

A Fuzzy Optimal Base Stock System for Tolerant Clients

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Abstract:

Base stock is an amount of stock that a company requires to maintain in order to handle an unexpected huge demand. Tolerant clients are the clients who are tolerant to the company's delayed supply, who come back to the same company even if their demands are not met within their expected time. This implies that the clients are solely relied on that company. Marketing manager requires to maintain on hand to satisfy such tolerant clients' demands with some delay, no longer than expected by that tolerant client. This paper diagnosis the performance measures of optimal base stock system for tolerant clients in fuzzy environment. The fuzzy numbers can be modified to crisp number with the help of fuzzy ranking methods. Here the used fuzzy ranking method is prominent for de-fuzzification. The famous Triangular fuzzy number and Trapezoidal fuzzy number methods are played a major role for de-fuzzification. At last, the optimization of base stock is verified with numerical examples in fuzzy environment.

Keywords: Base stock, Tolerant clients, Fuzzy environment, Fuzzy ranking, De-fuzzification.

1. Introduction

In the theory of stock management, the well-known base stock system for tolerant clients is an important model and these models have been considered by many researchers. Base Stock system for tolerant clients in the theory of stock management is an absorbing method of ordering techniques. Initially the stocks starting with 'B' number of items. Whenever a client order for 'r' units is received, a stock replenishment order for 'r' units is placed instantly. Replenishment orders are full filled after the lead time 'L'. The client's demand is met with viable from the supply on hand. If the total unfilled client's demand surplus the supply on hand, then assume that the client will not cancel the orders but the clients wait till their requirement is fulfilled. The sum of stock on hand and order placed is constant in time and equal to 'B' called as Base Stock.

The detailed study of the base stock system for tolerant clients has been considered by Gaver[8]. The very basic model had been discussed by Hanssman [9]. Ramanarayanan et al., [18] have discussed the model in which the lead time was assumed to be a random variable. Suresh Kumar [26] have proposed the model by using the change of distribution property. Baimei Yang et al [1] derived a

single period inventory model with two suppliers. Ramathilagam et al., [19] proposed an optimal reserve inventory model with the assumption that it undergoes parametric change. Chung – Ho Chen [6] have developed the optimum production run length using SCBZ property. ShirajulIslam Ukil et al., [25] have proposed a production inventory model with the assumptions that the production and rate of demand are in linear trend. The concept of SCBZ property was basically discussed by RajaRao and Talwalker [17]. Henry et al [10, 11, 12, 13] have discussed the optimum base stock model with the assumption that the lead time random variable has the SCBZ property and assuming that the truncation point itself a random variable which follows exponential distribution.

Kaufmann [14] explained in detail the theory of fuzzy subsets. Campos L et al., [2], Cheng CH [3], Chu TC [4] and Chen et al., [5] are developed their models by applying the ranking fuzzy numbers. The theory of fuzzy sets and fuzzy logic was developed by Klir et al., [15] and Zadeh [33, 34]. Modarres et al., [16] have explained in detail in the performance ratio by using the concept of fuzzy ranking numbers. Ramesh et al [20, 21] have applied the very famous wingspans fuzzy ranking method for developing M/M/1/N fuzzy queuing model. Syamala et al., [27, 28, 29, 30] discussed about some fuzzy concepts. Wang Y et al., [31], Deng Y et al., [7] and Yager R R [32] are introduced the revised fuzzy ranking methods for their articles.

Sachithanantham et al., [22, 23, 24] have discussed the modified version of the Base Stock model in which the lead time was assumed to be a random variable and which satisfies the so called SCBZ property. In the sense that after the truncation point the lead time takes parametric change. The change point known as the truncation point and is also assumed to be a random variable and based on these assumptions, optimal Base Stock has been obtained. Also it is assumed that the lead time random variable follows exponential distribution which satisfies the SCBZ property and the change point is itself a random variable which has the mixed exponential distribution. Under this assumption, the optimal Base Stock is obtained. This article considers the above said model and verified their performance in fuzzy environment. The fuzzy numbers can be modified to crisp number with the help of Wingspans fuzzy ranking method. Here the well-known Triangular fuzzy numbers and Trapezoidal fuzzy numbers are played for de-fuzzification. Finally, the optimization of base stock is verified with a numerical example and to validate of our proposed method.

2. Preliminaries

Definition 1: Base Stock

Base stock is an amount of stock that marketing requires to maintain on hand to satisfy clients' demands with some delay, no longer than expected by clients.

Definition 2: Tolerant Clients

Tolerant clients are clients who are tolerant to the company's delayed supply. They come back to the same company even if their demands are not met within their expected time, which implies the clients are solely relied on that company.

Definition 3: Fuzzy Number

A fuzzy number is a generalization of a regular real number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each possible value has its own weight between 0 and 1.

Definition 4: Triangular Fuzzy Number

A triangular fuzzy number $\tilde{A}(x_1, x_2, x_3; 1)$ is dictated by a membership function

$$\mu_{\tilde{A}}(z) = \begin{cases} \frac{z - a_1}{a_2 - a_1}, & a_1 \leq z \leq a_2 \\ 1, & z = a_2 \\ \frac{z - a_3}{a_2 - a_3}, & a_2 \leq z \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

Definition 5: Trapezoidal Fuzzy Number

A trapezoidal fuzzy number $\tilde{A}(a_1, a_2, a_3, a_4; 1)$ is dictated by a membership function

$$\mu_{\tilde{A}}(z) = \begin{cases} \frac{z - a_1}{a_2 - a_1}, & a_1 \leq z \leq a_2 \\ 1, & a_2 \leq z \leq a_3 \\ \frac{z - a_4}{a_3 - a_4}, & a_3 \leq z \leq a_4 \\ 0, & \text{otherwise} \end{cases}$$

3. Model Description

The model which is assumed that the lead time random variable follows an exponential distribution, which satisfies the so called SCBZ property and it is also assumed that the truncation point itself a random variable which has the mixed exponential distribution with parameter τ and δ . Based on the above stated assumptions the optimal base stock model considered by Sachithanatham et al., and the derived model is

$$\frac{d}{h+d} - \left\{ \frac{\lambda \beta \left[1 - e^{-\mu B \left(\frac{\theta + \tau}{\lambda + \theta + \tau} \right)} \right]}{(\theta + \tau)(\lambda + \theta + \tau)} \right\} \left\{ \theta - \frac{\theta^* \tau}{(\theta + \tau - \theta^*)} \right\} = \left\{ \frac{\lambda(1-\beta) \left[1 - e^{-\mu B \left(\frac{\theta + \delta}{\lambda + \theta + \delta} \right)} \right]}{(\theta + \delta)(\lambda + \theta + \delta)} \right\} \left\{ \theta - \frac{\delta \theta^*}{(\theta + \delta - \theta^*)} \right\} +$$

$$\left\{ \frac{\lambda \left[1 - e^{-\mu B \left(\frac{\theta^*}{\lambda + \theta^*} \right)} \right]}{(\lambda + \theta^*)} \right\} \left\{ \frac{\tau \beta}{(\theta + \tau - \theta^*)} + \frac{\delta(1-\beta)}{(\theta + \delta - \theta^*)} \right\} \text{-----(1)}$$

Where h: Inventory holding cost / unit / time, d: Shortage cost / unit / time.

B: The Base Stock level, λ : Parameter of inter arrival time distribution.

μ : Parameter of demand distribution, τ, δ : Parameters of mixed exponential distribution.

Here we have to verify the performance of optimum base stock model in fuzzy environment by using Wingspans ranking function method.

4. Wingspans Ranking Function Method

Let a membership function $\phi_{\tilde{A}}$ of a fuzzy number $\tilde{A}$ which has the core point a_0 . Then

$W_A = a_0 - \frac{1}{2} \int_{-\infty}^{a_0} \phi_{\tilde{A}}(x) dx + \frac{1}{2} \int_{a_0}^{\infty} \phi_{\tilde{A}}(x) dx$ is called the Wingspans center of $\tilde{A}$. This wingspans center

based on the area between horizontal real axis and the curve of the membership function, also it is symmetric.

The ranking function for triangular fuzzy number $\tilde{A} = [a_l, a_0, a_r]$ with respect to its wingspans center is

$$R(\tilde{A}) = \frac{1}{2}a_0 + \frac{1}{4}(a_l + a_r).$$

The ranking function for trapezoidal fuzzy number $\tilde{A} = [a_l, a_b, a_c, a_r]$ with respect to its wingspans

center is $R(\tilde{A}) = \frac{1}{4}(a_l + a_b + a_c + a_r)$

We use this proposed fuzzy ordering technique, which de-fuzzifies the fuzzy numerals in the Optimum Base Stock model. We have used these fuzzy numerals as the fuzzy parameters.

5. Practical Examples

Steel authority of India Ltd., Salem branch is one of the major sources from which various stainless steel vessel making companies get their supply. Consider the sales during marriage seasons or any festival seasons, in Steel authority of India Ltd where in the demand from their clients exceeds the stock level, there forth they could not satisfy their clients when they place excess orders. In such cases the marketing manager decides to maintain the optimum base stock level in-order to satisfy the tolerant clients. If the given situation is applied to a normal environment the above equation (1) is already derived and verified by numerical example. Here the same situation is applied in fuzzy environment and their performances are measured.

For Triangular fuzzy number

Under the triangular fuzzy numbering, the optimal base stock is measured by using the following illustration. Let us assume the values of various parameters in equation (1) and its performance were analysed in Fuzzy environment.

Illustration 1.1

For $\tilde{d}=[2,3,4], \tilde{\lambda}=[1,2,3], \tilde{\beta}=[0.2,0.5,0.8], \tilde{\tau}=[0.5,1,1.5], \tilde{\delta}=[0.5,1.5,2.5], \tilde{\theta} = [0.2,0.5,0.8],$

$\tilde{\theta}^*=[0.5,1,1.5], \tilde{\mu}=[0.5,1.5,2.5]$, the optimal value of $\hat{B}$ is obtained and the variations in $\hat{B}$ for the changes in the value of $\tilde{h}$ are listed.

Now the ranking index of $\tilde{h}$ is $R(\tilde{h}) = R(4,5,6) = \frac{1}{2}(5) + \frac{1}{4}(4+6) = 5$, $R(\tilde{h}) = R(9,10,11) = 10$,

$R(\tilde{h}) = R(14,15,16) = 15$, $R(\tilde{h}) = R(19,20,21) = 20$

By using the above de-fuzzified numbers for the stock holding cost and similarly the de-fuzzified values of all other parameters, the optimal base stock $\hat{B}$ is obtained by solving the equation (1) and presented in the table below.

Table 1.1: The variations in $\hat{B}$ for the changes in the value of $\tilde{h}$

$\tilde{h}$	$\tilde{\delta}$	$\tilde{\mu}$	$\tilde{d}$	$\tilde{\lambda}$	$\tilde{\beta}$	$\tilde{\tau}$	$\tilde{\theta}$	$\tilde{\theta}^*$	$\hat{B}$ $\times 10^{-4}$
[4,5,6]	[0.5,1.5,2.5] 	[0.5,1.5,2.5] 	[2,3,4] 	[1,2,3] 	[0.2,0.5,0.8] 	[0.5,1,1.5] 	[0.2,0.5,0.8] 	[0.5,1,1.5] 	1620 7
[9,10,11]	[0.5,1.5,2.5] 	[0.5,1.5,2.5] 	[2,3,4] 	[1,2,3] 	[0.2,0.5,0.8] 	[0.5,1,1.5] 	[0.2,0.5,0.8] 	[0.5,1,1.5] 	8770
[14,15,16] 	[0.5,1.5,2.5] 	[0.5,1.5,2.5] 	[2,3,4] 	[1,2,3] 	[0.2,0.5,0.8] 	[0.5,1,1.5] 	[0.2,0.5,0.8] 	[0.5,1,1.5] 	6040
[19,20,21] 	[0.5,1.5,2.5] 	[0.5,1.5,2.5] 	[2,3,4] 	[1,2,3] 	[0.2,0.5,0.8] 	[0.5,1,1.5] 	[0.2,0.5,0.8] 	[0.5,1,1.5] 	4610

Table 1.1 determines that if the de-fuzzified values of stock holding cost increased then the optimal base stock $\hat{B}$ decreased.

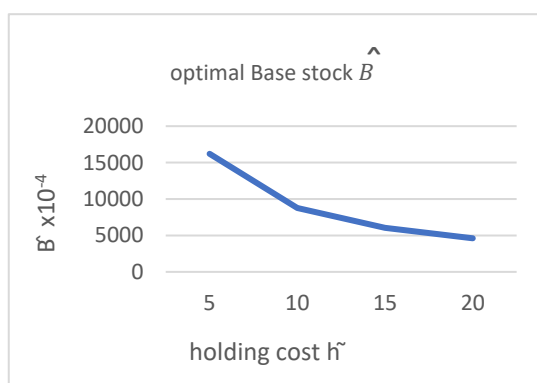


Fig.1.1: The variations in $\hat{B}$ for the changes in the value of $\tilde{h}$

From the graph, it is observed that if the values of the shortage cost $\tilde{h}$ increases, then the optimum base stock is reduced.

Illustration 1.2

For $\tilde{h}=[4,5,6]$, $\tilde{\lambda}=[1,2,3]$, $\tilde{\beta}=[0.2,0.5,0.8]$, $\tilde{\tau}=[0.5,1,1.5]$, $\tilde{\delta}=[0.5,1.5,2.5]$,

$\tilde{\theta} = [0.2,0.5,0.8]$, $\tilde{\theta}^*=[0.5,1,1.5]$, $\tilde{\mu}=[0.5,1.5,2.5]$, the optimal value of $\hat{B}$ is obtained and the variations in $\hat{B}$ for the changes in the value of $\tilde{d}$ are listed.

Now the ranking index of $\tilde{d}$ is $R(\tilde{d}) = R(2.5,3.0,3.5) = \frac{1}{2}(3) + \frac{1}{4}(2.5+3.5) = 3$,

$R(\tilde{d}) = R(4.5,5.0,5.5) = 5$, $R(\tilde{d}) = R(6.5,7.0,7.5) = 7$, $R(\tilde{d}) = R(8.5,9.0,9.5) = 9$

By using the above de-fuzzified numbers for the stock shortage cost and similarly the de-fuzzified values of all other parameters, the optimal base stock $\hat{B}$ is obtained by solving the equation (1) and presented in the table below.

Table 1.2: The variations in $\hat{B}$ for the changes in the value of $\tilde{d}$

$\tilde{d}$	$\tilde{\delta}$	$\tilde{\mu}$	$\tilde{h}$	$\tilde{\lambda}$	$\tilde{\beta}$	$\tilde{\tau}$	$\tilde{\theta}$	$\tilde{\theta}^*$	$\hat{B}$ $\times 10^{-4}$
[2.5,3.0,3.5 1]	[0.5,1.5,2.5 1]	[0.5,1.5,2.5 1]	[4,5,6 1]	[1,2,3 1]	[0.2,0.5,0.8 1]	[0.5,1,1.5 1]	[0.2,0.5,0.8 1]	[0.5,1,1.5 1]	1620 7
[4.5,5.0,5.5 1]	[0.5,1.5,2.5 1]	[0.5,1.5,2.5 1]	[4,5,6 1]	[1,2,3 1]	[0.2,0.5,0.8 1]	[0.5,1,1.5 1]	[0.2,0.5,0.8 1]	[0.5,1,1.5 1]	2516 0
[6.5,7.0,7.5 1]	[0.5,1.5,2.5 1]	[0.5,1.5,2.5 1]	[4,5,6 1]	[1,2,3 1]	[0.2,0.5,0.8 1]	[0.5,1,1.5 1]	[0.2,0.5,0.8 1]	[0.5,1,1.5 1]	3383 0
[8.5,9.0,9.5 1]	[0.5,1.5,2.5 1]	[0.5,1.5,2.5 1]	[4,5,6 1]	[1,2,3 1]	[0.2,0.5,0.8 1]	[0.5,1,1.5 1]	[0.2,0.5,0.8 1]	[0.5,1,1.5 1]	4297 0

Table 1.2 determines that if the de-fuzzified values of stock shortage cost increased then the optimal base stock $\hat{B}$ increased.

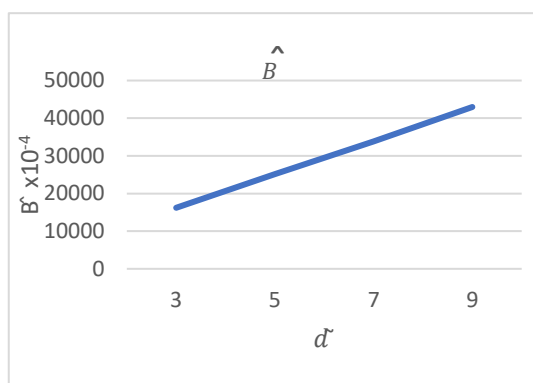


Fig 1.2: The variations in $\hat{B}$ for the changes in the value of $\tilde{d}$

From the graph, it is observed that if the values of the shortage cost $\tilde{d}$ increases, then the optimum base stock is increased.

Illustration 1.3

For $\tilde{h}=[4,5,6]$, $\tilde{d}=[2,3,4]$, $\tilde{\beta}=[0.2,0.5,0.8]$, $\tilde{\tau}=[0.5,1,1.5]$, $\tilde{\delta}=[0.5,1.5,2.5]$,

$\tilde{\theta} = [0.2,0.5,0.8]$, $\tilde{\theta}^*=[0.5,1,1.5]$, $\tilde{\mu}=[0.5,1.5,2.5]$, the optimal value of $\hat{B}$ is obtained and the variations in $\hat{B}$ for the changes in the value of $\tilde{\lambda}$ are listed.

Now the ranking index of $\tilde{\lambda}$ is $R(\tilde{\lambda}) = R(1.8, 2.0, 2.2) = \frac{1}{2}(2) + \frac{1}{4}(1.8 + 2.2) = 2.0$,

$R(\tilde{\lambda}) = R(2.3, 2.5, 2.7) = 2.5$, $R(\tilde{\lambda}) = R(2.8, 3.0, 3.2) = 3.0$, $R(\tilde{\lambda}) = R(3.3, 3.5, 3.7) = 3.5$

By using the above de-fuzzified numbers for the inter arrival time and similarly the de-fuzzified values of all other parameters, the optimal base stock $\hat{B}$ is obtained by solving the equation (1) and are presented in the table below.

Table 1.3: The variations in $\hat{B}$ for the changes in the value of $\tilde{\lambda}$

$\tilde{\lambda}$	$\tilde{\delta}$	$\tilde{\mu}$	$\tilde{h}$	$\tilde{d}$	$\tilde{\beta}$	$\tilde{\tau}$	$\tilde{\theta}$	$\tilde{\theta}^*$	$\hat{B}$ $\times 10^{-4}$
[1.8,2.0,2.2] 1	[0.5,1.5,2.5] 1	[0.5,1.5,2.5] 1	[4,5,6] 1	[2,3,4] 1	[0.2,0.5,0.8] 1	[0.5,1,1.5] 1	[0.2,0.5,0.8] 1	[0.5,1,1.5] 1	1620 7
[2.3,2.5,2.7] 1	[0.5,1.5,2.5] 1	[0.5,1.5,2.5] 1	[4,5,6] 1	[2,3,4] 1	[0.2,0.5,0.8] 1	[0.5,1,1.5] 1	[0.2,0.5,0.8] 1	[0.5,1,1.5] 1	1793 0
[2.8,3.0,3.2] 1	[0.5,1.5,2.5] 1	[0.5,1.5,2.5] 1	[4,5,6] 1	[2,3,4] 1	[0.2,0.5,0.8] 1	[0.5,1,1.5] 1	[0.2,0.5,0.8] 1	[0.5,1,1.5] 1	1986 0
[3.3,3.5,3.7] 1	[0.5,1.5,2.5] 1	[0.5,1.5,2.5] 1	[4,5,6] 1	[2,3,4] 1	[0.2,0.5,0.8] 1	[0.5,1,1.5] 1	[0.2,0.5,0.8] 1	[0.5,1,1.5] 1	2191 0

Table.1.3 determines that if the de-fuzzified values of inter arrival time increased then the optimal base stock $\hat{B}$ increased.

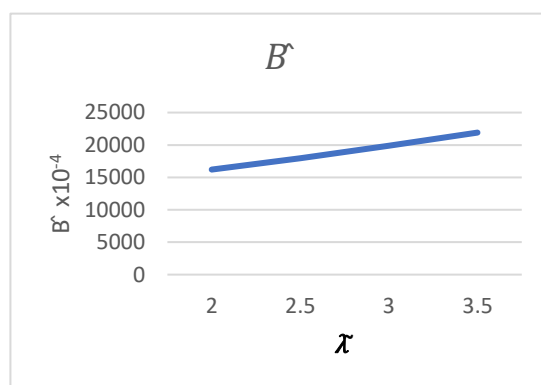


Fig 1.3: The variations in $\hat{B}$ for the changes in the value of $\tilde{d}$

From the graph, it is observed that if the values of the shortage cost $\tilde{d}$ increases, then the optimum base stock is increased.

Illustration 1.4

For $\tilde{h}=[4,5,6]$, $\tilde{d}=[2,3,4]$, $\tilde{\beta}=[0.2,0.5,0.8]$, $\tilde{\tau}=[0.5,1,1.5]$, $\tilde{\delta}=[0.5,1.5,2.5]$,

$\tilde{\theta} = [0.2,0.5,0.8]$, $\tilde{\theta}^*=[0.5,1,1.5]$, $\tilde{\lambda} = [1,2,3]$, the optimal value of $\hat{B}$ is obtained and the variations in $\hat{B}$ for the changes in the value of $\tilde{\mu}$ are listed. Now the ranking index of $\tilde{\mu}$ is

$$R(\tilde{\mu}) = R(1.3, 1.5, 1.7) = \frac{1}{2}(1.5) + \frac{1}{4}(1.3 + 1.7) = 1.5, \quad R(\tilde{\mu}) = R(1.8, 2.0, 2.2) = 2.0,$$

$$R(\tilde{\mu}) = R(2.3, 2.5, 2.7) = 2.5, \quad R(\tilde{\mu}) = R(2.8, 3.0, 3.2) = 3.0$$

By using the above de-fuzzified numbers for the demand and similarly the de-fuzzified values of all other parameters, the optimal base stock $\hat{B}$ is obtained by solving the equation (1) and presented in the table below.

Table 1.4: The variations in $\hat{B}$ for the changes in the value of $\tilde{\mu}$

$\tilde{\mu}$	$\tilde{\delta}$	$\tilde{\lambda}$	$\tilde{h}$	$\tilde{d}$	$\tilde{\beta}$	$\tilde{\tau}$	$\tilde{\theta}$	$\tilde{\theta}^*$	$\hat{B}$ $\times 10^{-4}$
[1.3,1.5,1.7] 1	[0.5,1.5,2.5] 1	[0.5,1.5,2.5] 1	[4,5,6] 1	[2,3,4] 1	[0.2,0.5,0.8] 1	[0.5,1,1.5] 1	[0.2,0.5,0.8] 1	[0.5,1,1.5] 1	1620 7
[1.8,2.0,2.2] 1	[0.5,1.5,2.5] 1	[0.5,1.5,2.5] 1	[4,5,6] 1	[2,3,4] 1	[0.2,0.5,0.8] 1	[0.5,1,1.5] 1	[0.2,0.5,0.8] 1	[0.5,1,1.5] 1	1216 0
[2.3,2.5,2.7] 1	[0.5,1.5,2.5] 1	[0.5,1.5,2.5] 1	[4,5,6] 1	[2,3,4] 1	[0.2,0.5,0.8] 1	[0.5,1,1.5] 1	[0.2,0.5,0.8] 1	[0.5,1,1.5] 1	9720
[2.8,3.0,3.2] 1	[0.5,1.5,2.5] 1	[0.5,1.5,2.5] 1	[4,5,6] 1	[2,3,4] 1	[0.2,0.5,0.8] 1	[0.5,1,1.5] 1	[0.2,0.5,0.8] 1	[0.5,1,1.5] 1	8100

Table 1.4 determines that if the de-fuzzified values of demand increased then the optimal base stock $\hat{B}$ decreased.

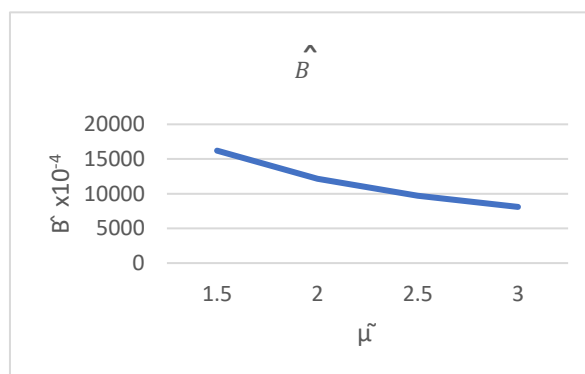


Fig 1.4: The variations in $\hat{B}$ for the changes in the value of $\tilde{d}$

From the graph, it is observed that if the values of the shortage cost $\tilde{d}$ increase, then the optimum base stock is increases.

For Trapezoidal fuzzy number

Let us assumes the values of various parameters in equation (1) and its performance were analyzed in Fuzzy environment.

Illustration 2.1

For $\tilde{d}=[1,2,3,4]$, $\tilde{\lambda}=[1,2,3,4]$, $\tilde{\beta}=[0.2,0.5,0.8,1.1]$, $\tilde{\tau}=[0.5,1.0,1.5,2.0]$, $\tilde{\delta}=[0.5,1.5,2.5,3.5]$, $\tilde{\theta}=[0.2,0.5,0.8,1.1]$, $\tilde{\theta}^*=[0.5,1.0,1.5,2.0]$, $\tilde{\mu}=[0.5,1.0,1.5,2.0]$, the optimal value of $\hat{B}$ is obtained and the variations in $\hat{B}$ for the changes in the value of $\tilde{h}$ are listed.

Now the ranking index of $\tilde{h}$ is

$$R(\tilde{h}) = R(3,4,5,6) = \frac{1}{4}(3 + 4 + 5 + 6) = 4.5, \quad R(\tilde{h}) = R(8,9,10,11) = 9.5,$$

$$R(\tilde{h}) = R(13,14,15,16) = 14.5, R(\tilde{h}) = R(18,19,20,21) = 19.5$$

By using the above de-fuzzified numbers for the stock holding cost and similarly the de-fuzzified values of all other parameters, the optimal base stock $\hat{B}$ is obtained by solving the equation (1) and presented in the table below.

Table 2.1: The variations in $\hat{B}$ for the changes in the value of $\tilde{h}$

$\tilde{h}$	$\tilde{\delta}$	$\tilde{\mu}$	$\tilde{d}$	$\tilde{\lambda}$	$\tilde{\beta}$	$\tilde{\tau}$	$\tilde{\theta}$	$\tilde{\theta}^*$	$\hat{B}$ $\times 10^{-4}$
[3,4,5,6]	[2,3,4,5 	[0.5,1.0,1.5,2. 0]	[1,2,3,4 	[1,2,3,4 	[4,5,6,7 	[3,4,5,6 	[0.5,1.0,1.5,2. 0]	[3,4,5,6 	2284 0
[8,9,10,11]	[2,3,4,5 	[0.5,1.0,1.5,2. 0]	[1,2,3,4 	[1,2,3,4 	[4,5,6,7 	[3,4,5,6 	[0.5,1.0,1.5,2. 0]	[3,4,5,6 	8210
[13,14,15,16 	[2,3,4,5 	[0.5,1.0,1.5,2. 0]	[1,2,3,4 	[1,2,3,4 	[4,5,6,7 	[3,4,5,6 	[0.5,1.0,1.5,2. 0]	[3,4,5,6 	5180
[18,19,20,21 	[2,3,4,5 	[0.5,1.0,1.5,2. 0]	[1,2,3,4 	[1,2,3,4 	[4,5,6,7 	[3,4,5,6 	[0.5,1.0,1.5,2. 0]	[3,4,5,6 	3800

Table 2.1 determines that if the values of the holding cost $\tilde{h}$ increased then the optimum base stock is decreased.

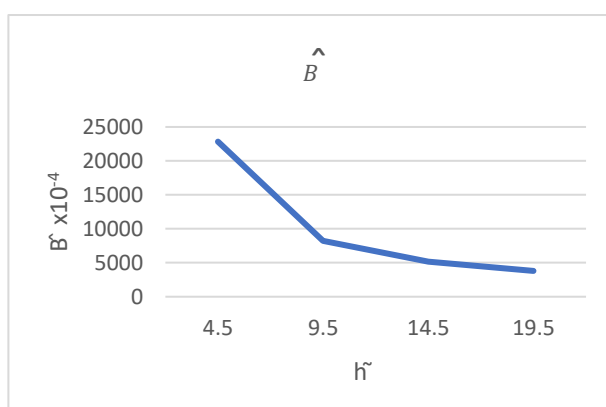


Fig.2.1: The variations in $\hat{B}$ for the changes in the value of $\tilde{h}$

From the graph, it is observed that if the values of the holding cost $\tilde{h}$ increased then the optimum base stock is decreased.

Illustration 2.2

For $\tilde{h}=[3,4,5,6], \tilde{\lambda}=[1,2,3,4], \tilde{\beta}=[0.2,0.5,0.8,1.1], \tilde{\tau}=[0.5,1.0,1.5,2.0], \tilde{\delta}=[0.5,1.5,2.5,3.5],$

$\tilde{\theta} = [0.2,0.5,0.8,1.1], \tilde{\theta}^*=[0.5,1.0,1.5,2.0], \tilde{\mu}=[0.5,1.0,1.5,2.0]$, the optimal value of $\hat{B}$ is obtained and the variations in $\hat{B}$ for the changes in the value of $\tilde{d}$ are listed.

Now the ranking index of $\tilde{d}$

$$R(\tilde{d}) = R(1,2,3,4) = \frac{1}{4}(1 + 2 + 3 + 4) = 2.5, R(\tilde{d}) = R(3,4,5,6) = 4.5,$$

$$R(\tilde{d}) = R(5,6,7,8) = 6.5, R(\tilde{d}) = R(7,8,9,10) = 8.5$$

By using the above de-fuzzified numbers for the stock holding cost and similarly the de-fuzzified values of all other parameters, the optimal base stock $\hat{B}$ is obtained by solving the equation (1) and presented in the table below.

Table 2.2: The variations in $\hat{B}$ for the changes in the value of $\tilde{d}$

$\tilde{d}$	$\tilde{\delta}$	$\tilde{\mu}$	$\tilde{h}$	$\tilde{\lambda}$	$\tilde{\beta}$	$\tilde{\tau}$	$\tilde{\theta}$	$\tilde{\theta}^*$	$\hat{B} \times 10^{-4}$
[1,2,3,4]	[2,3,4,5] 1	[0.5,1.0,1.5,2.0] 1	[3,4,5,6] 1	[1,2,3,4] 1	[4,5,6,7] 1	[3,4,5,6] 1	[0.5,1.0,1.5,2.0] 1	[3,4,5,6] 1	17200
[3,4,5,6]	[2,3,4,5] 1	[0.5,1.0,1.5,2.0] 1	[3,4,5,6] 1	[1,2,3,4] 1	[4,5,6,7] 1	[3,4,5,6] 1	[0.5,1.0,1.5,2.0] 1	[3,4,5,6] 1	44090
[5,6,7,8]	[2,3,4,5] 1	[0.5,1.0,1.5,2.0] 1	[3,4,5,6] 1	[1,2,3,4] 1	[4,5,6,7] 1	[3,4,5,6] 1	[0.5,1.0,1.5,2.0] 1	[3,4,5,6] 1	10999 0
[7,8,9,10] 1	[2,3,4,5] 1	[0.5,1.0,1.5,2.0] 1	[3,4,5,6] 1	[1,2,3,4] 1	[4,5,6,7] 1	[3,4,5,6] 1	[0.5,1.0,1.5,2.0] 1	[3,4,5,6] 1	14998 0

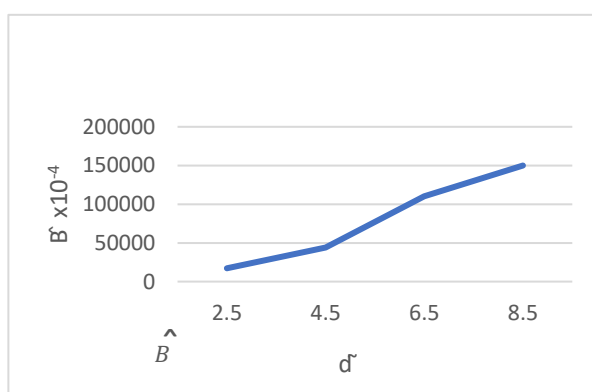


Fig 2.2: The variations in $\hat{B}$ for the changes in the value of $\tilde{d}$

From the graph, it is observed that if the values of the shortage cost $\tilde{d}$ increases, then the optimum base stock is increased.

Illustration 2.3

For $\tilde{h}=[3,4,5,6]$, $\tilde{d}=[1,2,3,4]$, $\tilde{\beta}=[0.2,0.5,0.8,1.1]$, $\tilde{\tau}=[0.5,1.0,1.5,2.0]$, $\tilde{\delta}=[0.5,1.5,2.5,3.5]$, $\tilde{\theta} = [0.2,0.5,0.8,1.1]$, $\tilde{\theta}^*=[0.5,1.0,1.5,2.0]$, $\tilde{\mu}=[0.5,1.0,1.5,2.0]$, the optimal value of $\hat{B}$ is obtained and the variations in $\hat{B}$ for the changes in the value of $\tilde{\lambda}$ are listed.

Now the ranking index of $\tilde{\lambda}$

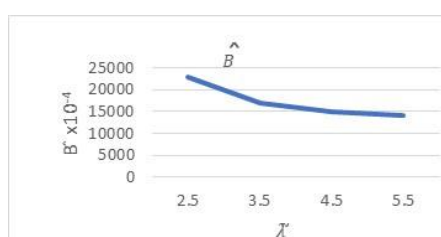
$$R(\tilde{\lambda}) = R(1,2,3,4) = \frac{1}{4}(1 + 2 + 3 + 4) = 2.5, \quad R(\tilde{\lambda}) = R(2,3,4,5) = 3.5,$$

$$R(\tilde{\lambda}) = R(3,4,5,6) = 4.5, \quad R(\tilde{\lambda}) = R(4,5,6,7) = 5.5$$

By using the above de-fuzzified numbers for the stock holding cost and similarly the de-fuzzified values of all other parameters, the optimal base stock $\hat{B}$ is obtained by solving the equation (1) and presented in the table below.

Table 2.3: The variations in $\hat{B}$ for the changes in the value of $\tilde{d}$

$\tilde{\lambda}$	$\tilde{\delta}$	$\tilde{\mu}$	$\tilde{h}$	$\tilde{d}$	$\tilde{\beta}$	$\tilde{\tau}$	$\tilde{\theta}$	$\tilde{\theta}^*$	$\hat{B}$ $\times 10^{-4}$
[1,2,3,4] 	[2,3,4,5] 	[0.5,1.0,1.5,2.0] 	[3,4,5,6] 	[1,2,3,4] 	[4,5,6,7] 	[3,4,5,6] 	[0.5,1.0,1.5,2.0] 	[3,4,5,6] 	2284 0
[2,3,4,5] 	[2,3,4,5] 	[0.5,1.0,1.5,2.0] 	[3,4,5,6] 	[1,2,3,4] 	[4,5,6,7] 	[3,4,5,6] 	[0.5,1.0,1.5,2.0] 	[3,4,5,6] 	1686 0
[3,4,5,6] 	[2,3,4,5] 	[0.5,1.0,1.5,2.0] 	[3,4,5,6] 	[1,2,3,4] 	[4,5,6,7] 	[3,4,5,6] 	[0.5,1.0,1.5,2.0] 	[3,4,5,6] 	1486 0
[4,5,6,7] 	[2,3,4,5] 	[0.5,1.0,1.5,2.0] 	[3,4,5,6] 	[1,2,3,4] 	[4,5,6,7] 	[3,4,5,6] 	[0.5,1.0,1.5,2.0] 	[3,4,5,6] 	1399 0

**Fig 2.3:** Variations in $\hat{B}$ for the changes in the value of $\tilde{\lambda}$

From the graph, it is observed that if the values of $\tilde{\lambda}$ increased then the optimum base stock is reduced.

Illustration 2.4

For $\tilde{d}=[1,2,3,4]$, $\tilde{\lambda}=[1,2,3,4]$, $\tilde{\beta}=[0.2,0.5,0.8,1.1]$, $\tilde{\tau}=[0.5,1.0,1.5,2.0]$, $\tilde{\delta}=[0.5,1.5,2.5,3.5]$,

$\tilde{\theta} = [0.2,0.5,0.8,1.1]$, $\tilde{\theta}^*=[0.5,1.0,1.5,2.0]$, $\tilde{\mu}=[0.5,1.0,1.5,2.0]$, the optimal value of $\hat{B}$ is obtained and the variations in $\hat{B}$ for the changes in the value of $\tilde{\mu}$ are listed. Now the ranking index of $\tilde{\mu}$ is

$$R(\tilde{\mu}) = R(0.5,1.0,1.5,2.0) = \frac{1}{4}(0.5 + 1.0 + 1.5 + 2.0) = 1.25, \quad R(\tilde{\mu}) = R(1.0,1.5,2.0,2.5) = 1.75,$$

$$R(\tilde{\mu}) = R(1.5,2.0,2.5,3.0) = 2.25, \quad R(\tilde{\mu}) = R(2.0,2.5,3.0,3.5) = 2.75$$

By using the above de-fuzzified numbers for the stock holding cost and similarly the de-fuzzified values of all other parameters, the optimal base stock $\hat{B}$ is obtained by solving the equation (1) and presented in the table below.

Table 2.4: The variations in $\hat{B}$ for the changes in the value of $\tilde{\mu}$

$\tilde{\mu}$	$\tilde{\delta}$	$\tilde{h}$	$\tilde{d}$	$\tilde{\lambda}$	$\tilde{\beta}$	$\tilde{\tau}$	$\tilde{\theta}$	$\tilde{\theta}^*$	$\hat{B}$ $\times 10^{-4}$
[0.5,1.0,1.5,2.0] 	[2,3,4,5] 	[3,4,5,6] 	[1,2,3,4] 	[1,2,3,4] 	[4,5,6,7] 	[3,4,5,6] 	[0.5,1.0,1.5,2.0] 	[3,4,5,6] 	2284 0
[1.0,1.5,2.0,2.5] 	[2,3,4,5] 	[3,4,5,6] 	[1,2,3,4] 	[1,2,3,4] 	[4,5,6,7] 	[3,4,5,6] 	[0.5,1.0,1.5,2.0] 	[3,4,5,6] 	1641 0
[1.5,2.0,2.5,3.0] 	[2,3,4,5] 	[3,4,5,6] 	[1,2,3,4] 	[1,2,3,4] 	[4,5,6,7] 	[3,4,5,6] 	[0.5,1.0,1.5,2.0] 	[3,4,5,6] 	1272 0
[2.0,2.5,3.0,3.5] 	[2,3,4,5] 	[3,4,5,6] 	[1,2,3,4] 	[1,2,3,4] 	[4,5,6,7] 	[3,4,5,6] 	[0.5,1.0,1.5,2.0] 	[3,4,5,6] 	1039 0

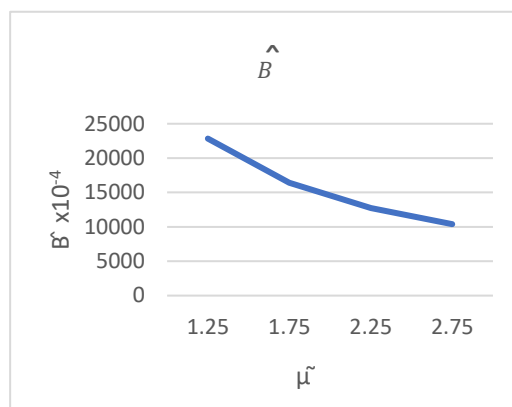


Fig 2.4: The variations in $\hat{B}$ for the changes in the value of $\tilde{\mu}$

From the graph, it is observed that if the values of $\tilde{\mu}$ increased then the optimum base stock is decreased.

6. Findings and Conclusions

From the tables and graphs, it is observed that the performance of base stock is verified under fuzzy environments and it is found that, as the holding cost $\tilde{h}$ increases, the optimal Base Stock $\hat{B}$ decreases. As the shortage cost $\tilde{d}$ increases, the optimal Base Stock $\hat{B}$ increases. As $\tilde{\lambda}$, the parameter of inter arrival time distribution increases, the optimal Base Stock $\hat{B}$ decreases. As $\tilde{\mu}$, the parameter of demand distribution increases, the optimal Base Stock $\hat{B}$ decreases. Hence the performance of optimal base stock system for tolerant clients in fuzzy environment is verified by using our proposed ranking procedures. It is observed that the findings of both methods are reflecting the same as in Fuzzy Environment. Based on these results, the manager can hold of the supreme conclusion in the sense that the maintenance of optimal base stock under fuzzy environment. We come to a head that our preferred ordering procedure is a best representative and this paper furnishes feasible facts for future stock management.

References

- [1] Baimei Yang, Lihui Sui & Peipei Zhu (2014), 'Research on optimal policy of single-period inventory management with two suppliers', The scientific World Journal, Vol. 2014, pp. 1-5.
- [2] Campos L, Gonzalez A (1989) A subjective approach for ranking fuzzy numbers. Fuzzy Sets and Systems 29(2): 145
- [3] Cheng C H (1998) A new approach for ranking fuzzy numbers by distance method. Fuzzy Sets and Systems 95(3): 307
- [4] Chu T C, Tsao C T (2002) Ranking fuzzy numbers with an area between the centroid point and original point. Computers and Mathematics with Applications 43(1-2): 111
- [5] Chen S J, Chen S M (2007) Fuzzy risk analysis based on the ranking of generalized trapezoidal fuzzy numbers. Applied Intelligence 26(1): 1
- [6] Chung-Ho Chen (2010), 'Optimum production run length and process mean setting', Tamkang, Journal of Science and Engineering, vol. 13, no. 2, pp. 157-164
- [7] Deng Y, Liu Q (2005) A TOPSIS-based centroid-index ranking method of fuzzy numbers and its applications in decision making. Cybernetics and Systems 36(6): 581
- [8] Gaver, DP (1959), 'On Base Stock Level Inventory Control', Operations Res, vol. 7, pp. 689- 703
- [9] Hanssmann, F (1962), 'Operations Research in Production and inventory control', John Wiley and sons, Inc, New York.
- [10] Henry L., Selvaraj C & Sachithanantham. S., (2012). A Model for optimal Base stock System for patient clients with lead time satisfies SCBZ property, bulletin of pure and Applied sciences, Vol. 31.E (No. 1), PP 119-128.

- [11] Henry, L, Ramathilakam, S & Sachithanantham, S (2016), 'Optimal reserve inventory between two machines with repair time having SCBZ property –reference to truncation point of the repair time', *Acta Ciencia Indica*, vol. XLII M, no. 3, pp. 227-236.
- [12] Henry, L, Selvaraj, C & Sachithanantham, S (2011), 'A model for base stock system for patient customers with lead time having a change of distribution', *International journal of Ultra Scientist For Physical Sciences*, vol. 23, no. 3, pp. 783-789.
- [13] Henry, L, Selvaraj, C & Sachithanantham, S (2012), 'A model for optimal base stock System for patient customers with lead time satisfying SCBZ property', *Bulletin of Pure and Applied Sciences*, Vol. 31E, no. 1, pp. 119-128.
- [14] Kaufmann. A, (1975)., *Introduction to the Theory of Fuzzy Subsets*, Volume 1, New York: Academic Press.
- [15] Klir G. J and Yuwan B (2005) , "Fuzzy Sets and Fuzzy Logic Theory and Applications", Prentice Hall of India.
- [16] Modarres M, SadiNezhad S (2001) Ranking fuzzy numbers by preference ratio. *Fuzzy Sets and Systems* 118(3): 429.
- [17] Raja Rao, B & Talwalker, S (1990), 'Setting the Clock Back to Zero property of a Family of life Distributions', *Journal of Statistical Planning and inference*, Vol.24,pp. 347-352.
- [18] Ramanarayanan. R, Ramachandran. V. and Sathiyamoorthi. R, (1998). Base Stock system for patient clients when inter arrival times of demands are dependent. *ASR*. Vol. 2. No.1, PP 1-6.
- [19] Ramathilagam. S, Henry. L and Sachithanantham. S (2014): A model for optimal Reserve stock between two machines in series with repair time undergoes a parametric change. *Ultra- scientist*, Vol.26 (3) P:227-237.
- [20] Ramesh R and Hari Ganesh A., (2019)"M/M/1/N Fuzzy Queueing Models with Discouraged Arrivals under Wingspans Fuzzy Ranking Method", *International Journal of Applied Engineering Research*, Volume 14, Number 4.
- [21] Ramesh. R, Seenivasan. M, Performance measures of two heterogeneous servers queueing models under triangular fuzzy trapezoidal approach, *Malaya Journal of Matematika*, Vol. 5, No. 1, 392-396, 2020.
- [22] Sachithanantham. S, Ganesan.V and sathiyamoorthi. R., (2006) Optimal Reserves for Two Machines with Repair time having SCBZ Property. *Bulletin of Pure and Applied Sciences*, Vol.25E(No.2), PP.287-297.
- [23] Sachithanantham. S, Ganesan.V and Sathiyamoorthi. R, (2007) A model for Optimal Reserve stock Between Two Machines in Series. *Journal of Indian Acad. Math*. Vol.29.No.1 pp 59-70.
- [24] Sachithanantham.S, Ganesan.V and Sathiyamoorthi. R., (2008). A model of Base Stock System for patient clients with lead time distribution undergoing parametric change, *Journal of Ultra scientist of physical sciences*. Vol.20(No.3) Dec 2008.
- [25] Shirajul Islam Ukil & Sharif Uddin, MD (2016), 'A production inventory model of constant production rate demand of level dependent linear trend', *American Journal of Operations Research*, Vol. 6, pp. 61-70.
- [26] Suresh Kumar R.(2006), Shock Model when the threshold has a change of Distribution after a change point. *Journal of Indian Acada Math*, Vol.28, (No.1):pp.73-84.
- [27] Syamala. P, Balasubramanian K.R, "Distance in intuitionistic fuzzy graphs", *The International Journal of Analytical and Experimental Modal Analysis*, Vol. - XI, Issue -X, 349-364, 2019.
- [28] Syamala. P, Balasubramanian K.R, "On lexicographic products of two intuitionistic fuzzy graphs", *J. Math. Comput. Sci.*, Volume -10, Number - 1, 136-149, 2019.
- [29] Syamala. P, Balasubramanian K.R, "Perfectly Regular and Perfectly Edge-Regular Intuitionistic Fuzzy Graphs", *International Journal of Engineering and Advanced Technology*, Volume-9, Issue-1, 2019.
- [30] Syamala. P, Ramesh. R, Seenivasan. M and Singaravel. R, "3D Based CT Scan Retrieval Queueing Models by Fuzzy Ordering Approach", *Second International Conference on Electrical, Electronics, Information and Communication Technologies (ICEEICT)*, IEEE Explore pp.01-05, doi: 10.1109/ICEEICT56924.2023.10157305, 2023.
- [31] Wang Y J, Lee H S (2008) The revised method of ranking fuzzy numbers with an area between the centroid and original points. *Computers and Mathematics with Applications* 55(9).
- [32] Yager R R (1981) A procedure for ordering fuzzy subsets of the unit interval. *Informational Sciences* 24(2): 143.
- [33] Zadeh L.A, (1965) Fuzzy sets, *Information and control*. Vol.8 pp.338-353.
- [34] Zadeh L. A, (1978), Fuzzy sets as a basis for a theory of possibility, *fuzzy sets and systems* 1, 3-28.