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On Neutrosophic Transitivity and Absorbent Filters of Basic Logic Algebras

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Abstract:

The vital objective of this article is to explore the neutrosophic nature of transitive and absorbent filters in Basic Logic (BL) algebras. We establish the notion of neutrosophic transitive and absorbent filters in BL-algebras with suitable illustrations and examine a few of their properties. Also, we prove that every neutrosophic transitive filter in BL-algebras is a neutrosophic filter. In addition, we confer some necessary and sufficient conditions for a neutrosophic filter to be a transitive filter and an extension property. Further, we obtain (i) Every neutrosophic associative filter is an absorbent filter. (ii) C is a neutrosophic positive implicative filter if and only if it is a neutrosophic absorbent filter. (iii) If C is a neutrosophic absorbent filter, then it is a neutrosophic fantastic filter. In the future, the above research can be extended to deductive filters. Moreover, these filters can be applied in fields such as information technology and systems.

Keywords: BL-algebra; Filter; Neutrosophic filter; Neutrosophic transitive filter; Neutrosophic absorbent filter.

1. Introduction

The Greek term for knowledge of neutral thought is neutrosophy. It is predicated on an examination of both opposing arguments and the neutralities that exist between them. Since there is uncertainty in everything in the world, the neutrosophic has emerged and found a home in science.

In order to investigate, from a semantic perspective, the logical system whose propositional value is given in a lattice. Lattice implication algebras were introduced and some of their features were addressed by Y. Xu [1]. The idea of filters was first presented by Y. Xu and K.Y. Qin [2]. The concepts of transitive and absorbent filters were established and their features were examined by M. Sambasiva Rao [3] in lattice implication algebras.

The authors were inspired to investigate this idea in Basic Logic (BL)algebras by this. Recently, the authors studied the neutrosophication of filters of BL-algebras [4]. They then developed the concept to fantastic, positive implicative and associative filters [5, 6].

Our major contributions:

➢ The ideas of neutrosophic transitive and absorbent filters are applied in BL-algebras. We obtain a few equivalent requirements for a neutrosophic filter to be neutrosophic transitive and absorbent. Finally, we establish the relationship among the neutrosophic transitive, absorbent and associative filters in BL-algebras.

2. Preliminaries

In this part, few of the definitions and findings from the literature are referred to progress the major conclusions.

Definition 2.1[7,8] A BL-algebra $(\mathcal{G}, \vee, \wedge, \circ, \rightarrow, 0,1)$ of type $(2,2,2,2,0,0)$ such that the subsequent requirements are persuaded for all $\alpha_5, \beta_5, \gamma_5 \in \mathcal{G}$,

(i) $(\mathcal{G}, \vee, \wedge, 0,1)$ is a bounded lattice,
(ii) $(\mathcal{G}, \circ, 1)$ is a commutative monoid,
(iii) $'\circ'$, $'\rightarrow'$ is an adjoint pair, that is, $\gamma_5 \leq \alpha_5 \rightarrow \beta_5$ if and only if $\alpha_5 \circ \gamma_5 \leq \beta_5$ for all α_5, $\beta_5, \gamma_5 \in \mathcal{G}$,
(iv) $\alpha_5 \wedge \beta_5 = \alpha_5 \circ (\alpha_5 \rightarrow \beta_5)$,
(v) $(\alpha_5 \rightarrow \beta_5) \vee (\beta_5 \rightarrow \alpha_5) = 1$.

Proposition 2.2[9,10] The succeeding requirements are persuaded in a BL- algebra $\mathcal{G}$ for all $\alpha_5, \beta_5, \gamma_5 \in \mathcal{G}$,

(i) $\beta_5 \rightarrow (\alpha_5 \rightarrow \gamma_5) = \alpha_5 \rightarrow (\beta_5 \rightarrow \gamma_5) = (\alpha_5 \circ \beta_5) \rightarrow \gamma_5$,
(ii) $1 \rightarrow \alpha_5 = \alpha_5$,
(iii) $\alpha_5 \leq \beta_5$ if and only if $\alpha_5 \rightarrow \beta_5 = 1$,
(iv) $\alpha_5 \vee \beta_5 = ((\alpha_5 \rightarrow \beta_5) \rightarrow \beta_5) \wedge ((\beta_5 \rightarrow \alpha_5) \rightarrow \alpha_5)$,
(v) $\alpha_5 \leq \beta_5$ implies $\beta_5 \rightarrow \gamma_5 \leq \alpha_5 \rightarrow \gamma_5$,
(vi) $\alpha_5 \leq \beta_5$ implies $\gamma_5 \rightarrow \alpha_5 \leq \gamma_5 \rightarrow \beta_5$,
(vii) $\alpha_5 \rightarrow \beta_5 \leq (\gamma_5 \rightarrow \alpha_5) \rightarrow (\gamma_5 \rightarrow \beta_5)$,
(viii) $\alpha_5 \rightarrow \beta_5 \leq (\beta_5 \rightarrow \gamma_5) \rightarrow (\alpha_5 \rightarrow \gamma_5)$,
(ix) $\alpha_5 \leq (\alpha_5 \rightarrow \beta_5) \rightarrow \beta_5$,
(x) $\alpha_5 \circ (\alpha_5 \rightarrow \beta_5) = \alpha_5 \wedge \beta_5$,
(xi) $\alpha_5 \circ \beta_5 \leq \alpha_5 \wedge \beta_5$
(xii) $\alpha_5 \rightarrow \beta_5 \leq (\alpha_5 \circ \gamma_5) \rightarrow (\beta_5 \circ \gamma_5)$,
(xiii) $\alpha_5 \circ (\beta_5 \rightarrow \gamma_5) \leq \beta_5 \rightarrow (\alpha_5 \circ \gamma_5)$,
(xiv) $(\alpha_5 \rightarrow \beta_5) \circ (\beta_5 \rightarrow \gamma_5) \leq \alpha_5 \rightarrow \gamma_5$,
(xv) $(\alpha_5 \circ \alpha_5^*) = 0$.

Definition 2.3[11,12] A neutrosophic subset C of the universe U is a triple (T_C, I_C, F_C) where $T_C: U \rightarrow [0,1]$, $I_C: U \rightarrow [0,1]$ and $F_C: U \rightarrow [0,1]$ represents truth membership, indeterminacy and false membership functions respectively where $0 \leq T_C(\alpha_5) + I_C(\alpha_5) + F_C(\alpha_5) \leq 3$ for all $\alpha_5 \in U$.

Definition 2.4[4] A neutrosophic set C of an algebra $\mathcal{G}$ is called a neutrosophic filter, if it persuades the requirements:

(i) $T_C(\alpha_5) \leq T_C(1)$, $I_C(\alpha_5) \geq I_C(1)$ and $F_C(\alpha_5) \geq F_C(1)$,

(ii) $\min\{T_C(\alpha_5 \rightarrow \beta_5), T_C(\alpha_5)\} \leq T_C(\beta_5)$, $\min\{I_C(\alpha_5 \rightarrow \beta_5), I_C(\alpha_5)\} \geq I_C(\beta_5)$ and $\min\{F_C(\alpha_5 \rightarrow \beta_5), F_C(\alpha_5)\} \geq F_C(\beta_5)\}$ for all $\alpha_5, \beta_5 \in \mathcal{G}$.

Proposition 2.5[4] Let C be a neutrosophic filter of $\mathcal{G}$ if and only if

(i) If $\alpha_5 \leq \beta_5$ then $T_C(\alpha_5) \leq T_C(\beta_5)$, $I_C(\alpha_5) \geq I_C(\beta_5)$ and $F_C(\alpha_5) \geq F_C(\beta_5)$,

(ii) $T_C(\alpha_5 \circ \beta_5) \geq \min\{T_C(\alpha_5), T_C(\beta_5)\}$, $I_C(\alpha_5 \circ \beta_5) \leq \min\{I_C(\alpha_5), I_C(\beta_5)\}$ and $F_C(\alpha_5 \circ \beta_5) \leq \min\{F_C(\alpha_5), F_C(\beta_5)\}$ for all $\alpha_5, \beta_5 \in \mathcal{G}$.

Proposition 2.6[4, 5]Let C be a neutrosophicfilter of $\mathcal{G}$ for all$\alpha_5, \beta_5, \gamma_5 \in \mathcal{G}$ then the following hold.

(i) $T_C(\alpha_5 \to \beta_5) = T_C(1)$, then $T_C(\alpha_5) \leq T_C(\beta_5)$, $I_C(\alpha_5 \to \beta_5) = I_C(1)$, then $I_C(\alpha_5) \geq I_C(\beta_5)$, $F_C(\alpha_5 \to \beta_5) = F_C(1)$, then $F_C(\alpha_5) \geq F_C(\beta_5)$

(ii) $T_C(\alpha_5 \wedge \beta_5) = \min\{T_C(\alpha_5), T_C(\beta_5)\}$, $I_C(\alpha_5 \wedge \beta_5) = \min\{I_C(\alpha_5), I_C(\beta_5)\}$, $F_C(\alpha_5 \wedge \beta_5) = \min\{F_C(\alpha_5), F_C(\beta_5)\}$

(iii) $T_C(\alpha_5 \circ \beta_5) = \min\{T_C(\alpha_5), T_C(\beta_5)\}$, $I_C(\alpha_5 \circ \beta_5) = \min\{I_C(\alpha_5), I_C(\beta_5)\}$, $F_C(\alpha_5 \circ \beta_5) = \min\{F_C(\alpha_5), F_C(\beta_5)\}$

(iv) $T_C(0) = \min\{T_C(\alpha_5), T_C(\alpha_5^*)\}$, $I_C(0) = \min\{I_C(\alpha_5), I_C(\alpha_5^*)\}$, $F_C(0) = \min\{F_C(\alpha_5), F_C(\alpha_5^*)\}$

Definition 2.7[5] Let C be called a neutrosophic fantastic filter of $\mathcal{G}$, if it persuades the subsequent requirements for all $\alpha_5, \beta_5, \gamma_5 \in \mathcal{G}$,

(i) $T_C(1) \geq T_C(\alpha_5), I_C(1) \leq I_C(\alpha_5), F_C(1) \leq F_C(\alpha_5)$.

(ii) $\min\{T_C(\alpha_5 \to \beta_5), T_C(\alpha_5)\} \leq T_C(\beta_5)$,$\min\{I_C(\alpha_5 \to \beta_5), I_C(\alpha_5)\} \geq I_C(\beta_5)$ and $\min\{F_C(\alpha_5 \to \beta_5), F_C(\alpha_5)\} \geq F_C(\beta_5)\}$.

(iii) $T_C\big((\alpha_5 \to \beta_5) \to \beta_5\big) \to \alpha_5) \geq \min\big\{T_C\big(\gamma_5 \to (\beta_5 \to \alpha_5)\big), T_C(\gamma_5)\big\}$,
$I_C\big((\alpha_5 \to \beta_5) \to \beta_5\big) \to \alpha_5) \leq \min\{I_C\big(\gamma_5 \to (\beta_5 \to \alpha_5)\big), I_C(\gamma_5)\}$,
$F_C\ ((\alpha_5 \to \beta_5) \to \beta_5) \to \alpha_5) \leq \min\{F_C(\gamma_5 \to (\beta_5 \to \alpha_5)), F_C(\gamma_5)\}$.

Proposition 2.8[5] Let C be a neutrosophic fantastic filter of $\mathcal{G}$ if and only if

$T_C\Big(\big((\alpha_5 \to \beta_5) \to \beta_5\big) \to \alpha_5\Big) \geq T_C(\beta_5 \to \alpha_5), I_C\big(\big((\alpha_5 \to \beta_5) \to \beta_5\big) \to \alpha_5\big) \leq I_C(\beta_5 \to \alpha_5)andF_C\Big(\big((\alpha_5 \to \beta_5) \to \beta_5\big) \to \alpha_5\Big) \leq F_C(\beta_5 \to \alpha_5)$for all $\alpha_5, \beta_5 \in \mathcal{G}$.

Definition 2.9[6] Let C be a neutrosophic filter of a BL-algebra $\mathcal{G}$. C is called a neutrosophic positive implicative filter if it persuades the following,

(i) $T_C(\alpha_5) \leq T_C(1)$, $I_C(\alpha_5) \geq I_C(1)$ and $F_C(\alpha_5) \geq F_C(1)$,

(ii) $\min\{T_C(\alpha_5 \to ((\beta_5 \to \gamma_5) \to \beta_5), T_C(\alpha_5)\} \leq T_C(\beta_5)$,
$\min\{I_C(\alpha_5 \to ((\beta_5 \to \gamma_5) \to \beta_5), I_C(\alpha_5)\} \geq I_C(\beta_5)$,$\min\{F_C(\alpha_5 \to ((\beta_5 \to \gamma_5) \to \beta_5), F_C(\alpha_5)\} \geq F_C(\beta_5)$ for all $\alpha_5, \beta_5, \gamma_5 \in \mathcal{G}$.

Definition 2.10[6] Let C be a neutrosophic filter of a BL-algebra $\mathcal{G}$. C is called a neutrosophic associative filter if it satisfies the following,

(i) $T_C(\alpha_5) \leq T_C(1)$, $I_C(\alpha_5) \geq I_C(1)$ and $F_C(\alpha_5) \geq F_C(1)$,

(ii) $\min\{T_C(\alpha_5 \to (\beta_5 \to \gamma_5)), T_C(\alpha_5 \to \beta_5)\} \leq T_C(\gamma_5)$,
$\min\{I_C(\alpha_5 \to (\beta_5 \to \gamma_5)), I_C(\alpha_5 \to \beta_5)\} \geq I_C(\gamma_5)$, $\min\{F_C(\alpha_5 \to (\beta_5 \to \gamma_5)), F_C(\alpha_5 \to \beta_5)\} \geq F_C(\gamma_5)$ for all $\alpha_5, \beta_5, \gamma_5 \in \mathcal{G}$.

Proposition 2.11[6] Let C be a neutrosophic filter of $\mathcal{G}$. Then, C is a neutrosophic associative filter if and only if it satisfies $T_C((\alpha_5 \to \beta_5) \to \gamma_5) \geq T_C(\alpha_5 \to (\beta_5 \to \gamma_5))$, $I_C((\alpha_5 \to \beta_5) \to \gamma_5) \leq I_C(\alpha_5 \to (\beta_5 \to \gamma_5))$,$F_C((\alpha_5 \to \beta_5) \to \gamma_5) \leq F_C(\alpha_5 \to (\beta_5 \to \gamma_5))$ for all $\alpha_5, \beta_5, \gamma_5 \in \mathcal{G}$.

Proposition 2.12[6] Everyneutrosophic positive implicative filter of $\mathcal{G}$ is a neutrosophic fantastic filter.

3. Neutrosophic transitive filter

Here, we put forward the conception of a neutrosophic transitive filter and confer its features with illustrations.

Definition 3.1 Let C be called a neutrosophic transitive filter of $\mathcal{G}$, if it persuades the subsequent requirements for all $\alpha_5, \beta_5, \gamma_5 \in \mathcal{G}$,

(i) $T_C(1) \geq T_C(\alpha_5), I_C(1) \leq I_C(\alpha_5), F_C(1) \leq F_C(\alpha_5)$.

(ii) $\min\{T_C(\alpha_5 \to \beta_5), T_C(\alpha_5)\} \leq T_C(\beta_5)$, $\min\{I_C(\alpha_5 \to \beta_5), I_C(\alpha_5)\} \geq I_C(\beta_5)$ and $\min\{F_C(\alpha_5 \to \beta_5), F_C(\alpha_5)\} \geq F_C(\beta_5)\}$.

(iii) $T_C(\alpha_5 \to \gamma_5) \geq \min\{T_C(\alpha_5 \to \beta_5), T_C(\beta_5 \to \gamma_5)\}$,
$I_C(\alpha_5 \to \gamma_5) \leq \min\{I_C(\alpha_5 \to \beta_5), I_C(\beta_5 \to \gamma_5)\}$,
$F_C(\alpha_5 \to \gamma_5) \leq \min\{F_C(\alpha_5 \to \beta_5), F_C(\beta_5 \to \gamma_5)\}$.

Example 3.2 Let $C = \{0, \epsilon_1, \mu_1, \rho_1, 1\}$. The binary operations $\circ$ and $\to$ are given by the subsequent tables (3.1) and (3.2).

Table 3.1: $'\circ'$ Operation

$\circ$	0	ϵ_1	μ_1	ρ_1	1
0	0	0	0	0	0
ϵ_1	0	ϵ_1	ρ_1	ρ_1	ϵ_1
μ_1	0	ρ_1	μ_1	ρ_1	μ_1
ρ_1	0	ρ_1	ρ_1	ρ_1	ρ_1
1	0	ϵ_1	μ_1	ρ_1	1

Table 3.2: $'\to'$ Operation

$\to$	0	ε_1	μ_1	ρ_1	1
0	1	1	1	1	1
ε_1	0	1	μ_1	μ_1	1
μ_1	0	ϵ_1	1	ϵ_1	1
ρ_1	0	1	1	1	1
1	0	ϵ_1	μ_1	ρ_1	1

Then, $(\mathcal{G}, \vee, \wedge, \circ, \to, 0, 1)$ is a BL- algebra. Define a neutrosophic set C of $\mathcal{G}$ as follows:

$$C = \{(0, [0.5,0.7,0.7]), (\epsilon_1, [0.5,0.7,0.7]), (\mu_1, [0.5,0.7,0.7]),$$
$$(\rho_1, [0.5,0.7,0.7]), (1, [0.6,0.7,0.7])\}.$$

It is evident that C assures the conditions (i) and (ii) of the definition 3.1 and hence is a neutrosophic transitive filter.

Proposition 3.3 Every neutrosophic transitive filter of a BL-algebra $\mathcal{G}$ is a neutrosophic filter with respect to 1.

Proof: Let C be a neutrosophic transitive filter of a BL-algebra$\mathcal{G}$.
Taking $\alpha_5 = 1$ in (iii) of the Definition 3.1, we get

$$T_C(1 \to \gamma_5) \geq \min\{T_C(1 \to \beta_5), T_C(\beta_5 \to \gamma_5)\}$$

$$T_C(\gamma_5) \geq \min\{T_C(\beta_5), T_C(\beta_5 \to \gamma_5)\} \text{for all } \alpha_5, \beta_5 \in \mathcal{G}.$$

Similarly, we can prove for I_C, F_C.

The converse part may not be true. This can be proved by an example.

Example 3.4 Let $D = \{0, \epsilon_1, \mu_1, \rho_1, 1\}$. The bi-fold operations are specified by the tables (3.1) and (3.2).

Let $D = \{(0, [0.3,0.7,0.7]), (\epsilon_1, [0.5,0.7,0.7]), (\mu_1, [0.3,0.7,0.7]),$

$(\rho_1, [0.3,0.7,0.7]), (1, [0.4,0.6,0.6])\}$.

Here, D is not a neutrosophic transitive filter.

Since, $T_D(1) = 0.4 \ngeq 0.5 = T_D(\varepsilon_1)$.

Proposition 3.5 Let C be a neutrosophic filter of a BL-algebra $\mathcal{G}$. Then, C is a neutrosophic transitive filter of $\mathcal{G}$ if and only if it satisfies the following conditions for all $\alpha_5, \beta_5, \gamma_5 \in \mathcal{G}$, $T_C\big((\alpha_5 \to \beta_5) \to (\alpha_5 \to \gamma_5)\big) \geq \min\{T_C((\alpha_5 \to \beta_5) \to (\beta_5 \to \gamma_5), T_C((\alpha_5 \to \beta_5)\}$,

$$I_C\big((\alpha_5 \to \beta_5) \to (\alpha_5 \to \gamma_5)\big) \leq \min\{I_C((\alpha_5 \to \beta_5) \to (\beta_5 \to \gamma_5), I_C((\alpha_5 \to \beta_5)\}$$
$$F_C\big((\alpha_5 \to \beta_5) \to (\alpha_5 \to \gamma_5)\big) \leq \min\{F_C((\alpha_5 \to \beta_5) \to (\beta_5 \to \gamma_5), F_C((\alpha_5 \to \beta_5)\}$$

for all $\alpha_5, \beta_5, \gamma_5 \in \mathcal{G}$.

Proof: Let C be a neutrosophic filter of a BL-algebra $\mathcal{G}$.

Assume that C is a neutrosophic transitive filter of $\mathcal{G}$.

$$\begin{aligned} &T_C\big((\alpha_5 \to \beta_5) \to (\alpha_5 \to \gamma_5)\big) \\ &\geq \min\{T_C((\alpha_5 \to \beta_5) \to (\beta_5 \to \gamma_5)), T_C((\beta_5 \to \gamma_5) \to (\alpha_5 \to \gamma_5))\}, \\ &\geq \min\{T_C((\alpha_5 \to \beta_5) \to (\beta_5 \to \gamma_5)), T_C(\alpha_5 \to \beta_5)\}, \end{aligned}$$

Similarly, we can prove for I_C, F_C.

Conversely, consider $T_C(\alpha_5 \to \gamma_5) \geq T_C\big((\alpha_5 \to \beta_5) \to (\alpha_5 \to \gamma_5)\big)$

$$\begin{aligned} &\geq \min\{ T_C(\alpha_5 \to \beta_5), T_C\big((\alpha_5 \to \beta_5) \to (\beta_5 \to \gamma_5)\big)\} \\ &\geq \min\{ T_C(\alpha_5 \to \beta_5), T_C(\beta_5 \to \gamma_5)\} \text{for all } \alpha_5, \beta_5, \gamma_5 \in \mathcal{G}. \end{aligned}$$

Similarly, we can prove for I_C, F_C.

Hence, C is a neutrosophic transitive filter of$\mathcal{G}$.

Proposition 3.6 Let C be a neutrosophic filterof$\mathcal{G}$. Then, C is a neutrosophic transitive filter of $\mathcal{G}$ if it satisfies $T_C((\alpha_5 \to \beta_5) \to \gamma_5) \geq T_C(\alpha_5 \to (\beta_5 \to \gamma_5))$, $I_C((\alpha_5 \to \beta_5) \to \gamma_5) \leq I_C(\alpha_5 \to (\beta_5 \to \gamma_5))$,$F_C((\alpha_5 \to \beta_5) \to \gamma_5) \leq F_C(\alpha_5 \to (\beta_5 \to \gamma_5))$ for all $\alpha_5, \beta_5, \gamma_5 \in \mathcal{G}$.

Every neutrosophic transitive filter of a BL-algebra$\mathcal{G}$ is an associative filter.

Proof: Let$C$ be a neutrosophicfilter of $\mathcal{G}$ satisfying the given condition.

Then, $\min\{T_C(\alpha_5 \to \beta_5), T_C(\beta_5 \to \gamma_5)\} \leq \min\{T_C(\alpha_5 \to \beta_5), T_C(\alpha_5 \to (\beta_5 \to \gamma_5))\}$

$$\begin{aligned} &\leq \min\{T_C(\alpha_5 \to \beta_5), T_C((\alpha_5 \to \beta_5) \to \gamma_5)\} \\ &\leq T_C(\gamma_5) \\ &\leq T_C(\alpha_5 \to \gamma_5) \end{aligned}$$

$$T_C(\alpha_5 \to \gamma_5) \geq \min\{T_C(\alpha_5 \to \beta_5), T_C(\beta_5 \to \gamma_5)\}$$

Similarly, we can prove for I_C, F_C.

Hence, C is a neutrosophic transitive filter of$\mathcal{G}$.

Corollary 3.7 Let C be a neutrosophic filter of a BL-algebra $\mathcal{G}$. If C is a neutrosophic associative filter, then it is a neutrosophic transitive filter.
Proof: By the proposition 3.6 and 2.11, the proof is obvious.

Proposition 3.8 Each neutrosophic filter C of a BL-algebra $\mathcal{G}$ is a neutrosophic transitive filter if it satisfies$T_C(\gamma_5) \geq \min\{T_C(\alpha_5 \to \beta_5), T_C(\alpha_5 \to (\beta_5 \to \gamma_5))\}$

$$I_C(\gamma_5) \leq \min\{I_C(\alpha_5 \to \beta_5), I_C(\alpha_5 \to (\beta_5 \to \gamma_5))\}$$
$$F_C(\gamma_5) \leq \min\{F_C(\alpha_5 \to \beta_5), F_C(\alpha_5 \to (\beta_5 \to \gamma_5))\} \text{for all } \alpha_5, \beta_5, \gamma_5 \in \mathcal{G}.$$

Proof: Let$C$ be a neutrosophic filter of a BL-algebra $\mathcal{G}$.
Assume that $T_C(\gamma_5) \geq \min\{T_C(\alpha_5 \to \beta_5), T_C(\alpha_5 \to (\beta_5 \to \gamma_5))\}$.
Consider $T_C(\alpha_5 \to \gamma_5) \geq T_C(\gamma_5)$

$$\geq \min\{T_C(\alpha_5 \to \beta_5), T_C((\alpha_5 \to \beta_5) \to \gamma_5)\}$$
$$\geq \min\{T_C(\alpha_5 \to \beta_5), T_C(\alpha_5 \to (\beta_5 \to \gamma_5))\}$$
$$\geq \min\{T_C(\alpha_5 \to \beta_5), T_C(\beta_5 \to \gamma_5)\}$$

Similarly, we can prove for I_C, F_C.
Hence, C is a neutrosophic transitive filter of$\mathcal{G}$.

Proposition 3.9 Let C and D be two neutrosophic filters of $\mathcal{G}$ and C be a neutrosophic transitive filter of $\mathcal{G}$. If $C \subseteq D$, then $T_C(1) = T_D(1)$, $I_C(1) = I_D(1), F_C(1) = F_D(1)$ and D is also a neutrosophic transitive filter.

Proof: Let C be a neutrosophic transitive filter of $\mathcal{G}$.

$$T_D((\beta_5 \to \gamma_5) \to ((\alpha_5 \to \beta_5) \to (\alpha_5 \to \gamma_5)))$$
$$\geq T_C((\beta_5 \to \gamma_5) \to ((\alpha_5 \to \beta_5) \to (\alpha_5 \to \gamma_5)))$$
$$= T_C((\alpha_5 \to \beta_5) \to ((\beta_5 \to \gamma_5) \to (\alpha_5 \to \gamma_5)))$$
$$= T_C((\alpha_5 \to \beta_5) \to (\alpha_5 \to (\beta_5 \to \gamma_5) \to \gamma_5))$$
$$= T_C((\alpha_5 \to \beta_5) \to ((\gamma_5 \to \beta_5) \to (\alpha_5 \to \beta_5)))$$
$$= T_C((\gamma_5 \to \beta_5) \to ((\alpha_5 \to \beta_5) \to (\alpha_5 \to \beta_5)))$$
$$= T_C((\gamma_5 \to \beta_5) \to 1)$$
$$= T_C(1)$$
$$= T_D(1)$$

Since D is a neutrosophic filter,

$$T_D((\alpha_5 \to \beta_5) \to (\alpha_5 \to \gamma_5)) \geq \min\{T_D(\beta_5 \to \gamma_5), T_D((\beta_5 \to \gamma_5) \to ((\alpha_5 \to \beta_5) \to (\alpha_5 \to \gamma_5)))\}$$
$$= T_D(\beta_5 \to \gamma_5) \geq \min\{T_D(\alpha_5 \to \beta_5), T_D((\alpha_5 \to \beta_5) \to (\beta_5 \to \gamma_5))\}$$

Similarly, we can prove for I_D, F_D.
Hence, D is a neutrosophic transitive filter.

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4. Neutrosophic absorbent filter

Here, we put forward the conception of a neutrosophic absorbent filter and confer its features with illustrations.

Definition 4.1 Let C be called a neutrosophic absorbent filter of $\mathcal{G}$, if it persuades the subsequent requirements for all $\alpha_5, \beta_5, \gamma_5 \in \mathcal{G}$,

(i) $T_C(1) \geq T_C(\alpha_5), I_C(1) \leq I_C(\alpha_5), F_C(1) \leq F_C(\alpha_5)$.

(ii) $\min\{T_C(\alpha_5 \to \beta_5), T_C(\alpha_5)\} \leq T_C(\beta_5)$, $\min\{I_C(\alpha_5 \to \beta_5), I_C(\alpha_5)\} \geq I_C(\beta_5)$ and $\min\{F_C(\alpha_5 \to \beta_5), F_C(\alpha_5)\} \geq F_C(\beta_5)\}$.

(iii) $T_C(\alpha_5) \geq T_C\big((\alpha_5 \to \beta_5) \to \alpha_5\big)$,

$I_C(\alpha_5) \leq I_C\big((\alpha_5 \to \beta_5) \to \alpha_5\big)$,

$F_C(\alpha_5) \leq F_C\big((\alpha_5 \to \beta_5) \to \alpha_5\big)$

Example 4.2 Let $C = \{0, \epsilon_1, \mu_1, \rho_1, 1\}$. The bi-fold operations are given by the subsequent tables (4.1) and (4.2).

Table 4.1: $'\circ'$ OperationTable 4.2: $' \to '$ Operation

$\circ$	0	ϵ_1	μ_1	1
0	0	ϵ_1	μ_1	μ_1
ϵ_1	0	0	0	ϵ_1
μ_1	0	0	0	μ_1
1	0	1	1	0

$\to$	0	ϵ_1	μ_1	1
0	0	0	0	0
ϵ_1	ϵ_1	0	0	ϵ_1
μ_1	ϵ_1	1	0	μ_1
1	1	1	0	0

Then, $(\mathcal{G}, \vee, \wedge, \circ, \to, 0, 1)$ is a BL- algebra.

Consider a neutrosophic set C:

$$C = \{(0, [0.5,0.7,0.7]), (\epsilon_1, [0.5,0.7,0.7]), (\mu_1, [0.5,0.7,0.7]), (\rho_1, [0.5,0.7,0.7]), (1, [0.6,0.7,0.7])\}.$$

It is evident that C assures the definition 4.1. Hence, C is a neutrosophic absorbent filter.

Proposition 4.3 Every neutrosophic associative filter of $\mathcal{G}$ is a neutrosophic absorbent filter.

Proof: Let C be a neutrosophic associative filter of $\mathcal{G}$.

$$\begin{aligned} T_C(\alpha_5) = T_C(1 \to \alpha_5) &= T_C(((\alpha_5 \to \beta_5) \to 1) \to \alpha_5) \\ &\geq T_C((\alpha_5 \to \beta_5) \to (1 \to \alpha_5)) \\ &= T_C(1 \to \big((\alpha_5 \to \beta_5) \to \alpha_5\big)) \\ &= T_C((\alpha_5 \to \beta_5) \to \alpha_5) \end{aligned}$$

Therefore, $T_C(\alpha_5) \geq T_C((\alpha_5 \to \beta_5) \to \alpha_5)$

Similarly, we can prove for I_C, F_C.
Hence, C is a neutrosophic absorbent filter of$\mathcal{G}$.
Proposition 4.4 A neutrosophic filter C of a BL-algebra $\mathcal{G}$ is a neutrosophic positive implicative filter if and only if it is a neutrosophic absorbent filter.
Proof: Assume that$C$ is a neutrosophic positive implicative filter of $\mathcal{G}$.
From the Definition 2.9, we have $T_C(\alpha_5) \geq \min\{T_C\left(1 \to \left((\alpha_5 \to \beta_5) \to \alpha_5\right)\right), T_C(1)\}$

$$= \min\{T_C\left((\alpha_5 \to \beta_5) \to \alpha_5\right), T_C(1)\}$$
$$= T_C((\alpha_5 \to \beta_5) \to \alpha_5)$$

$T_C(\alpha_5) \geq T_C((\alpha_5 \to \beta_5) \to \alpha_5)$for all $\alpha_5, \beta_5 \in \mathcal{G}$.
Similarly, we can prove for I_C, F_C.
Hence, C is a neutrosophic absorbent filter of$\mathcal{G}$.
Conversely, C is a neutrosophic absorbent filter of $\mathcal{G}$.

$$T_C(\alpha_5) \geq T_C((\alpha_5 \to \beta_5) \to \alpha_5)$$
$$\geq \min\{T_C\left(\gamma_5 \to \left((\alpha_5 \to \beta_5) \to \alpha_5\right)\right), T_C(\gamma_5)\}$$
$$\text{Therefore, } T_C(\alpha_5) \geq \min\left\{T_C(\gamma_5 \to \left((\alpha_5 \to \beta_5) \to \alpha_5\right)\right), T_C(\gamma_5)\}.$$

Similarly, we can prove for I_C, F_C.
Hence, C is a neutrosophic positive implicative filter of $\mathcal{G}$.

Corollary 4.5 Let C be a neutrosophic filter of a BL-algebra $\mathcal{G}$. If C is a neutrosophic absorbent filter, then it is a neutrosophic fantastic filter of $\mathcal{G}$.
Proof: Let C be a neutrosophic absorbent filter of a BL-algebra $\mathcal{G}$.
Then, by the Proposition 4.4, C is a neutrosophic positive implicative filter of $\mathcal{G}$.
Then from the Proposition 2.12, C is a neutrosophic fantastic filter of $\mathcal{G}$.

5. Conclusion

In the current study, we have put forward the notions of neutrosophic transitive and absorbent filters in Basic Logic algebras and looked into a few associated features. Additionally, we have proved that every neutrosophic transitive filter in BL-algebras is a neutrosophic filter and a neutrosophic associative filter. In addition, we confer some necessary and sufficient condition, extension property for a neutrosophic filter to be a transitive filter. The purpose of this article is two fold, first is to introduce the notions in BL-algebra and then to explore their relationship among various filters. Further, we have obtained (i) Everyneutrosophic associative filter is an absorbent filter. (ii) C is a neutrosophic positive implicative filter if and only if it is a neutrosophic absorbent filter. (iii) If C is a neutrosophic absorbent filter, then it is a neutrosophic fantastic filter. In the future, the above research can be extended to ultra and deductive filters.

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