Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 31 No. 6s (2024)

Elzaki Transform Homotopy Analysis Techniques for Solving Fractional (2+1)-D and (3+1)-D Nonlinear Schrodinger Equations

Sandeep Sharma[1], Inderdeep Singh[2]
[1,2]Sant Baba Bhag Singh University, Jalandhar-144030, Punjab, India
Email: [1]sandeepsharma200@gmail.com , [2]inderdeeps.ma.12@gmail.com

Article History:

Received: *08-06-2024*

Revised: *09-07-2024*

Accepted: *29-07-2024*

Abstract:
In this research, New homotopy analysis method for solving the fractional (2+1) D and (3+1) D non-linear Schrödinger equations by Elzaki. To solve these equations , the Elzaki transform is applied jointly to the Homotopy analysis method (HAM). This has proved efficient in tackling fractional calculus and nonlinear dynamics since correct solutions are offered and they converge at a faster rate. The accuracy of the proposed technique has been corroborated by analyzing various examples for which the latter were used for solving high-dimensional non-linear Schrodinger equation, which indicates that the technique is quite resilient as well as efficient; thus, making it an effective tool in theoretical physics and other applied sciences.

Keywords: Elzaki Transform, Homotopy Analysis Method, (2+1)-D- and (3+1)-D nonlinear fractional Schrodinger equations.

1. Introduction

This research paper is devoted to finding the semi-analytical solutions of (2+1)-D and (3+1)-D nonlinear fractional Schrodinger equations of the form:

$$iw_t^{\alpha}(\Omega) + a\Delta^2 w(\Omega) + \alpha(\Omega)w(\Omega) - \beta|w|^2 w(\Omega) = 0, \tag{1}$$

with initial condition $w(\Omega, 0) = w_0(\Omega)$ and $i^2 = -1$. Here, Ω is either (x, y) or (x, y, z), a, β are constants and α is a function of variables x, y and z.
Elzaki integral transform has been introduced in [1] for solving differential equations. In [2], various applications of Elzaki transform had been used for solving several mathematical models in the PDE (Partial Differential Equations) form. In [3], the authors have presented a comparison study between Laplace and Elzaki transforms. For solving various differential equations, a new transform called Sumudu transform-based technique had been utilized in [4]. A brief discussion about integral transform for solving differential equations had been represented in [5]. Homotopy analysis approaches had been applied for solving generalized Benjamin-Bona-Mahony equation and fifth-order KdV eqns in [6-7]. Homotopy analysis addressed nonlinear issues in science and engineering [8]. A comparison study has been presented in [9] for solving various differential equations. For this purpose, Homotopy analysis method and Homotoy perturbation method have been used. In [10], fractional Kdv-Burgers-Kuramoto eqn had been solved with the help of Homotopy analysis approach. Homotopy analysis [11] finds semi-analytical solutions for nonlinear fractional differential equations. Linear along with nonlinear fractional diffusion- wave eqns had been solved by using Homotopy analysis approach in [12]. In [13], homotopy analysis solves linear and nonlinear Schrodinger equations. To

solve heat radiation equations, Homotopy analysis was devised [14]. 2D Schrodinger eqns were solved utilizing a compact boundary value approach [18]. In [19], decomposition solves cubic Schrodinger equations.
This research paper is constituted as follows: Section 2 consists of the basic definitions of fractional calculus and the basic properties of the Elzaki transform. "Homotopy analysis approach has been discussed in Section 3. The suggested scheme on the basis of the combination of Homotopy analysis as well as the Elzaki transform approach for solving (2+1)-D and (3+1)-D nonlinear fractional Schrodinger equations in Section 4. Test experiments have been performed to solve nonlinear (2+1)-D and (3+1)-D fractional" Schrodinger equations in Section 5. The conclusion has been discussed in Section 6.

2. Basic Of Fractional Calculus And Elzaki Transform

This section covers fractional calculus basics.

Definition 2.1. A real function $h(t) \in C_\mu,\ t > 0,\ \mu \in \mathcal{R}$ if $\exists\ q \in \mathcal{R}; (q > \mu)$, s.t $h(t) = t^q m(t)$, where $m(t) \in C[0, \infty)$ & $h(t) \in C_\mu^n$ if $h^{(n)} \in C_\mu, n \in N$.

Definition 2.2. The Caputo fractional derivative of $h(\tau)$ is written as:

$$\frac{\partial^\alpha}{\partial\tau^\alpha} h(\tau) = J^{(n-\alpha)} \frac{\partial^n}{\partial\tau^n} h(\tau) = \frac{1}{\Gamma(n-\alpha)} \int_0^\tau (\tau - \Omega)^{n-\alpha-1} h^n(\Omega) d\Omega,$$

where $h \in C_{-1}^n, n-1 < \alpha \leq n, n \in \mathbb{N}, \tau > 0$. Here, $\frac{\partial^\alpha}{\partial\tau^\alpha}$ is Caputo derivative operator & Γ as Gamma function.

Definition 2.3. The function $g_1(t)$ Elzaki transform has been expressed as:

$$E\{g_1(t)\} = v \int_0^\infty g_1(t). e^{-\frac{t}{v}} dt, \qquad t > 0$$

Definition 2.4. For 2 parameters a & b, the Mittag-Leffler function is defined as:

$$E_{a,b}(\tau) = \sum_{n=0}^{\infty} \frac{\tau^n}{\Gamma(an+b)}, \qquad a, b > 0$$

SOME BASIC PROPERTIES

- The Caputo fractional derivative Elzaki transform is:

$$E\left\{\frac{\partial^\alpha}{\partial\tau^\alpha} h(\tau)\right\} = \frac{E\{h(\tau)\}}{v^\alpha} - \sum_{k=0}^{n-1} v^{k-\alpha+2} h^k(0), \quad n-1 < k \leq n$$

- Below are the Elzaki transformations of certain partial derivatives:

a) $E\left[\frac{\partial}{\partial t} f(x,t)\right] = \frac{E[f(x,t)]}{v} - v. f(x,0),$

b) $E\left[\frac{\partial^2}{\partial t^2} f(x,t)\right] = \frac{1}{v^2} E[f(x,t)] - f(x,0) - v.\frac{\partial f}{\partial t}(x,0),$

c) $E\left[\frac{\partial}{\partial x} f(x,t)\right] = \frac{d}{dx} E[f(x,t)],$

d) $E\left[\frac{\partial^2}{\partial x^2} f(x,t)\right] = \frac{d^2}{dx^2} E[f(x,t)].$

- The Elzaki transform of certain functions is provided in the list:

$$E(1) = v^2, \qquad E(t) = v^3, \qquad E(t^n) = n!\, v^{n+2},$$

$$E(e^{at}) = \frac{v^2}{1-av}, \quad E(\sin at) = \frac{av^3}{1+a^2v^2}$$

3. Homotopy Analysis Method [15-17]

Take into account the subsequent nonlinear differential equation

$$N[w(\Omega,t)] = 0 \tag{2}$$

Where $w(\Omega,t)$ as an unknown function, N as a nonlinear operator, and Ω may be $\{x,y\}$ or $\{x,y,z\}$. The variables $x,y,z,$ and t as the temporal and spatial independent variables, correspondingly. Utilizing the classical Homotopy method "(invented by Liao)

$$(1-p)L[\,\varphi(\Omega,t;p) - w_0(\Omega,t)] = phN[\varphi(\Omega,t;p)] \tag{3}$$

Where h is a nonzero auxiliary parameter, $p \in [0,1]$ is an embedding parameter, L is an auxiliary linear operator, $\varphi(\,\Omega,t;p)$ as an unknown function and $w_0(\Omega,t)$ is as $w(\Omega,t)$ initial guess . If $p = 0$ & $p = 1$, it holds

$$\varphi(\,\Omega,t;0) = w_0(\Omega,t),$$

and

$$\varphi(\,\Omega,t;1) = w(\Omega,t)$$

Therefore as p rises from 0-1, solution $\varphi(\,\Omega,t;p)$ which has been differs from the initial guess $w_0(\Omega,t)$ to solution $w(\Omega,t)$. Expanding $\varphi(\,\Omega,t;p)$ n Taylor series regarding p , then we have

$$\varphi(\,\Omega,t;p) = w_0(\Omega,t) + \sum_{m=1}^{\infty} w_m\,(\Omega,t)p^m \tag{4}$$

Where,

$$w_m(\Omega,t) = \frac{1}{m!}\,\frac{\partial^m \varphi(\Omega,t;p)}{\partial p^m}\bigg|_{p=0}$$

If the auxiliary linear operator, auxiliary parameter h, initial guess, and auxiliary function which have been appropriately selected, then the series (4) converges at $p = 1$ and we get

$$w(\Omega,t) = w_0(\Omega,t) + \sum_{m=1}^{\infty} w_m\,(\Omega,t), \tag{5}$$

This should be a valid solution to the original nonlinear eqn. The governing eqn could be derived from the 0-order deformation eqn (3) based on definition (5).

Define the vector

$$\overrightarrow{w_n} = \{w_0(\Omega,t), w_1(\Omega,t), w_2(\Omega,t) \ldots \ldots .. w_n(\Omega,t)\}$$

Differentiating the zero- order deformation eqn (3), m −times regarding embedding parameter p. After that putting $p = 0$ and then dividing it with $m!$, then the mth-order deformation eqn is:

$$L\,[w_m(\Omega,t) - \chi_m\, w_{m-1}\,(\Omega,t)] = h\, R_m[w_{m-1}\,(\Omega,t)]$$

where

$$R_m(\overrightarrow{w_{m-1}}) = \frac{1}{m-1!}\,\frac{\partial^{m-1} N[\varphi(\Omega,t;p)}{\partial p^{m-1}}\quad\Big|_{p=0}$$

and

$$\chi_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 1 \end{cases}.$$

4. Elzaki Transform Homotopy Analysis Method

Rewrite" Equation (1) as:

$$w_t^{\alpha}(\Omega) = i\{a\Delta^2 w(\Omega) + \alpha(\Omega)w(\Omega) - \beta w^2\overline{w}\}.$$

Taking Elzaki transform both sides, we obtain

$$E\{w_t^{\alpha}(\Omega)\} = iE\{a\Delta^2 w(\Omega) + \alpha(\Omega)w(\Omega) - \beta w^2\overline{w}\}.$$

Using applications of Elzaki transform as well as an initial condition, we obtain

$$E\{w(\Omega,t)\} = v^2 w_0(\Omega) + v^{\alpha} iE\{a\Delta^2 w(\Omega) + \psi(\Omega)w(\Omega) - \beta w^2\overline{w}\}.$$

Taking the nonlinear part as:

$$R[\varphi(\Omega,t;p)] = E(\varphi) - v^2 w_0(\Omega) - v^{\alpha} iE\{a\Delta^2\varphi(\Omega) + \psi(\Omega)\varphi(\Omega) - \beta\varphi^2\overline{\varphi}\}.$$

We formulate the zero-order deformation eqn under the given assumption. $H(x,y,z,t) = 1$, we have

$$(1-p)E\{\varphi(\Omega,t) - w_0(\Omega,t)\} = phR[\varphi(\Omega,t;p)].$$

When $p = 0$ & $p = 1$, we get,

$$\begin{cases} \varphi(\Omega,t;0) = w_0(\Omega,0) \\ \varphi(\Omega,t;1) = w(\Omega,t). \end{cases}$$

Hence, we obtain the eqn of deformation of order m.

$$E\{w_m(\Omega,t) - \chi_m w_{m-1}(\Omega,t)\} = hR_m(\overrightarrow{w_{m-1}}(\Omega,t)).$$

Inverse Elzaki transforms both sides, we obtain,

$$w_m(\Omega,t) - \chi_m w_{m-1}(\Omega,t) = E^{-1}\left\{hR_m\left(\overrightarrow{w_{m-1}}(\Omega,t)\right)\right\}.$$

From the above Eqon, we get

$$w_1(\Omega,t) = -E^{-1}\{R_1(\overrightarrow{w_0}(\Omega,t))\},$$
$$w_2(\Omega,t) = w_1(\Omega,t) - E^{-1}\{R_2(\overrightarrow{w_1}(\Omega,t))\},$$
$$w_3(\Omega,t) = w_2(\Omega,t) - E^{-1}\{R_3(\overrightarrow{w_2}(\Omega,t))\},$$
$$\vdots$$

Therefore, the solution is:

$$w(\Omega,t) = w_0 + w_1 + w_2 + \cdots$$

5. Test examples:

In "this Section, we will perform some test examples to find semi-analytical solutions of nonlinear fractional (2+1)-D and (3+1)-D nonlinear fractional" Schrodinger equations.

Example 1: Consider the (3+1)-D fractional nonlinear Schrodinger eqn of form

$$iw_t^\alpha + w_{xx} + w_{yy} + w_{zz} + 4|w|^2 w = 0, \tag{6}$$

with initial condition $w(x,y,z,0) = e^{i(x+y+z)}$. The problem $(\alpha = 1) the$ exact solution is:

$$w(x,y,z,t) = e^{i(x+y+z+t)}$$

Rewrite the given problem as:

$$iw_t^\alpha = -(w_{xx} + w_{yy} + w_{zz} + 4w^2\overline{w}),$$

It implies

$$w_t^\alpha = i(w_{xx} + w_{yy} + w_{zz} + 4w^2\overline{w}) \tag{7}$$

Taking Elzaki transform to both the sides of Eqn (7), we get,

$$E[w_t^\alpha] = E[i(w_{xx} + w_{yy} + w_{zz} + 4w^2\overline{w})]$$

This implies

$$E[w(x,y,z,t)] = \sum_{i=0}^{n-1} v^{i+2}\, w^{(i)}(x,y,z,0) + v^\alpha iE[w_{xx} + w_{yy} + w_{zz} + 4w^2\overline{w}]$$

After applying initial conditions, we get

$$E[w(x,y,z,t)] = v^2.e^{i(x+y+z)} + v^\alpha iE[w_{xx} + w_{yy} + w_{zz} + 4w^2\overline{w}]$$

Or

$$E[w(x,y,z,t)] - v^2.e^{i(x+y+z)} - v^\alpha iE[w_{xx} + w_{yy} + w_{zz} + 4w^2\overline{w}] = 0$$

The nonlinear component is defined as:

$$R[\varphi(x,y,z,t;p)] = E[\varphi] - v^2.e^{i(x+y+z)} - v^\alpha iE[\varphi_{xx} + \varphi_{yy} + \varphi_{zz} + 4\varphi^2\overline{\varphi}] \tag{8}$$

We formulate the zero-order deformation eqn under the given assumption $H(x,y,z,t) = 1$, we "have

$$(1-p)E\{\varphi(x,y,z,t)-w_0(x,y,z,t)\}=phR[\varphi(x,y,z,t;p)]$$

When $p=0$ & $p=1$, we have

$$\begin{cases}\varphi(x,y,z,t;0)=w_0(x,y,z,0)\\ \varphi(x,y,z,t;1)=w(x,y,z,t)\end{cases}$$

So, the *m*th-order deformation eqn

$$E\{w_m(x,y,z,t)-\chi_m w_{m-1}(x,y,z,t)\}=hR_m(\overrightarrow{w_{m-1}}(x,y,z,t)) \quad (9)$$

By inverse Elzaki transform both sides, we obtain

$$w_m(x,y,z,t)-\chi_m w_{m-1}(x,y,z,t)=E^{-1}\left\{hR_m\left(\overrightarrow{w_{m-1}}(x,y,z,t)\right)\right\} \quad (10)$$

From Equation (10) (Taking $h=-1$), we obtain

$$w_1(x,y,z,t)=-E^{-1}\left\{R_1\left(\overrightarrow{w_0}(x,y,z,t)\right)\right\},$$

$$w_2(x,y,z,t)=w_1(x,y,z,t)-E^{-1}\left\{R_2\left(\overrightarrow{w_1}(x,y,z,t)\right)\right\},$$

$$w_3(x,y,z,t)=w_2(x,y,z,t)-E^{-1}\left\{R_3\left(\overrightarrow{w_2}(x,y,z,t)\right)\right\},$$

$$\vdots$$

Where,”

$$R_1\left(\overrightarrow{w_0}(x,y,z,t)\right)=E[w_0]-v^2.e^{i(x+y+z)}-v^\alpha iE\left[(w_0)_{xx}+(w_0)_{yy}+(w_0)_{zz}+4{w_0}^2\overline{w_0}\right],$$

$$R_2\left(\overrightarrow{w_1}(x,y,z,t)\right)=E[w_1]-v^\alpha iE\left[(w_1)_{xx}+(w_1)_{yy}+(w_1)_{zz}+4{w_0}^2\overline{w_1}+8w_0\overline{w_0}w_1\right],$$

$$R_3\left(\overrightarrow{w_2}(x,y,z,t)\right)=E[w_2]$$
$$-v^\alpha iE\left[(w_2)_{xx}+(w_2)_{yy}+(w_2)_{zz}+4{w_0}^2\overline{w_2}+8w_0\overline{w_1}w_1+8w_0\overline{w_0}w_2+4\overline{w_0}{w_1}^2\right],$$

$$\vdots$$

After simplifications, we obtain

$$R_1\left(\overrightarrow{w_0}(x,y,z,t)\right)=-i.\,v^{\alpha+2}e^{i(x+y+z)},$$

$$R_2\left(\overrightarrow{w_1}(x,y,z,t)\right)=e^{i(x+y+z)}(iv^{\alpha+2}+v^{\alpha+3}),$$

$$R_3\left(\overrightarrow{w_2}(x,y,z,t)\right)=e^{i(x+y+z)}\{-v^{\alpha+3}-iv^{\alpha+4}\},$$

$$\vdots$$

Therefore,

$$w_1(x,y,z,t)=i\frac{t^\alpha}{(\alpha)!}e^{i(x+y+z)},$$

$$w_2(x,y,z,t)=i^2\frac{t^{\alpha+1}}{(\alpha+1)!}e^{i(x+y+z)},$$

$$w_3(x,y,z,t) = i^3 \frac{t^{\alpha+2}}{(\alpha+2)!} e^{i(x+y+z)},$$

$$\vdots$$

For $\alpha = 1$, the solution is:

$$w(x,y,z,t) = w_0 + w_1 + w_2 + w_3 + \cdots$$

Or

$$w(x,y,z,t) = e^{i(x+y+z)} \left\{ 1 + (it) + \frac{(it)^2}{2!} + \cdots \right\} = e^{i(x+y+z+t)}$$

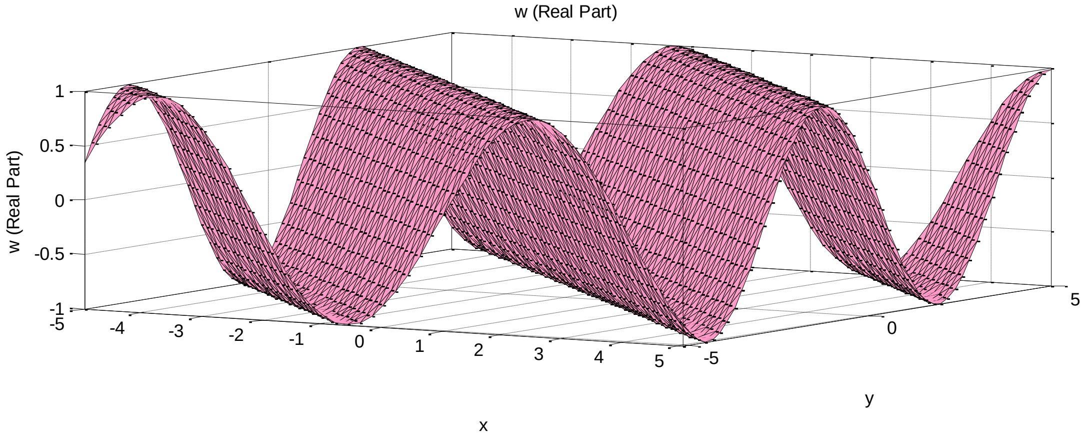

Figure 1: Physical behavior of solutions of real part for $z = 2$ and $t = 0.5$

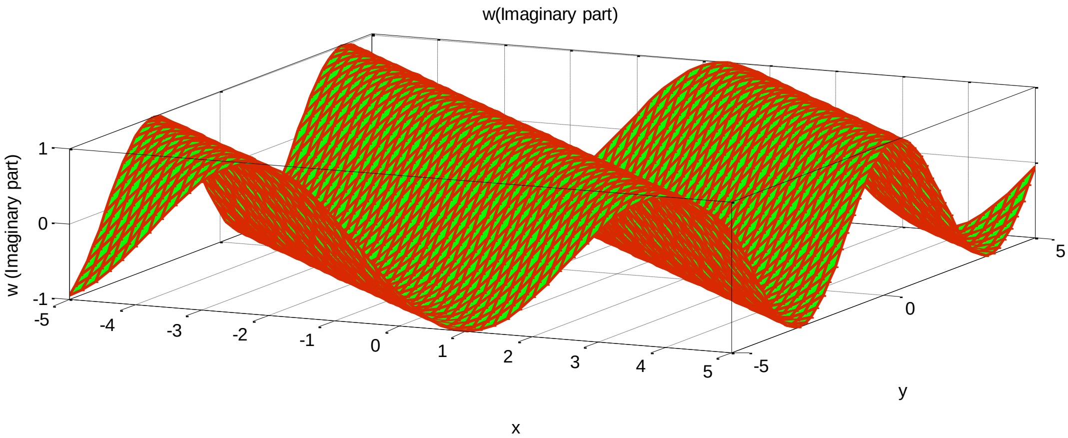

Figure 2: Physical behavior of solutions of imaginary part for $z = 2$ and $t = 0.5$

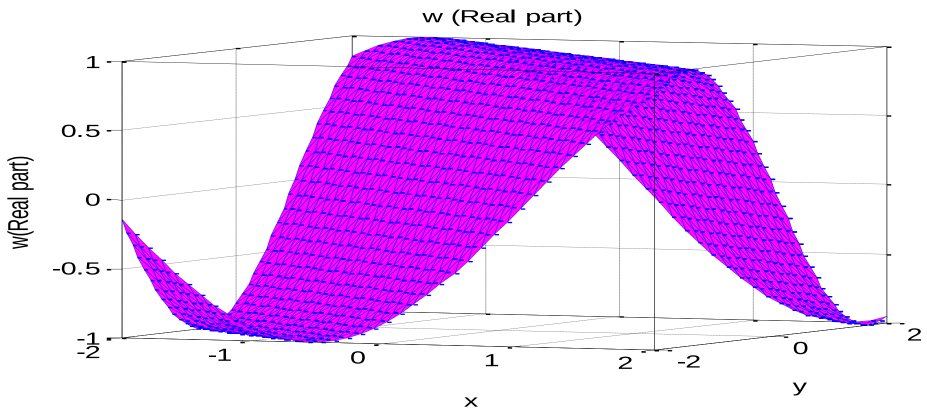

Figure 3: Physical behavior of solutions of real part for $z = 10$ and $t = 2$

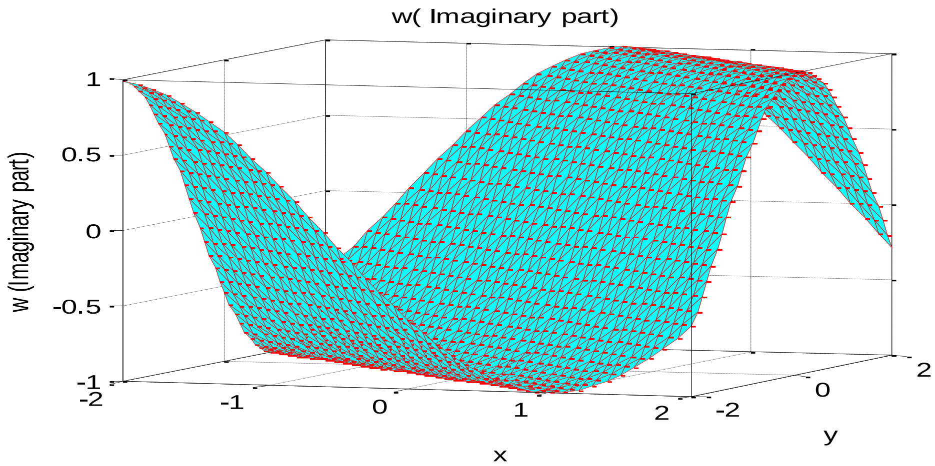

Figure 4: Physical behavior of solutions of imaginary part for $z = 10$ and $t = 2$

Figures 1 & 2 show the real & the imaginary part solutions' physical behavior of Example 1 at $z = 2$, $t = 0.5$ respectively. Figures 3 & 4 show the real & the imaginary part solution's physical behavior of Example 1 at $z = 10,\ t = 2$ respectively.

Example 2: Consider the (2+1)-D fractional nonlinear Schrodinger eqn of the form

$$iw_t^\alpha = -\frac{1}{4}w_{xx} - \frac{1}{4}w_{yy} - w\sin^2 x \sin^2 y + |w|^2 w, \tag{11}$$

with initial "condition $w(x, y, z, 0) = \sin x \sin y$. The exact solution to the problem $(\alpha = 1)$ is:

$$w(x, y, t) = e^{-it/2} \sin x \sin y$$

Rewrite the given problem as:

$$w_t^\alpha = i\left(\frac{1}{4}w_{xx} + \frac{1}{4}w_{yy} + w\sin^2 x \sin^2 y - w^2\overline{w}\right) \tag{12}$$

Taking Elzaki transform to both sides of Eqn (12), we obtain

$$E[w_t^\alpha] = E\left[i\left(\frac{1}{4}w_{xx} + \frac{1}{4}w_{yy} + w\sin^2 x \sin^2 y - w^2\overline{w}\right)\right]$$

This implies

$$E[w(x,y,t)] = \sum_{i=0}^{n-1} v^{i+2}\, w^{(i)}(x,y,0) + v^{\alpha} iE\left[\frac{1}{4}w_{xx} + \frac{1}{4}w_{yy} + w\sin^2 x\,\sin^2 y - w^2\overline{w}\right]$$

After applying initial conditions, we obtain

$$E[w(x,y,t)] = v^2.\sin x\sin y + v^{\alpha} iE\left[\frac{1}{4}w_{xx} + \frac{1}{4}w_{yy} + w\sin^2 x\,\sin^2 y - w^2\overline{w}\right]$$

Or

$$E[w(x,y,t)] - v^2.\sin x\sin y - v^{\alpha} iE\left[\frac{1}{4}w_{xx} + \frac{1}{4}w_{yy} + w\sin^2 x\,\sin^2 y - w^2\overline{w}\right] = 0$$

The nonlinear component is:

$$R[\varphi(x,y,t;p)] = E[\varphi] - v^2.\sin x\sin y - v^{\alpha} iE\left[\frac{1}{4}\varphi_{xx} + \frac{1}{4}\varphi_{yy} + \varphi\sin^2 x\,\sin^2 y - \varphi^2\overline{\varphi}\right] \quad (13)$$

We build the zero-order deformation eqn with the assumption $H(x,y,t) = 1$,

$$(1-p)E\{\varphi(x,y,t) - w_0(x,y,t)\} = phR[\varphi(x,y,t;p)]$$

When $p = 0$ & $p = 1$, we get

$$\begin{cases}\varphi(x,y,t;0) = w_0(x,y,0)\\ \varphi(x,y,t;1) = w(x,y,t)\end{cases}$$

Therefore, the mth-order deformation eqn

$$E\{w_m(x,y,t) - \chi_m w_{m-1}(x,y,t)\} = hR_m(\overrightarrow{w_{m-1}}(x,y,t)) \quad (14)$$

Inverse Elzaki transforms both sides and gives

$$w_m(x,y,t) - \chi_m w_{m-1}(x,y,t) = E^{-1}\left\{hR_m\left(\overrightarrow{w_{m-1}}(x,y,t)\right)\right\} \quad (15)$$

From Equation (15) (Taking $h = -1$), we obtain

$$w_1(x,y,t) = -E^{-1}\left\{R_1\left(\overrightarrow{w_0}(x,y,t)\right)\right\},$$

$$w_2(x,y,t) = w_1(x,y,t) - E^{-1}\left\{R_2\left(\overrightarrow{w_1}(x,y,t)\right)\right\},$$

$$w_3(x,y,t) = w_2(x,y,t) - E^{-1}\left\{R_3\left(\overrightarrow{w_2}(x,y,t)\right)\right\},$$

$$\vdots$$

where

$$R_1\left(\overrightarrow{w_0}(x,y,t)\right) = E[w_0] - v^2.\sin x\sin y$$

$$-v^{\alpha} iE\left[\frac{1}{4}(w_0)_{xx} + \frac{1}{4}(w_0)_{yy} + w_0\sin^2 x\,\sin^2 y - {w_0}^2\overline{w_0}\right],$$

$$R_2\left(\overrightarrow{w_1}(x,y,t)\right) = E[w_1] - v^{\alpha} iE\left[\frac{1}{4}(w_1)_{xx} + \frac{1}{4}(w_1)_{yy} + w_1\sin^2 x\,\sin^2 y - {w_0}^2\overline{w_1} - 2w_0\overline{w_0}w_1\right],$$

$$R_3\left(\overrightarrow{w_2}(x,y,t)\right) = E[w_2]$$

$$-v^{\alpha} iE\left[\frac{1}{4}(w_2)_{xx}+\frac{1}{4}(w_2)_{yy}+w_2\sin^2 x\,\sin^2 y-{w_0}^2\overline{w_2}-2w_0\overline{w_1}w_1-2w_0\overline{w_0}w_2-\overline{w_0}{w_1}^2\right],$$

$$\vdots$$

After simplifications, we obtain

$$R_1\left(\overrightarrow{w_0}(x,y,t)\right)=i.\,v^{\alpha+2}\sin x\sin y,$$

$$R_2\left(\overrightarrow{w_1}(x,y,t)\right)=\sin x\sin y\left(-\frac{i}{2}v^{\alpha+2}+\frac{1}{4}v^{\alpha+3}\right),$$

$$R_3\left(\overrightarrow{w_2}(x,y,t)\right)=\sin x\sin y\left\{-\frac{1}{4}v^{\alpha+3}-\frac{i}{8}v^{\alpha+4}\right\},$$

$$\vdots$$

Therefore,

$$w_1(x,y,z,t)=-i\frac{\left({}^{t}\!/_{2}\right)^{\alpha}}{\alpha!}\sin x\sin y,$$

$$w_2(x,y,z,t)=i^2\frac{\left({}^{t}\!/_{2}\right)^{\alpha+1}}{(\alpha+1)!}\sin x\sin y,$$

$$w_3(x,y,z,t)=-i^3\frac{\left({}^{t}\!/_{2}\right)^{\alpha+2}}{(\alpha+2)!}\sin x\sin y,$$

$$\vdots$$

For $\alpha=1$, the solution” is:

$$w(x,y,z,t)=w_0+w_1+w_2+w_3+\cdots$$

Or

$$w(x,y,z,t)=\sin x\sin y\left\{1+\left(\frac{it}{2}\right)+\frac{\left(\frac{it}{2}\right)^2}{2!}+\cdots\right\}=e^{-it/2}\sin x\sin y$$

which is totally equal to the exact solution.

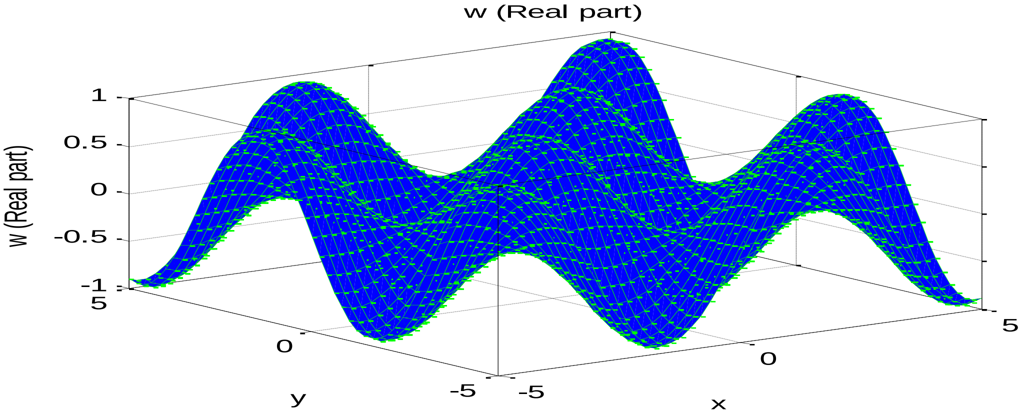

Figure 5: Physical behavior of real part solution at $t = 0.5$

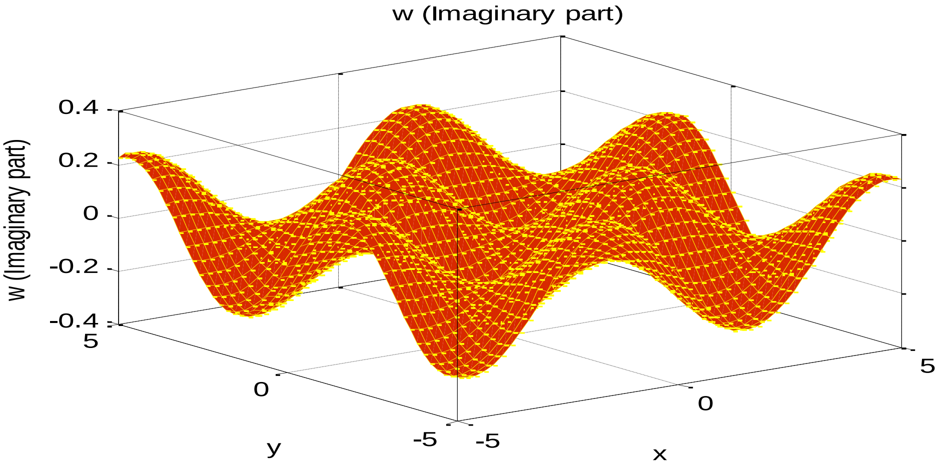

Figure 6: Physical behavior of imaginary part solution at $t = 0.5$

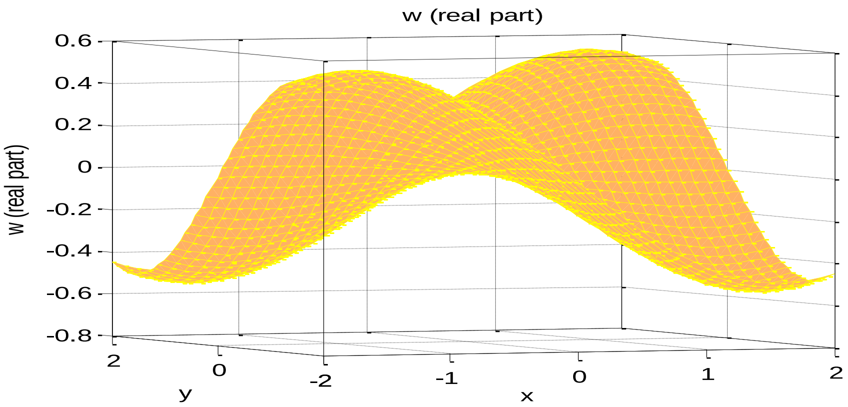

Figure 7: Physical behavior of the solution of real part at $t = 2$

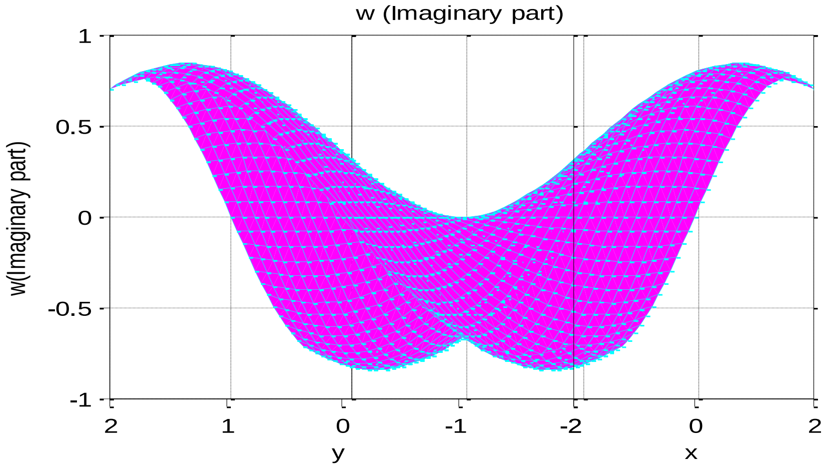

Figure 8: Physical behavior of the solution of imaginary part at $t = 2$

Figures 5 & 6 show the real & imaginary part solutions physical behavior of Example 2 at $t = 0.5$ respectively. Figures 7 & 8 show the real & imaginary part solutions physical behavior of Example 2 at $t = 2$ respectively.

6. Conclusion

The numerical data suggests that the Elzaki transform-based Homotopy analysis technique provides accurate solutions for solving (2+1)-D and (3+1)-D nonlinear fractional Schrodinger eqns. In the future, this approach will be valid for various applications of sciences and engineering.

References

[1] Elzaki, T.M. The new integral transform "Elzaki Transform" Global Journal of Pure and Applied Mathematics. 2011, 1: 57-64.

[2] Elzaki, T.M. and Elzaki, S. M. Application of new transform "Elzaki Transform" to partial differential equations, Global Journal of Pure and Applied Mathematics. 2011, 1: 65-70.

[3] Elzaki, T.M. and Elzaki, S. M. On the Connections between Laplace and Elzaki transforms, Advances in Theoretical and Applied Mathematics. 2011, 6(1): 1-11.

[4] Eltayeb, H. and Kilicman, A. A note on the Sumudu transforms and differential equations, Applied Mathematical Sciences. 2010, 4(22): 1089-1098

[5] Kilicman A. and ELtayeb, H. A note on Integral transform and partial differential equation, Applied Mathematical Sciences. 2010, 4(3):109-118.

[6] Abbasbandy, S. Homotopy analysis method for generalized Benjamin-Bona-Mahony equation, Z. Angew. Math. Phys. 2008, 59: 51-62.

[7] Abbasbandy, S. and Zakaria, F.S. Soliton solutions for the fifth-order K-dV equation with the homotopy analysis method, Nonlinear Dyn. 2008, 51: 83-87.

[8] Liao, S.J. On the homotopy analysis method for nonlinear problems, Appl. Math. Comput. 2004, 147: 499–513.

[9] Liao, S.J. Comparison between the homotopy analysis method and homotopy perturbation method, Appl. Math. Comput. 2005, 169 :1186–1194.

[10] Song, L. and Zhang, H. Application of Homotopy analysis method to fractional Kdv-Burgers-Kuramoto equation, Physics Letters A. 2007, 367: 88-94.

[11] Ganjiani, M. Solution of nonlinear fractional differential equation using Homotopy Analysis method, Applied Mathematical Modeling. 2010, 34: 1634-1641.

[12] Jafari, H. and Seifi, S. Homotopy analysis method for solving linear and nonlinear fractional diffusion-wave equation. Comun. Nonlin. Sci. Num. Sim. 2009, 14(5): 2006-2012.

[13] Alomari, A.K., Noorani, M.S.M. and Nazar, R. Explicit series solutions of some linear and nonlinear Schrodinger equations via the Homotopy analysis method, Communications in Nonlinear Science and Numerical Simulation. 2009, 14(4): 1196–1207.

[14] Biazar, J. and Ghanbari, B. HAM solution of some initial value problems arising in heat radiation equations, Journal of King Saud University-Science. 2012, 24(2): 161–165.

[15] Liao, S.J. Beyond Perturbation: Introduction to the Homotopy Analysis Method, Chapman & Hall, CRC Press, Boca Raton, Fla, USA, 2003.

[16] Liao, S.J. Notes on the homotopy analysis method: some definitions and theorems, Communications in Nonlinear Science and Numerical Simulation. 2009, 14(4): 983–997.

[17] Liao, S.J. A new branch of solutions of boundary-layer flows over an impermeable stretched plate, International Journal of Heat and Mass Transfer. 2005, 48(12): 2529–2539.

[18] Mohebbi, A. and Dehghan, M. The use of compact boundary value method for the solution of two-dimensional Schrodinger equation, Journal of Computational and Applied Mathematics, 2009, 225(1): 124–134.

[19] Khuri, S.A. A new approach to the cubic Schrodinger equation: an application of the decomposition technique, Applied Mathematics and Computation. 1998, 97: 251–254.