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Ranking Fuzzy Numbers by Defuzzification using Volumes

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Abstract:

Fuzzy numbers (FNs) are used to represent uncertain and ambiguous data and are mainly useful in decision-making. Ranking fuzzy numbers is an important concept in fuzzy set theory and has various applications in decision-making, data analysis, artificial intelligence, and optimization problems. To overcome the shortcomings in some of the existing fuzzy ranking methods, in this paper, we introduce a new ranking method for ranking generalized trapezoidal fuzzy numbers (GTrFNs) by a defuzzification technique using a score function defined using the volume of the solid obtained by revolving the left and right inverse membership functions of GTrFN about a vertical line. This score represents the defuzzified value of the GTrFN and is used to rank FNs. The proposed ranking method can rank different types of FNs in an effective manner and can be implemented in real-time applications like multicriteria decision-making and risk analysis.

Keywords: Generalized Trapezoidal fuzzy numbers; Defuzzification; Volume of the solid; Score Function

1 Introduction

Zadeh (1965) introduced the fuzzy set theory, which determines the impreciseness and ambiguity in decision-making problems. Fuzzy number (FN) ranking is an important aspect of decision-making, giving the best alternative among options. Ranking FNs is crucial in various fields like decision-making, data analysis, linear programming, risk analysis, and supply chain management. Several methods have been proposed over the years to rank FNs, each with its own benefits and constraints. Jain proposed the concept of ranking FNs (1976). Ranking FNs using the centroid concept was initiated by Yager (1978). Cheng (1998) proposed a ranking approach using the distance method. Later, Yao & Wu (2000) ranked FNs based on the decomposition principle and signed distance. Chen & Lu (2001) introduced an approximate approach for ranking FNs considering the left & right spreads at each α-level of FN. Later, Wang & Lee (2008) suggested an updated approach to Chu & Tsao's (2002) ranking method by considering the FNs based on the area between the centroid and the original points of an FN. Using distance minimization, Asady & Zendehnam (2007) ranked FNs. Abbasbandy & Hajjari (2009) proposed to rank FNs based on their left and right spreads. Additionally, Chen & Chen (2009) proposed to rank FNs according to their heights and spreads, the improved distance minimization approach for ranking FNs was proposed by Asady (2011), Nejad and Maschinchi (2011) presented a novel FN ranking approach on regions of the left and right sides

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utilizing the deviation degree method, which can successfully rank various FNs and their images. Later, Chen et al. (2012) proposed a novel ranking algorithm for ranking generalized fuzzy numbers (GFNs) with varying left and right heights, Yu et al. (2013) suggested an epsilon deviation-based ranking function, Eslamipoor et al. (2015) suggested a novel ranking algorithm for GFNs based on Euclidean distance, Rezvani (2015) proposed ranking generalized exponential fuzzy numbers based on variance. Chutia (2017) developed a modified epsilon deviation approach for ranking FNs. Dombi & Jónás (2020) proposed a new ranking algorithm to rank FNs using a probability-based preference intensity index method. Using radius, midpoint, and left & right spread values of TrFNs, Ponnialagan et al. (2018) proposed a complete ranking method. Patra (2022) introduced the ranking of generalized trapezoidal fuzzy numbers (GTrFNs), considering FNs' mean position, area, and perimeter as major factors. Hop (2022) proposed a new ranking method using relative relationships and shape characteristics of FNs. Prasad &Sinha (2022) introduced a ranking index using the left & right limits of the α-cut integral and mode area integral of the FN. Additionally, Jeevaraj (2022) proposed an improved ranking principle on GTrFNs and discussed the drawback of the Marimuthu and Mahapatra (2021) ranking approach. A setback in ranking FNs was introduced by Sotoudeh-Anvari & Sotoudeh-Anvari (2022), which indicates the most confusing state. They addressed some of the drawbacks of the existing ranking methods by giving counter-examples and studied the setback in fuzzy risk assessment in diabetes prediction. Later, Bihari et al. (2023a) ranked GTrFNs based on diagonal distance and mean also applied to supplier selection problem. Bihari et al. (2023b) introduced a new geometric approach using centroids to rank GTrFNs and applied it to a Multi-criteria decision-making (MCDM) problem in selecting the best security guard. Also, Bihari et al. (2024c) introduced a complete ranking function based on diagonal distance scores to rank GrTrFNs. It also addressed the drawbacks of Marimuthu & Mahapatra's (2021) approach and introduced the cocoso approach to solving MCDM problems.

In this paper, we introduce a new ranking method for ranking GTrFNs by a defuzzification technique using a score function defined using the volume of the solid obtained by revolving the left and right inverse membership functions of GTrFN about a vertical line. This score represents the defuzzified value of the GTrFN and is used to rank FNs. The rest of the paper is divided into six sections: The definitions related to the study are given in Section 2, and the proposed method is introduced in Section 3. Section 4 presents some properties, and some reasonable properties defined by Wang & Kerre (2001) are given in Section 5. Section 6 presents some numerical examples, Section 7 presents the comparative study, and Section 8 concludes the paper.

2 Preliminaries

The definitions of GFNs in this section are drawn from (Zimmermann, 2013).

Definition 2.1 If S is a universe of discourse and s be any particular element of S. The fuzzy set $\bar{Z}$ defined on S is a collection of ordered pairs,

$$\bar{Z} = \{(s, \mu_{\bar{Z}}(s)) | s \in S\}$$
(1)

where $\mu_{\bar{Z}}: S \to [0,1]$

Definition 2.2 A FN $\bar{P}$ shown in Fig.1 is a fuzzy subset of real line R with a membership function (MF) $f_{\bar{P}}$ satisfying the below properties:

1. $f_{\bar{P}}$ is a continuous from R to $[0, t]$,

2. $f_{\bar{P}}$ is strictly increasing on $[r_1, r_2]$,

3. $f_{\bar{P}}(x) = t$, for all $x \in [r_2, r_3]$,

4. $f_{\bar{P}}$ is strictly decreasing on $[r_3, r_4]$,

5. $f_{\bar{P}}(x) = 0$, otherwise

The MF of $f_{\bar{P}}$ can be expressed as:

$$f_{\bar{P}} = \begin{cases} f_{\bar{P}}^{L}(x); & r_1 \le x \le r_2, \\ t; & r_2 \le x \le r_3, \\ f_{\bar{P}}^{R}(x); & r_3 \le x \le r_4, \\ 0; & otherwise. \end{cases}$$
(2)

where $f_{\bar{P}}^{L} : [r_1, r_2] \to [0, t]$, and $f_{\bar{P}}^{R} : [r_3, r_4] \to [0, t]$.

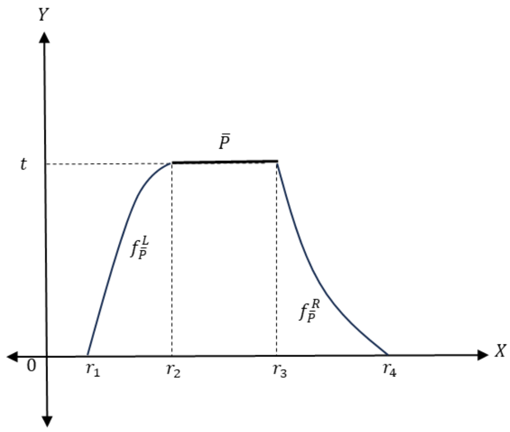


Fig .1. GFN representation

Definition 2.3 A GTrFN $\bar{P} = (r_1, r_2, r_3, r_4; t)$, shown in Fig. 2, is a fuzzy subset of the real line R with MF defined as follows:

$$f_{\bar{P}}(x) = \begin{cases} t\left(\frac{x - r_1}{r_2 - r_1}\right), & if\ r_1 \le x \le r_2, \\ t, & if\ r_2 \le x \le r_3, \\ t\left(\frac{r_4 - x}{r_4 - r_3}\right), & if\ r_3 \le x \le r_4, \\ 0, & otherwise. \end{cases} \quad (3)$$

here r_1, r_2, r_3, r_4 are real numbers, and $0 \le t \le 1$. If $t = 1$, then $\bar{P}$ is called a trapezoidal fuzzy number (TrFN), and if $r_2 = r_3$, then $\bar{P} = (r_1, r_2, r_3; t)$ is called a generalized triangular fuzzy number (GTFN).

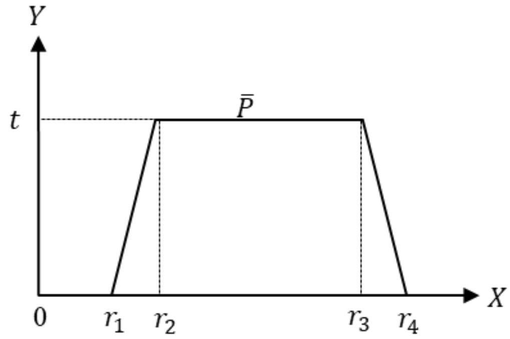


Fig. 2. MF of GTrFN $\bar{P}$.

Definition 2.4 Thomas et al. (2014) The volume of the solid generated by revolving the region between the y-axis and the graph of a continuous function $x = f^{-1}(y) \geq 0, L \leq 0 \leq y \leq w$, about a vertical line $y = L$ is

$$V = \int_0^w 2\pi(y - L)f^{-1}(y)dy \quad (4)$$

3 Proposed Method

This section presents the new method to rank FNs by defuzzification using the volume. A score function that represents the defuzzified value of a TrFN is defined by using the volume of the solid obtained by revolving the images of left and right membership functions (MFs) of the TrFN about a vertical line.

The volumes of the solid obtained by revolving the image of the left membership function (MF) of the GTrFN $\bar{P} = (r_1, r_2, r_3, r_4; t)$ about the vertical line $y = L$ is given by:

$$LV = \int_0^t 2\pi y f_L^{-1}(y)dy. \quad (5)$$

The volumes of the solid obtained by revolving the image of the right membership function (MF) about the vertical line $y = L$ is given by:

$$RV = \int_0^t 2\pi y f_R^{-1}(y)dy. \quad (6)$$

The score function represents the defuzzified value of the GTrFN $\bar{P} = (r_1, r_2, r_3, r_4; t)$, is defined as:

$$score(\bar{P}) = LV + RV = \int_0^t 2\pi y f_L^{-1}(y)dy + \int_0^t 2\pi y f_R^{-1}(y)dy \quad (7)$$

$$= 2\pi\left[\frac{r_1 t^2}{2} + \frac{(r_2 - r_1)t^2}{3}\right] + 2\pi\left[\frac{r_4 t^2}{2} - \frac{(r_4 - r_3)t^2}{3}\right] \quad (8)$$

$$score(\bar{P}) = \frac{2\pi t^2}{3}[(r_1 + r_4) + 2(r_2 + r_3)] \quad (9)$$

Ranking order

If $\bar{P}_1$ and $\bar{P}_2$ are two GTrFNs, then by using the above score, the ranking order is defined as follows:

i) $\bar{P}_1$ is less preferred to $\bar{P}_2$, expressed as $\bar{P}_1 \prec \bar{P}_2$, if $score(\bar{P}_1) < score(\bar{P}_2)$.

ii) $\bar{P}_1$ is more preferred to $\bar{P}_2$, expressed as $\bar{P}_1 \succ \bar{P}_2$, if $score(\bar{P}_1) > score(\bar{P}_2)$.

iii) $\bar{P}_1$ is equal to $\bar{P}_2$, expressed as $\bar{P}_1 \approx \bar{P}_2$, if $score(\bar{P}_1) = score(\bar{P}_2)$.

4 Properties

1) Let $\bar{I} = (i_1, i_2, i_3, i_4; t)$ & $\bar{J} = (j_1, j_2, j_3, j_4; t)$ be two GTrFNs, then

i) $score(\bar{I} + \bar{J}) = score(\bar{I}) + score(\bar{J})$
ii)$score(\bar{I} - \bar{J}) = score(\bar{I}) - score(\bar{J})$

Proof: i) Given $\bar{I} = (i_1, i_2, i_3, i_4; t)$ & $\bar{J} = (j_1, j_2, j_3, j_4; t)$ are two GTrFNs. From Eq. (9) we have

$$score(\bar{P}) = \frac{\pi}{3} t^2((r_1 + r_4) + 2(r_2 + r_3))$$

Now, $score(\bar{I} + \bar{J}) = \frac{\pi}{3} t^2((i_1 + j_1 + i_4 + j_4) + 2(i_2 + j_2 + i_3 + j_3))$

$$\Rightarrow \frac{\pi}{3} t^2\big((i_1 + i_4) + 2(i_2 + i_3)\big) + \frac{\pi}{3} t^2((i_1 + i_4) + 2(i_2 + i_3))$$

$$\therefore score(\bar{I} + \bar{J}) = score(\bar{I}) + score(\bar{J})$$

Similarly

ii) $score(\bar{I} - \bar{J}) = \frac{\pi}{3} t^2((i_1 - j_1 + i_4 - j_4) + 2(i_2 - j_2 + i_3 - j_3))$

$$\Rightarrow \frac{\pi}{3} t^2\big((i_1 + i_4) + 2(i_2 + i_3)\big) - \frac{\pi}{3} t^2((i_1 + i_4) + 2(i_2 + i_3))$$

$$\therefore score(\bar{I} + \bar{J}) = score(\bar{I}) - score(\bar{J})$$

2) Let $\bar{I} = (i_1, i_2, i_3, i_4; t)$ be a GTrFN. Then

i) $score(k\bar{I}) = kscore(\bar{I})$
ii) $score(-\bar{I}) = -score(\bar{I})$

Proof: i) Given $\bar{I} = (i_1, i_2, i_3, i_4; t)$ be a GTrFN. From the above Eq. (9) we have

$$score(\bar{P}) = \frac{\pi}{3} t^2((r_1 + r_4) + 2(r_2 + r_3))$$

Now, $$score(k\bar{I}) = \frac{\pi}{3} t^2((ki_1 + ki_4) + 2(ki_2 + ki_3))$$

$$\Rightarrow score(k\bar{I}) = k\frac{\pi}{3} t^2((i_1 + i_4) + 2(i_2 + i_3))$$

$$\therefore score(k\bar{I}) = kscore(\bar{I})$$

ii) Given $\bar{I} = (i_1, i_2, i_3, i_4; t)$ be a GTrFN. From the above Eq. (9) we have

$$score(\bar{P}) = \frac{\pi}{3} t^2((r_1 + r_4) + 2(r_2 + r_3))$$

Now, $$score(-\bar{I}) = \frac{\pi}{3} t^2((-i_1 - i_4) + 2(-i_2 - i_3))$$

$$\Rightarrow score(-\bar{I}) = \frac{\pi}{3} t^2(-(i_1 + i_4) - 2(i_2 + i_3))$$

$$\Rightarrow score(-\bar{I}) = -\frac{\pi}{3} t^2((i_1 + i_4) + 2(i_2 + i_3))$$

$$\therefore score(-\bar{I}) = -score(\bar{I})$$

3) Let $I = (0,0,0,0; t)$ be a GTrFN, then $score(I) = 0$.

Proof: Given $\bar{I} = (0,0,0,0; t)$ be a GTrFN. From the above Eq. (9) we have

$$score(\bar{P}) = \frac{\pi}{3} t^2((r_1 + r_4) + 2(r_2 + r_3))$$

Now, $$score(I) = \frac{\pi}{3} t^2\big((0+0) + 2(0+0)\big) = 0$$

$$\therefore score(I) = 0$$

5 Reasonable properties

In this section, we present some reasonable properties Wang &Kerre (2001)

Let $\boldsymbol{A}$ be the ordering approach and $\boldsymbol{B}$ be the set of fuzzy quantities for which the method $\boldsymbol{A}$ can be applied. $\boldsymbol{M}$ is a finite subset of $\boldsymbol{B}$ and $\bar{X}, \bar{Y}, \bar{Z}$ are elements of $\boldsymbol{M}$.

$\boldsymbol{P_1}$) For an arbitrary finite subset $\boldsymbol{M}$ of $\boldsymbol{B}$ and $\bar{X} \in \boldsymbol{B}$ and $\bar{X} \in \boldsymbol{M}$, $\bar{X} \succcurlyeq \bar{X}$ by $\boldsymbol{A}$ on $\boldsymbol{M}$.

$\boldsymbol{P_2}$) For an arbitrary finite subset $\boldsymbol{M}$ of $\boldsymbol{B}$ and $(\bar{X}, \bar{Y}) \in \boldsymbol{M^2}$, $\bar{X} \succcurlyeq \bar{Y}$ and $\bar{Y} \succcurlyeq \bar{X}$ by $\boldsymbol{A}$ on $\boldsymbol{M}$**,** we should have $\bar{X} \backsim \bar{Y}$ by $\boldsymbol{A}$ on $\boldsymbol{M}$.

$\boldsymbol{P_3}$) For an arbitrary finite subset $\boldsymbol{M}$ of $\boldsymbol{B}$ and $(\bar{X}, \bar{Y}, \bar{Z}) \in \boldsymbol{M^3}$, $\bar{X} \succcurlyeq \bar{Y}$ and $\bar{Y} \succcurlyeq \bar{Z}$ by $\boldsymbol{A}$ on $\boldsymbol{M}$**,** we should have $\bar{X} \gtrsim \bar{Z}$ by $\boldsymbol{A}$ on $\boldsymbol{M}$.

$\boldsymbol{P_4}$) For an arbitrary finite subset $\boldsymbol{M}$ of $\boldsymbol{B}$ and $(\bar{X}, \bar{Y}) \in \boldsymbol{M^2}$, $\inf supp(\bar{X}) > \sup supp(\bar{Y})$, we should have $\bar{X} \gtrsim \bar{Y}$ by $\boldsymbol{A}$ on $\boldsymbol{M}$.

$\boldsymbol{P_4'}$) For an arbitrary finite subset $\boldsymbol{M}$ of $\boldsymbol{B}$ and $(\bar{X}, \bar{Y}) \in \boldsymbol{M^2}$, $\inf supp(\bar{X}) > \sup supp(\bar{Y})$, we should have $\bar{X} \succ \bar{Y}$ by $\boldsymbol{A}$ on $\boldsymbol{M}$.

$\boldsymbol{P_5}$) Let $\boldsymbol{B}$ and $\boldsymbol{B'}$ be two arbitrary finite sets of fuzzy quantities in which $\boldsymbol{A}$ can be applied and $\bar{X}$ & $\bar{Y}$ are in $\boldsymbol{B} \cap \boldsymbol{B'}$**.** We obtain the ranking order $\bar{X} \succ \bar{Y}$, by $\boldsymbol{A}$ on $\boldsymbol{B'}$ iff $\bar{X} \succ \bar{Y}$ by $\boldsymbol{A}$ on $\boldsymbol{B}$.

$\boldsymbol{P_6}$) Let $\bar{X}, \bar{Y}, \bar{X} + \bar{Z}$ & $\bar{Y} + \bar{Z}$ be the elements of $\boldsymbol{B}$**.** If $\bar{X} \gtrsim \bar{Y}$ by $\boldsymbol{A}$ on $\{\bar{X}, \bar{Y}\}$, then $\bar{X} + \bar{Z} \gtrsim \bar{Y} + \bar{Z}$ by $\boldsymbol{A}$ on $\{\bar{X} + \bar{Z}, \bar{Y} + \bar{Z}\}$.

$\boldsymbol{P_6'}$) If $\bar{X} \succ \bar{Y}$ by $\boldsymbol{A}$ on $\{\bar{X}, \bar{Y}\}$, then $\bar{X} + \bar{Z} \succ \bar{Y} + \bar{Z}$ by $\boldsymbol{A}$ on $\{\bar{X} + \bar{Z}, \bar{Y} + \bar{Z}\}$ when $\bar{Z} \neq 0$.

$\boldsymbol{P_7}$) Let $\bar{X}, \bar{Y}, \bar{X}\bar{Z}$ & $\bar{Y}\bar{Z}$ be the elements of $\boldsymbol{B}$ and $\bar{Z} \geq 0$**.** $\bar{X} \gtrsim \bar{Y}$ by $\boldsymbol{A}$ on $\{\bar{X}, \bar{Y}\}$, then $\bar{X}\bar{Z} \gtrsim \bar{Y}\bar{Z}$ by $\boldsymbol{A}$ on $\{\bar{X}\bar{Z}, \bar{Y}\bar{Z}\}$.

6 Numerical examples

1) Consider two GTrFNs $Q_1 = (0.1,0.2,0.3,0.5; 1)$ & $Q_2 = (0.1,0.3,0.4,0.6; 1)$ taken from Le & Chu (2023), shown in Fig. 3

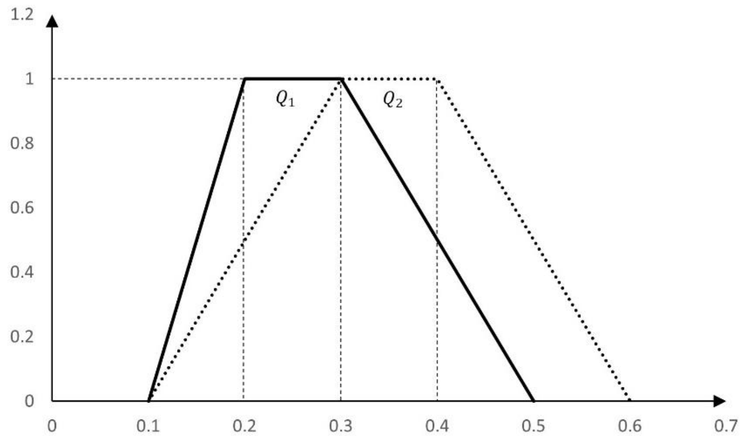


Fig.3 $Q_1 = (0.1,0.2,0.3,0.5; 1.0)$, $Q_2 = (0.1,0.3,0.4,0.5; 1.0)$

By applying the proposed method, we get $score(Q_1) = 1.6761$, $score(Q_2) = 2.2$, so, the ranking order is $Q_1 \prec Q_2$ and our result matches with Le & Chu's (2023) method.

2) Consider two GTrFNs $Q_1 = (0.1,0.2,0.3,0.5; 1)$ & $Q_2 = (-0.5, -0.3, -0.2, -0.1; 1)$ taken from Le & Chu (2023), shown in Fig. 4

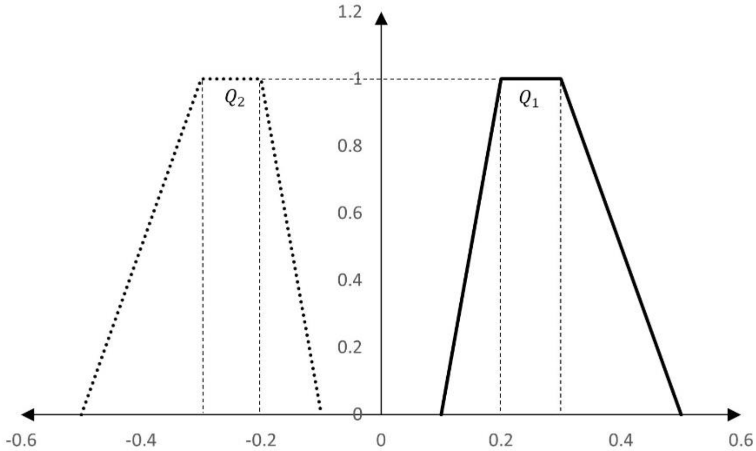


Fig.4 $Q_1 = (0.1,0.2,0.3,0.5; 1.0)$, $Q_2 = (-0.5, -0.3, -0.2, -0.1; 1.0)$

By applying the proposed method, we get $score(Q_1) = 1.6761$, $score(Q_2) = -1.6761$, so, the ranking order is $Q_2 \prec Q_1$ and our result matches with Le & Chu's (2023) method.

3) Consider three GTrFNs $Q_1 = (1,2,3,5; 0.7)$ & $Q_2 = (2,4,6,7; 0.6)$, $Q_3 = (4,6,6,8; 0.8)$ taken from Bihari et al. (2023), shown in Fig. 5

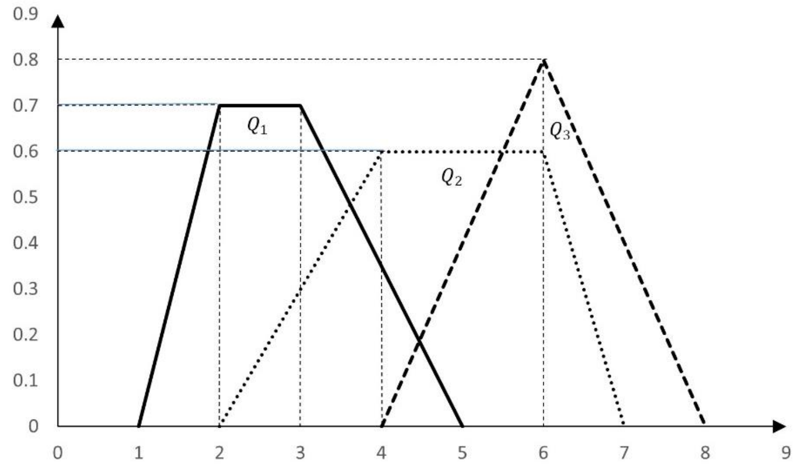


Fig.5 $Q_1 = (1,2,3,5; 0.7)$, $Q_2 = (2,4,6,7; 0.6)$, $Q_3 = (4,6,6,8; 0.8)$

By applying the proposed method, we get $score(Q_1) = 8.2133$, $score(Q_2) = 10.9371$, $score(Q_3) = 24.1371$, so the ranking order is $Q_1 \prec Q_2 \prec Q_3$ and our result matches with Bihari et al. (2023) method.

4) Consider two FNs $Q_1 = (2,3,8; 0.8)$ & $Q_2 = (2,4,6,8; 0.7)$ taken from Bihari et al. (2023), shown in Fig. 6

By applying the proposed method, we get $score(Q_1) = 14.7504$, $score(Q_2) = 15.4$, so the ranking order is $Q_1 \prec Q_2$ and our result matches with Bihari et al. (2023) method.

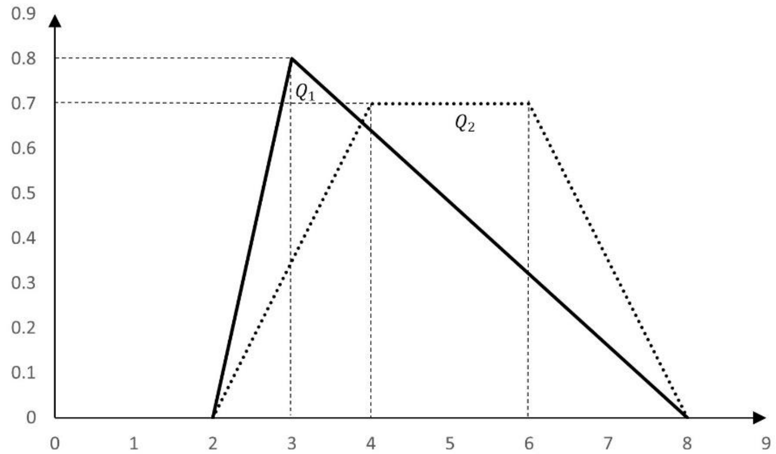


Fig.6 $Q_1 = (2,3,3,8; 0.8)$, $Q_2 = (2,4,6,8; 0.7)$

5) Consider the following fuzzy sets taken from Haji et al. (2014) shown in Fig. 7.

$$g_{Q_1} = \begin{cases} x-2, & 2 \leq x \leq 4 \\ \dfrac{6-x}{2}, & 4 \leq x \leq 6 \\ 0, & othewise \end{cases}$$

$$g_{Q_2} = \begin{cases} x-3, & 3 \leq x \leq 5 \\ 6-x, & 5 \leq x \leq 6 \\ 0, & othewise \end{cases}$$

$$g_{Q_3} = \begin{cases} x - 3, & 3 \leq x \leq 4 \\ 1, & 4 \leq x \leq 5 \\ \dfrac{7 - x}{2}, & 5 \leq x \leq 7 \\ 0, & othewise \end{cases}$$

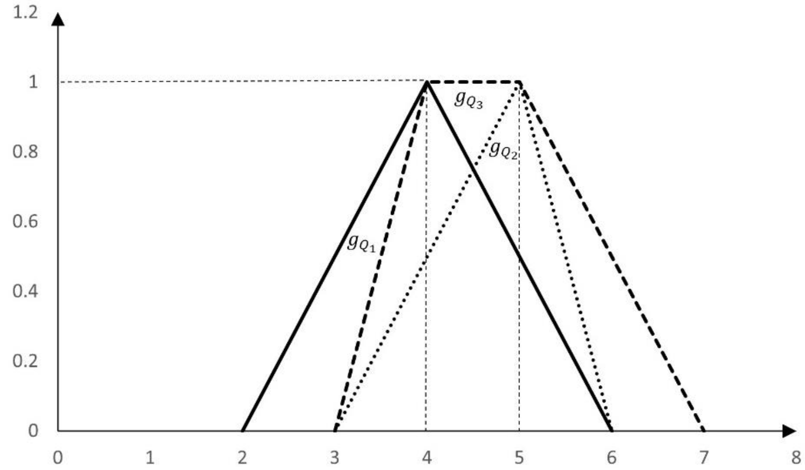


Fig.7 $g_{Q_1} = (2,4,4,6; 1.0)$, $g_{Q_2} = (3,5,5,6; 1.0)$, $g_{Q_3} = (3,4,5,7; 1.0)$

By applying the proposed method, we get $score(Q_1) = 47.1$, $score(Q_2) = 56.52$, $score(Q_3) = 59.66$, so the ranking order is $Q_1 \prec Q_2 \prec Q_3$ and our result matches with Haji et al. (2014) method.

6) Consider four FNs $Q_1 = (0.1,0.2,0.2,0.3; 1)$ & $Q_2 = (0.3,0.4,0.4,0.5; 1)$, $Q_3 = (0.6,0.7,0.8; 1)$, $Q_4 = (0.8,0.9,0.9,1.0; 1)$ taken from Ponnialagan et al. (2017) shown in Fig. 8.

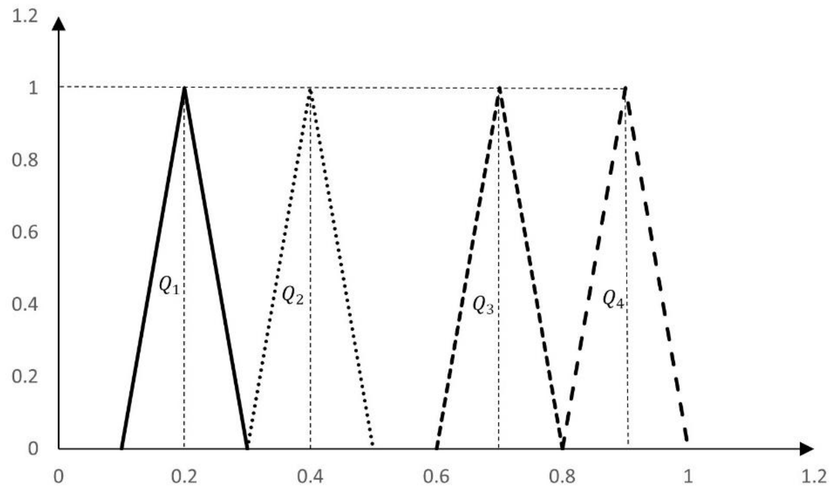


Fig.8 $Q_1 = (0.1,0.2,0.2,0.3; 1.0)$, $Q_2 = (0.3,0.4,0.4,0.5; 1.0)$,

$Q_3 = (0.6,0.7,0.7,0.8; 1.0)$, $Q_4 = (0.8,0.9,0.9,1.0; 1.0)$

By applying the proposed method, we get $score(Q_1) = 1.2571$, $score(Q_2) = 2.5142$, $score(Q_3) = 4.4$, $Q_4 = 5.6571$, so the ranking order is $Q_1 \prec Q_2 \prec Q_3 \prec Q_4$ and our result matches with Ponnialagan et al. (2017) method.

7) Consider three FNs $Q_1 = (0.1,0.3,0.3,0.8;1)$ & $Q_2 = (0.4,0.5,0.5,0.6;1)$, $Q_3 = (1,1,1,1;1)$,taken from Ponnialagan et al. (2017) shown in Fig. 9.

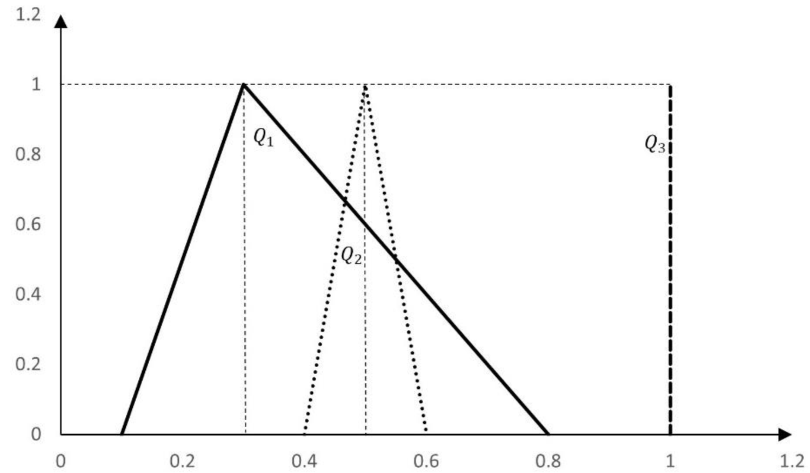


Fig.9 $Q_1 = (0.1,0.3,0.3,0.8;1.0)$, $Q_2 = (0.4,0.5,0.5,0.6;1.0)$, $Q_3 = (1,1,1,1;1.0)$

By applying the proposed method, we get $score(Q_1) = 2.2$, $score(Q_2) = 3.1428$, $score(Q_3) = 6.2857$, so, the ranking order is $Q_1 \prec Q_2 \prec Q_3$ and our result matches with Ponnialagan et al. (2017) method.

7 Comparative study

7.1) Consider the following fuzzy sets taken from Patra (2022), shown in Fig. 10.

a) Set I $Z_1 = (0.1,0.2,0.2,0.3;1.0)$, $Z_2 = (0.1,0.2,0.2,0.3;0.8)$

b) Set II $Z_1 = (1,1,1,1;1.0)$, $Z_2 = (1,1,1,1;0.8)$, $Z_3 = (1,1,1,1;0.5)$

c) Set III $Z_1 = (5,6,6,7;1.0)$, $Z_2 = (5.9,6,6,7;1.0)$, $Z_3 = (6,6,6,7;1.0)$

d) Set IV $Z_1 = (0.4,0.5,0.5,1.0;1.0)$, $Z_2 = (0.4,0.7,0.7,1.0;1.0)$, $Z_3 = (0.4,0.9,0.9,1.0;1.0)$

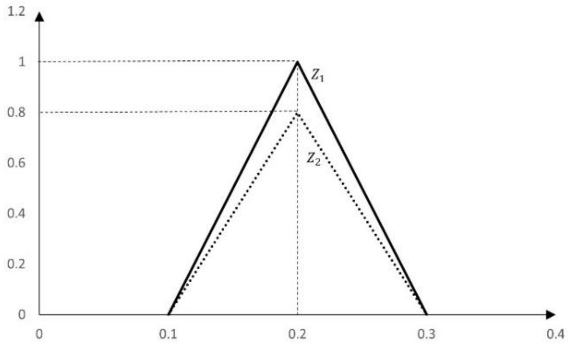
e) Set V $Z_1 = (0.2,0.5,0.5,0.8;1.0)$, $Z_2 = (0.4,0.5,0.5,0.8;1.0)$

Table 1 Comparative study taken from Patra (2022)

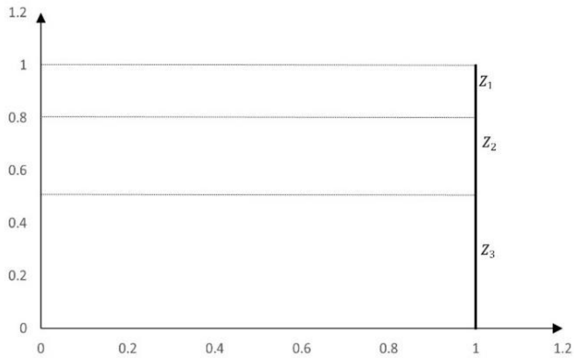
Methods	**Set I**		**Set II**			**Set III**			**Set IV**			**Set V**	
	Z_1	Z_2	Z_1		Z_3	Z_1	Z_2	Z_3	Z_1	Z_2	Z_3	Z_1	Z_2
Yager (1978)	0.2	0.2	-	- Z_2	-	6	6.3		0.6333	0.7	0.7666	0.5	0.5
Wang et al. (2006)	0.3887	0.3333	-	-	-	6.0092	6.3088	6.3421	0.7157	0.775	0.8359	0.35	0.45
Chen & Sanguansat (2011)	0.2	0.1882	1	0.9412	0.8	0.8571	0.8892	0.8928	0.6	0.7	0.8	0.5	0.5
Chen & Chen (2009)	0.1849	0.1479	1	0.8	0.5	0.7676	0.8279	0.8333	0.4721	0.562	0.6295	0.4016	0.462
Nasseri et al. (2013)	0.89	0.7118	2.5	2.32	2.125	12	12.766	12.853	1.6227	1.817	2.0227	1.4174	1.49
Rezvani (2015)	0.0116	0.0115	-	-	-	8.952	10.179	10.327	0.1366	0.137	0.1366	0.0782	0.063
Asady (2010)	0.1666	0.1666	0	0	0	0.6666	0.7101	0.7142	0.1818	0.375	0.8	0.3745	0.375

Yu et al. (2013)	1	1	1	1	1	0.0343	12.972	57.601	0.0467	1	21.396	0.1176	0.074
Abbasbandy & Hajjari (2009)	0.2	0.2	1	1	1	6	6.075	6.0833	0.5333	0.7	0.8666	0.5	0.5
Chutia (2017)	5.5597	0.213	60.523	1.0805	0.0017	0.0291	13.101	67.996	0.0426	0.934	23.431	0.1263	9.748
Patra (2022)	0.2	0.131	1	0.8	0.5	6	2.4977	2.212	0.6	0.691	0.8	0.5	0.455
Proposed method	1.2571	0.8045	6.2857	4.0228	1.5714	37.714	38.657	38.7619	3.5619	4.4	5.238	3.1428	3.352

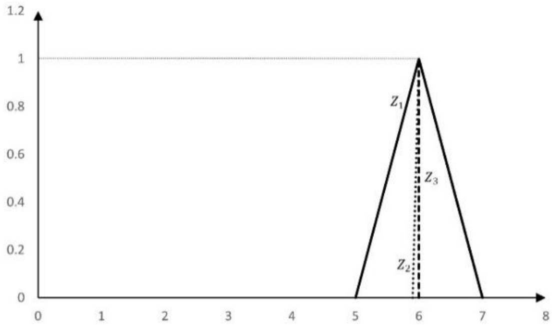
1) For Set I from Table 1, we can see that Yager (1978), Asady (2010), Yu et al. (2013), and Abbasbandy & Hajjari (2009) couldn't give the correct ranking order. From Fig.10, we can see that Z_1 & Z_2 has the same support, but the core is different due to different heights, so the ranking order should be $Z_1 > Z_2$. Our results match with all the other methods.
2) For Set II from Table 1, we can see that Yager (1978), Rezvani (2015), and Wang et al. (2006) failed to rank the FNs. Asady (2010), Yu et al. (2013). Abbasbandy & Hajjari (2009), gave incorrect ranking order. Our ranking order is $Z_1 > Z_2 > Z_3$ which matches with the other methods.
3) For Set III from Table 1, we can see that Patra (2022) couldn’t give the correct ranking order i.e. $Z_1 < Z_2 < Z_3$, and all other methods' results match with the proposed method's result as the x-coordinate centroid values of the FNs are the same as the order of the proposed method.
4) For Set IV from Table 1, we can see that, the proposed method results match with all the other methods i.e. $Z_1 < Z_2 < Z_3$.
5) For Set V from Table 1, we can see that Yager (1978), Chen & Sanguansat (2011), Rezvani (2015), Asady (2010), Yu et al. (2013), Abbasdandy & Hajjari (2009), Patra (2022) couldn’t give the correct ranking order i.e. $Z_1 < Z_2$, and all other methods' results match with the proposed method's result as the x-coordinate centroid values of the FNs are the same as the order of the proposed method.



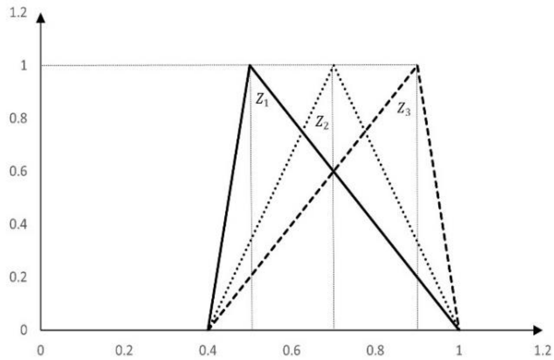
Set I $Z_1 = (0.1,0.2,0.2,0.3; 1.0)$, $Z_2 = (0.1,0.2,0.2,0.3; 0.8)$



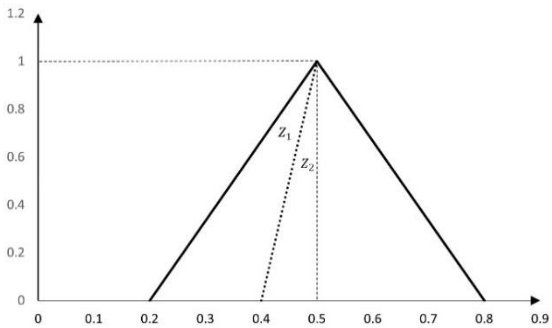
Set II $Z_1 = (1.0,1.0,1.0,1.0; 1.0)$, $Z_2 = (1.0,1.0,1.0,1.0; 0.8)$, $Z_3 = (1.0,1.0,1.0,1.0; 0.5)$



Set III $Z_1 = (5,6,6,7; 1.0)$, $Z_2 = (5.9,6,6,7; 1.0)$, $Z_3 = (6,6,6,7; 1.0)$



Set IV $Z_1 = (0.4,0.5,0.5,1.0; 1.0)$, $Z_2 = (0.4,0.7,0.7,1.0; 1.0)$, $Z_3 = (0.4,0.9,0.9,1.0; 1.0)$



Set V $Z_1 = (0.2,0.5,0.5,0.8; 1.0)$, $Z_2 = (0.4,0.5,0.5,0.8; 1.0)$

Fig. 10 Fuzzy Sets taken from Patra (2022)

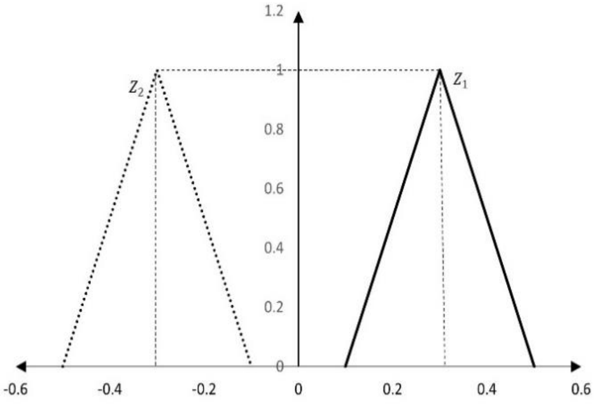
7.2) Consider the following fuzzy sets taken from Cheng et al. (2022), shown in Fig. 11.

a) Set I $Z_1 = (0.1,0.3,0.3,0.5; 1.0)$, $Z_2 = (-0.5,-0.3,-0.3,-0.1; 1.0)$
b) Set II $Z_1 = (0.1,0.2,0.4,0.5; 1.0)$, $Z_2 = (1,1,1,1; 1.0)$
c) Set III $Z_1 = (0.1,0.3,0.3,0.5; 0.8)$, $Z_2 = (0.1,0.3,0.3,0.5; 1.0)$
d) Set IV $Z_1 = (0.1,0.3,0.3,0.5; 1)$, $Z_2 = (0.3,0.5,0.5,0.7; 1)$
e) Set V $Z_1 = (0,0.4,0.6,0.8; 1)$, $Z_2 = (0.2,0.5,0.5,0.9; 1)$, $Z_3 = (0.1,0.6,0.7,0.8; 1.0)$

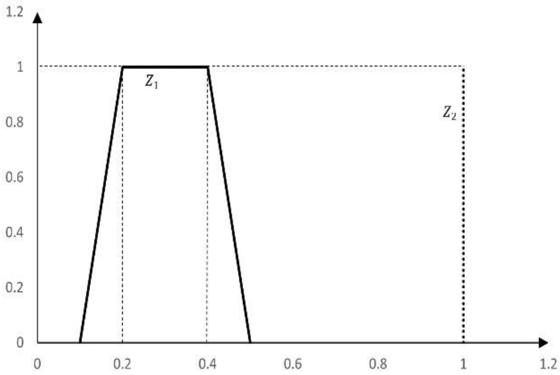
Table 2 Comparative study table taken from Cheng et al. (2022)

Methods	**Set I**		**Set II**		**Set III**		**Set IV**		**Set V**		
	Z_1	Z_2	Z_1	Z_2	Z_1	Z_2	Z_1	Z_2	Z_1	Z_2	Z_3
Chen et al. (2012)	0.2553	-0.2533	0.2533	1	0.2462	0.2553	0.2553	0.4444	0.4	0.4667	0.5057
Baker & Gegoy (2014)	0.0867	-0.0867	0.1096	0.3333	0.0715	0.0867	0.0867	0.1444	0.1197	0.1363	0.1452
Madhuri et al. (2014)	0.5774	0.5774	0.5885	-	0.4934	0.5774	0.5774	0.7024	0.6794	0.7052	0.7684
Wang (2015)	0.25	0	0.5	1	-	0.5	0.25	0.75	0.4615	0.5119	0.5275
Jiang (2015)	0.2882	-0.2882	0.2869	1	0.2306	0.2882	0.2882	0.4804	0.4146	0.4898	0.5103
Wu et al. (2018)	0.5906	-0.5906	0.5884	1	0.5332	0.5906	0.5906	0.7014	0.6506	0.7071	0.7003
Barazandeh & Ghazanfari (2021)	0.25	-0.25	0.2833	1	0.23	0.25	0.25	0.4167	0.396	0.4444	0.4594
Cheng et al. (2022)	0.74	0.26	0.36	1	0.332	0.34	0.34	0.5	0.47	0.5214	0.5348
Proposed method	1.8857	-1.8857	1.8857	6.2857	1.2068	1.8857	1.8857	3.1428	2.934	3.247	3.66

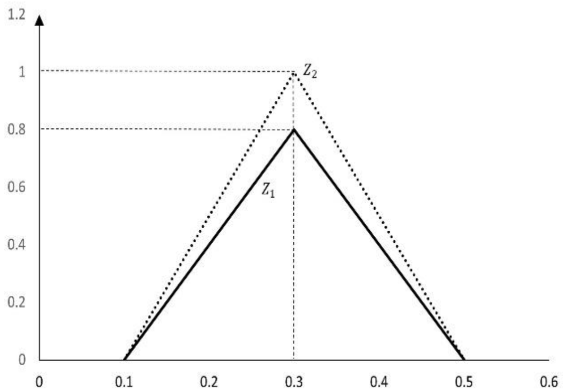
1) For Set I from Table 2, we can see that Baker & Gegoy (2014) couldn't give the correct ranking order i.e. $Z_2 < Z_1$ and the proposed method's result matches with all the other methods.
2) For Set II from Table 2, we can see that Madhuri et al. (2014) couldn't rank the FNs. The ranking order is $Z_1 < Z_2$ and the proposed method's result matches with all the other methods.
3) For Set III from Table 2, we can see that the proposed method results match with all other methods i.e. $Z_1 < Z_2$.
4) For Set IV from Table 2, we can see that the proposed method results match with all other methods i.e. $Z_1 < Z_2$.
5) For Set V from Table 2, we can see that Wu et al. (2018) couldn't give the correct ranking order i.e. $Z_1 < Z_2 < Z_3$ and the proposed method's result matches with all the other methods.



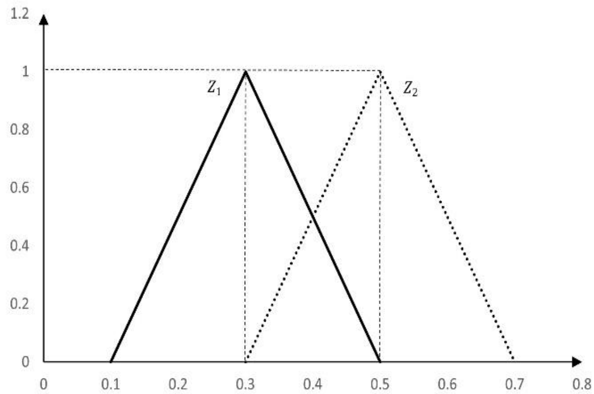
Set I $Z_1 = (0.1,0.3,0.3,0.5; 1.0)$, $Z_2 = (-0.5,-0.3,-0.3,-0.1; 1.0)$



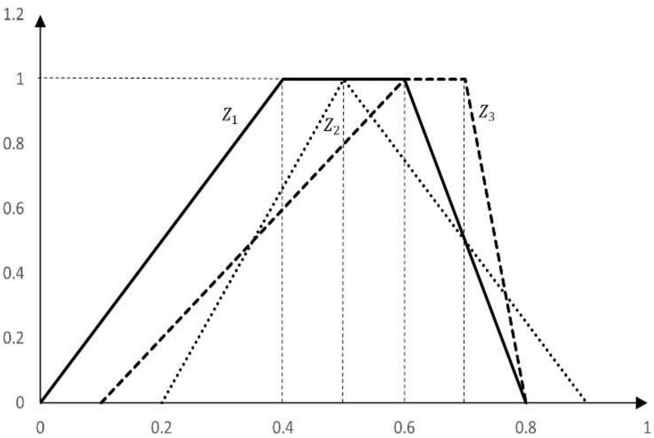
Set II $Z_1 = (0.1,0.2,0.4,0.5; 1.0)$, $Z_2 = (1.0,1.0,1.0,1.0; 1.0)$



Set III $Z_1 = (0.1,0.3,0.3,0.5; 0.8)$, $Z_2 = (0.1,0.3,0.3,0.5; 1.0)$



Set IV $Z_1 = (0.1,0.3,0.3,0.5; 1.0)$, $Z_2 = (0.3,0.5,0.5,0.7; 1.0)$



Set V $Z_1 = (0,0.4,0.6,0.8; 1.0)$, $Z_2 = (0.2,0.5,0.5,0.9; 1.0)$,
$Z_3 = (0.1,0.6,0.7,0.8; 1.0)$

Fig. 11 Fuzzy Sets taken from Cheng et al. (2022)

8 Conclusion

In this paper, we proposed a new method to rank GTrFNs using the concept of defuzzification by a score function using the volume of solid by revolving the images of left and right membership functions about a vertical line. The ranking score obtained is the defuzzified value of GTrFN and is used to select the best alternative from the available alternatives. The proposed method overcomes the limitations of some of the existing methods, and it can rank different types of FNs along with their images and crisp numbers. The proposed method can be applied to many applications, such as risk assessment, decision-making, and optimization problems.

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