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Bipolar Vague α Generalized Continuous Mappings in Topological Spaces

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Abstract:

In this paper we have introduced bipolar vague α generalized continuous mappings in topological spaces and investigated some of their properties. Also, we have provided some characterization of bipolar vague α generalized continuous mappings in topological spaces.

Keywords: Bipolar vague sets, bipolar vague topology, bipolar vague α generalized closed sets, bipolar vague α generalized continuous mappings and bipolar vague α generalized irresolute mappings.

1. Introduction

Fuzzy set was introduced by L.A.Zadeh [11] in 1965. The concept of fuzzy topology was introduced by C.L.Chang [3] in 1968. The generalized closed sets in general topology were first introduced by N.Levine [9] in 1970. K.Atanassov [2] in 1986 introduced the concept of intuitionistic fuzzy sets. The notion of vague set theory was introduced by W.L.Gau and D.J.Buehrer [7] in 1993. D.Coker [6] in 1997 introduced intuitionistic fuzzy topological spaces. Bipolar- valued fuzzy sets, which was introduced by K.M.Lee [8] in 2000 is an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [-1,1]. A new class of generalized bipolar vague sets was introduced by S.Cicily Flora and I.Arockiarani [4] in 2016. F.Prishka and L.Mariapresenti [10] introduced bipolar vague α generalized closed sets in topological spaces. In continuation of our research work we have introduced bipolar vague α generalized continuous mappings in topological spaces, bipolar vague α generalized irresolute mappings in topological spaces and investigated some of their properties. Also, we have provided some characterization of bipolar vague α generalized continuous mappings in topological spaces.

2. Preliminaries

Here in this paper the bipolar vague topological spaces are denoted by (X, BV_τ). Also, the bipolar vague interior, bipolar vague closure of a bipolar vague set A are denoted by BVInt(A) and BVCl(A). The complement of a bipolar vague set A is denoted by A^c and the empty set and whole sets are denoted by $0_\sim$ and $1_\sim$ respectively.

Definition 2.1: [8] Let X be the universe. Then a bipolar valued fuzzy sets, A on X is defined by positive membership function μ_A^+, that is μ_A^+: X→ [0,1], and a negative membership function μ_A^-, that is μ_A^-: X→ [-1,0]. For the sake of simplicity, we shall use the symbol A = $\{\langle x, \mu_A^+(x), \mu_A^-(\mathrm{x})\rangle : x \in X\}$.

Definition 2.2: [8] Let A and B be two bipolar valued fuzzy sets then their union, intersection and complement are defined as follows:

(i) $\mu_{A\cup B}^+$= max $\{\mu_A^+(x), \mu_B^+ x)\}$
(ii) $\mu_{A\cup B}^-$= min $\{\mu_A^-(x), \mu_B^- x)\}$
(iii) $\mu_{A\cap B}^+$= min $\{\mu_A^+(x), \mu_B^+ x)\}$
(iv) $\mu_{A\cap B}^-$= max $\{\mu_A^-(x), \mu_B^- x)\}$
(v) $\mu_{A^c}^+$(x) = 1-$\mu_A^+(x)$ and $\mu_{A^c}^-$(x) = -1-$\mu_A^-(x)$ for all $x \in X$.

Definition 2.3: [7] A vague set A in the universe of discourse U is a pair of (t_A, f_A) where t_A: U→[0,1], f_A: U→[0,1] are the mapping such that $t_A + f_A \leq 1$ for all $u \in U$. The function t_A and f_A are called true membership function and false membership function respectively. The interval $[t_A, 1 - f_A]$ is called the vague value of u in A, and denoted by $\nu_A(u)$, that is $\nu_A(u) = [t_A(u), 1 - f(u)]$.

Definition 2.4: [7] Let A be a non-empty set and the vague set A and B in the form A = $\{\langle x, t_A(x), 1 - f_A(x)\rangle : \mathrm{x} \in \mathrm{X}\}$, B = $\{\langle x, t_B(x), 1 - f_B(x)\rangle : \mathrm{x} \in \mathrm{X}\}$. Then

(i) A ⊆ B if and only if $t_A(x) \leq t_B(x)$ and $1 - f_A(x) \leq 1 - f_B(x)$
(ii) A ∪ B = $\left\{\langle \max(t_A(x), t_B(x)), \frac{\max(1-f_A(x), 1-f_B(x))\rangle}{\mathrm{x}} \in \mathrm{X}\right\}$.
(iii) A ∩ B = $\left\{\langle \min(t_A(x), t_B(x)), \frac{\min(1-f_A(x), 1-f_B(x))\rangle}{\mathrm{x}} \in \mathrm{X}\right\}$.
(iv) $\mathrm{A^c} = \{\langle x, f_A(x), 1 - t_A(x)\rangle : \mathrm{x} \in \mathrm{X}\}$.

Definition 2.5: [1] Let X be the universe of discourse. A bipolar-valued vague set A in X is an object having the form A = {⟨x, $[t_A^+(x), 1 - f_A^+(x)], [-1 - f_A^-(x), t_A^-(x)]\rangle$: x ∈ X} where $[t_A^+, 1 - f_A^+]$: X→[0,1] and $[-1 - f_A^-, t_A^-]$: X→[-1,0] are the mapping such that $t_A^+(x) + f_A^+(x) \leq$ 1 and -1≤ $t_A^- + f_A^-$. The positive membership degree $[t_A^+(x), 1 - f_A^+(x)]$ denotes the satisfaction region of an element x to the property corresponding to a bipolar-valued set A and the negative membership degree $[-1 - f_A^-(x), t_A^-(x)]$ denotes the satisfaction region of x to some implicit counter property of A. For a sake of simplicity, we shall use the notion of bipolar vague set $\nu_A^+ = [t_A^+, 1 - f_A^+]$ and $\nu_A^- = [-1 - f_A^-, t_A^-]$.

Definition 2.6: [5] A bipolar vague set A = $[\nu_A^+, \nu_A^-]$ of a set U with ν_A^+= 0 implies that t_A^+ = 0, $1 - f_A^+$= 0 and ν_A^-= 0 implies that t_A^- = 0, $-1 - f_A^-$ = 0 for all x ∈ U is called zero bipolar vague set and it is denoted by 0.

Definition 2.7: [5] A bipolar vague set A = $[\nu_A^+, \nu_A^-]$ of a set U with ν_A^+= 1 implies that t_A^+ = 1, $1 - f_A^+$= 1 and ν_A^-= -1 implies that t_A^- = -1, $-1 - f_A^-$ = -1 for all x ∈ U is called unit bipolar vague set and it is denoted by 1.

Definition 2.8: [4] Let A = ⟨x, $[t_A^+, 1 - f_A^+], [-1 - f_A^-, t_A^-]\rangle$ and ⟨x, $[t_B^+, 1 - f_B^+], [-1 - f_B^-, t_B^-]\rangle$ be two bipolar vague sets then their union, intersection and complement are defined as follows:

(i) A ∪ B = $\left\{\langle x, [t^+_{A\cup B}(x), 1-f^+_{A\cup B}(x)], \frac{[-1-f^-_{A\cup B}(x), t^-_{A\cup B}(x)]\rangle}{x} \in X\right\}$ where
$t^+_{A\cup B}(x)$ = max $\{t^+_A(x), t^+_B(x)\}$, $t^-_{A\cup B}(x)$ = min $\{t^-_A(x), t^-_B(x)\}$ and
$1-f^+_{A\cup B}(x)$ = max $\{1-f^+_A(x), 1-f^+_B(x)\}$,
$-1-f^-_{A\cup B}(x)$ = min $\{-1-f^-_A(x), -1-f^-_B(x)\}$.

(ii) A ∩ B = $\left\{\langle x, [t^+_{A\cap B}(x), 1-f^+_{A\cap B}(x)], \frac{[-1-f^-_{A\cap B}(x), t^-_{A\cap B}(x)]\rangle}{x} \in X\right\}$ where
$t^+_{A\cap B}(x)$ = min $\{t^+_A(x), t^+_B(x)\}$, $t^-_{A\cap B}(x)$ = max $\{t^-_A(x), t^-_B(x)\}$ and
$1-f^+_{A\cap B}(x)$ = min $\{1-f^+_A(x), 1-f^+_B(x)\}$,
$-1-f^-_{A\cup B}(x)$ = max $\{-1-f^-_A(x), -1-f^-_B(x)\}$.

(iii) $A^c = \{\langle x, [f^+_A(x), 1-t^+_A(x)], [-1-t^-_A(x), f^-_A(x)]\rangle / x \in X\}$.

Definition 2.9: [4] Let A and B be two bipolar vague sets defined over a universe of discourse X. We say that A ⊆ B if and only if $t^+_A(x) \le t^+_B(x)$, $1-f^+_A(x) \le 1-f^+_B(x)$ and $t^-_A(x) \ge t^-_B(x)$, $-1-f^-_A(x) \ge 1-f^-_B(x)$ for all x ∈ X.

Definition 2.10: [4] A bipolar vague topology (BVT) on a non-empty set X is a family BV_τ of bipolar vague set in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in BV_\tau$
(ii) $G_1 \cap G_2 \in BV_\tau$, for any $G_1, G_2 \in BV_\tau$
(iii) $\cup G_i \in BV_\tau$, for any arbitrary family $\{G_i : G_i \in BV_\tau, I \in I\}$.

In this case the pair (X, BV_τ) is called a bipolar vague topological space and any bipolar vague set (BVS) in BV_τ is known as bipolar vague open set in X. The complement A^c of a bipolar vague open set (BVOS) A in a bipolar vague topological space (X, BV_τ) is called a bipolar vague closed set (BVCS) in X.

Definition 2.11: [4] Let (X, BV_τ) be a bipolar vague topological space $A = \langle x, [t^+_A, 1-f^+_A], [-1-f^-_A, t^-_A]\rangle$ be a bipolar vague set in X. Then the bipolar vague interior and bipolar vague closure of A are defined by,

BVInt(A) = ∪ {G: G is a bipolar vague open set in X and G ⊆ A},

BVCl(A) = ∩ {K: K is a bipolar vague closed set in X and A⊆ K}.

Note that BVCl(A) is a bipolar vague closed set and BVInt(A) is a bipolar vague open set in X. Further,

(i) A is a bipolar vague closed set in X if and only if BVCl(A) = A,
(ii) A is a bipolar vague open set in X if and only if BVInt(A) = A.

Definition 2.12: [4] Let (X, BV_τ) be a bipolar vague topological space. A bipolar vague set A in (X, BV_τ) is said to be a generalized bipolar vague closed set if BVCl(A) ⊆ G whenever A⊆ G and G is bipolar vague open. The complement of a generalized bipolar vague closed set is generalized bipolar vague open set.

Definition 2.13: [4] Let (X, BV_τ) be a bipolar vague topological space and A be a bipolar vague set in X. Then the generalized bipolar vague closure and generalized bipolar vague interior of A are defined by, GBVCl(A) = ∩ {G: G is a generalized bipolar vague closed set in X and A⊆ G},

GBInt(A) = ∪ {G: G is a generalized bipolar vague open set in X and A ⊇ G}.

Definition 2.14: [10] A bipolar vague set A of a bipolar vague topological space X, is said to be

(i) a bipolar vague α-open set if A ⊆ BVInt(BVCl(BVInt(A)))
(ii) a bipolar vague pre-open set if A ⊆ BVInt(BVCl(A))
(iii) a bipolar vague semi-open set if A ⊆ BVCl(BVInt(A))
(iv) a bipolar vague semi-α-open set if A ⊆ BVCl(αBVInt(A))
(v) a bipolar vague regular-open set BVInt(BVCl(A)) = A
(vi) a bipolar vague β-open set A ⊆ BVCl(BVInt(BVCl(A))).

Definition 2.15: [10] A bipolar vague set A of a bipolar vague topological space X, is said to be

(i) a bipolar vague α-closed set if BVCl(BVInt(BVCl(A))) ⊆ A
(ii) a bipolar vague pre-closed set if BVCl(BVInt(A)) ⊆ A
(iii) a bipolar vague semi-closed set if BVInt(BVCl(A)) ⊆ A
(iv) a bipolar vague semi-α-closed set if BVInt(αBVCl(A)) ⊆ A
(v) a bipolar vague regular-closed set if BVCl(BVInt(A)) = A
(vi) a bipolar vague β-closed set if BVInt(BVCl(BVInt(A))) ⊆ A.

Definition 2.16: [10] Let A be a bipolar vague set of a bipolar vague topological space (X, BV_τ). Then the bipolar vague α interior and bipolar vague α closure are defined as

BV_αInt(A) = ∪ {G: G is a bipolar vague α-open set in X and G ⊆ A},

BV_αCl(A) = ∩ {K: K is a bipolar vague α-closed set in X and A⊆ K}.

Definition 2.17: [10] A bipolar vague set A in a bipolar vague topological space X, is said to be a bipolar vague α generalized closed set if BV_αCl(A) ⊆ U whenever A⊆ U and U is a bipolar vague open set in X. The complement A^c of a bipolar vague α generalized closed set A is a bipolar vague α generalized open set in X.

Definition 2.18: [4] Let (X, BV_τ) and (Y, BV_σ) be two bipolar vague topological spaces and f : X→ Y be a function. Then φ is said to be bipolar vague continuous if and only if the preimage of each bipolar vague open set in Y is a bipolar vague open set in X.

Definition 2.19: [4] A map f : (X, BV_τ) → (Y, BV_σ) is said to be generalized bipolar vague continuous if the inverse image of every bipolar vague open set in (Y, BV_σ) is a generalized vague open set in (X, BV_τ).

Definition 2.20: [4] Let f be a mapping from a bipolar vague topological space (X, BV_τ) into a bipolar vague topological space (Y, BV_σ). Then f is said to be a bipolar vague generalized irresolute mapping if the inverse image of every bipolar vague generalized closed set in (Y, BV_σ) is a bipolar vague generalized closed set in (X, BV_τ).

3. Bipolar Vague α Generalized Continuous Mappings in Topological Spaces

In this section we have introduced bipolar vague α generalized continuous mappings and investigated some of their properties. Also, we have established the relation between the newly introduced mappings and already existing mappings.

Definition 3.1: Let (X, BV_τ) and (Y, BV_σ) be two bipolar vague topological spaces. Then the mapping f : (X, BV_τ) $\rightarrow$ (Y, BV_σ) is called

(i) a bipolar vague α continuous if the inverse image of every bipolar vague closed set in (Y, BV_σ) is a bipolar vague α-closed set in (X, BV_τ).
(ii) a bipolar vague pre continuous if the inverse image of every bipolar vague closed set in (Y, BV_σ) is a bipolar vague pre-closed set in (X, BV_τ).
(iii) a bipolar vague semi continuous if the inverse image of every bipolar vague closed set in (Y, BV_σ) is a bipolar vague semi-closed set in (X, BV_τ).

Definition 3.2: Let (X, BV_τ) and (Y, BV_σ) be two bipolar vague topological spaces. A mapping f : (X, BV_τ) $\rightarrow$ (Y, BV_σ) is called a bipolar vague α generalized continuous mapping if f^{-1}(B) is a bipolar vague α generalized closed set in (X, BV_τ) for every bipolar vague closed set B of (Y, BV_σ).

Example 3.3: Let X = {a,b} and Y = {u,v}. Then $\tau = \{0_\sim, A, 1_\sim\}$ and $\sigma = \{0_\sim, B, 1_\sim\}$ are bipolar vague topologies on X and Y respectively, where A = ⟨x, [0.5, 0.5] [-0.5, -0.5], [0.5, 0.5] [-0.5, -0.5]⟩ and B = ⟨y, [0.7, 0.6] [-0.9, -0.9], [0.6, 0.6] [-0.5, -0.5]⟩. Define a mapping f : (X, BV_τ) $\rightarrow$ (Y, BV_σ) by f(a) = u and f(b) = v. Here the bipolar vague set B^c = ⟨y, [0.4, 0.3] [-0.1, -0.1], [0.4, 0.4] [-0.5, -0.5]⟩ is a bipolar vague closed set in Y. Then f^{-1} (B^c) = ⟨x, [0.4, 0.3] [-0.1, -0.1], [0.4, 0.4] [-0.5, -0.5]⟩ is a bipolar vague α generalized closed set in (X, BV_τ) as f^{-1} (B^c) ⊆ A and BV_αCl(f^{-1} (B^c)) = f^{-1} (B^c) ∪ BVCl(BVInt(BVCl(f^{-1} (B^c)))) = A^c ⊆ A, where A is a bipolar vague open set in X. Therefore, f is a bipolar vague α generalized continuous mapping.

Proposition 3.4: Every bipolar vague continuous mapping is a bipolar vague α generalized continuous mapping but not conversely in general.

Proof: Let f : (X, BV_τ) $\rightarrow$ (Y, BV_σ) be a bipolar vague continuous mapping. Let A be a bipolar vague closed set in Y. Then f^{-1}(A) is a bipolar vague closed set in X. Since every bipolar vague closed set is a bipolar vague α generalized closed set in X [10], f^{-1}(A) is a bipolar vague α generalized closed set in X. Hence f is a bipolar vague α generalized continuous mapping.

Example 3.5: Let X = {a,b} and Y = {u,v}. Then $\tau = \{0_\sim, A, 1_\sim\}$ and $\sigma = \{0_\sim, B, 1_\sim\}$ are bipolar vague topologies on X and Y respectively, where A = ⟨x, [0.2, 0.3] [-0.4, -0.4], [0.5, 0.5] [-0.4, -0.4]⟩ and B = ⟨y, [0.4, 0.4] [-0.4, -0.4], [0.6, 0.6] [-0.4, -0.4]⟩. Define a mapping f : (X, BV_τ) $\rightarrow$ (Y, BV_σ) by f(a) = u and f(b) = v. Here the bipolar vague set B^c = ⟨y, [0.6, 0.6] [-0.6, -0.6], [0.4, 0.4] [-0.6, -0.6]⟩ is a bipolar vague closed set in Y. Then f^{-1} (B^c) = ⟨x, [0.6, 0.6] [-0.6, -0.6], [0.4, 0.4] [-0.6, -0.6]⟩ is a bipolar vague α generalized closed set in (X, BV_τ) as f^{-1} (B^c) ⊆ $1_\sim$ and BV_αCl(f^{-1} (B^c)) = f^{-1} (B^c) ∪ BVCl(BVInt(BVCl(f^{-1} (B^c)))) = A^c ⊆ $1_\sim$, where A^c is a bipolar vague closed set in X. Therefore, f is a bipolar vague α generalized continuous mapping but since f^{-1} (B^c) is not a bipolar vague closed set in X as BVCl(f^{-1} (B^c)) = A^c ≠ f^{-1} (B^c), f is not a bipolar vague continuous mapping.

Proposition 3.6: Every bipolar vague α continuous mapping is a bipolar vague α generalized continuous mapping but not conversely in general.

Proof: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague α continuous mapping. Let A be a bipolar vague closed set in Y. Then f^{-1}(A) is a bipolar vague α-closed set in X. Since every bipolar vague α-closed set is a bipolar vague α generalized closed set in X [10], f^{-1}(A) is a bipolar vague α generalized closed set in X. Hence f is a bipolar vague α generalized continuous mapping.

Example 3.7: Let X = {a,b} and Y = {u,v}. Then $\tau = \{0_\sim, A, 1_\sim\}$ and $\sigma = \{0_\sim, B, 1_\sim\}$ are bipolar vague topologies on X and Y respectively, where A = ⟨x, [0.2, 0.3] [-0.3, -0.3], [0.5, 0.5] [-0.4, -0.4]⟩ and B = ⟨y, [0.3, 0.3] [-0.3, -0.3], [0.7, 0.7] [-0.5, -0.5]⟩. Define a mapping f : (X, BV_τ) → (Y, BV_σ) by f(a) = u and f(b) = v. Here the bipolar vague set B^c = ⟨y, [0.7, 0.7] [-0.7, -0.7], [0.3, 0.3] [-0.5, -0.5]⟩ is a bipolar vague closed set in Y. Then f^{-1} (B^c) = ⟨x, [0.7, 0.7] [-0.7, -0.7], [0.3, 0.3] [-0.5, -0.5]⟩ is a bipolar vague α generalized closed set in (X, BV_τ) as f^{-1} (B^c) $\subseteq 1_\sim$ and BV_αCl(f^{-1} (B^c)) = f^{-1} (B^c) ∪ BVCl(BVInt(BVCl(f^{-1} (B^c)))) = A^c $\subseteq 1_\sim$, where A^c is a bipolar vague closed set in X. Therefore, f is a bipolar vague α generalized continuous mapping but since $\mathrm{BVCl}\left(\mathrm{BVInt}\left(\mathrm{BVCl}\left(f^{-1}\,(B^c)\right)\right)\right)$ = A^c $\not\subseteq$ f^{-1} (B^c). Hence f is not a bipolar vague α continuous mapping.

Remark 3.8: Every bipolar vague semi continuous mapping and bipolar vague α generalized continuous mapping are independent to each other in general.

Example 3.9: In Example 3.4, f is a bipolar vague α generalized continuous mapping but since $\mathrm{BVInt}\left(\mathrm{BVCl}\left(f^{-1}\,(B^c)\right)\right)$ = BVInt(A^c) = A $\not\subseteq$ f^{-1} (B^c) = ⟨x, [0.4, 0.3] [-0.1, -0.1], [0.4, 0.4] [-0.5, -0.5]⟩, f^{-1} (B^c) is not a bipolar vague semi-closed set in X. Hence f is not a bipolar vague semi continuous mapping.

Example 3.10: Let X = {a,b} and Y = {u,v}. Then $\tau = \{0_\sim, A, 1_\sim\}$ and $\sigma = \{0_\sim, B, 1_\sim\}$ are bipolar vague topologies on X and Y respectively, where A = ⟨x, [0.4, 0.3] [-0.2, -0.2], [0.5, 0.5] [-0.5, -0.5]⟩ and B = ⟨y, [0.7, 0.6] [-0.8, -0.8], [0.5, 0.5] [-0.5, -0.5]⟩. Define a mapping f : (X, BV_τ) → (Y, BV_σ) by f(a) = u and f(b) = v. Here the bipolar vague set B^c = ⟨y, [0.4, 0.3] [-0.2, -0.2], [0.5, 0.5] [-0.5, -0.5]⟩ is a bipolar vague closed set in Y. But f^{-1} (B^c) = ⟨x, [0.4, 0.3] [-0.2, -0.2], [0.5, 0.5] [-0.5, -0.5]⟩ is not a bipolar vague α generalized closed set in (X, BV_τ) as f^{-1} (B^c) ⊆ A and BV_αCl(f^{-1} (B^c)) = f^{-1} (B^c) ∪ BVCl(BVInt(BVCl(f^{-1} (B^c)))) = A^c $\not\subseteq$ A, where A^c is a bipolar vague closed set in X. Therefore, f is not a bipolar vague α generalized continuous mapping but since $\mathrm{BVInt}\left(\mathrm{BVCl}\left(f^{-1}\,(B^c)\right)\right)$ = A $\subseteq$ f^{-1} (B^c) is a bipolar vague semi-closed set in X. Hence f is a bipolar vague semi continuous mapping.

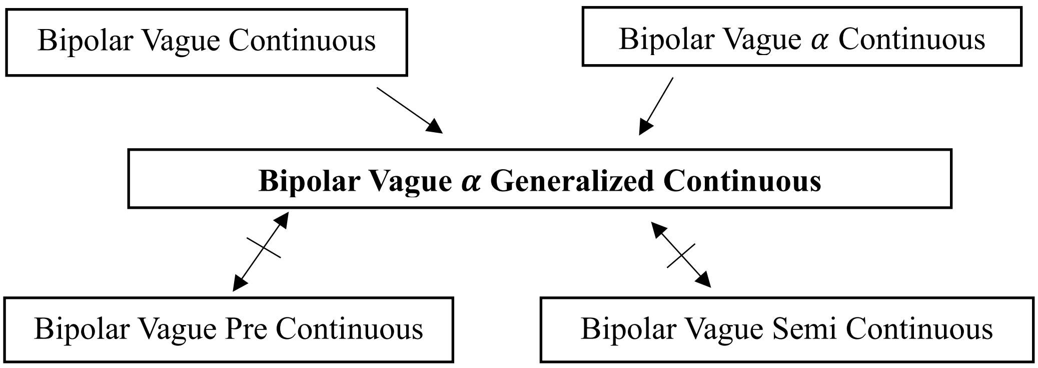
Remark 3.11: Every bipolar vague pre continuous mapping and bipolar vague α generalized continuous mapping are independent to each other in general.

Example 3.12: Let X = {a,b} and Y = {u,v}. Then $\tau = \{0_\sim, A, 1_\sim\}$ and $\sigma = \{0_\sim, B, 1_\sim\}$ are bipolar vague topologies on X and Y respectively, where A = ⟨x, [0.1, 0.1] [-0.4, -0.4], [0.6, 0.3] [-0.5, -0.5]⟩ and B = ⟨y, [0.2, 0.2] [-0.5, -0.5], [0.7, 0.3] [-0.5, -0.5]⟩. Define a mapping f

: (X, BV_τ) $\to$ (Y, BV_σ) by f(a) = u and f(b) = v. Here the bipolar vague set B^c = ⟨y, [0.8, 0.8] [-0.5, -0.5], [0.7, 0.3] [-0.5, -0.5]⟩ is a bipolar vague closed set in Y. Then f^{-1} (B^c) = ⟨x, [0.8, 0.8] [-0.5, -0.5], [0.7, 0.3] [-0.5, -0.5]⟩ is a bipolar vague α generalized closed set in (X, BV_τ) as f^{-1} (B^c) $\subseteq 1_\sim$ and $BV_\alpha Cl(f^{-1}$ (B^c)) = f^{-1} (B^c) ∪ BVCl(BVInt(BVCl(f^{-1} (B^c)))) = $A^c \subseteq 1_\sim$, where A^c is a bipolar vague closed set in X. Therefore, f is a bipolar vague α generalized continuous mapping. Since $\mathrm{BVCl}\left(\mathrm{BVInt}\left(f^{-1}\,(B^c)\right)\right) = A^c \not\subseteq f^{-1}$ (B^c), f^{-1} (B^c) is not a bipolar vague pre-closed set in X. Hence f is not a bipolar vague pre continuous mapping.

Example 3.13: Let X = {a,b} and Y = {u,v}. Then $\tau = \{0_\sim, A, 1_\sim\}$ and $\sigma = \{0_\sim, B, 1_\sim\}$ are bipolar vague topologies on X and Y respectively, where A = ⟨x, [0.5, 0.4] [-0.3, -0.2], [0.5, 0.5] [-0.3, -0.2]⟩ and B = ⟨y, [0.8, 0.7] [-0.8, -0.8], [0.5, 0.5] [-0.8, -0.8]⟩. Define a mapping f : (X, BV_τ) $\to$ (Y, BV_σ) by f(a) = u and f(b) = v. Here the bipolar vague set B^c = ⟨y, [0.3, 0.2] [-0.2, -0.2], [0.5, 0.5] [-0.2, -0.2]⟩ is a bipolar vague closed set in Y. Then f^{-1} (B^c) = ⟨x, [0.3, 0.2] [-0.2, -0.2], [0.5, 0.5] [-0.2, -0.2]⟩ is not a bipolar vague α generalized closed set in (X, BV_τ) as f^{-1} (B^c) ⊆ A and $BV_\alpha Cl(f^{-1}$ (B^c)) = f^{-1} (B^c) ∪ BVCl(BVInt(BVCl(f^{-1} (B^c)))) = $A^c \not\subseteq$ A, where A^c is a bipolar vague closed set in X. Therefore, f is not a bipolar vague α generalized continuous mapping. Since $\mathrm{BVCl}\left(\mathrm{BVInt}\left(f^{-1}\,(B^c)\right)\right) = 0_\sim \subseteq f^{-1}$ (B^c), f^{-1} (B^c) is a bipolar vague pre-closed set in X. Hence f is a bipolar vague pre continuous mapping.

The relation between various types of bipolar vague continuity is given in the following diagram:



Proposition 3.14: A mapping f : (X, BV_τ) $\to$ (Y, BV_σ) is a bipolar vague α generalized continuous if and only if the inverse image of each bipolar vague open set in Y is a bipolar vague α generalized open set in X.

Proof: Necessity: Let A be a bipolar vague open set in Y. This implies A^c is a bipolar vague closed set in Y. Since f is a bipolar vague α generalized continuous, f^{-1}(A^c) is a bipolar vague α generalized closed set in X. Since f^{-1}(A^c) = $(f^{-1}(A))^c$, f^{-1}(A) is a bipolar vague α generalized open set in X.

Sufficiency: Let A be a bipolar vague closed set in Y. This implies A^c is a bipolar vague open set in Y. By hypothesis, f^{-1}(A^c) is a bipolar vague α generalized open set in X. Since f^{-1}(A^c) = $(f^{-1}(A))^c$, f^{-1}(A) is a bipolar vague α generalized closed set in X. Hence f is a bipolar vague α generalized continuous mapping.

Proposition 3.15: If f : (X, BV_τ) → (Y, BV_σ) is a bipolar vague α generalized continuous mapping and g : (Y, BV_σ) → (Z, BV_δ) is a bipolar vague continuous mapping, then $g \circ f$: (X, BV_τ) → (Z, BV_δ) is a bipolar vague α generalized continuous mapping.

Proof: Let A be a bipolar vague closed set in Z. Then g^{-1}(A) be a bipolar vague closed set in Y, by hypothesis. Since f is a bipolar vague α generalized continuous mapping, f^{-1}(g^{-1}(A)) is a bipolar vague α generalized closed set in X. Hence $g \circ f$ is a bipolar vague α generalized continuous mapping.

Definition 3.16: Let (X, BV_τ) be a bipolar vague topological space. The bipolar vague alpha generalized closure (B$V_{\alpha g}$Cl(A)) for any bipolar vague set A is defined as follows:

BV_αCl(A) = ∩ {K: K is a bipolar vague α generalized closed set in X and A⊆ K}. If A is a bipolar vague α generalized closed set, then B$V_{\alpha g}$Cl(A) = A.

Proposition 3.17: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague α generalized continuous mapping. Then the following conditions are hold:

(i) f(B$V_{\alpha g}$Cl(A)) ⊆ BVCl(f(A)), for every bipolar vague set A in X.
(ii) B$V_{\alpha g}$Cl(f^{-1}(B)) ⊆ f^{-1}(BVCl(B)), for every bipolar vague set B in Y.

Proof: (i) Since BVCl(f(A)) is a bipolar vague closed set in Y and f is a bipolar vague α generalized continuous mapping, then f^{-1}(BVCl(f(A))) is a bipolar vague α generalized closed set in X. That is B$V_{\alpha g}$Cl(f^{-1}(BVCl(f(A)))) = f^{-1}(BVCl(f(A))). Now, f(B$V_{\alpha g}$Cl(f^{-1}(BVCl(f(A)))) = ff^{-1}(BVCl(f(A))) ⊆ BVCl(f(A)). Then f(B$V_{\alpha g}$Cl(A)) ⊆ f(B$V_{\alpha g}$Cl($f^{-1}f$(A))) ⊆ f(B$V_{\alpha g}$Cl(f^{-1}(BVCl(f(A)))) ⊆ BVCl(f(A)). Therefore f(B$V_{\alpha g}$Cl(A)) ⊆ BVCl(f(A)), for every bipolar vague set A in X.

(ii) Replacing A by f^{-1}(B) in (i), we get f(B$V_{\alpha g}$Cl(f^{-1}(B))) ⊆ BVCl(f(f^{-1}(B))) ⊆ BVCl(B). Hence B$V_{\alpha g}$Cl(f^{-1}(B)) ⊆ f^{-1}(f(B$V_{\alpha g}$Cl(f^{-1}(B)))) ⊆ f^{-1}(BVCl(B)), for every bipolar vague set B in Y.

Definition 3.18: A bipolar vague topological space (X, BV_τ) is said to be bipolar vague αa $T_{1/2}$(B$V_{\alpha a}T_{1/2}$) space if every bipolar vague α generalized closed set in X is a bipolar vague closed set in X.

Definition 3.19: A bipolar vague topological space (X, BV_τ) is said to be bipolar vague αb $T_{1/2}$(B$V_{\alpha b}T_{1/2}$) space if every bipolar vague α generalized closed set in X is a bipolar vague generalized closed set in X.

Proposition 3.20: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague α generalized continuous mapping, then f is a bipolar vague continuous mapping, if X is a B$V_{\alpha a}T_{1/2}$ space.

Proof: Let A be a bipolar vague closed set in Y. Then f^{-1}(A) is a bipolar vague α generalized closed set in X, by hypothesis. Since X is a B$V_{\alpha a}T_{1/2}$ space, f^{-1}(A) is a bipolar vague closed set in X. Hence f is a bipolar vague continuous mapping.

Proposition 3.21: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague α generalized continuous mapping, then f is a bipolar vague generalized continuous mapping, if X is a B$V_{\alpha b}T_{1/2}$ space.

Proof: Let A be a bipolar vague closed set in Y. Then f^{-1}(A) is a bipolar vague α generalized closed set in X, by hypothesis. Since X is a B$V_{\alpha b}T_{1/2}$ space, f^{-1}(A) is a bipolar vague generalized closed set in X. Hence f is a bipolar vague generalized continuous mapping.

Proposition 3.22: Let f : (X, BV_τ) → (Y, BV_σ) be a mapping from a bipolar vague topological space X into a bipolar vague topological space Y. Then the following conditions are equivalent if X is a B$V_{\alpha a}T_{1/2}$ space:

(i) f is a bipolar vague α generalized continuous mapping.
(ii) If B is a bipolar vague open set in Y, f^{-1}(B) is a bipolar vague α generalized closed set in X.
(iii) f^{-1}(BVInt(B)) ⊆ BVInt(BVCl(BVInt(f^{-1}(B)))) for every bipolar vague set B in Y.

Proof: (i) ⟹ (ii) is obviously true.

(ii) ⟹ (iii). Let B be any bipolar vague open set in Y. The BVInt(B) is a bipolar vague open set in Y. Then f^{-1}(BVInt(B)) is a bipolar vague α generalized open set in X. Since X is a B$V_{\alpha a}T_{1/2}$ space, f^{-1}(BVInt(B)) is a bipolar vague open set in X. Therefore, f^{-1}(BVInt(B)) = BVInt(f^{-1}(BVInt(B))) ⊆ BVInt(BVCl(BVInt(f^{-1}(B)))).

(iii) ⟹ (i). Let B be a bipolar vague closed set in Y. Then its complement B^c is a bipolar vague open set in Y. By hypothesis, f^{-1}(BVInt(B^c)) ⊆ BVInt(BVCl(BVInt(f^{-1}(B^c)))). This implies f^{-1}(B^c) ⊆ BVInt(BVCl(BVInt(f^{-1}(B^c)))). Hence f^{-1}(B^c) is a bipolar vague α-open set in X. Since every bipolar vague α-open set is a bipolar vague α generalized open set, f^{-1}(B^c) is a bipolar vague α generalized open set in X. Therefore, f^{-1}(B) is a bipolar vague α generalized closed set in X. Hence f is a bipolar vague α generalized continuous mapping.

Proposition 3.23: Let f : (X, BV_τ) → (Y, BV_σ) be a mapping from a bipolar vague topological space X into a bipolar vague topological space Y. Then the following conditions are equivalent if X is a B$V_{\alpha a}T_{1/2}$ space:

(i) f is a bipolar vague α generalized continuous mapping.
(ii) If f^{-1}(B) is a bipolar vague α generalized closed set in X, for every bipolar vague closed set B in Y.
(iii) BVCl(BVInt(BVCl(f^{-1}(A)))) ⊆ f^{-1}(BVCl(A)) for every bipolar vague set B in Y.

Proof: (i) ⟹ (ii) is obviously true.

(ii) ⟹ (iii). Let A be any bipolar vague set in Y. Then BVCl(A) is a bipolar vague closed set in Y. By hypothesis, f^{-1}(BVCl(A)) is a bipolar vague α generalized closed set in X. Since X is a B$V_{\alpha a}T_{1/2}$ space, f^{-1}(BVCl(A)) is a bipolar vague closed set in X. Therefore, BVCl(f^{-1}(BVCl(A))) = f^{-1}(BVCl(A)). Now BVCl(BVInt(BVCl((f^{-1}(A)))) ⊆ BVCl(BVInt(BVCl((f^{-1}(BVCl(A)))) ⊆ f^{-1}(BVCl(A)).

(iii) ⟹ (i). Let A be a bipolar vague closed set in Y. Then by hypothesis, BVCl(BVInt(BVCl(f^{-1}(BVCl(A))))) ⊆ f^{-1}(BVCl(A)) = f^{-1}(A). This implies f^{-1}(A) is a bipolar vague α-closed set in X and hence it is a bipolar vague α generalized closed set in X. Therefore, f is a bipolar vague α generalized continuous mapping.

4. Bipolar Vague α Generalized Irresolute Mappings in Topological Spaces

In this section we have introduced bipolar vague α generalized irresolute mappings and studied some of their properties.

Definition 4.1: A mapping f : (X, BV_τ) → (Y, BV_σ) is called a bipolar vague α generalized irresolute mapping if f^{-1}(A) is a bipolar vague α generalized closed set in (X, BV_τ) for every bipolar vague α generalized closed set A of (Y, BV_σ).

Proposition 4.2: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague α generalized irresolute mapping, then f is a bipolar vague α generalized continuous mapping but not conversely.

Proof: Let f be a bipolar vague α generalized irresolute mapping. Let A be any bipolar vague closed set in Y. Since every bipolar vague closed set is a bipolar vague α generalized closed set [10], A is a bipolar vague α generalized closed set in Y. By hypothesis, f^{-1}(A) is a bipolar vague α generalized closed set in X. Hence f is a bipolar vague α generalized continuous mapping.

Example 4.3: Let X = {a,b} and Y = {u,v}. Then $\tau = \{0_\sim, A, 1_\sim\}$ and $\sigma = \{0_\sim, B, 1_\sim\}$ are bipolar vague topologies on X and Y respectively, where A = ⟨x, [0.1, 0.3] [-0.3, -0.3], [0.6, 0.3] [-0.3, -0.3]⟩ and B = ⟨y, [0.3, 0.1] [-0.1, -0.1], [0.5, 0.6] [-0.1, -0.1]⟩. Define a mapping f : (X, BV_τ) → (Y, BV_σ) by f(a) = u and f(b) = v. Then f is a bipolar vague α generalized continuous mapping but not a bipolar vague α generalized irresolute mapping. Since the bipolar vague set M = ⟨y, [0.1, 0.3] [-0.2, -0.2], [0.6, 0.2] [-0.2, -0.2]⟩ is a bipolar vague α generalized closed set in Y but f^{-1} (M) is not a bipolar vague α generalized closed set in X as f^{-1} (M) = ⟨x, [0.1, 0.3] [-0.2, -0.2], [0.6, 0.2] [-0.2, -0.2]⟩ ⊆ A but BV_αCl(f^{-1} (M)) = f^{-1} (M) ∪ BVCl(BVInt(BVCl(f^{-1} (M)))) = A^c ⊄ A. Hence f is not a bipolar vague α generalized irresolute mapping.

Proposition 4.4: Let f : (X, BV_τ) → (Y, BV_σ) and g : (Y, BV_σ) → (Z, BV_δ) be any two bipolar vague α generalized irresolute mappings, then $g \circ f$: (X, BV_τ) → (Z, BV_δ) is a bipolar vague α generalized irresolute mapping.

Proof: Let A be a bipolar vague α generalized closed set in Z. Then g^{-1} (A) is a bipolar vague α generalized closed set in Y. Since f is a bipolar vague α generalized irresolute mapping, $f^{-1}(g^{-1}$ (A)) is a bipolar vague α generalized closed set in X. Hence $g \circ f$ is a bipolar vague α generalized irresolute mapping.

Proposition 4.5: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague α generalized irresolute mapping and g : (Y, BV_σ) → (Z, BV_δ) be a bipolar vague α generalized continuous mapping, then $g \circ f$: (X, BV_τ) → (Z, BV_δ) is a bipolar vague α generalized continuous mapping.

Proof: Let A be a bipolar vague closed set in Z. Then g^{-1} (A) is a bipolar vague α generalized closed set in Y, by hypothesis. Since f is a bipolar vague α generalized irresolute mapping, $f^{-1}(g^{-1}$ (A)) is a bipolar vague α generalized closed set in X. Hence $g \circ f$ is a bipolar vague α generalized continuous mapping.

Proposition 4.6: Let f : (X, BV_τ) → (Y, BV_σ) be a bipolar vague α generalized irresolute mapping and g : (Y, BV_σ) → (Z, BV_δ) be a bipolar vague continuous mapping, then $g \circ f$: (X, BV_τ) → (Z, BV_δ) is a bipolar vague α generalized continuous mapping.

Proof: Let A be a bipolar vague closed set in Z. Then g^{-1} (A) is a bipolar vague closed set in Y. Since every bipolar vague closed set is a bipolar vague α generalized closed set [10], g^{-1} (A) is a bipolar vague α generalized closed set in Y. Therefore $f^{-1}(g^{-1}$ (A)) is a bipolar vague α generalized closed set in X, by hypothesis. Hence $g \circ f$ is a bipolar vague α generalized continuous mapping.

Proposition 4.7: A mapping f : (X, BV_τ) → (Y, BV_σ) is a bipolar vague α generalized irresolute mapping if and only if the inverse image of each bipolar vague α generalized open set in Y is a bipolar vague α generalized open set in X.

Proof: Necessity: Let A be a bipolar vague α generalized open set in Y. Then A^c is a bipolar vague α generalized closed set in Y. Since f is a bipolar vague α generalized irresolute, $f^{-1}(A^c)$ is a bipolar vague α generalized closed set in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, f^{-1} (A) is a bipolar vague α generalized open set in X.

Sufficiency: Let A be a bipolar vague α generalized closed set in Y. This implies A^c is a bipolar vague α generalized open set in Y. By hypothesis, $f^{-1}(A^c)$ is a bipolar vague α generalized open set in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, f^{-1} (A) is a bipolar vague α generalized closed set in X. Hence f is a bipolar vague α generalized irresolute mapping.

Proposition 4.8: Let f : (X, BV_τ) → (Y, BV_σ) be a mapping from a bipolar vague topological space X into a bipolar vague topological space Y. Then the following conditions are equivalent if X and Y are B$V_{\alpha a}T_{1/2}$ spaces:

(i) f is a bipolar vague α generalized irresolute mapping.
(ii) f^{-1} (B) is a bipolar vague α generalized open set in X for each bipolar vague α generalized open set in Y.
(iii) BVCl(f^{-1} (B)) ⊆ f^{-1}(BVCl(B)) for each bipolar vague set B of Y.

Proof: (i) ⟹ (ii) is obviously true from the Proposition 4.7.

(ii) ⟹ (iii). Let B be any bipolar vague set in Y and B ⊆ BVCl(B). Then f^{-1} (B) ⊆ f^{-1}(BVCl(B)). Since BVCl(B) is a bipolar vague closed set in Y, f^{-1}(BVCl(B)) is a bipolar vague α generalized closed set in X, by hypothesis. Since X is a B$V_{\alpha a}T_{1/2}$ space, f^{-1}(BVCl(B)) is a bipolar vague closed set in X. Hence BVCl(f^{-1} (B)) ⊆ BVCl(f^{-1}(BVCl(B))) = f^{-1}(BVCl(B)).

(iii) ⟹ (i). Let B be a bipolar vague α generalized closed set in Y. Since Y is a B$V_{\alpha a}T_{1/2}$ space, B is a bipolar vague closed set in Y and BVCl(B) = B. Hence f^{-1} (B) = f^{-1}(BVCl(B)) ⊇ BVCl(f^{-1} (B)). But f^{-1} (B) ⊆ BVCl(f^{-1} (B)). Therefore, BVCl(f^{-1} (B)) = f^{-1} (B). This implies f^{-1} (B) is a bipolar vague closed set and hence it is a bipolar vague α generalized closed set in X. Thus f is a bipolar vague α generalized irresolute mapping.

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