Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 31 No. 7s (2024)

Concepts of Bi-Ternary Semi Groups

[1]N. Sandhya Rani, *[2]G. Srinivasa Rao, *[3]Ch. Ramprasad

[1]Research Scholar, Department of Mathematics & Statistics, School of Applied Sciences & Humanities, VFSTR Deemed to be University, Vadlamudi, Guntur, A.P., India. Email: n.sandhayarani@rguktrkv.ac.in

[1]Mathematics Mentor, RGUKT, R.K.Valley, Idupulapaya, A.P., India.

[2]Associate Professor, Department of Mathematics & Statistics, School of Applied Sciences & Humanities, VFSTR Deemed to be University, Vadlamudi, Guntur, A.P., India.Email:gsrinulakshmi77@gmail.com

[3]Associate Professor, Department of Mathematics, VVIT, Guntur, A.P., India.Email: ramprasadchegu1984@gmail.com

Article History:

Received: *01-06-2024*

Revised: *03-07-2024*

Accepted: *29-07-2024*

Abstract:

In this paper we made an attempt to study the algebraic structure of bi -groups, bi- ternary semi group ,bi-ternary sub semi group , ideals in bi ternary semi group and discussed some of its properties with counter examples.

Keywords: Group, Bi-groups, Bi-Ternary Semi Groups.

Introduction

Bi-groups are a particularly useful tool since they provide solutions to a significant difficulty that all groups encounter, namely that the union of two subgroups does not create any algebraic framework, but they do make a good bi-algebraic framework. The bi-group research was conducted from 1994 to 1996. Maggu were the initial one to use a syntax for bi-groups. Vasantha Kandaswamy and Meiyappan expanded this theory in 1997. Several individuals propose adjustments to some of Maggu's already proven outcomes. These conclusions included supplementary bi-group characterisation results. However, Vasantha Kandaswamy has lately researched the idea of bi-algebraic framework.Agboola and Akinola investigated bi-cosets in a bivector space..I expanded upon it the ternary operation is one of the operations in algebraic frame work that I used, and then used the same idea in every way that I could. G. Srinivasa Rao et.al[1-2, 13-18] discussed about ternary semirings, ordered ternary semirings and gamma semirings and their properties.

1. Preliminaries:

Definition 1.1: A non-empty set$G$and$'*'$is a binary operation on G if it satisfies the following conditions, then algebraic structure$(G,*)$ is called a **group**.

$i)$ $a,b \in G \Longrightarrow a*b \in G$

$ii)$ $a*(b*c) = (a*b)*c \forall a,b,c \in G$

$iii)$ If $e \in G$Such that $a*e = e*a = a \forall a \in G$

$iv)$ meant for each $a \in G$ present exist part $b \in G$ such with the aim of $a*b = b*a = e$

where$b = a^{-1}$

Definition 1.2: A non-empty group $G$and $' * '$ is a dual procedure on G if it satisfies the following conditions, then algebraic structure$(G,*)$ be call a **semi group**.

$i) a, b \in G \Longrightarrow a * b \in G$

$ii) a * (b * c) = (a * b) * c \forall a, b, c \in G$

Definition 1.3: A non-empty group be a T. Then T be understood to exist a **ternary semi group** from $T \times T \times T \rightarrow T$ which map $[x_1 x_2 x_3] \rightarrow x_1 x_2 x_3$ satisfy the condition:$[(x_1 x_2 x_3) x_4 x_5] = [x_1 (x_2 x_3 x_4) x_5] = [x_1 x_2 (x_3 x_4 x_5)]$ for all $x_i \in T, i = 1 to 5$.

Definition 1.4: A non-empty set $(G, +, \cdot)$ with dual action two are $' + '$ & $' \cdot '$ be call a **bi-group** if here be real two suitable sub sets G_1 & G_2 of G follow

$i)$ $G = G_1 \cup G_2$

$ii)$ $(G_1 +)$is a group.

$iii)$$(G_1, \cdot)$ is group.

Example 1.5: Let $G = \{\text{set of integers }\} \cup \{i, -i\}$ be a **bi group**.

Definition 1.6:A non-empty set $(G, +, \cdot)$ with dual action two are $' + '$ & $' \cdot '$ is call a **bi semi group** if here be real two suitable sub sets G_1 & G_2 of G as follow

$i) G = G_1 \cup G_2$

$ii)(G_1, +)$ be semi group.

$iii)(G_1, \cdot)$ be semi group.

Example 1.7: let $G = (Z^+, +) \cup (\{1, -1, i, -i\}, \cdot)$ is a **bi semi group**.

2. Main Results:

Definition 2.1: A non-empty set $T = T_1 \cup T_2$ be a **bi ternary semi group**. Then both T_1 and T_2 are satisfy the conditions of ternary semi group. i.e. $[(x_1 x_2 x_3) x_4 x_5] = [x_1 (x_2 x_3 x_4) x_5] = [x_1 x_2 (x_3 x_4 x_5)] \forall x_i \in T_1$

Similarly $[(abc)de] = [a(bcd)e] = [ab(cde)] \forall a, b, c, d, e \in T_2$

set of non-empty is T with ternary growth be a mark of [ ] is said to be a **Bi-Ternary semi group** if $T = T_1 \cup T_2$ someplace T_1 & T_2 be proper subsets of T such that

$i)T_1$ is ternary semi group.

$ii)T_2$ is ternary semi group.

Example 2.2: Let $T = \{i, -i\} \cup \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ then T is **bi ternary semi group** under complex multiplication and matrix multiplication.

Example 2.3: Let $T = \mathbb{C} \cup Z$ is **of bi ternary semi group**.

Example 2.4: Let $T = C_0^- \cup Z$ is **of bi ternary semi group**.

Example 2.5: Let $T = T_1 \cup T_2$ where if $T_1 = (C,\cdot)$ and $T_2 = (3\boldsymbol{Z}, [\ ])$ are twoternary semi group so that T is **Bi ternary semi group.**

Definition 2.6: A nonempty sub set $S = S_1 \cup S_2$ of a bi ternary semi group T be understood to be a **bi ternary sub semi group,** when if both $S_1 \& S_2$ satisfy the condition: $a_1 b_1 c_1 \forall a_1, b_1, c_1 \in S_1$

$a_2 b_2 c_2 \forall a_2, b_2, c_2 \in S_2$

Note 2.7: A non-blank sub group $S = S_1 \cup S_2$ of a bi-digit group T be tell to be a bi ternary sub semi group if and only if$S_1 S_1 S_1 \subseteq S_1$

$S_2 S_2 S_2 \subseteq S_2$

Example 2.8: Let $T = T_1 \cup T_2$ where $T_1 = (C,\cdot)$ and $T_2 = (\boldsymbol{Z}, [\ ])$ are two ternary semi groups. Let $S = S_1 \cup S_2$ where $S_1 = (\boldsymbol{C}_0^-, [\ ])$ and $S_2 = (3\boldsymbol{Z}, [\ ])$ are two ternary sub semi-groups of T.So,$S \subset T$ is ternary sub semi group of T.

Theorem 2.9: The non-empty intersection of two Bi-ternary sub semi groups of a Biternary semi group T is a Bi-ternary sub semi-group of T.

Proof. : Let S_1, S_2 be two Bi-ternary sub semi groups of T. let $a, b, c \in S_1 \cap S_2$

$a, b, c \in S_1 \cap S_2$ Then $a, b, c \in S_1$ *and* $a, b, c \in S_2$

$a, b, c \in S_1$, S_1 is a Bi-ternary sub semi group of T , $abc \in S_1$

$a, b, c \in S_2$, S_2 is a Bi-ternary sub semi group of T, $abc \in S_2$

So, that$abc \in S_1$, $abc \in S_2$ then $abc \in S_1 \cap S_2$

Therefore $S_1 \cap S_2$ is a Bi-ternary sub semi group of T.

Theorem 2.10: The meeting point of some relations of Bi-ternary sub semi groups of T is the Bi ternary sub semi group of T.

Proof : Let $\{S_\alpha\}_{\alpha \in \Delta}$ be a family of bi ternary sub semi groups of T, And $S = \bigcap_{\alpha \in \Delta} S_\alpha$.

Let $a, b, c \in S \Rightarrow a, b, c \in \bigcap_{\alpha \in \Delta} S_\alpha \Rightarrow a, b, c \in S_\alpha \ \forall \ \alpha \in \Delta$

$a, b, c \in S_\alpha$, S_α is a bi ternary sub semi group T then $abc \in S_\alpha$

Now, $abc \in S_\alpha$,$\forall \alpha \in \Delta \Rightarrow abc \in \bigcap_{\alpha \in \Delta} S_\alpha$

Then $abc \in S$. Therefore S is a bi ternary sub semi group of T.

Theorem 2.11: Let T be a bi ternary semi grouping & A be a non-empty sub set of T. Then $< A >=$ the intersection of all bi ternary sub semi groups of Tcontaining$A$.

Proof : Letting Δ be a group comprising all bi-ternary primary semi-groups of T containing A.

bi ternary be T, sub semi group of `Tcontaining$A$. $T \in \Delta$, accordingly $\Delta \neq \emptyset$

Agree $S^* = \bigcap_{s \in \Delta} S$

Giving that $A \subseteq S \ \forall \ S \in \Delta$ and $A \subseteq S^*$

Since S^* is the intersection of bi ternary sub semi groups of T.

So,S^* is a bi ternary sub semi group of$T$.

Since $S^* \subseteq S$ for all $S \in \Delta, S^*$ is the smallest bi ternary sub semi group of T containing A.

Hence $S^* = < A >$

Definition 2.12: A bi-ternary semi group T be thought concurrent if for every $a, b, c \in T$, then wehave $abc = bca = cab = acb = bac = cba$

3.Ideals In Biternary Semi Groups:

Left Ideals in Bi-ternary semi groups

Definition 3.1: A nonempty sub set $A = A_1 \cup A_2$ of a bi-ternary semi group T be thought concurrent **left ideal** of T if both A_1 and A_2 are Left ideals of T_1 and T_2 such that if $b, c \in T_1, a \in A_1$ implies $bca \in A_1$

Similarly, $b, c \in T_2, a \in A_2$implies $bca \in A_2$.

Note 3.2: A unfilled auxiliary set called A of a biternary semigroup T is considered that it's left idealistic if just if $TTA \subseteq A$.

Example 3.3:$T = T_1 \cup T_2$ Be a Bi ternary semi group Where $T_1 = \left\{\begin{bmatrix} a & 0 \\ b & c \end{bmatrix}, a,\ b,\ c \ \in Z\right\}$, $T_2 = \left\{\begin{bmatrix} x & y \\ 0 & z \end{bmatrix},\ x\ ,\ y,\ z \in Z\right\}$are ternary semi groups of T. $I = I_1 \cup I_2$ where $I_1 = \left\{\begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix},\ a,\ b \in Z\right\}, I_2 = \left\{\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix},\ a \ \in \ Z\right\}$ are left ideals of $T_1 \cap T_2$respectively. Thus I is the left idealistic of Bi ternary semi group T.

Theorem 3.4: The intersection of any two left ideals be a bi-ternary semi group T be left T is idealistic.

Proof. : Let A, B left idealistic of two be$T$. Let $a \in A \cap B$ and$b, c \in T$.

If $a \in A \cap B$ then $a \in A$ and $a \in B$

$a \in A$; $b,\ c \in T$, A be left idealistic T then $bca \in A$.

$a \in B; b, c \in T$, B be left idealistic of T then $bca \in B$

therefore, $bca \in A, bca \in B \Longrightarrow bca \in A \cap B$.

Therefore $A \cap B$ is a left ideals of T.

Theorem 3.5: T's left ideally suited represents the non-empty intersection of any set of left ideals bi-ternary semi groups.

Proof: Let $A_\alpha, \alpha \in \Delta$ be group to be left idealistic of T & let $A = \bigcap_{\alpha\in\Delta} A_\alpha$

Let$a \in A; b, c \in T$. Now $a \in A, a \in \bigcap_{\alpha\in\Delta} A_\alpha \Longrightarrow a \in A_\alpha$ for every$\alpha \in \Delta$.

$a \in A_\alpha; b, c \in T$, A_α be left idealistic of left of $T \Longrightarrow bca \in A_\alpha$

$bca \in A_\alpha$ for all $\alpha \in \Delta \Longrightarrow bca \in \bigcap_{\alpha\in\Delta} A_\alpha \Longrightarrow bca \in A$.

Therefore A be left idealistic of bi ternary semi group T.

Theorem 3.6: The unification of other two left ideals of bi-ternary semi group T is a left ideal of T.

Proof: Let I_1, I_2 idealistic left two of bi ternary semi group$T$.

Let $a \in I_1 \cup I_2 \Longrightarrow a \in I_1$ or $a \in I_2$ or both and $\alpha, \beta \in T$

$\alpha, \beta \in T, a \in I_1 \Longrightarrow \alpha\beta a \in I_1$

$\alpha, \beta \in T, a \in I_2 \Longrightarrow \alpha\beta a \in I_2$

$\alpha\beta a \in I_1, \alpha\beta a \in I_2 \Longrightarrow \alpha\beta a \in I_1 \cup I_2$

$\alpha\beta a \in I_1 \cup I_2$

Then$I_1 \cup I_2$ is left ideal of T.

Theorem 3.7: Any family combined of left idealistic of bi-ternary semi group T be left ideal of T.

Proof.: Agree to $A_\alpha, \alpha \in \Delta$ be a related of left ideals of T and let $A = \bigcup_{\alpha \in \Delta} A_\alpha$

Clearly A is a non empty sub set of$T$.

Let$a \in A; b, c \in T$. Now $a \in A, a \in \bigcup_{\alpha \in \Delta} A_\alpha$ then $a \in A_\alpha$ for some $\alpha \in \Delta$

$a \in A_\alpha; b, c \in T$, A_α be left idealistic of $T \Longrightarrow bca \in A_\alpha$

$bca \in A_\alpha$ for all $\alpha \in \Delta \Longrightarrow bca \in \bigcup_{\alpha \in \Delta} A_\alpha \Longrightarrow bca \in A$.

Therefore A be left idealistic of bi ternary semi group T.

Lateral Ideals in Bi-ternary semi groups:

Definition 3.8: A non − blank auxiliary set $A = A_1 \cup A_2$ of a **bi-ternary semi group**T be **lateral ideal** of T if both A_1 and A_2 are Lateral ideals of T_1 and T_2 such that if $b, c \in T_1, a \in A_1$ implies $bac \in A_1$

Similarly, $b, c \in T_2, a \in A_2$ implies $bac \in A_2$.

Note 3.9: A non-empty auxiliary set A of a **bi-ternary semi group**T be to **lateral ideal** if and only if $TAT \subseteq A$.

Example 3.10: $T = T_1 \cup T_2$ Be a Bi-ternary semi group Where$T_1 = \left\{\begin{bmatrix} a & 0 \\ b & c \end{bmatrix},\ a,\ b,\ c\ \in\ Z\right\}$,

$T_2 = \left\{\begin{bmatrix} x & y \\ 0 & z \end{bmatrix},\ x,\ y,\ z\ \in\ Z\right\}$ are ternary semi groups of T. $I = I_1 \cup I_2$ where

$I_1 = \left\{\begin{bmatrix} 0 & 0 \\ b & 0 \end{bmatrix}, b \in Z\right\}$, $I_2 = \left\{\begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}, a \in Z\right\}$ are lateral ideals of T_1&T_2respectively. Thus **I** is the lateral Ideal of Bi ternary semi group **T**.

Theorem 3.11: The connection by two lateral ideals of a bi-ternary semi group T be a lateral idealistic of T.

Proof: Agree A, B be two lateral perfects of T. Be in agreement $a \in A \cap B$ and $b, c \in T$.

If $a \in A \cap B$ then $a \in A$ and $a \in B$

$a \in A; b, c \in T$, A is a lateral perfect of T then$bac \in A$.

$a \in B; b, c \in T$, B is a lateral perfect of T then $bac \in B$

Hence, $bac \in A, bac \in B \Longrightarrow bac \in A \cap B$.

Therefore $A \cap B$ is a on the side perfect of T.

Theorem 3.12: The non-empty joint of any family of lateral ideals bi ternary semi group T is a lateral perfect of T.

Proof: Agree $A_\alpha, \alpha \in \Delta$ be a combined of on the side perfect of T and let $A = \bigcap_{\alpha\in\Delta} A_\alpha$

Let $a \in A; b, c \in T$. Now $a \in A, a \in \bigcap_{\alpha\in\Delta} A_\alpha \Longrightarrow a \in A_\alpha$ for every$\alpha \in \Delta$.

$a \in A_\alpha; b, c \in T$, A_α were perfect on side perfect & $T \Longrightarrow bac \in A_\alpha$

$bca \in A_\alpha$ for all $\alpha \in \Delta \Longrightarrow bac \in \bigcap_{\alpha\in\Delta} A_\alpha \Longrightarrow bac \in A$.

thus A be a lateral idealistic of bi ternary semi group T.

Theorem 3.13: The amalgamation by two lateral ideals of bi-ternary semi grouping T is a on the side perfect of T.

Proof : Agree to I_1, I_2 any two on the side perfect of bi-ternary semi-grouping T.

Let $a \in I_1 \cup I_2 \Longrightarrow a \in I_1$ or $a \in I_2$ or both and $\alpha, \beta \in T$

$\alpha, \beta \in T, a \in I_1 \Longrightarrow \alpha a \beta \in I_1$

$\alpha, \beta \in T, a \in I_2 \Longrightarrow \alpha a \beta \in I_2$

$\alpha a \beta \in I_1, \alpha a \beta \in I_2 \Longrightarrow \alpha a \beta \in I_1 \cup I_2$

$\alpha a \beta \in I_1 \cup I_2$

Then$I_1 \cup I_2$ is lateral ideal of T.

Theorem 3.14: The combined be all related of lateral ideals be bi-ternary semi grouping T is a lateral ideal of T.

Proof: Agree to $A_\alpha, \alpha \in \Delta$ become a group of transverse ideals. of T and let $A = \bigcup_{\alpha\in\Delta} A_\alpha$

Clearly A is a non-blank sub set of T.

Let $a \in A; b, c \in T$. Now $a \in A, a \in \bigcup_{\alpha\in\Delta} A_\alpha$ then $a \in A_\alpha$ for some $\alpha \in \Delta$

$a \in A_\alpha; b, c \in T$, A_α is a transverse ideals of $T \Longrightarrow bac \in A_\alpha$

$bca \in A_\alpha$ for all $\alpha \in \Delta \Longrightarrow bac \in \bigcup_{\alpha\in\Delta} A_\alpha \Longrightarrow bac \in A$.

thus A is a lateral ideals of bi ternary semi group T.

Right Ideals in Bi-ternary semi groups:

Definition 3.15: A non-empty sub set $A = A_1 \cup A_2$ of become a family of lateral ideas. A bi-ternary semi-group T is considered the proper optimum of T if equally A_1 and A_2 are right ideals of T_1 and T_2 such that if $b, c \in T_1, a \in A_1$ implies $abc \in A_1$

Note 3.16: A non-empty auxiliary set. A of a bi-ternary semi-group T is considered to be correct if just if $ATT \subseteq A$.

Example 3.17:$T = T_1 \cup T_2$ Be a Bi ternary semi group Where $T_1 = \left\{\begin{bmatrix} a & 0 \\ b & c \end{bmatrix},\ a,\ b,\ c \in Z\right\}$, $T_2 = \left\{\begin{bmatrix} x & y \\ 0 & z \end{bmatrix},\ x,\ y,\ z\ \in\ Z\right\}$ are ternary semi groups of T. $I = I_1 \cup I_2$ where

$I_1 = \left\{\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix},\ a,\ b\ \in\ Z\ \right\}, I_2 = \left\{\begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}, a \in Z\right\}$ are right ideals of T_1& T_2respectively. Thus $I$is the right Ideal of Bi ternary semi group T.

Theorem 3.18: T's right ideal is the non-empty confluence of multiple right conceptions of a bi-ternary semi group.

Proof: Agree to I_1, I_2 be two ideals of bi-ternary semi group T.

Let $a, b \in I_1 \cap I_2$&$\alpha, \beta \in T$

Also $\alpha, \beta \in T, a \in I_1 \Longrightarrow \alpha\beta a \in I_1$

$\alpha, \beta \in T, a \in I_2 \Longrightarrow \alpha\beta a \in I_2$

$\alpha\beta a \in I_1, \alpha\beta a \in I_2 \Longrightarrow \alpha\beta a \in I_1 \cap I_2$

Hence $I_1 \cap I_2$ is the right ideal of Bi ternary semi group$T$.

Theorem 3.19: The non-empty intersections of any group of right conceptions be Bi-ternary semi grouping T is a right ideal of T..

Theorem 3.20: The amalgamation of neither of the right ideals be bi-digit semi grouping T were right picture of T.

Proof : Agree I_1, I_2 be multiple (two) right ideals of bi ternary semi group$T$.

Let $a \in I_1 \cup I_2 \Longrightarrow a \in I_1$ or $a \in I_2$ or both and $\alpha, \beta \in T$

Also$\alpha, \beta \in T, a \in I_1 \Longrightarrow \alpha\beta a \in I_1$

$\alpha, \beta \in T, a \in I_2 \Longrightarrow \alpha\beta a \in I_2$

$\alpha\beta a \in I_1, \alpha\beta a \in I_2 \Longrightarrow \alpha\beta a \in I_1 \cup I_2$

$\alpha, \beta \in T, a \in I_1 \cup I_2, \alpha\beta a \in I_1 \cup I_2$

Then$I_1 \cup I_2$ is rightideal of$T$.

Definition 3.21: A nonblank set A be **bi ternary semi group**T is said to be **ternary ideal** or just an perfect of T if

$b, c \in T, a \in A$Then , $bca \in A$, $bac \in A$, $abc \in A$.

Definition 3.22: An Ideal $A = A_1 \cup A_2$be a **Bi-ternary semi group** T be maximal perfect. Both A_1&A_2 are **maximal** ideals of T_1&T_2 provided that A_1&A_2 any good ideals of T_1&T_2. Hence A is **maximal ideal** of Bi ternary semi group$T$.

Definition3.23: An ideal A be **bi-ternary semi group** T, the **Principal Bi ternary ideal** generated by a if $A_1 \& A_2$ are ideals of $T_1 \& T_2$ generated by $\{a\}$ for some $a \in T$. is denoted by $\langle a \rangle$.

Definition 3.24: An ideal A of a **bi ternary semi group**T is said to be **globally idempotent**.If $\boldsymbol{A^3 = A}$.

Definition 3.25:A **bi ternary semi group**T is said to be **globally idempotent**.$\boldsymbol{T^3 = T}$.

Theorem 3.26: If T is a bi ternary semi group with unity 1 then the union of all proper ideals of T is the unique maximal ideal of T.

Proof: Let M be the union of all proper ideals of T. Since 1 is not an element of any proper ideal of T. $1 \notin M$. Therefore M is aproper sub set of T. By theorem union of all ideals of T is an ideal of T.M is an ideal of T. Thus M is a proper ideal of T. Since M contains all proper ideals of , M is a maximal ideal of T. If W is any maximal ideal of , then

$W \subseteq M \subseteq T$ and hence $W = M$. Therefore $M$is the unique maximal ideal of T.

Definition 3.27: Let T be a **bi ternary semi group** and A be non empty subset of T. The smallest left ideal of Tcontaining A is call left perfect of T**generated by** $\boldsymbol{A}$.

Theorem 3.28: The intersection of all left ideals of T that include A is the left perfect of a bi modal semigroup T created by a non-empty auxiliary set A.

Proof: Agree of Δ a put the left ideals of T contaiing A. Given that T is a left perfect and includes A, $T \in \Delta$. So $\Delta \neq \emptyset$. Agree $S^* = \bigcap_{s \in \Delta} S$. Seeing as A is a resulting put of S for all $S \in \Delta, A \subseteq S^*$. S^* is the left ideal of S. Let P be the left ideal of T containing$A$.P be the left perfect of T. Clearly $A \subseteq P$.Therefore$P \in \Delta \Longrightarrow S^* \subseteq P$ & hence S^* be the left perfect of Tcontaining A.

Definition 3.34: Preservative auxiliary semi group Q of a **bi-ternary semiring** T is called **quasi ideal** of bi ternary semiring T then $Q = Q_1 \cup Q_2$ mutually Q_1 & Q_2 are quasi standards of T_1& T_2 if

$Q_1 T_1 T_1 \cap (T_1 Q_1 T_1 + T_1 T_1 Q_1 T_1 T_1) \cap T_1 T_1 Q_1 \subseteq Q_1$

$Q_2 T_2 T_2 \cap (T_2 Q_2 T_2 + T_2 T_2 Q_2 T_2 T_2) \cap T_2 T_2 Q_2 \subseteq Q_2$. Again Q is a **quasi-ideal** of T.

Definition 3.35: A apt perfect of $P = P_1 \cup P_2$ of a commutative **bi ternary semi group**T is **prime ideal** if, together P_1 & P_2 were main ideals of $T_1 \& T_2$ correspondingly if X_1, Y_1, Z_1 are ideals of T_1andX_2, Y_2, Z_2 are ideals ofT_2 such that

$X_1 Y_1 Z_1 \subseteq P_1 \Longrightarrow X_1 \subseteq P_1 or Y_1 \subseteq P_1 or\ Z_1 \subseteq P_1$

$X_2 Y_2 Z_2 \subseteq P_2 \Longrightarrow X_2 \subseteq P_2 or Y_2 \subseteq P_2 or\ Z_2 \subseteq P_2$

Definition 3.36: A put perfect by $P = P_1 \cup P_2$ of a **commutative bi ternary semi group**T is **completely prime ideal** if, together P_1 & P_2 are completely prime ideals of $T_1 \& T_2$ separately if x_1, y_1, z_1 are of T_1andx_2, y_2, z_2 are ofT_2 such that

$x_1 y_1 z_1 \in P_1 \Longrightarrow x_1 \in P_1 or y_1 \in P_1 or\ z_1 \in P_1$

$x_2 y_2 z_2 \in P_2 \Longrightarrow x_2 \in P_2 or y_2 \in P_2 or\ z_2 \in P_2$

Theorem 3.37: Let T be a **bi ternary semi group** with identity. Then T's maximum notions are all fundamental concepts.

Proof: Agree P be a ultimate perfect of T. Let A, B, C be ideals of T such that$ABC \subseteq P$.Suppose that A, B not contained in P. Then $A \cup P = T$ and $B \cup P = T$. As $e \in T$ so $e \in A \cup P$ and $e \in B \cup P$ implies $e \in A$ or $e \in P$ and $e \in B$ or $e \in P$. Since $e \notin P$ so $e \in A$ and $e \in B \Rightarrow A = T$ and $B = T$. Now since $e \in T$, So $TTC = C$ and $C = TTC = ABC \subseteq P$ suggests $C \subseteq P$. Henceforth P is a main idealistic of T.

4.HOMOMARPHISM ON BI TERNARY SEMI GROUP:

Definition 4.1: Let $(S,\cdot,*)$ $where\ S = (S_1,\cdot) \cup (S_2,*)$ and $(T,\circ,\otimes) = (T_1,\circ) \cup (T_2,\otimes)$ be two **bi ternary semi group**s. A plan $\emptyset = \emptyset_1 \cup \emptyset_2$ on or after S to T is **bi ternary semi group homomorphism**. If $\emptyset_1$preserves homomorphism from S_1toT_1 , $\emptyset_2$ preserves homomorphism from $S_2$to T_2 such that

1) $\emptyset(a \cdot b \cdot c) = \emptyset(a) \circ \emptyset(b) \circ \emptyset(c)$

2) $\emptyset(a * b * c) = \emptyset(a) \otimes \emptyset(b) \otimes \emptyset(c)$ for all $a, b, c \in S$ & $\emptyset(a), \emptyset(b), \emptyset(c) \in T$.

Definition 4.2: A bi ternary semi group homomorphism $\emptyset: S \to T$ is onto homomorphism is called **Epimorphism**

$\emptyset: S \to T$.

Definition 4.3: A bi ternary semi group homomorphism $\emptyset: S \to T$ is one-to-one homomorphism is called **Monomorphism** $\emptyset: S \to T$.

Definition 4.4: A bi ternary semi group homomorphism $\emptyset: S \to T$is both one-to-one and onto (i.e. Bijective) homomorphism is called **Isomorphism**$\emptyset: S \to T$. it says that S is isomorphic to T. $S \cong T$.

Definition 4.5: A bi ternary isomorphism $\emptyset$ is defined from same bi ternary semiring onto itself is called an **Automorphism.**

Definition 4.6: A mapping $\emptyset: S \to T$ is bi ternary semi group homomorphism the kernel of the homomorphism $ker\emptyset$ is defined as $ker\emptyset = \{e \in S: \emptyset(e) = e'\}$ Where $e' \in T$ is the identity of T.

Theorem 4.7: Let $S = S_1 \cup S_2$ and$T = T_1 \cup T_2$ be two bi ternary semi groups with $\emptyset$ is a bi ternary semi group homomorphism from $S \to T$ then $ker\emptyset = ker\emptyset_1 \cup ker\emptyset_2$ is ideal of the set S.

Proof: Given that S is a bi ternary semi group.

Let $ker\emptyset = \{a \in \emptyset: \emptyset(a) = 0'\}$

We prove that $ker\emptyset$ is an ideal of S.

We first prove $ker\emptyset$ is a non empty set.

Let us assume 0 is the 0(zero) section of S, also $0'$ is the 0(zero) section of T.

Let $0 \in S \Rightarrow 0 \in S_1 \cup S_2 \Longrightarrow 0 \in S_1 \& 0 \in S_2 \Rightarrow \emptyset_1(0) = 0' \& \emptyset_2(0) = 0'$

Thus $0 \in ker\emptyset$ .Hence $ker\emptyset$ is non empty.

Let $x, y \in ker\emptyset$ by the definition of $ker\emptyset$

If $x \in ker\emptyset \Rightarrow \emptyset(x) = 0'\&\emptyset(y) = 0'$

Let $a, b \in ker\emptyset$ & $s \in S$

Since $a, b \in ker\emptyset \Rightarrow \emptyset(\text{a}) = 0'\& \emptyset(\text{b}) = 0'$

Consider,$\emptyset(abs) = \emptyset(a)\emptyset(b)\emptyset(s) = 0'.0'.\emptyset(s) = \emptyset(s) = 0'$

$\therefore abs \in ker\emptyset$

Similarly $sab \in ker\emptyset , asb \in ker\emptyset$

Hence $\forall a, b \in ker\emptyset \&s \in S \; then \; abs \in ker\emptyset , sab \in ker\emptyset , asb \in ker\emptyset$

So $ker\emptyset$ is an ideal of S.

Theorem 4.8: The homomorphic image of an ideal is an ideal. If $\emptyset: S \rightarrow T$ is an bi ternary semi group onto homomorphism and A is an ideal of S then $\emptyset(A)$ is an ideal of T.

Proof: Perfect 0 & $0'$ be the additive identity of S & T respectively.

$$\emptyset(A) = \{\mu' \in T: \exists\mu \in S \; s.t \; \emptyset(\mu) = \mu'\}$$

$0 \in A \; s.t \; \emptyset(0) \in \emptyset(A) \Rightarrow 0' \in \emptyset(A)$ where $\emptyset(0) = 0' \in T$

$\therefore \emptyset(A)$ is a non-empty.

$\therefore \emptyset(A) \subseteq T$

Let $a' , b' \in \emptyset(A) \exists a, b \in A$

Such that$\emptyset(a) = a' , \emptyset(b) = b'$

Let $a' , b' \in \emptyset(A)\& \; t \in T.$

We prove $a'b't \in \emptyset(A)$

Since $\emptyset$ is onto $\exists s \in S \;\; s.t \; \emptyset(s) = t$

Consider, $a'b't = \emptyset(a)\emptyset(b)\emptyset(s) = \emptyset(abs) \in \emptyset(A)$

Since $a, b \in A \; \&s \in S \; \Rightarrow abs \in A$ has a perfect of S.

$\therefore a'b't \in \emptyset(A)$

Similarly $ta'b' \in \emptyset(A)\&a'tb' \in \emptyset(A)$

Hence $\emptyset(A)$ is the left, on the side, right perfect of T.

Hereafter $\emptyset(A)$ be idealistic of T.

Definition 5.1: Bi ternary semi field: Let T is a bi ternary semi ring, T said of T is a bi-digit semi field condition T_1 is a ternary semi field & T_2 is a ternary semi field everyplace $T = T_1 \cup T_2$.i.e. equally T_1 & T_2 stand shifting ternary semi rings thru non zero(0) element has multiplicative inverse.

References:

[1] Rao, D. M., & Srinivasa Rao, G. (2014). Concepts on Ternary semirings. *International Journal of Modern Sciences and Engineering Technology*, *1*(7), 105-110.

[2] Rao, D. M., & Srinivasa Rao, G. (2015). Structure of Certain Ideals in Ternary Semirings. *International Journal of Innovative Science and Modern Engineering (IJISME)*, *3*, 49-56.

[3] Sarala, Y., Anjaneyulu, A., & Rao, D. M. (2013). Ternary semi groups. *International Journal of Mathematics Sciences, Technology and Humanities*, *76*, 848-859.

[4] Dixit, V. N., & Dewan, S. (1995). A note on quasi and bi-ideals in ternary semi groups. *International Journal of Mathematics and Mathematical Sciences*, *18*, 501-508.5. Kar, S., & Maity, B. K. (2011). Some ideals of ternary semi groups. *Annals of the Alexandru Ioan Cuza University-Mathematics*, *57*(2), 247-258.

[5] Chinram, R., Baupradist, S., & Saelee, S. (2012). Minimal and maximal bi-ideals in ordered ternary semi groups. *Int. J. Phys. Sci*, *7*, 2674-2681.

[6] Daddi, V. R., & Pawar, Y. S. (2012). On Ordered Ternary Semi groups. *Kyungpook Mathematical Journal*, *52*(4).

[7] Sarala, Y., Anjaneyulu, A., & Rao, M. D. (2019), Prime radicals in ternary semi groups. *International Journal of Innovative Technology and Exploring Engineering*, *8*(6 Special Issue 4), 1403-1404.

[8] Iampan, A. (2007). Lateral ideals of ternary semi groups. *Український математичний вісник*.

[9] Choosuwan, P., & Chinram, R. (2012). A study on quasi-ideals in ternary semi groups. *Int. J. Pure Appl. Math*, *77*(5), 639-647

[10] Vasantha Kandasamy W.B., Bialgebraic Structures and Smarandache Bialgebraic Structures, American Research Press, Rehoboth, 2003.

[11] Vasantha Kandasamy W.B., Bivector Spaces, U. Sci. Phy. Sci., (11) (1999), 186-190.

[12] G. Srinivasa Rao, D. Madhusudhanarao and P. Siva Prasad, **Simple Ternary Semi-rings**, The Global Journal of Mathematics & Mathematical Sciences, 9(2) (2016), 185-196.

[13] D. Madhusudhana Rao, G. Srinivasa Rao, **Special Elements in ternary semi rings**, International Journal of Engineering Research and Applications, 4(11) (2014), 123-130.

[14] G. Srinivasa Rao, D. Madhusudhana Rao, **A Study on Ternary Semi rings**, Int. J. of Math. Archive, 5(12) (2014), 24-30.

[15] G. Srinivasa Rao, D. Madhusudhana Rao, **Characteristics of Ternary Semi rings**, Int.J. of Engg. Res. and Mgt., 2(1) (2015), 3-6.

[16] G. Srinivasa Rao, A. Nagamalleswara Rao, P.L.N. Varma, D.Madhusudhana Rao, Ch. Ramprasad, **Prime Bi-interior ideals in TGSR**, Malaya Journal of Mathematika, Vol.9, No.1, pp:542-546, 2021.

[17] G. Srinivasa Rao, A. Nagamalleswara Rao, P.L.N. Varma, D. Madhusudhana Rao, Ch. Ramprasad, **Bi-interior ideals in TGSR**, Advances in Mathematics Scientific Journal, 10 (2021), No.3, pp: 1183-1195.