

Influence of Cattaneo-Christov Heat Flux Model Rotating Maxwell Nanofluid with MHD and Double Stratification Across a Bidirectional Stretching Surface

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Abstract:

In this research, New homotopy analysis method for solving the fractional (2+1) D and (3+1) D non-linear Schrödinger equations by Elzaki. To solve these equations, the Elzaki transform is applied jointly to the Homotopy analysis method (HAM). This has proved efficient in tackling fractional calculus and nonlinear dynamics since correct solutions are offered and they converge at a faster rate. The accuracy of the proposed technique has been corroborated by analyzing various examples for which the latter were used for solving high-dimensional non-linear Schrodinger equation, which indicates that the technique is quite resilient as well as efficient; thus, making it an effective tool in theoretical physics and other applied sciences.

Keywords: Maxwell nanofluid, Rotating, Cattaneo-Christov heat flux model, Magnetic field, double stratification.

1. Introduction

Nanofluids are attempting to develop additional ways to enhance thermal system heat transfer efficiency. Applications of nanofluids are numerous in industrial, and cooling systems like aerodynamics, pipe and valves, electronic battery, air conditioning, ventilations, medicine, power and chemical engineering, and others. Metallic surface is used to construct fittings, reflectors, surface coatings and as a protector in areas where heat radiates. With this surface, the fluid of nanoparticles work as a cooling substance to reduce the heat in various industrial applications. The flow of a Maxwell nanofluid including gyrotactic microorganisms with a heat source/sink and a magnetic field is studied in three dimensions via a stretching surface by Aamir Ali. Convective heat transfer conditions with exponentially stretching sheet for rotating flow of nanofluids were discussed and compared by Ahmad. In rotating incompressible Maxwell nanofluid with CC-Model, chemical reaction and gyrotactic micro-organisms are discussed over a Riga plate. MHD rotating flow of Maxwell and tangent hyperbolic nanofluid with CC-Model over a bidirectional stretching sheet are considered and solved numerically by using finite element analysis.

Fouzia Rehman considered single and multi-walls in micropolar nanofluid to the base fluid with MHD and convective conditions within the parallel rotating plates. Hassan Waqas motivated to improve the heat transfer on nanofluid and hybrid nanofluid flow between rotating plates. Hayat elaborated stretching surface exponentially along with Brownian motion and thermophoresis in the rotating Maxwell fluid flow. Influence of thermal radiation, Iskander Tlili reveals the effect on Maxwell fluid and entropy analysis. Mustafa and Muhammed Naveed Khan examined the variable thermal conductivity by applying CC-Model in the rotating Maxwell fluid. In Muhammad Awais revealed the features, properties of nanofluid to provide the heat transfer performance. Effect of activation energy on rotating Maxwell fluid with double stratification conditions by comparing the linear and exponential stretching surface.

Rahimah Jusoh suggested convective boundary conditions over a stretching/shrinking surface on Maxwell fluid to analyze the heat transfer rate. Ramaiah and Shafique discussed MHD rotating nanofluid with chemical reaction and activation energy by using CC-Model. Considering the solid permeable plate parallel to the stretching surface to analyze the fluid motion effects through various nanoparticles in a rotating frame. Maxwell fluid flow with double stratification, thermal radiation and magnetic field considered by Tariq Hussain. Hayat discussed the rotating flow of Maxwell nanofluid with thermophoresis and Brownian motion, and extend to convective Darcy-Forchheimer with porous medium. Wang explored Maxwell fluid with bio-convective flow by using stretching surface in exponential form with slip effect. Wasim Jamshed investigated Maxwell nanofluid with its properties by comparing the fluids to reveal the boundary layer thickness. Muhammad Ramzan illustrated Maxwell nanoliquid with gyrotactic micro-organisms and heat source/sink on Newtonian heating.

Here, aforementioned studies and detailed review of literature on rotating Maxwell nanofluid by linearly and exponentially stretching surface with various boundary conditions and heat transfer models. Therefore, the aim of this paper to explore the Maxwell condition to the MHD nanofluid flow due to rotating frame across a bidirectional stretching surface. Solving this paper numerically by bvp5c in MATLAB technique which represents quantitative values of Nusselt and Sherwood parameter, and graphs for velocity, thermal and concentration profiles by highlights the model to various impacts of implanted parameters.

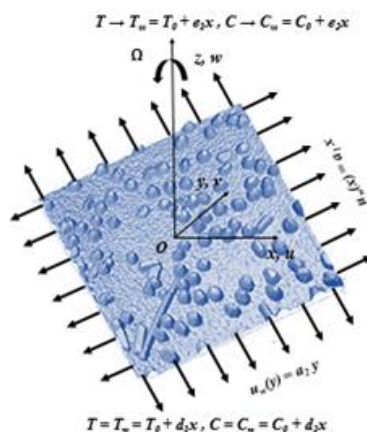


Figure 1. Schematic configuration of the problem

2. Mathematical demonstration

Consider the steady, incompressible, three-dimensional rotating Maxwell fluid flow across bi-directional stretching surface with double stratification and magnetic strength. The metal sheet is stretched along x and y directions, the fluid motion is in the path of z -axis which is normal to the surface and it has constant angular velocity ω (say). The motion of fluid is on the z direction which is perpendicular to it and it resides at $z \leq 0$. The stretching velocities of the fluid flow directions are $u_w = a_1x$ (x -direction) and $w_w = a_2y$ (y -direction), (a_1, a_2 are positive real numbers). With the use of CC model, the equation of energy is updated to simulate thermal impacts. Velocity field $V = (u_1, v_1, w_1)$ is considered to this problem and, while surface temperature and concentration is denoted by T_w and C_w , then the ambient temperature and concentration is T_∞ and C_∞ respectively in Figure 1. Governing equations of the considered problem as follows:

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \quad (1)$$

$$u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y} + w_1 \frac{\partial u_1}{\partial z} - 2\omega v_1 = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u_1}{\partial z^2} - \lambda_1 \left[u_1^2 \frac{\partial^2 u_1}{\partial x^2} + v_1^2 \frac{\partial^2 u_1}{\partial y^2} + w_1^2 \frac{\partial^2 u_1}{\partial z^2} + 2u_1 v_1 \frac{\partial^2 u_1}{\partial x \partial y} + 2v_1 w_1 \frac{\partial^2 u_1}{\partial y \partial z} + 2u_1 w_1 \frac{\partial^2 u_1}{\partial x \partial z} - 2\omega \left(u_1 \frac{\partial v_1}{\partial x} + v_1 \frac{\partial v_1}{\partial y} + w_1 \frac{\partial v_1}{\partial z} \right) + 2\omega \left(v_1 \frac{\partial u_1}{\partial x} - u_1 \frac{\partial u_1}{\partial y} \right) - \frac{\sigma_{nf} B_0^2}{\rho_{nf}} \left(u_1 + \lambda_1 w_1 \frac{\partial u_1}{\partial z} \right) \right] \quad (2)$$

$$u_1 \frac{\partial v_1}{\partial x} + v_1 \frac{\partial v_1}{\partial y} + w_1 \frac{\partial v_1}{\partial z} + 2\omega u_1 = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 v_1}{\partial z^2} - \lambda_1 \left[u_1^2 \frac{\partial^2 v_1}{\partial x^2} + v_1^2 \frac{\partial^2 v_1}{\partial y^2} + w_1^2 \frac{\partial^2 v_1}{\partial z^2} + 2u_1 v_1 \frac{\partial^2 v_1}{\partial x \partial y} + 2v_1 w_1 \frac{\partial^2 v_1}{\partial y \partial z} + 2u_1 w_1 \frac{\partial^2 v_1}{\partial x \partial z} - 2\omega \left(u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y} + w_1 \frac{\partial u_1}{\partial z} \right) + 2\omega \left(v_1 \frac{\partial v_1}{\partial x} - u_1 \frac{\partial v_1}{\partial y} \right) - \frac{\sigma_{nf} B_0^2}{\rho_{nf}} \left(v_1 + \lambda_1 w_1 \frac{\partial v_1}{\partial z} \right) \right] \quad (3)$$

$$u_1 \frac{\partial T}{\partial x} + v_1 \frac{\partial T}{\partial y} + w_1 \frac{\partial T}{\partial z} = \frac{k_{nf}}{(\rho c_p)_{nf}} \frac{\partial^2 T}{\partial z^2} - \lambda_2 \left[u_1^2 \frac{\partial^2 T}{\partial x^2} + v_1^2 \frac{\partial^2 T}{\partial y^2} + w_1^2 \frac{\partial^2 T}{\partial z^2} + 2u_1 v_1 \frac{\partial^2 T}{\partial x \partial y} + 2v_1 w_1 \frac{\partial^2 T}{\partial y \partial z} + 2u_1 w_1 \frac{\partial^2 T}{\partial x \partial z} + \left(u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y} + w_1 \frac{\partial u_1}{\partial z} \right) \frac{\partial T}{\partial x} + \left(u_1 \frac{\partial v_1}{\partial x} + v_1 \frac{\partial v_1}{\partial y} + w_1 \frac{\partial v_1}{\partial z} \right) \frac{\partial T}{\partial y} + \left(u_1 \frac{\partial w_1}{\partial x} + v_1 \frac{\partial w_1}{\partial y} + w_1 \frac{\partial w_1}{\partial z} \right) \frac{\partial T}{\partial z} \right] \quad (4)$$

$$u_1 \frac{\partial C}{\partial x} + v_1 \frac{\partial C}{\partial y} + w_1 \frac{\partial C}{\partial z} = D \frac{\partial^2 C}{\partial z^2} \quad (5)$$

With associated boundary conditions,

$$\text{At } z=0, u_1 = a_1x, v_1 = a_2y, w_1 = 0, T_w = T = d_1x + T_0, C_w = C = d_2x + C_0$$

$$\text{As } z \rightarrow \infty, u_1 \rightarrow 0, v_1 \rightarrow 0, T \rightarrow T_\infty = T_0 + e_1x, C \rightarrow C_\infty = C_0 + e_2x \quad (6)$$

Thermophysical properties for Maxwell nanofluid.

$$\mu_{nf} = \frac{\mu_{bf}}{(1-\phi)^{2.5}}, (\rho c_p)_{nf} = (1-\phi)(\rho c_p)_{bf} + \phi(\rho c_p)_{np}, \frac{\rho_{nf}}{\rho_{bf}} = (1-\phi) + \phi \frac{\rho_{np}}{\rho_{bf}},$$

$$\frac{\sigma_{nf}}{\sigma_{bf}} = \left[1 + \frac{3 \left(\frac{\sigma_{np}}{\sigma_{bf}} - 1 \right) \phi}{\left(\frac{\sigma_{np}}{\sigma_{bf}} + 2 \right) - \left(\frac{\sigma_{np}}{\sigma_{bf}} - 1 \right) \phi} \right], \quad \frac{k_{nf}}{k_{bf}} = \left[\frac{(k_{np} + 2k_{bf}) - 2\phi(k_{bf} - k_{np})}{(k_{np} + 2k_{bf}) + \phi(k_{bf} - k_{np})} \right] \quad (7)$$

Here, ϕ is nanoparticle volume fraction. Also, the subscripts bf, np and nf represents basefluid, nanoparticle and nanofluid.

The dimensionless transformation of the mathematical model are:

$$u_1 = a_1 x f'(\zeta), \quad v_1 = a_1 y g'(\zeta), \quad w_1 = -\sqrt{a_1 \nu} [f(\zeta) + g(\zeta)], \quad (8)$$

$$\zeta = \sqrt{\frac{a_1}{\nu}} z, \quad \theta(\zeta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\zeta) = \frac{C - C_\infty}{C_w - C_\infty}$$

By this transformation, Eqn (1) satisfies. While Eqs (2) to (5) are:

$$f'''' \left(\frac{1}{\Pi_1 \Pi_2} - \beta_1 (f + g)^2 \right) - f'^2 + 2\lambda \beta_2 g' + 2\beta_1 f' f'' (f + g) + (f + g) f'' + 2\beta_1 \beta_2 \lambda (g'^2 - (f + g) g' - f' g')$$

$$- \frac{\Pi_4}{\Pi_2} M (f' - \beta_1 (f + g) f'') = 0 \quad (9)$$

$$g'''' \left(\frac{1}{\Pi_1 \Pi_2} - \beta_1 (f + g)^2 \right) + g'' (f + g) - g'^2 - 2\beta_1 (f + g) g' g'' - 2 \frac{\lambda}{\beta_2} f' + 2 \frac{\beta_1 \lambda}{\beta_2} ((f + g) f'' - f'^2 + f' g')$$

$$+ \frac{\Pi_4}{\Pi_2} M (\beta_1 (f + g) g'' - g') = 0 \quad (10)$$

$$\theta'' \left(\frac{\Pi_5}{\Pi_3 \text{Pr}} - \beta_3 (f + g)^2 \right) + (f + g) \theta' (1 - \beta_3 (f' + g')) = 0 \quad (11)$$

$$\Phi'' + (f + g) \text{Sc} \Phi' = 0 \quad (12)$$

Subjected to the boundary conditions are:

$$\text{At } \zeta = 0, \quad f' = 1, \quad g' = S, \quad \theta = 1 - S_1, \quad \Phi = 1 - S_2$$

$$\text{As } \zeta \rightarrow \infty, \quad f' = g' = 0, \quad \theta = 0, \quad \Phi = 0 \quad (13)$$

Where Π_i 's, $i = 1, 2 \dots 5$ in Eqs (9) - (12) shows the Thermophysical properties for the Maxwell nanofluid,

$$\Pi_1 = (1 - \phi)^{2.5}, \quad \Pi_2 = \left(1 - \phi + \phi \frac{\rho_{np}}{\rho_{bf}} \right), \quad \Pi_3 = \left(1 - \phi + \phi \frac{(\rho c_p)_{np}}{(\rho c_p)_{bf}} \right),$$

$$\Pi_4 = \left[1 + \frac{3 \left(\frac{\sigma_{np}}{\sigma_{bf}} - 1 \right) \phi}{\left(\frac{\sigma_{np}}{\sigma_{bf}} + 2 \right) - \left(\frac{\sigma_{np}}{\sigma_{bf}} - 1 \right) \phi} \right], \quad \Pi_5 = \left[\frac{(k_{np} + 2k_{bf}) - 2\phi(k_{bf} - k_{np})}{(k_{np} + 2k_{bf}) + \phi(k_{bf} - k_{np})} \right]$$

The parameters are defined here by $\beta_1 = \lambda_1 a_1$ is the Deborah relaxation number, $M = \sigma_{bf} B_0^2 / \rho_{bf} a_1$ magnetic parameter, $\lambda = \omega / a_1$ rotation parameter, $\beta_2 = \lambda_2 a_1$ Deborah retardation number, $S = a_2 / a_1$ stretching ratio, $Sc = \nu_{bf} / D$ Schmidt number, $Pr = \nu_{bf} (\rho c_p)_{bf} / k_{bf}$ Prandtl number, $S_2 = e_2 / d_2$ concentration stratification and $S_1 = e_1 / d_1$ thermal stratification.

We can use Fourier and Fick's law to determine what Nusselt and Sherwood number are:

$$Nu_x = \frac{x q_w}{k_{bf} (T_w - T_0)}, \quad Sh_x = \frac{x j_w}{D(C_w - C_0)}, \quad (14)$$

$$q_w = -k_{nf} \left(\frac{\partial T}{\partial z} \right)_{z=0}, \quad j_w = -D \left(\frac{\partial C}{\partial z} \right)_{z=0}$$

where q_w, j_w are the wall heat and mass flux respectively. With the help of transformations, Eqn (14) takes the form,

$$\frac{Nu_x}{\sqrt{Re_x}} = -\theta'(0) \phi_5, \quad \frac{Sh_x}{\sqrt{Re_x}} = -\Phi'(0), \quad (15)$$

where $Re_x = a_1 x^2 / \nu_{bf}$, Reynolds number.

3. Mathematical Solution

By setting down the PDE Equations (9) through (12) as an initial value problem with boundary conditions, we want to: $f = H(1)$, $f' = H(2)$, $f'' = H(3)$, $g = H(4)$, $g' = H(5)$, $g'' = H(6)$, $\theta = H(7)$, $\theta' = H(8)$, $\Phi' = H(9)$, $\Phi'' = H(10)$ and then, we obtain the first-order equation system then it solve through `bvp4c` Matlab solver.

4. Discussion of the findings

By adjusting parameter values, we may examine the fluid effects on velocity, temperature, and concentration profiles. The defined values of the parameters are $M=0.2$, $\lambda=0.2$, $\beta_1 = \beta_2 = 0.1$, $S=0.1$, $Pr=1$, $S_1=0.8$, $S_2=0.8$, $Sc=0.8$, $\phi=0.05$ (nanofluid), and $\phi=0.0$ (base fluid). In graphs and tables, the numerical calculations are performed for nanofluid and base fluid (Water) with the various parameters. Table 1 represents the values of properties of nanofluids and base fluid.

Impact of several parameters on $f'(\zeta)$ and $g'(\zeta)$ profile

By amplifying the magnetic parameter, Figure 2 shows that the velocity of fluid tends to decrease, but in Figure 3 velocity of the fluid increases. Moreover, Figure 2 clarifies that the thickness of nanofluid momentum boundary layer is less than the base fluid. Whereas, in Figure 3 boundary layer thickness of base fluid is greater than the nanofluid but by slowly increasing of ζ , the thickness of boundary layer turns vice-versa in a particular value of ζ . With an increment in Maxwell parameter the velocity profile tends to decay and it causes the reduction in momentum boundary layer in Figure 4 and 5. Especially in Figure 5 velocity profile shows cross over point nearly $\zeta = 3$ to 4, velocity increases before $\zeta = 3$ and after $\zeta = 4$ velocity starts decreasing. While increasing the rotation parameter, the velocity profile and boundary layer thickness is decayed in Figure 6 and 7. Moreover, the velocity of the fluid decline, but in y-direction, the fluid velocity have some changes after a critical point of x-direction.

Effect of various parameters on $\theta(\zeta)$ profile

The temperature difference for variations of Prandtl number (Pr), thermal stratification parameter (S_1) and retardation Deborah number (β_3) are examined in the Figures 8-10. Higher Pr produces weaker thermal diffusivity of both nanofluid and base fluid which gives degeneration in thermal energy. In Figures 9 and 10, the qualitatively similar effect is noted for different values of S_1 and β_3 . By rising the values of S_1 and β_3 , temperature sketch and related boundary layer becomes thinner. Physically, due to heat flux model and double stratification, the temperature of the fluid in the surface decreases. Therefore, the temperature distribution decays and as a result boundary layer becomes thinner.

Impact of various parameters on concentration $\Phi(\zeta)$ profile

The effects of the solutal stratification (S_2) and Schmidt number (Sc) on the $\Phi(\zeta)$ profile is shown in Figures 11 and 12. It is described that the different values of solutal stratification parameter and Schmidt number, reduce the solutal diffusivity sketch as well as the boundary layer thickness also reduced. The thickness of nanofluid concentration boundary is more than the base fluid. This concludes that, when the temperature difference between the wall and surrounding environment grows larger, then the thickness of concentration boundary decreases.

Effect of several parameters on $Re_x^{-1/2}Nu_x$ and $Re_x^{-1/2}Sh_x$ profile

The volume proportion of nanoparticles influences the Nusselt number, which determines the rate of heat transmission in the surface. Figure 13 and 14, the heat transfer in Magnetite fluid decreases when the values of rotation rate and retardation number rises, and heat transfer rate rises in Pr and nanoparticle volume fraction with Alumina fluid. The mass transfer rate expressed by the effects of rotation on the Sherwood number in Figure 15. This inspected that the rising of rotation parameter and Schmidt number, which gives decrease in the mass transfer rate with Magnetite nanofluid. For high values of rotation parameter and solutal stratification, the mass transfer rate decreases with nanofluid in Figure 16. Finally, as Pr , β_3 with nanofluid grows, heat transfer rate increases and thermal boundary layer deepens.

Tables 2 and 3 shows the patterns in heat and mass transmission rates with increasing the several parameters and the Nusselt and Sherwood numbers are compared with different nanofluids and parameters are $M=0.2$, $\beta_1 = \beta_2 = 0.1$, $\lambda=0.2$, $Pr=10$, $Sc=2.5$, $S=0.1$, $S_1=0.8$, $S_2=0.8$ and $\phi=0.05$ (nanofluid). According to Table 2, the local Nusselt number's magnitude grows as the Prandtl number increases. When Deborah relaxation number and rotation parameter increases, Nusselt decreases. When the Schmidt number and stretching ratio rise, Table 3 shows a dramatic increase in the local Sherwood number.

5. Conclusion

A numerical analysis for rotating Maxwell nanofluid along with magneto-hydrodynamic, non-Fourier heat flux model and double stratification. The numerical method of bvp5c MATLAB algorithm is used to solve the mathematical model. The key observation of this paper is highlighted as below:

- Strong magnetic impact slows the flow rate.

- The fluid velocity is reduced due to enhancement in rotation parameter and Deborah relaxation number. In other words, the cooling process suffers adverse as fluid rotates.
- The temperature gradient diminishes, while rising of Pr , S_1 and β_3 .
- Higher values of Sc and S_2 are associated with weaker concentration.
- Higher values of the nanoparticle volume fraction with Pr result in an increase in the heat and mass transfer rate; otherwise, the rate degrades.
- In contrast to the temperature and concentration drawing, where the boundary layer thickness of the nanofluid dominates base fluid, the velocity profile shows a thickness of the nanofluid boundary layer that is less than base fluid.

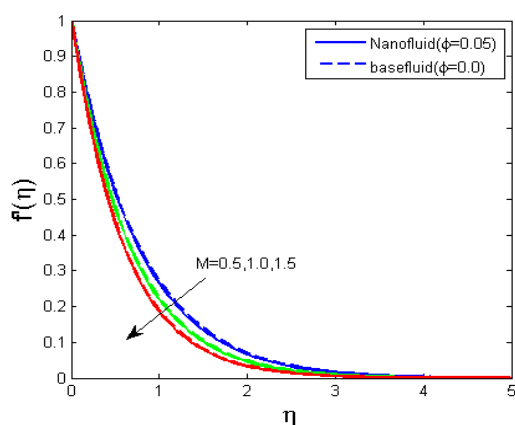


Figure 2: Impact of M on velocity profile

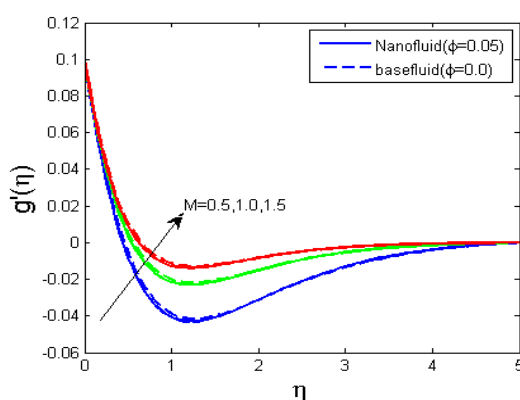


Figure 3: Impact of M on velocity profile

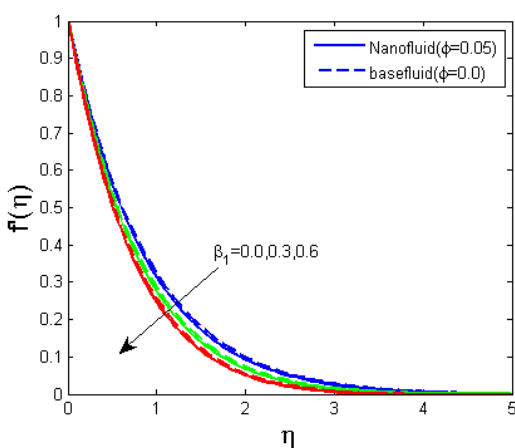


Figure 4: Impact of β_1 on velocity profile

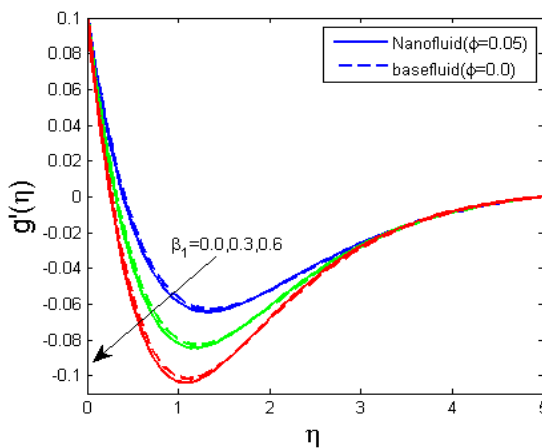


Figure 5: Impact of β_1 on velocity profile

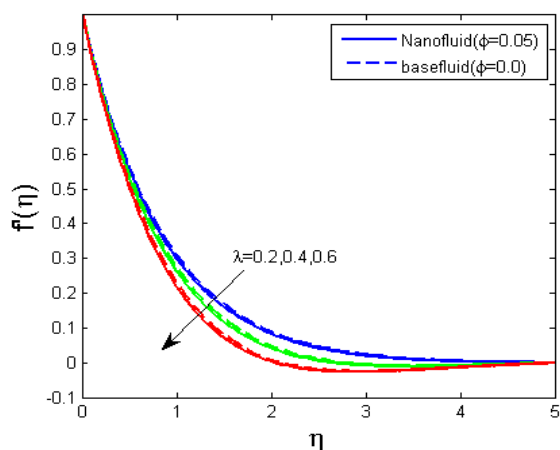


Figure 6: Impact of λ on velocity profile

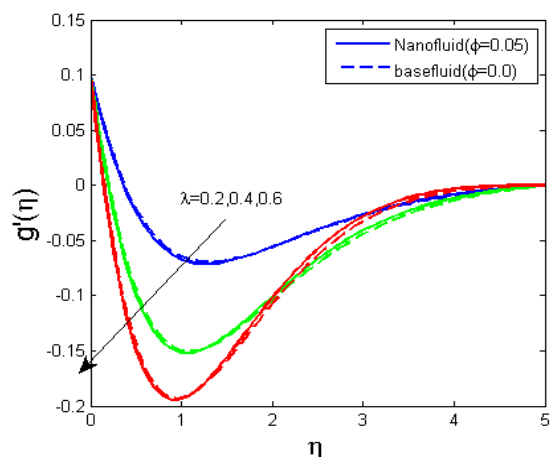


Figure 7: Impact of λ on velocity profile

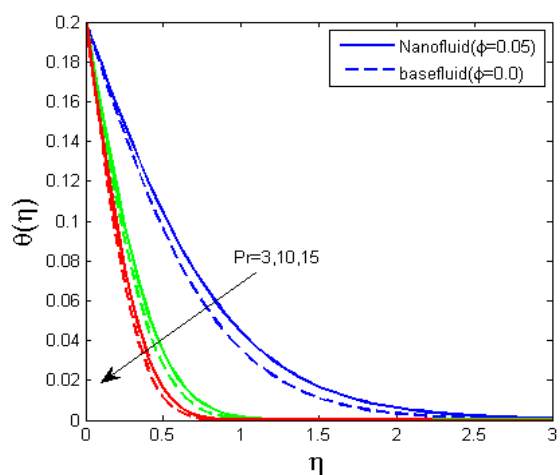


Figure 8: Impact of Pr on temperature profile

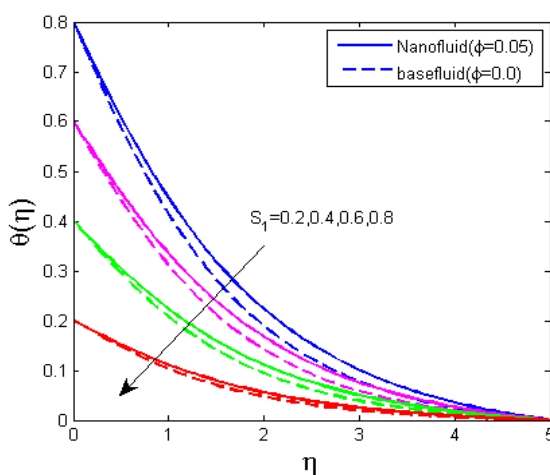


Figure 9: Impact of S_1 on temperature profile

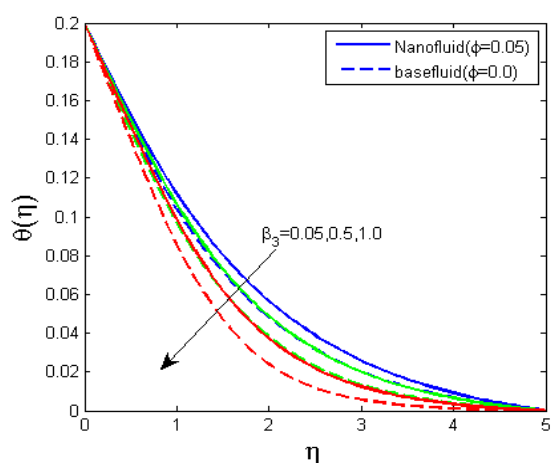


Figure 10: Impact of β_3 on temperature profile

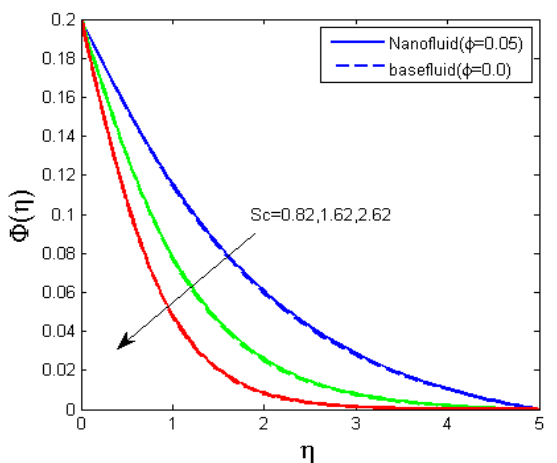


Figure 11: Impact of Sc on Concentration profile

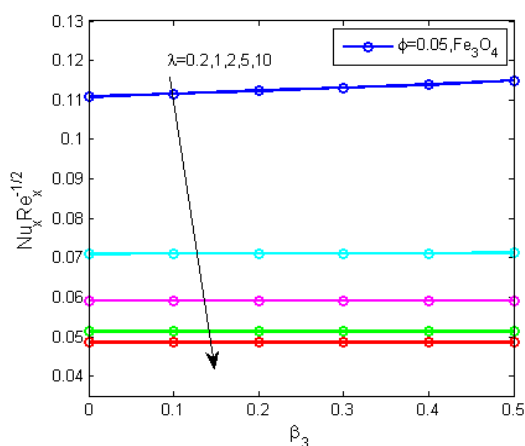
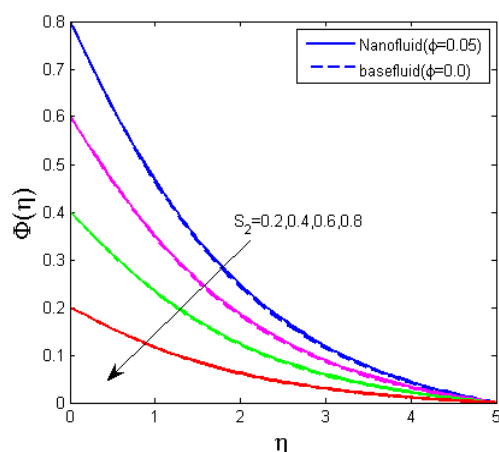


Figure 12: Impact of S_2 on Concentration profile. Figure 13: Impact of β_3 and λ on heat transfer rate

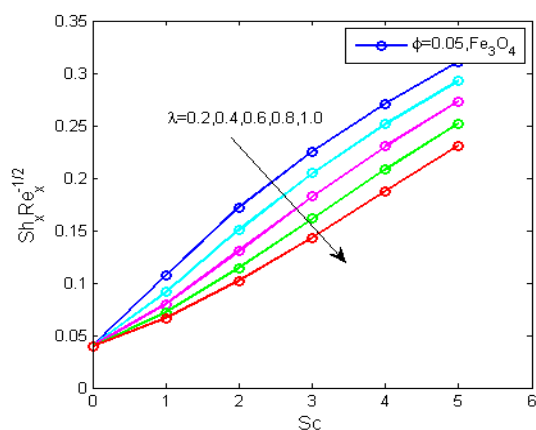
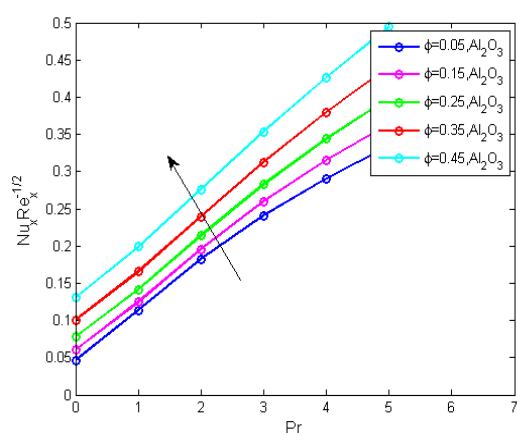
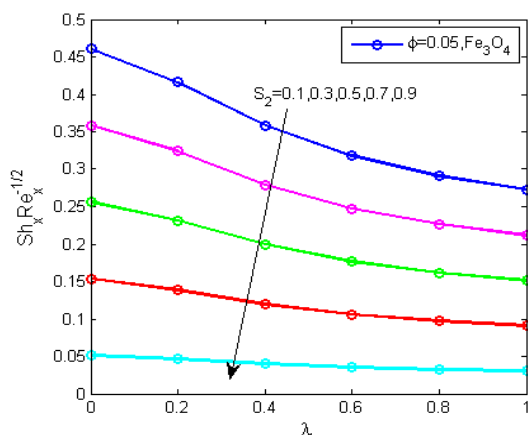


Figure 14: Impact of Pr and ϕ on heat transfer rate. Figure 15: Impact of Sc and λ on mass transfer rate



Properties	ρ (kg / m^3)	C_p ($J / kg K$)	k ($W / m K$)	σ (S / m)
Water	997.1	4179	0.613	5.5×10^6
Copper	8933	385	401	59.6×10^6
Alumina	3970	765	40	35×10^6
Magnetite	5180	670	9.7	0.74×10^5

Figure 16: Impact of S_2 and λ on mass transfer rate **Table 1.** Thermophysical properties of nanofluid.

Table 2 Numerical values of local Nusselt number $-\theta'(0)$ for various values of λ , Pr , β .

$-\theta'(0)$					
λ	β_1	Pr	Cu - Water	Al ₂ O ₃ - Water	Fe ₃ O ₄ - Water
0.1	0.1	10	0.509430	0.513045	0.509059
0.2			0.499952	0.505101	0.500915
0.6			0.456986	0.469109	0.464188
0.8			0.434806	0.451455	0.446057
	0.05		0.501755	0.506668	0.502504
	0.15		0.498157	0.503543	0.499334
	0.25		0.494589	0.500455	0.496199
	0.35		0.491044	0.497399	0.493094
		5	0.321044	0.334080	0.330906
		10	0.499952	0.505101	0.500915
		15	0.631682	0.635960	0.630981
		25	0.742548	0.746134	0.740480

Table 3 Numerical values of local Sherwood number $-\Phi'(0)$ for various values of λ , S , Sc .

$-\Phi'(0)$					
λ	S	Sc	Cu - Water	Al ₂ O ₃ - Water	Fe ₃ O ₄ - Water
0.1	0.1	10	0.204007	0.209256	0.207933
0.2			0.194780	0.201603	0.199890
0.6			0.146775	0.159987	0.156576
0.8			0.127230	0.141136	0.137444
	0.2		0.209608	0.215819	0.214256
	0.3		0.222392	0.228337	0.226840
	0.4		0.233864	0.239703	0.238233
	0.5		0.244400	0.250222	0.248756
		5	0.305799	0.312459	0.310793
		10	0.462331	0.468681	0.467091
		15	0.582001	0.588198	0.586645
		25	0.682715	0.688820	0.687288

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