

A Fuzzy Log-Normal Distribution Model for the Rainfall level of Namakkal District

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Abstract:

Models of mathematics can accurately and precisely obtain solutions to many complex unsolvable problems in all fields. In this paper using a few mathematical models we can take the rainfall levels in Namakkal district from 2014 to 2018 and find the highest rainfall year by the fuzzy mean and the fuzzy variance in the Log-Normal distribution.

Keywords: Log - Normal distribution, Mean, Variance, Mathematical Modelling, Fuzzy Set

2010 AMS subject classification: 01-06, 62E10, 62E86, 97M10.

1. Introduction

A model may help to explain a system and to study the effect of different components, and to make predictions about behavior [1]. The Namakkal area is referred to as 'Thiruvaraikkal' in the inscriptions on the north-western and southern walls of the Paladaina temple located on the hill. Namakkal is also known as the 'City of Chickens' and the 'Egg City' as the eggs sent to most parts of the country is produced in Namakkal. Namakkal district was separated from Salem district and emerged as a separate district from 01.01.1997. In Namakkal district, there are eight circles namely Namakkal, Rasipuram, Kollimalai, Chenthamangalam, Paramathi Vellore, Tiruchengode, Kumarapalayam and Mohanur. Namakkal district is bounded on the north by Salem district, on the south by Karur district, on the east by Trichy and Salem districts and on the west by Erode district. According to the 2011 census, the population of Namakkal district is 17, 26,601. This includes 8, 69,280 males and 8, 57,321 females [6]. Some authors [2, 3, 4] instructed with some distributions in their books.

The geographical area of Namakkal district is 3368.21 sq. Km. Is. The district is located between 11.00 ' and 11.360 ' north latitude and 77.28' and 78.300' east longitude. The main waterfalls of the district are Kaviriyaru, Ayyaru, Karippottan River and Thirumanimuttaru. Of these, the Cauvery River flows in a south-southwesterly direction, bordering the district. The famous Cauvery River

flows through the Paramathi Vellore Circle. Thus irrigated lands are located in Paramathi and Mohanur Unions.

Agriculture continues to be an important part of the economy of the district. 70% of the people are engaged in agriculture and its related occupations as their livelihood. The total area of the district is 3363.35 sq km. The total crop area is 3.367 lakh hectares. Sustainable agricultural production, increasing productivity in sustainable agriculture, meeting the demand for food in line with the growing population, meeting the demand for raw materials for agro-based industries and providing employment to the rural population have always been key principles and principles. The total geographical area of Namakkal district is 3, 36,719 hectares. Of this, the area under net crop cultivation is 1, 41,537 hectares. Of these, 60,939 hectares are irrigated. The remaining 80,598 hectares are irrigated area. The Pallipalayam area is irrigated on an area of 4585 hectares through the Mato East Coast Canal. The average annual rainfall is 716.54 mm [5]. Due to the cultivation of various crops in Namakkal district, agriculture is the main occupation of the people of this district. The district receives rainfall in all seasons. Most rainfall, however, is available through the northeast monsoon. Syamala et. al [7, 8, 9, 10] discussed about some fuzzy concepts.

In this paper, we can find out about calculating the maximum rainfall year in Namakkal district by using fuzzy mean and the fuzzy variance in the Log-Normal distribution using mathematical models with rainfall patterns for 2014-2018.

2. Fuzzy Log-Normal Distribution

One way to quality a random variable follows a log-normal distribution is to say that its logarithm is normally distributed [4]. The p.d.f of log-normal distribution is given by

$$f(n; \mu, \sigma) = \frac{1}{n\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln n - \mu}{\sigma}\right)^2}$$

where the variable $n > 0$ and the parameter μ and $\sigma > 0$ all are real numbers.

It is sometimes denoted $\Lambda(\mu, \sigma^2)$ in the same spirit as we often a normally distributed variable by $N(\mu, \sigma^2)$ [2]. The c.d.f of the log-normal distribution is given by

$$F(n) = \frac{1}{\sigma\sqrt{2\pi}} \int_0^n \frac{1}{t} e^{-\frac{1}{2}\left(\frac{\ln t - \mu}{\sigma}\right)^2} dt$$

The mean of the log-normal distribution is given by $\mu_k^1 = E(n^k) = e^{k\mu + \frac{k^2\sigma^2}{2}}$

The α cut of fuzzy mean is $\bar{E}(n) = \{\bar{E}_l(n), \bar{E}_u(n)\}$

where $\bar{E}_l(n) = \min \left\{ e^{\bar{\mu} + \frac{\bar{\sigma}^2}{2}} \right\}$ and $\bar{E}_u(n) = \max \left\{ e^{\bar{\mu} + \frac{\bar{\sigma}^2}{2}} \right\}$.

The α cut of fuzzy variance is $\bar{V}(n) = \{\bar{V}_l(n), \bar{V}_u(n)\}$

where $\bar{V}_l(n) = \min \left\{ e^{2\bar{\mu} + \bar{\sigma}^2} (e^{\bar{\sigma}^2} - 1) \right\}$ and $\bar{V}_u(n) = \max \left\{ e^{2\bar{\mu} + \bar{\sigma}^2} (e^{\bar{\sigma}^2} - 1) \right\}$.

3. Application

Let us take the Namakkal district rainfall levels from 2014 to 2018 [3].

Table.3.A Rainfall levels for 2014

Months	Jan	Feb	Mar	April	May	June	July	Aug	Sep	Oct	Nov	Dec
Rainfall levels (mm)	0	0.6	0	21.4	96.2	35.7	34.8	67.3	81.9	195.5	44.8	11.3

Table.3.B Rainfall levels for 2015

Months	Jan	Feb	Mar	April	May	June	July	Aug	Sep	Oct	Nov	Dec
Rainfall levels (mm)	24.5	1.1	2.7	107.8	75.4	45.2	23.6	65.7	164.3	115	162.3	31.9

Table.3.C Rainfall levels for 2016

Months	Jan	Feb	Mar	April	May	June	July	Aug	Sep	Oct	Nov	Dec
Rainfall levels (mm)	0	0	0	5.3	85.6	35.4	125.2	49.4	31.3	24.2	7.4	25.9

Table.3.D Rainfall levels for 2017

Months	Jan	Feb	Mar	April	May	June	July	Aug	Sep	Oct	Nov	Dec
Rainfall levels (mm)	6.3	0	16.8	26.9	98	24	45.1	115.2	194.8	149.8	43.6	63.1

Table.3.E Rainfall levels for 2018

Months	Jan	Feb	Mar	April	May	June	July	Aug	Sep	Oct	Nov	Dec
Rainfall levels (mm)	0	26.8	14	13.6	155.8	33.2	41.5	61.1	120.9	106.5	63.7	12.4

4. Results

The double parameter of the log-normal distribution for Table.3.A is

$$\sigma = 0.57667, \mu = 4.1952.$$

Let the matching fuzzy triangular numbers are

$$\bar{\sigma} = [0.34547, 0.57667, 0.80787] \text{ and } \bar{\mu} = [3.7415, 4.1952, 4.6489]$$

and the matching α - cut are given by

$$\bar{\sigma} = [0.34547 + 0.2312\alpha, 0.80787 - 0.2312\alpha] \text{ and}$$

$$\bar{\mu} = [3.7415 + 0.4537\alpha, 4.6489 - 0.4537\alpha]$$

Table 4.A Fuzzy Mean and variance value for lower and upper alpha values

α	low σ	low μ	up σ	up μ	$E_l(x)$	$E_u(x)$	$V_l(x)$	$V_u(x)$
0	0.34547	3.7415	0.80787	4.6489	44.75373	144.7817	253.8945	19298.03
0.1	0.36859	3.78687	0.78475	4.60353	47.21914	135.8357	324.458	15705.73
0.2	0.39171	3.83224	0.76163	4.55816	49.847	127.5107	412.0517	12782.51
0.3	0.41483	3.87761	0.73851	4.51279	52.64925	119.7599	520.5086	10402.46
0.4	0.43795	3.92298	0.71539	4.46742	55.63876	112.5403	654.5131	8463.763
0.5	0.46107	3.96835	0.69227	4.42205	58.82946	105.8125	819.7913	6883.961
0.6	0.48419	4.01372	0.66915	4.37668	62.2364	99.54013	1023.346	5596.23
0.7	0.50731	4.05909	0.64603	4.33131	65.87584	93.68962	1273.746	4546.353
0.8	0.53043	4.10446	0.62291	4.28594	69.76539	88.23012	1581.479	3690.304
0.9	0.55355	4.14983	0.59979	4.24057	73.92409	83.13319	1959.391	2992.299
1	0.57667	4.1952	0.57667	4.1952	78.37258	78.37258	2423.228	2423.228

The double parameter of the log-normal distribution for Table.3.B is

$$\sigma = 0.65456, \mu = 4.3452.$$

Let the matching fuzzy triangular numbers are

$$\bar{\sigma} = [0.42336, 0.65456, 0.88576] \text{ and } \bar{\mu} = [3.8915, 4.3452, 4.7989]$$

and the matching α - cut are given by

$$\bar{\sigma} = [0.42336 + 0.2312\alpha, 0.88576 - 0.2312\alpha] \text{ and}$$

$$\bar{\mu} = [3.8915 + 0.4537\alpha, 4.7989 - 0.4537\alpha]$$

Table 4.B Fuzzy Mean and variance value for lower and upper alpha values

α	low σ	low μ	up σ	up μ	$E_l(x)$	$E_u(x)$	$V_l(x)$	$V_u(x)$
0	0.42336	3.8915	0.88576	4.7989	53.57684	179.6814	563.4755	38466.97
0.1	0.44648	3.93687	0.86264	4.75353	56.63019	168.2757	707.4666	31279.98
0.2	0.4696	3.98224	0.83952	4.70816	59.88955	157.6783	884.9611	25444.96
0.3	0.49272	4.02761	0.8164	4.66279	63.37038	147.8273	1103.46	20704.03
0.4	0.51584	4.07298	0.79328	4.61742	67.08936	138.6658	1372.141	16849.36
0.5	0.53896	4.11835	0.77016	4.57205	71.06457	130.1416	1702.243	13713.31
0.6	0.56208	4.16372	0.74704	4.52668	75.31558	122.2068	2107.535	11160.47
0.7	0.5852	4.20909	0.72392	4.48131	79.86355	114.8171	2604.894	9081.355
0.8	0.60832	4.25446	0.7008	4.43594	84.73143	107.9319	3215.026	7387.357
0.9	0.63144	4.29983	0.67768	4.39057	89.94409	101.5138	3963.351	6006.681
1	0.65456	4.3452	0.65456	4.3452	95.52848	95.52848	4881.101	4881.101

The double parameter of the log-normal distribution for Table.3.C is

$$\sigma = 1.0416, \mu = 3.9543.$$

Let the matching fuzzy triangular numbers are

$$\bar{\sigma} = [0.8104, 1.0416, 1.2728] \text{ and } \bar{\mu} = [3.5006, 3.9543, 4.4070]$$

and the matching α - cut are given by

$$\bar{\sigma} = [0.8104 + 0.2312\alpha, 1.2728 - 0.2312\alpha] \text{ and}$$

$$\bar{\mu} = [3.5006 + 0.4537\alpha, 4.4070 - 0.4537\alpha]$$

Table 4.C Fuzzy Mean and variance value for lower and upper alpha values

α	low σ	low μ	up σ	up μ	$E_l(x)$	$E_u(x)$	$V_l(x)$	$V_u(x)$
0	0.8104	3.5006	1.2728	4.408	46.01531	184.5665	1966.037	138071.1
0.1	0.83352	3.54597	1.24968	4.36263	49.0749	171.3109	2416.1	110549.4
0.2	0.85664	3.59134	1.22656	4.31726	52.36592	159.0923	2969.923	88628.67
0.3	0.87976	3.63671	1.20344	4.27189	55.90752	147.8242	3651.929	71144.36
0.4	0.90288	3.68208	1.18032	4.22652	59.72055	137.4277	4492.455	57179.18
0.5	0.926	3.72745	1.1572	4.18115	63.82775	127.8306	5529.222	46009.63
0.6	0.94912	3.77282	1.13408	4.13578	68.25389	118.9673	6809.197	37064.2
0.7	0.97224	3.81819	1.11096	4.09041	73.02598	110.7778	8390.929	29890.69
0.8	0.99536	3.86356	1.08784	4.04504	78.1735	103.2071	10347.5	24130.86
0.9	1.01848	3.90893	1.06472	3.99967	83.72861	96.20529	12770.24	19500.42
1	1.0416	3.9543	1.0416	3.9543	89.72642	89.72642	15773.46	15773.46

The double parameter of the log-normal distribution for Table.3.D is

$$\sigma = 0.70643, \mu = 4.3393.$$

Let the matching fuzzy triangular numbers are

$$\bar{\sigma} = [0.47523, 0.70643, 0.93763] \text{ and } \bar{\mu} = [3.8856, 4.3393, 4.7930]$$

and the matching α - cut are given by

$$\bar{\sigma} = [0.47523 + 0.2312\alpha, 0.93763 - 0.2312\alpha] \text{ and}$$

$$\bar{\mu} = [3.8856 + 0.4537\alpha, 4.7930 - 0.4537\alpha]$$

Table 4.D Fuzzy Mean and variance value for lower and upper alpha values

α	low σ	low μ	up σ	up μ	$E_l(x)$	$E_u(x)$	$V_l(x)$	$V_u(x)$
0	0.47523	3.8856	0.93763	4.793	54.5175	187.2744	753.0841	49410.82
0.1	0.49835	3.93097	0.91451	4.74763	57.69361	175.1765	938.3589	40134.62
0.2	0.52147	3.97634	0.89139	4.70226	61.08739	163.9478	1166.133	32617.59
0.3	0.54459	4.02171	0.86827	4.65689	64.7154	153.5209	1445.926	26520.79
0.4	0.56771	4.06708	0.84515	4.61152	68.59553	143.8339	1789.403	21571.84
0.5	0.59083	4.11245	0.82203	4.56615	72.74717	134.8303	2210.864	17551.54
0.6	0.61395	4.15782	0.79891	4.52078	77.19134	126.4578	2727.856	14283.33
0.7	0.63707	4.20319	0.77579	4.47541	81.9508	118.6686	3361.928	11624.8
0.8	0.66019	4.24856	0.75267	4.43004	87.05024	111.4188	4139.556	9460.951
0.9	0.68331	4.29393	0.72955	4.38467	92.51643	104.6678	5093.308	7698.846
1	0.70643	4.3393	0.70643	4.3393	98.37844	98.37844	6263.272	6263.272

The double parameter of the log-normal distribution for Table.3.E is

$$\sigma = 0.52939, \mu = 4.2867.$$

Let the matching fuzzy triangular numbers are

$$\bar{\sigma} = [0.29819, 0.52939, 0.76059] \text{ and } \bar{\mu} = [3.8330, 4.2867, 4.7404]$$

and the matching α - cut are given by

$$\bar{\sigma} = [0.29819 + 0.2312\alpha, 0.76059 - 0.2312\alpha] \text{ and}$$

$$\bar{\mu} = [3.8330 + 0.4537\alpha, 4.7404 - 0.4537\alpha]$$

Table 4.E Fuzzy Mean and variance value for lower and upper alpha values

α	low σ	low μ	up σ	up μ	$E_l(x)$	$E_u(x)$	$V_l(x)$	$V_u(x)$
0	0.29819	3.833	0.76059	4.7404	48.30131	152.8793	216.9477	18308.66
0.1	0.32131	3.87837	0.73747	4.69503	50.90647	143.5899	281.842	14899.57
0.2	0.34443	3.92374	0.71435	4.64966	53.68083	134.937	362.9574	12122.59
0.3	0.36755	3.96911	0.69123	4.60429	56.63666	126.8734	463.9738	9859.662
0.4	0.39067	4.01448	0.66811	4.55892	59.7872	119.3554	589.3857	8015.078
0.5	0.41379	4.05985	0.64499	4.51355	63.14673	112.3429	744.6849	6511.193
0.6	0.43691	4.10522	0.62187	4.46818	66.7307	105.799	936.5822	5284.951
0.7	0.46003	4.15059	0.59875	4.42281	70.55579	99.68954	1173.281	4285.101
0.8	0.48315	4.19596	0.57563	4.37744	74.64003	93.9831	1464.815	3469.948
0.9	0.50627	4.24133	0.55251	4.33207	79.00291	88.65068	1823.457	2805.546
1	0.52939	4.2867	0.52939	4.2867	83.66552	83.66552	2264.233	2264.233

Table 4.F Fuzzy Mean value of the Rainfall levels for 2014 to 2018

α	$E_l(x)$					$E_u(x)$				
	2014	2015	2016	2017	2018	2014	2015	2016	2017	2018
0	44.7537 3	53.5768 4	46.0153 1	54.5175	48.3013 1	144.781 7	179.681 4	184.566 5	187.274 4	152.879 3
0.1	47.2191 4	56.6301 9	49.0749	57.6936 1	50.9064 7	135.835 7	168.275 7	171.310 9	175.176 5	143.589 9
0.2	49.847	59.8895 5	52.3659 2	61.0873 9	53.6808 3	127.510 7	157.678 3	159.092 3	163.947 8	134.937
0.3	52.6492 5	63.3703 8	55.9075 2	64.7154	56.6366 6	119.759 9	147.827 3	147.824 2	153.520 9	126.873 4
0.4	55.6387 6	67.0893 6	59.7205 5	68.5955 3	59.7872	112.540 3	138.665 8	137.427 7	143.833 9	119.355 4
0.5	58.8294 6	71.0645 7	63.8277 5	72.7471 7	63.1467 3	105.812 5	130.141 6	127.830 6	134.830 3	112.342 9
0.6	62.2364	75.3155 8	68.2538 9	77.1913 4	66.7307	99.5401 3	122.206 8	118.967 3	126.457 8	105.799
0.7	65.8758 4	79.8635 5	73.0259 8	81.9508	70.5557 9	93.6896 2	114.817 1	110.777 8	118.668 6	99.6895 4
0.8	69.7653 9	84.7314 3	78.1735	87.0502 4	74.6400 3	88.2301 2	107.931 9	103.207 1	111.418 8	93.9831
0.9	73.9240 9	89.9440 9	83.7286 1	92.5164 3	79.0029 1	83.1331 9	101.513 8	96.2052 9	104.667 8	88.6506 8
1	78.3725 8	95.5284 8	89.7264 2	98.3784 4	83.6655 2	78.3725 8	95.5284 8	89.7264 2	98.3784 4	83.6655 2

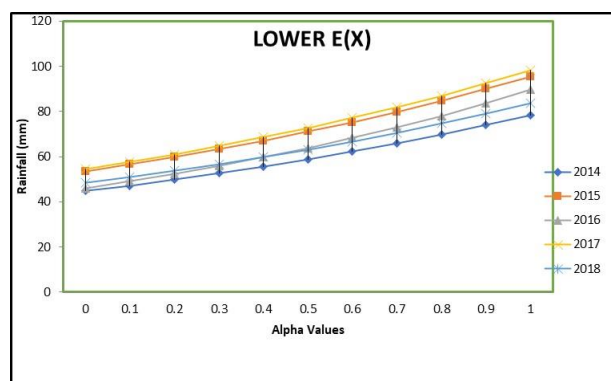


Fig.4.A Lower mean value of rainfall levels in 2014 - 2018

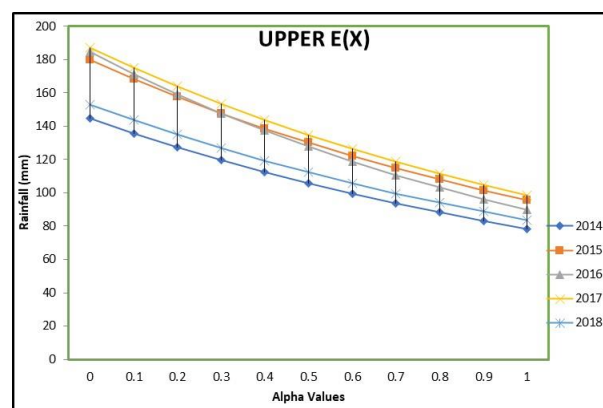


Fig.4.B Upper mean value of rainfall levels in 2014 - 2018

5. Conclusion

This paper gets the results through a mathematical model of rainfall levels in Namakkal district from 2014 to 2018. These results show that the alpha cut value continues to increase in the lower average and the alpha cut value decreases in the upper average. By this approach the conclusion holds that in 2017, Namakkal district received heavy rainfall.

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