

Thermal Radiation Effects of Convective Flow of a Hybrid Nanofluid with Different Base Fluids in Heat Transfer Over a Stretching Sheet

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Article History:

Received: 12-06-2024

Revised: 10-07-2024

Accepted: 01-08-2024

Abstract:

The study investigates a two-dimensional flow with uniform heat transfer processes in one direction, focusing on velocity profiles, temperature distribution, heat transfer characteristics, and potential effects of parameters on flow behavior of a hybrid nanofluid over a stretching sheet. Nanofluids are engineered colloidal suspensions of nanoparticles in a base fluid. In your case, TiO₂ and Cu nanoparticles are dispersed in a base fluid consisting of water (H₂O) and sodium citrate (NaC₆H₉O₇). This mixture is known as a hybrid nanofluid. Additionally, consideration is given to the components of the magnetic field and nonlinear thermal radiation. The Runge-Kutta fourth-order method is a numerical technique used to solve ordinary differential equations, offering a balance between accuracy and computational efficiency, by simplification of partial differential equations. The results illustrate that significant parameter such as the magnetic parameter, nanoparticles of solid volume fractions, Radiation parameter significantly influence momentum and thermal profiles. These analyses protest that raising the Radiation parameter grounds an increase in the hybrid nanofluid temperature. Moreover, the Casson parameter has decreased the velocity profile in TiO₂ - Cu/ sodium alginate hybrid nanofluids. Nonlinear thermal radiation effects become more pronounced at high temperatures. Studying their impact in hybrid nanofluids is essential for applications in high-temperature environments, such as in industrial processes or electronic devices.

Keywords: Stretching sheet, Lorentz Force, Non-linear thermal radiation, Hybrid nanofluid.

Nomenclature

B	magnetic field intensity, $\text{kg s}^{-2} \text{a}^{-1}$
C_f	skin friction coefficient
C_p	specific heat, $\text{J kg}^{-1} \text{K}^{-1}$
R	nonlinear radiation parameter
k	thermal conductivity, $\text{W m}^{-1} \text{K}^{-1}$
M	magnetic parameter
Nu_x	local Nusselt number
Pr	prandtl number

q_w	surface heat flux, wm^{-2}
Re	local reynolds number
F	Dimensionless fluid velocity
T	temperature of fluid
T_w	ambient thermal
T_∞	surface thermal
U	reference velocity, ms^{-1}
U_w	constant velocity,
U_∞	free stream momentum
u	momentum factor in the x way
x, r	cylindrical directs
Greek letters	
α	angle
β	thermal expansion coefficient, k^{-1}
ϕ_1, ϕ_2	nanoparticles of volume fraction
μ	dynamic viscosity, $\text{kgm}^{-1}\text{s}^{-1}$
ν	kinematic viscosity, m^2s^{-1}
σ	electrical conductivity, sm^{-1}
ρ	density, kgm^{-3}
θ_w	Temperature ratio parameter
τ_w	wall shear stress, nm^{-2}
<i>Subscripts</i>	
hnf	hybrid nanofluid
nf	nanofluid
f	fluid
s	solid

1 Introduction:

The study of nonlinear thermal radiation and viscous dissipation in hybrid nanofluids is a multidisciplinary field with applications in various thermal management systems. Ongoing research in this area is likely to contribute to the development of more efficient and advanced heat transfer technologies. In this research explained for developing advanced materials with improved thermal properties, which can have wide-ranging implications in various industries. The ability to enhance the thermal conductivity of fluids is crucial for optimizing heat transfer processes and improving the efficiency of thermal management systems [1]. This research may involve the development of mathematical models or equations that describe the viscoplastic flow over an inclined surface. This could include constitutive equations for viscoplastic materials, boundary conditions, and other relevant parameters [2]. The study investigates the impact of nanoparticles on natural convection in a cubic enclosure, examining heat transfer, fluid flow patterns, and thermal performance [3]. This research analyzes the effective thermal conductivity of a mixed convection flow of sodium alginate

and molybdenum disulphide nanoparticles using Maxwell Garnett and Brinkman models under specific conditions [4]-[5]. This research refers to a surface over which the fluid is flowing and stretching. Such sheets are often used as simplified models to represent industrial processes like extrusion or continuous casting [6]. The Casson model is a rheological model used to describe the flow behavior of certain fluids, particularly those with a yield stress. It's commonly employed in the study of non-Newtonian fluids as investigated in this research [7]. The study involves fluid flow, heat transfer, and electromagnetic effects, with sodium alginate as a base for nanofluids, known for their enhanced thermal properties due to nanoparticles [8]. This study explores the interaction of viscoelastic hybrid nanofluids in a porous space, aiming to enhance heat transfer efficiency and understand their behavior in specific applications [9]. This research to be focused on the free convection phenomenon, which occurs when a fluid is heated, becomes less dense, and rises due to buoyancy effects [10]. Fluid flow occurs through porous materials, indicating void spaces or pores, which are common in natural and engineered systems like soil and certain types of filters, as investigated in this research [11]. The paper investigates the natural convective flow of a nanofluid near a spinning cone, utilizing the Tiwari-Das nanofluid model, with specific findings and methodology available in the paper [12]. This research explores the influence of natural convection, thermal radiation, and nanoparticles on nanofluid flow over a non-linearly stretching surface, focusing on fluid dynamics and heat transfer properties [13]. This research explores mathematical modeling and computational analysis for solving complex fluid flow problems using magnetic fields, nanofluids, heat transfer, chemical reactions, and radiation [14]-[15]. This research investigates the Darcy-Forchheimer phenomenon, which refers to viscous and inertial effects in fluid flow through porous media [16]. The paper explores the impact of nanoparticles on forced convective heat transfer in a microchannel with asymmetric heating, emphasizing the importance of viscous dissipation in microscale fluid dynamics [17]. This research investigates magnetohydrodynamics (MHD) natural convection in a triangular cavity filled with a hybrid nanofluid of copper and aluminum oxide nanoparticles in water [18].

This research aims to understand how the combination of hybrid nanofluids, different base fluids, convective flow, thermal radiation, and Lorentz force affects heat transfer over a stretching sheet. Lorentz force is the force experienced by a charged particle in an electromagnetic field. Its inclusion implies that the study is considering the effects of an applied magnetic field on the nanofluid flow. This is often relevant in magnetohydrodynamics (MHD) studies. Thermal radiation refers to the electromagnetic radiation emitted by the fluid due to its temperature. Including thermal radiation in the study suggests an acknowledgment of the importance of radiative heat transfer mechanisms. The convective flow refers to the movement of fluid due to the temperature difference within the fluid. The stretching sheet implies that the fluid is flowing over a surface that is continuously stretching, which is a common scenario in boundary layer flows. This type of study is common in the field of fluid dynamics and heat transfer, particularly in applications related to nanotechnology, materials science, and energy systems. The outcomes of such research can have implications for improving heat transfer efficiency in various engineering applications.

2 Mathematical Formulation:

- Consider two dissimilar base fluids (water (H_2O), sodium alginate ($\text{NaC}_6\text{H}_9\text{O}_7$)) with two distinct hybrid nanoparticles (titanium oxide (TiO_2), copper (Cu)) and a uniform Lorentz force and nonlinear thermal radiation. The boundary layer flow should be stable, laminar, and convective along a stretching sheet.
- The aim of the study is to examine the impact of hybrid nanoparticles on the flow characteristics and heat transfer of the two base fluids, in addition to the effects of magnetic fields and nonlinear thermal radiation.
- A magnetic field would change the flow behavior by adding a Lorentz force term to the momentum equations and nonlinear thermal radiation to the energy equation.
- Setting a boundary temperature at the stretching sheet's surface would govern the surface temperature and impact the system's heat transfer properties.
- The physical sketch of present model is shown in Fig.I.
- Beneath these assumptions, the governing equations in a stable state [6]

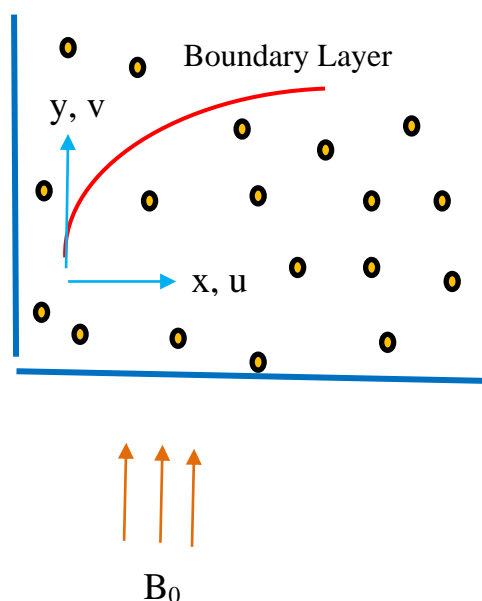


Fig.I Geometry of the problem

Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum Equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_{hnf} \left(1 + \frac{1}{\beta} \right) \mu_{hnf} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{hnf}}{\rho_{hnf}} B_0^2 u, \quad (2)$$

Energy Equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{hnf}}{(\rho C_p)_{hnf}} \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^*}{3k^*} \frac{\partial}{\partial y} \left(T^3 \frac{\partial T}{\partial y} \right) \frac{1}{(\rho C_p)_{hnf}} \quad (3)$$

2.1. Similarity Transformations and Boundary conditions:

The suitable set of variables are

$$u = cx f'(\eta), v = -(cv)^{\frac{1}{2}} f(\eta), \quad \eta = \left(\frac{c}{v}\right)^{\frac{1}{2}} y, T - T_0 = (T_w - T_\infty) \theta(\eta), \quad (4)$$

Relevant boundary circumstances are

$$\begin{aligned} u &= u_w = cx, & v &= 0, & T &= T_w & \text{at } y &= 0, \\ u &= 0, & T &= T_\infty & \text{at } y &= \infty. \end{aligned} \quad (5)$$

2.2. Hybrid nanofluid Properties:

$$\mu_{hnf} = \frac{\mu_f}{(1 - \phi_{np1})^{2.5} (1 - \phi_{np2})^{2.5}} \quad (6)$$

$$\rho_{hnf} = \{(1 - \phi_{np2})[(1 - \phi_{np1})\rho_f + \phi_{np1}\rho_{np1}]\} + \phi_{np2}\rho_{np2} \quad (7)$$

$$\alpha_{hnf} = \frac{k_{hnf}}{(\rho C_p)_{hnf}} \quad (8)$$

$$(\rho C_p)_{hnf} = \{(1 - \phi_{np2})[(1 - \phi_{np1})(\rho C_p)_f + \phi_{np1}(\rho C_p)_{np1}]\} + \phi_{np2}(\rho C_p)_{np2} \quad (9)$$

$$(\rho \beta_T)_{hnf} = \{(1 - \phi_{np2})[(1 - \phi_{np1})(\rho \beta_T)_f + \phi_{np1}(\rho \beta_T)_{np1}]\} + \phi_{np2}(\rho \beta_T)_{np2} \quad (10)$$

$$\frac{k_{hnf}}{k_f} = \frac{k_{np2} + 2k_{nf} - 2\phi_{np2}(k_{nf} - k_{np2})}{k_{np2} + 2k_{nf} + \phi_{np2}(k_{nf} - k_{np2})} \quad (11)$$

where

$$\frac{k_{nf}}{k_f} = \frac{k_{np1} + 2k_f - 2\phi_{np1}(k_f - k_{np1})}{k_{np1} + 2k_f + \phi_{np1}(k_f - k_{np1})} \quad (12)$$

$$\frac{\sigma_{hnf}}{\sigma_{nf}} = \frac{\sigma_{np2} + 2\sigma_{nf} - 2\phi_{np2}(\sigma_{nf} - \sigma_{np2})}{\sigma_{np2} + 2\sigma_{nf} + \phi_{np2}(\sigma_{nf} - \sigma_{np2})} \quad (13)$$

where

$$\frac{\sigma_{nf}}{\sigma_f} = \frac{\sigma_{np1} + 2\sigma_f - 2\phi_{np1}(\sigma_f - \sigma_{np1})}{\sigma_{np1} + 2\sigma_f + \phi_{np1}(\sigma_f - \sigma_{np1})} \quad (14)$$

2.3. Explanation for Momentum:

Using above equations (4) and (5), eqns. (1) – (2) can be transmuted as,

$$\frac{\left(1 + \frac{1}{\beta}\right)}{(1 - \phi_{np1})^{2.5}(1 - \phi_{np2})^{2.5}} f''' - \frac{\sigma_{hnf}}{\sigma_f} M f' + \left[(1 - \phi_{np2}) \left[1 - \phi_{np1} + \phi_{np1} \frac{\rho_{np1}}{\rho_f} \right] + \phi_{np2} \frac{\rho_{np2}}{\rho_f} \right] [f f'' - f'^2] = 0 \quad (15)$$

2.4. Temperature Distribution:

Using above equations (4) and (5), eqn (3) can be transmuted as,

$$\theta'' \left[\frac{k_{hnf}}{k_f} + R((\theta_w - 1)\theta + 1)^3 \right] + Pr f \theta' \left[(1 - \phi_{np2}) \left[1 - \phi_{np1} + \phi_{np1} \frac{(\rho C_p)_{s1}}{(\rho C_p)_f} \right] + \phi_{np2} \left(\frac{(\rho C_p)_{s2}}{(\rho C_p)_f} \right) \right] = 0 \quad (16)$$

and the related boundary restrictions are

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \\ f'(\infty) = 0, \quad \theta(\infty) = 0 \quad (17)$$

where,

$$M = \frac{\sigma_f B_0^2}{c \rho_f}, \quad Pr = \frac{\nu_f}{\alpha_f}, \quad Re = \frac{x u_w}{\nu_f}, \quad q_r = \frac{-16 \sigma^*}{3 k^*} \frac{\partial}{\partial y} \left(T^3 \frac{\partial T}{\partial y} \right)$$

2.5. Nusselt number and Skin friction Analysis:

The Heat transfer rate and Surface drag force, which is determined by,

The dimensionless form of C_f and Nu , are

$$C_f = \frac{\tau_w}{\rho_f u_w^2}, \quad Nu = \frac{x q_w}{k_f (T_w - T_\infty)} \quad (18)$$

$$\tau_w = \mu_{hnf} \left(1 + \frac{1}{\beta} \right) \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = q_r - k_{hnf} \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad (19)$$

$$Nu Re^{\frac{-1}{2}} = - \left(\frac{k_{hnf}}{k_f} \right) + R((\theta_w - 1)\theta + 1)^3 * \theta'. \quad (20)$$

$$C_f Re^{\frac{1}{2}} = \left(1 + \frac{1}{\beta}\right) \left(\frac{\mu_{hnf}}{\mu_f}\right) f''(0) \quad (21)$$

3. Physical Explanation:

The resulting ordinary differential equations expressed in Eqns. (15) & (16) confined to the boundary constraints (17) have been resolved by IVth order Runge-Kutta form with shooting manner with the support of BVP4C code in MATLAB software. Fig.II is mentioned the numerical solution for flow chart, we indicate $F = y_1$, $\theta = y_4$ for our present model and render a prime step of the system are

$$y_1' = y_2$$

$$y_2' = y_3$$

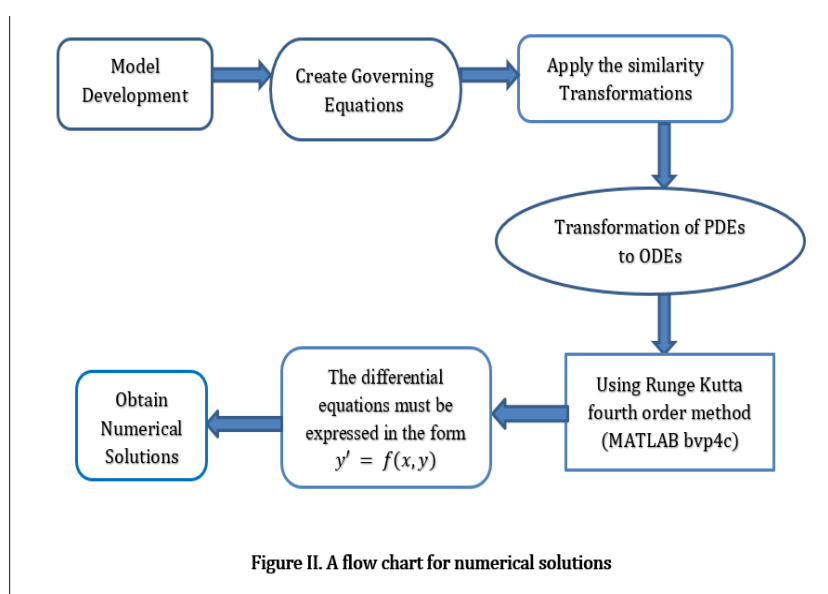
$$y_3' = \left(1 + \frac{1}{\beta}\right)^{-1} \left((1 - \phi_{np1})^{2.5} (1 - \phi_{np2})^{2.5}\right) \left\{ \frac{\sigma_{hnf}}{\sigma_f} M y_2 - \left[(1 - \phi_{np2}) \left[1 - \phi_{np1} + \phi_{np1} \frac{\rho_{np1}}{\rho_f} \right] + \phi_{np2} \frac{\rho_{np2}}{\rho_f} \right] [y_1 y_3 - y_2^2] \right\}$$

$$y_4' = y_5$$

$$y_5' = -Pr \left(\frac{k_{hnf}}{k_f} + \frac{1}{R((\theta_w - 1)y_4 + 1)^3} \right) \left[(1 - \phi_{np2}) \left[1 - \phi_{np1} + \phi_{np1} \frac{(\rho C_p)_{s1}}{(\rho C_p)_f} \right] + \phi_{np2} \left(\frac{(\rho C_p)_{s2}}{(\rho C_p)_f} \right) \right] [y_1 y_5]$$

and

$$y_1(0) = 0, y_2(0) = 1, y_4(0) = 1$$



4. Graphical and tabulated outcomes with discussion:

Figure I depict the items are placed of the conundrum. In regard to physical behaviour and attitude are explanations in the Figures. 1-8 have been drawn for the hybrid nanofluid flow, which consists of TiO_2 and Cu nanoparticles, (water (H_2O), sodium alginate ($\text{NaC}_6\text{H}_9\text{O}_7$)) are considered as base fluids to demonstrate the impression of numerous parameters on acceleration, thermal profiles. The base fluids' and the nanoparticles' thermophysical characteristics were described in Table 1. The values for surface drag force and nusselt number are given in Table 2.

4.1. Deviation of the Velocity Profile:

Fig.1. shows, with the use of water and sodium alginate mixing hybrid nanofluid, the velocity profile of the stretching sheet is observed to decrease. The decrease in the velocity profile could be attributed to the magnetic field's impact ($M = 1,2,3$) on the fluid, potentially altering its viscosity or flow pattern. The sodium alginate in the hybrid nanofluid may also contribute to the changes in the velocity profile due to its rheological properties.

As the radiation parameter ($R = 1,1.5,2$) increases, the velocity profile of the stretching sheet decreases. This suggests that the influence of radiation on the motion or flow of the water mixing hybrid nanofluid causes a reduction in the velocity profile of the stretching sheet. Also, the velocity profile of the stretching sheet increases with the radiation parameter when sodium alginate is used as the base fluid in the hybrid nanofluid. This implies that higher radiation levels enhance the velocity profile in the case of sodium alginate-based hybrid nanofluids, as explain Fig.2.

Fig.3.displays, when the base fluid is a water mixing hybrid nanofluid, the velocity profile of the stretching sheet decreases as the temperature ratio parameter ($\theta_w = 0.1,0.2,0.3$) increases. This could be due to changes in viscosity, thermal conductivity, or other fluid properties influenced by the temperature ratio. In contrast, when using a sodium alginate mixing hybrid nanofluid as the base fluid, the velocity profile of the stretching sheet increases. This suggests that the presence of sodium alginate, a natural polymer, in the nanofluid may lead to different thermal and flow characteristics compared to water, resulting in an increased velocity profile as the temperature ratio parameter rises.

An increasing Casson parameter ($\beta = 0.1,0.2,0.3$) generally leads to a decrease in the velocity of the fluid. Higher yield stress reduces fluid movement, causing slower velocity. Increased Casson parameter indicates shear-thinning behavior, decreasing viscosity. Shear forces influence fluid velocity in stretching sheets, with higher Casson parameters indicating greater fluid sensitivity, as appeared in Fig .4.

4.2. Deviation of the Thermal Profile:

Fig.5. explains, As the magnetic field parameter ($M = 1,2,3$) increases, there is an increase in the temperature profile of the stretching sheet. In this case, it's a hybrid nanofluid, meaning it's a combination of water and sodium alginate mixed with nanoparticles. The increase in the magnetic field parameter suggests that there's a stronger magnetic field applied to the system. Magnetic fields can affect the behavior of fluids, especially when they contain nanoparticles. The magnetic field may induce changes in the alignment and movement of nanoparticles within the

nanofluid.

As the radiation parameter ($R = 1, 1.5, 2$) increases, it becomes a more dominant mode of heat transfer, indicating its significant influence on heat transfer behavior. The presence of hybrid nanofluids, whether water-based or sodium alginate-based, further modifies the heat transfer characteristics of the system. The use of nanoparticles in these hybrid nanofluids contributes to changes in thermal conductivity, and the overall temperature increases on the stretching sheet, as exposed in Fig.6.

Fig.7. explains, as the temperature ratio parameter ($\theta_w = 0.1, 0.2, 0.3$) increases, the temperature profile of the stretching sheet in the context of water mixing hybrid nanofluid and sodium alginate mixing hybrid nanofluid is subject to changes in heat transfer, fluid flow, and thermal boundary layer characteristics. The specific effects will depend on the interplay of factors such as nanofluid properties, additive effects, and the overall thermal conditions imposed on the system. so finally as the temperature ratio parameter increases, it suggests that the temperature distribution on the sheet is increasing.

The increase in the Casson parameter ($\beta = 0.1, 0.2, 0.3$) could lead to a higher resistance to flow, affecting the boundary layer near the stretching sheet. This change in flow behavior, combined with the enhanced thermal properties of the sodium alginate mixing hybrid nanofluid. hybrid nanofluid, may contribute to an increase in the temperature profile along the stretching sheet. The nanoparticles in the hybrid nanofluid can potentially enhance heat transfer, resulting in a higher temperature gradient, explained in Fig.8.

Velocity Distributions:

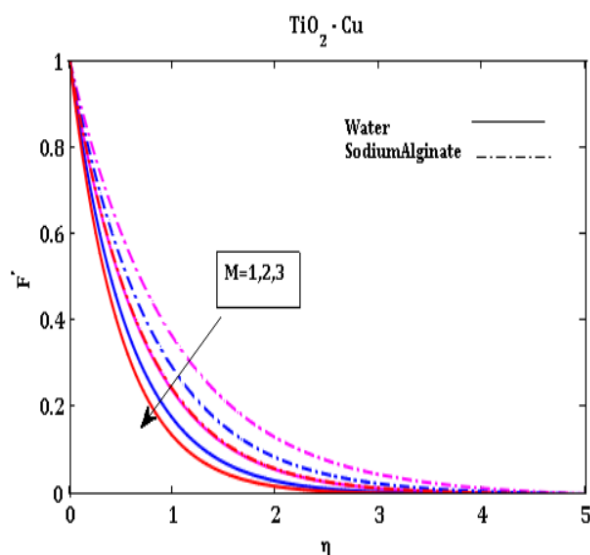


Fig. 1. The effects of $M = 1, 2, 3$ on f' .

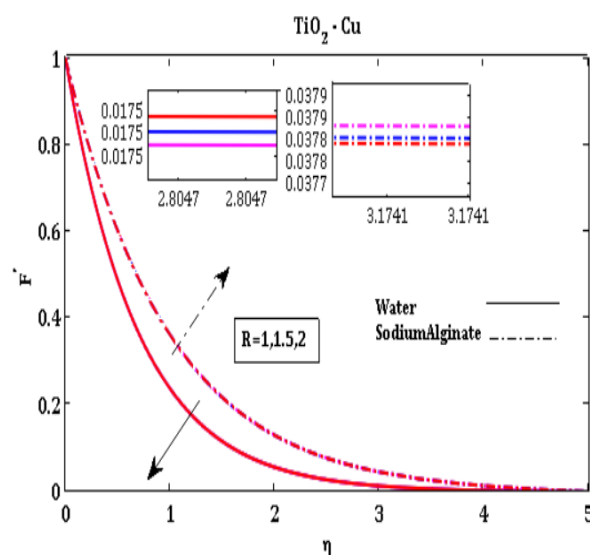


Fig. 2. The impacts of $R = 1, 1.5, 2$ on f'

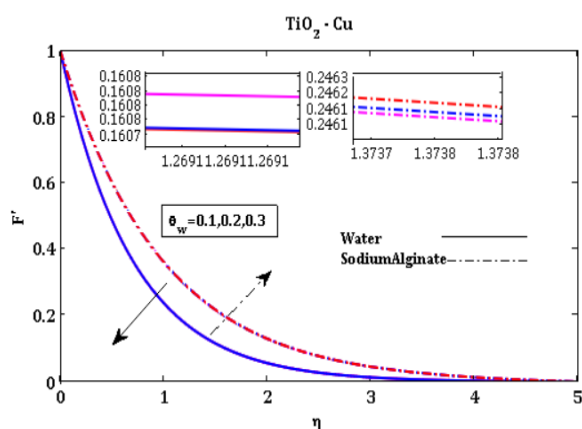


Fig. 3. The effects of $\theta_w = 0.1, 0.2, 0.3$ on f' .

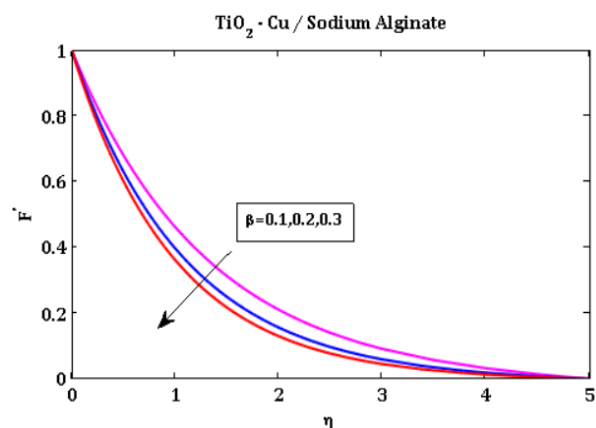


Fig. 4. The impacts of $\beta = 0.1, 0.2, 0.3$ on f' .

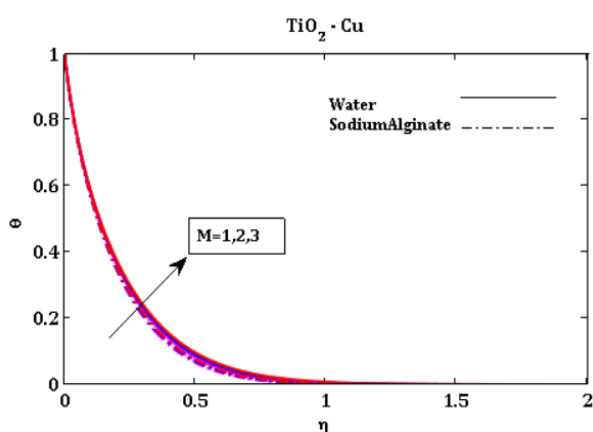


Fig. 5. The effects of $M = 1, 2, 3$ on θ .

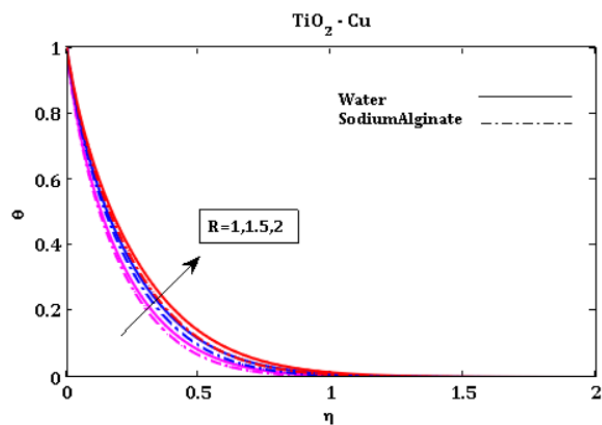


Fig. 6. The impacts of $R = 1, 1.5, 2$ on θ .

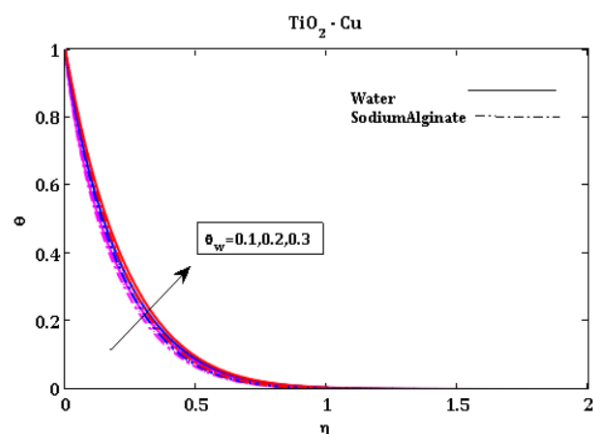


Fig. 7. The effects of $\theta_w = 0.1, 0.2, 0.3$ on θ .

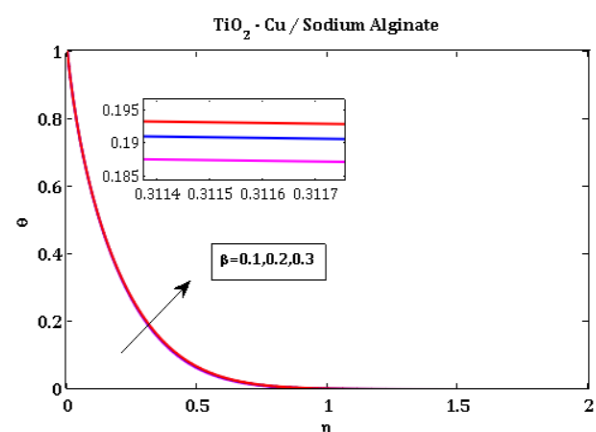


Fig. 8. The impacts of $\beta = 0.1, 0.2, 0.3$ on θ .

Table 1: Thermophysical characteristics of the base fluids and the nanoparticles [10],[12]

Base fluid and nanoparticle	$C_p/(J/(kg.K))$	$\rho/(kg/m^3)$	$k/(W/(m.K))$	$\beta/10^{-5}(1/K)$	$\sigma(sm^{-1})$
H ₂ O	4179	997	0.613	21	0.05
NaC ₆ H ₉ O ₇	4175	989	0.6376	99	2.6×10^{-4}
TiO ₂	686.2	4250	8.954	0.9	2.38×10^6
Cu	385	8933	400	1.67	5.96×10^7

Table 2: The values of C_f coefficient and Nu for M, β with surface temperature is listed below

M	R	θ_w	β	$C_f Re^{\frac{1}{2}}$		$Nu Re^{-\frac{1}{2}}$	
				TiO ₂ - Cu		TiO ₂ - Cu	
				H ₂ O	NaC ₆ H ₉ O ₇	H ₂ O	NaC ₆ H ₉ O ₇
1	1	0.1	1	-1.5309	-2.2377	7.2386	7.4772
2				-1.8479	-2.6540	7.1926	7.4472
3				-2.1220	-3.0246	7.1528	7.4204
1	1	0.1	1	-1.5309	-2.2377	7.2386	7.4772
	1.5			-1.5309	-2.2377	5.9887	6.1986
	2			-1.5309	-2.2377	5.2565	5.4500
1	1	0.1	1	-1.5309	-2.2377	7.2386	7.4772
		0.2		-1.5309	-2.2377	5.1425	5.3424
		0.3		-1.5309	-2.2377	4.3040	4.4915
1	1	0.1	0.4	-	-3.1450	-	7.5083
			0.7	-	-2.5075	-	7.4896
			1	-	-2.2377	-	7.4772

Conclusion:

A summary of their findings and a discussion of the implications of taking into account the Lorentz force and nonlinear thermal radiation were covered in the study's conclusion section on the convective flow of a hybrid nanofluid with different base fluids in heat transfer over a stretching sheet. It has been discussed that the following results and ramifications:

- This refers to the motion of a fluid driven by temperature differences within the fluid itself. Convection plays a significant role in heat transfer processes.
- This refers to a surface over which a fluid is flowing and the sheet is simultaneously stretched. The stretching sheet problem is a classical scenario in fluid mechanics and heat transfer.
- Thermal radiation is the emission of electromagnetic waves (radiation) due to the temperature of an object. In the context of heat transfer, thermal radiation plays a role alongside conduction and convection.
- The Lorentz force is the force experienced by a charged particle moving through an electromagnetic field. This force is perpendicular to both the velocity of the particle and the magnetic field.

➤ This type of study is common in the field of fluid dynamics and heat transfer, particularly in applications related to nanotechnology, materials science, and energy systems. The outcomes of such research can have implications for improving heat transfer efficiency in various engineering applications.

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