $N_{t_1} = N_{t_2}$
- (ii) There is no $\lambda \in X$ such that $t_1 \leq T_N(\lambda) \leq t_2$.

Proof: Let N be neutrosophic ideal of a KK-algebra X .

Assume that $N_{t_1} = N_{t_2}$ for $t_1 < t_2$ and that there exists $\lambda \in X$ such that $t_1 \leq T_N(\lambda) \leq t_2$ then N_{t_2} is proper subset of N_{t_1} . This is a contradiction.

Therefore, there is no $\lambda \in X$ such that $t_1 \leq T_N(\lambda) \leq t_2$

Conversely, suppose that there is no $\lambda \in X$ such that $t_1 \leq T_N(\lambda) \leq t_2$.

It follows that $t_1 < t_2$ then $N_{t_2} \subset N_{t_1}$.

Let $\lambda \in N_{t_1}$ then $T_N(\lambda) \geq t_1$ and $T_N(\lambda) \geq t_2$ because $T_N(\lambda)$ does not lie between t_1 and t_2 hence $\lambda \in N_{t_2}$. This implies that $N_{t_1} \subset N_{t_2}$ therefore $N_{t_1} = N_{t_2}$. ■

Proposition 3.12: The intersection of any set of neutrosophic ideals of KK-algebra X is also neutrosophic ideal.

Proof: For any $\lambda, y \in X$, let $\{N_i / i \in A\}$ be a family of neutrosophic ideals of KK-algebra X then, $i \in A$, $(\cap_{i \in A} T_{N_i})(0) = \inf\{T_{N_i}(0)\} \geq \inf\{T_{N_i}(\lambda)\} = \cap_{i \in A} T_{N_i}(\lambda)$

$$\begin{aligned} \cap_{i \in A} T_{N_i}(y) &= \inf T_{N_i}(\mu) \\ &\geq \inf (\min (T_{N_i}(\lambda * \mu), T_{N_i}(\lambda))) \end{aligned}$$

$$\begin{aligned}
&= \min (Inf T_{N_i}(\lambda * \mu), Inf T_{N_i}(\lambda)) \\
&= \min \{ \cap_{i \in A} T_{N_i}(\lambda * \mu), \cap_{i \in A} T_{N_i}(\lambda) \}.
\end{aligned}$$

Similarly, we can prove for I_N and F_N . ■

Definition 3.13: Let $(X, *, 0)$ and $(Y, *, 0')$ be any two KK-algebras and $f: (X, *, 0) \rightarrow (Y, *, 0')$ be a mapping between the corresponding nonempty sets X and Y . For every neutrosophic subset N of X , the neutrosophic subset N' of Y defined by

$$f(N)(\mu) = \begin{cases} \sup \{ T_N(\lambda) : \lambda \in f^{-1}(\mu) \} & \text{if } f^{-1}(\mu) = \{ \lambda \in X, f(\lambda) = \mu \} \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

is called as the image of N under f . Similarly, N' is the subset of Y then the neutrosophic subset $N = (N' \circ f)$

That is, $T_N(\lambda) = T_{N'}[f(\lambda)]$ for all $\lambda \in X$ is called pre image of N' under f .

Theorem 3.14: The neutrosophic ideal is also its homomorphic pre-image.

Proof: A homomorphism of KK-algebras, denoted by $f: (X, *, 0) \rightarrow (Y, *, 0')$, where N' is the neutrosophic ideal of Y and N is the pre image of N' under f , then $T_{N'}[f(\lambda)] = T_N(\lambda)$ for every $\lambda \in X$. Since $f(\lambda) \in Y$ and N' is neutrosophic ideal of Y it follows that

$$T_{N'}(0') \geq T_{N'}[f(\lambda)] = T_N(\lambda) \text{ for every } \lambda \in X \text{ where } 0' \text{ is the zero element of } Y.$$

$$\text{But, } T_{N'}(0') = T_{N'}(f(0)) = T_N(0) \text{ and so } T_N(0) \geq T_N(\lambda) \text{ for all } \lambda \in X.$$

$$\text{Let's now } \lambda, \mu \in X, \text{ then we obtain } T_N(\mu) \geq T_{N'}\{f(\mu)\}$$

$$\begin{aligned}
&\geq \min \{ T_{N'}\{f(\lambda) * f(\mu)\}, T_{N'}\{f(\lambda)\} \} \\
&= \min \{ T_{N'}\{f(\lambda * \mu)\}, T_{N'}\{f(\lambda)\} \} \\
&= \min \{ T_N(\lambda * \mu), T_N(\lambda) \}
\end{aligned}$$

$$\text{That is } T_N(\mu) \geq \min \{ T_N(\lambda * \mu), T_N(\lambda) \} \text{ for all } \lambda, \mu \in X.$$

Similarly, we can prove for I_N and F_N . ■

Definition 3.15 : A neutrosophic subset N of a set X has supremum property if for any subset A of X , there exist $a_0 \in A$ such that $T_N(a_0) = \sup \{ T_N(a) / a \in A \}$.

Theorem 3.16: Let $f: (X, *, 0) \rightarrow (Y, *, 0')$ be a homomorphism between KK-algebras X and Y respectively. For every neutrosophic ideal N in X with supremum property, then $f(T_N) = T_{N'}$ is a neutrosophic ideal of Y .

Proof: Let $(X, *, 0)$ and $(Y, *, 0')$ be any two KK-algebras and $f: (X, *, 0) \rightarrow (Y, *, 0')$ satisfies a homomorphism property, from the definition of 3.15,

$$\begin{aligned}
T_{N'}(\mu') &= f(T_N)(\mu') \\
&= \sup \{ T_N(\lambda) / \lambda \in f^{-1}(\mu') \} \text{ for all } \mu' \in Y \text{ (sup } \emptyset = 0)
\end{aligned}$$

$$\text{We have to prove } T_{N'}(\mu') \geq \min \{ T_{N'}(\lambda' * 0'), T_{N'}(\lambda') \} \text{ for all } \lambda', \mu' \in Y,$$

$$\text{Assume that the onto homomorphism of the KK-algebra is } f: (X, *, 0) \rightarrow (Y, *, 0'),$$

N is neutrosophic ideal of X with supremum property and N' is the image of N under f .

Since N is neutrosophic ideal of X , we have $T_N(0) \geq T_N(\lambda)$ for all $\lambda \in X$.

Note that $0 \in f^{-1}(0')$, where 0 and $0'$ are the zero elements of X and Y respectively.

$$\begin{aligned} \text{Thus we have } T_{N'}(0') &= \sup_{t \in f^{-1}(0')} T_N(t) \\ &= T_N(0) \\ &\geq T_N(\lambda) \text{ for all } \lambda \in X. \end{aligned}$$

Which implies that $T_{N'}(0') \geq \sup_{t \in f^{-1}(\lambda')} T_N(t) = T_{N'}(\lambda')$ for any $\lambda' \in Y$.

For any $\lambda', \mu' \in Y$, let $\lambda_0 \in f^{-1}(\lambda')''$, $\mu_0 \in f^{-1}(\mu')$ be such that

$$T_N(\lambda_0 * \mu_0) = T_{N'}[f(\lambda_0 * \mu_0)] = T_{N'}[f[(\lambda')'' * \mu']] = \sup_{\lambda_0 * \mu_0 \in f^{-1}(\lambda' * \mu')} T_N(\lambda_0 * \mu_0),$$

$$T_N(\lambda_0) = T_{N'}[f(\lambda_0)] = T_{N'}[f(\lambda')] = \sup_{\lambda_0 \in f^{-1}(\lambda')} T_N(\lambda_0).$$

$$\begin{aligned} \text{Then, } T_{N'}(\mu') &= \sup T_N(t) \\ &= T_{N_{t \in f^{-1}(\mu')}}(\mu_0) \\ &\geq \min\{T_N(\lambda_0 * \mu_0), T_N(\lambda_0)\} \\ &= \min\left\{\sup_{t \in f^{-1}(\lambda' * \mu')} T_N(t), \sup_{t \in f^{-1}(\lambda')} T_N(\lambda_0')\right\} \\ &= \min\{T_{N'}(\lambda' * \mu'), T_{N'}(\lambda')\} \end{aligned}$$

Thus, $T_{N'}(\mu') \geq \min\{T_{N'}(\lambda' * \mu'), T_{N'}(\lambda')\}$

Therefore, $T_{N'}$ is neutrosophic ideal of Y .

Similarly, we can prove for I_N and F_N . ■

4. Conclusions

This paper starts by discussing the concepts of KK-algebra and the neutrosophic ideal with appropriate examples. Then, we look into various fundamental aspects of the neutrosophic ideal in KK-algebra. We also need to deal with the homomorphism of image and inverse image of neutrosophic ideals in KK-algebra.

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