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A Common Fixed Point Result in Menger Space

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Abstract:

By using compatibility condition type (P) in probabilistic metric space, establish common fixed point results for four self-mappings with control function in [0,1]. The result of Chaudhary et. al [5] is a particular case of this new result and it extends and generalizes other similar results in the literature.

Keywords: Common fixed point, Menger Space, Compatible mappings, Compatible mappings of type (P).

Mathematics Subject Classification: 47H10, 54H25

1. Introduction:

Probabilistic metric space (PM space) is the idea of Karl's Menger [11], a significant generalization of M. Frechet's [3] metric space. If PM space includes Menger inequality, then it is called Menger space. This space becomes active after the significant work of B. Schweizer and A. Skalar [13], [16] and V.M. Sehgal and A.T. Barucha Reid [14].

In 1991, S. N. Mishra [12] introduced the notion of compatible mapping in the Menger space and then so many researchers worked in this space, defining weakly compatible mappings, different compatible mappings types like (A), (K), (P) etc. see references [[2], [6], [7], [8], [9], [10], [14], [16]]. Recently, Chaudhary et. al [5-6] have given notions of compatible mapping of type (P) and weakly compatible mappings of type (P).

This paper gives the new results in Menger space by using a control function $\varphi: [0,1] \rightarrow [0,1]$ in four self-mappings and also deduces some consequences.

2. Preliminaries:

Definition 2.1 [16]: If a function $M: \mathbb{R} \rightarrow \mathbb{R}^+$ is

(i) a non-decreasing function,
(ii) left continuous and
(iii) $\inf\{F(x): x \in \mathbb{R}\} = 0$, $\sup\{F(x): x \in \mathbb{R}\} = 1$

then M is said to be a ***distribution function***.

Definition 2.2 [4]: Let $M: Y \times Y \rightarrow L$ be a distribution function, L be the set of all distribution functions and Y be a non-empty set. Then, a pair (Y, M) is said to be ***probabilistic metric space*** (abbreviated as pm-Space) if the distribution function $M(p, q)$, where $(p, q) \in Y \times Y$, also denoted by $M_{p,q}$ satisfies following conditions:

(M1) $M_{p,q}(x) = 1$ for every $x > 0$ if and only if $p = q$,

(M2) $M_{p,q}(0) = 0$ for every $p, q \in K$,

(M3) $M_{p,q}(x) = M_{q,p}(x)$ for every $p, q \in K$, and

(M4) $M_{p,q}(x + y) = 1$ if and only if $M_{p,r}(x) = 1$ and $M_{r,q}(y) = 1$.

Here, $M_{p,q}(x)$ represents the value of distribution function $M_{p,q}\ at\ x \in \mathbb{R}$.

Definition 2.3 [4]: A function $t : [0,1] \times [0,1] \to [0,1]$ is referred to as a ***triangular norm*** (shortly t-norm)

if it satisfies the following conditions:

T1: $t\,(0,0) = 0$,

T2: $t\,(a,1) = a$ for all $a \in [0,1]$,

T3: $t\,(a,b) = t\,(b,a)$ for all $a, b \in [0,1]$,

T4: $if\ a \le c, b \le d$ then $t\,(a,b) \le t\,(c,d)$, and

T5: $t\,(t\,(a,b),c) = t\,\big(a, t\,(b,c)\big)$, where $a, b, c, d \in [0,1]$.

Definition 2.4 [2]: ***A*** probabilistic metric space (Y, M) is said to be Menger space (Y, M, t),

where t is a t-norm satisfying the following conditions:

(M5) $M_{p,q}(x + y) \ge t\left(M_{p,r}(x), M_{r,q}(y)\right)$ for every $p, q, r \in Y$ and $x, y \in \mathbb{R} > 0$.

Definition 2.5 [2]: A mapping $A: Y \to Y$ in Menger Space (K, F, t), is said to be ***continuous***

at a point $p \in Y$ if for every $\varepsilon > 0$ and $\lambda > 0$, there exist $\varepsilon_1 > 0$ and $\lambda_1 > 0$ such that

if $M_{p,q}\,(\varepsilon_1) > 1 - \lambda_1$ then $M_{Ap,Aq}(\varepsilon) > 1 - \lambda$.

Definition 2.6 [2]: Let (Y, M, t) be a Menger space and t be a continuous t-norm. Then,

(a) A sequence $\{y_n\}$ in Y is said to ***converge*** to a point y in Y if and only if for every $\varepsilon > 0$ and $\lambda > 0$, there exist an integer $N = N\,(\varepsilon, \lambda)$ such that $M_{y_n,y}(\varepsilon) > 1 - \lambda$ for all $n \ge N$. In this case, we write, $\lim\limits_{n\to\infty} y_n = y$.

(b) A sequence $\{y_n\}$ in Y is said to be a ***Cauchy sequence*** if for every $\varepsilon > 0$ and $\lambda > 0$, there exists an integer $N = N\,(\varepsilon, \lambda) > 0$ such that $M_{y_n,y_m}(\varepsilon) > 1 - \lambda$ for all $m, n \ge N$.

(c) A Menger space (Y, M, t) is said to be ***complete*** if every Cauchy sequence in Y converges to a point in Y.

Definition 2.7:[7] Common fixed point of self-mapping functions $A, B: Y \to Y$ is a point $y \in Y$ if

$$A(y) = B(y) = y.$$

Example 2.1: Let $A, B: \mathbb{R} \to \mathbb{R}$ be functions such that $A(y) = \frac{y^2}{4}$ and $B(y) = 2y - 4$,

then $y = 4$ is a common fixed point of A and B.

Definition 2.8:[12] Two mappings $A, B: Y \to Y$ are said to be ***compatible mappings*** in Menger space (Y, M, t) iff $\lim_{n\to\infty} F_{ABx_n, BAx_n}(x) = 1$ for all $x > 0$, whenever sequence $\{x_n\}$ in Y such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = y$ for some y in Y.

Definition 2.10: [15] Two mappings $A, B: Y \to Y$ are said to be ***weakly compatible (or coincidently commuting)*** in Menger space (Y, F, t) if they commute at their coincidence points, that is, if $Ax = Bx$ for some $x \in Y$ then $ABx = BAx$.

Definition 2.11:[6] Two mappings $A, B: Y \to Y$ are said to be ***compatible mappings of type (P)*** in Menger space (Y, M, t) iff $\lim_{n\to\infty} M_{AAx_n, BBx_n}(x) = 1 \; \forall \; x > 0$ whenever $\{x_n\}$ is a sequence in Y such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = y$ for some $y \; in \; Y$.

Definition 2.12: [5]Two mappings $A, B: Y \to Y$ are said to be ***weakly compatible mapping of type(P)*** in Menger Space (Y, M, t) iff $\lim_{n\to\infty} M_{AAx_n, BBx_n}(x) \geq M_{Ax_n, Bx_n}(x) \; \forall \; x > 0,$ whenever $\{x_n\}$ is a sequence in Y such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = y$ for some $y \; in \; Y$.

Example 2.2: Let (Y, d) be metric space where $Y = [0, 2]$with usual metric $d(x, y) = |x - y|$ and (Y, M) be PM space with

$$M_{x,y}(t) = \begin{cases} e^{\frac{d(x,y)}{t}}, & if \; t > 0, \\ 0, & if \; t = 0. \end{cases} \quad \text{for all } x, y \in Y.$$

We define A and B as:

$$A(x) = \begin{cases} 1 - x, & for \;\; x \in [0, 1/2) \\ 1, & for \; x \in [\frac{1}{2}, 2] \end{cases} \text{ and } B(x) = \begin{cases} x, & for \;\; x \in [0, 1/2) \\ 1, & for \; x \in \left[\frac{1}{2}, 2\right]. \end{cases}$$

Taking sequence $\{x_n\}$ in Y where $x_n = \frac{1}{2} - \frac{1}{n}$, $n \in N$. Then, (A, B) are weakly compatible mappings of type (P) and it is neither compatible mappings of type (P) nor compatible mappings.

Theorem 2.1[2]: Let (Y, M, t) be Menger space with the continuous $t - norm \; t$ and $A: Y \to Y$. Then, A is continuous at a point $y \in Y$ if and only if for every sequence $\{y_n\}$ in Y converging to a point y, then sequence $\{Ay_n\}$ converges to the point Ay, i.e. if $\{y_n\} \to y$ then it implies $\{Ay_n\} \to Ay$.

Proposition 2.1[9]: In Menger Space(Y, M, t), if $t\,(k, k) \geq k$ for all $k \in [0, 1]$ then $t(a, b) = min\,\{a, b\}$ for all $a, b \in [0, 1]$.

Lemma 2.1[15]: Let (Y, M, t) be a Menger space. If there exists $k \in (0, 1)$ such that for all $p, q \in Y$, $M_{p,q}(kx) \geq M_{p,q}(x)$ then $p = q$.

Proposition 2.2:[5] Let (Y, M, t) be a Menger space such that the t-norm t is continuous and $t\,(x, x) \geq x$ for all $x \in [0, 1]$ and $A, B: Y \to Y$ be continuous mappings. Then, A and B are weakly compatible mappings of type (P) if they are compatible mappings of type(P).

Proposition 2.3: [5]Let (Y, M, t) be a Menger space such that the t-norm t is continuous and $t\ (x, x)\ \geq x$ for all $x \in [0, 1]$ and $A, B: Y \to Y$ be continuous mappings. Then, A and B are compatible mappings of type (P) if they are weakly compatible mappings of type (P).

Proposition 2.4: [5]Let (Y, M, t) be a Menger space such that the t-norm t is continuous and $t\ (x, x)\ \geq x$ for all $x \in [0, 1]$ and $A, B: Y\ \to Y$ be mappings. If A and B are weakly compatible mappings of type (P) and $Ak\ =\ Bk$ for some $k\ \in K$, then, $AAk\ =\ ABk\ =\ BAk =\ BBk$.

Proposition 2.5:[5] Let (Y, M, t) be a Menger space such that the t-norm t is continuous and $t\ (x, x)\ \geq x$ for all $x \in [0, 1]$ and $A, B: Y\ \to\ Y$ be mappings. Let A and B be weakly compatible mappings of type (P) and $\lim_{n\to\infty} \mathrm{A}k_n = \lim_{n\to\infty} \mathrm{B}k_n =$ k for some $k \in Y$. Then

We have,

(i) $\lim_{n\to\infty} \mathrm{BB}k_n =$ Ak $\ if\ A\ is\ continuous\ at\ k,$

(ii) $\lim_{n\to\infty} \mathrm{AA}k_n =$ Bk $\ if\ B\ is\ continuous\ at\ k,$

(iii) $ABk\ =\ BAk\ and\ Ak\ =\ Bk\ if\ A\ and\ B\ are\ continuous\ at\ k.$

The following lemma needs to prove the main theorem:

Lemma 2.2[15]: Let $\{x_n\}$ be a sequence in Menger space (Y, M, t), where t is continuous t −norm and $t\ (x, x)\ \geq\ x$ for all $x\ \in [0, 1]$. If there exists a constant $k\ \in\ [0, 1]$ such that

$$M_{x_{n}, x_{n+1}}\ (kx)\ \geq M_{x_{n-1}, x_n}(x) \text{ for all } x\ >\ 0 \text{ and } n\ \in N,$$

then $\{x_n\}$ is a Cauchy sequence in Y.

3. Main Theorem:

Now, we prove our main theorem for weakly compatible mappings of type (P) in complete Menger space:

Theorem 3.1: Let (Y, M, t) be a complete Menger space with $t\ (x, y)\ =\ min\ \{x, y\}$ for all $x, y\ \in\ [0, 1]\ and\ A, B, S, T: Y\ \to Y$ be mappings such that

(3.1.1) $A\ (Y) \subset T\ (Y) and\ \ B\ (Y) \subset\ S\ (Y),$

(3.1.2) the pairs (A, S) and (B, T) are weakly compatible mappings of type (P),

(3.1.3) One of $A,\ S,\ B,\ T$ be continuous, and

(3.1.4) there exists a constant $\xi\ \in\ (0, 1)$ such that

$$M(Ax, By, \xi\, q) \geq \varphi\{\min\{M(Sx, Ax, q), M(Ty, By, q), M(\,Ty, Ax, rq),$$
$$M(Sx, By(2-r)q, M(Sx, Ty, q)\}\}$$

for all $x, y\ \in Y, r\ \in\ (0, 2)$ and $q\ >\ 0,\ where$ φ: [0,1] → [0,1] satisfies

(i) φ is continuous and non-decreasing on [0,1]

(ii) φ(n) > n for all n in [0,1]

noting that if $\varphi \in \Phi$, class of all mappings $\varphi: [0,1] \to [0,1]$ then $\varphi(0) = 0, \varphi(1) = 1$ and $\varphi(n) \geq n$ for all n in [0,1].

Then, A, B, S, T have a unique common fixed point in Y.

Proof: Consider $u_0 \in Y$. Since $A(Y) \subset T(Y)$, so there exists a point $u_1 in\, Y$ such that $Au_0 = Tu_1 = v_0$. Again, since $B(Y) \subset S(Y)$, so for u_1, we may choose u_2 in Y such that $Bu_1 = Su_2 = v_1$ and so on.

And inductively, we may construct sequence $\{u_n\}$ and $\{v_n\}$ in Y such that

$$Au_{2n} = Tu_{2n+1} = v_{2n} \text{, and } Bu_{2n+1} = Su_{2n+2} = v_{2n+1}, \quad for \quad n = 0, 1, 2, \ldots$$

Putting $x = u_{2n}$ and $y = u_{2n+1}$ $for\ all\ q > 0\ and\ r = 1 - p\ with\ p \epsilon (0,1)\ in$ (3.1.4), we get

$M(Au_{2n}, Bu_{2n+1}, \xi q) \geq \varphi\{\min\{M(Su_{2n}, Au_{2n}, q), M(Tu_{2n+1}, Bu_{2n+1}, q),$
$M(Tu_{2n+1}, Au_{2n}, ((1-p))q), M(Su_{2n}, Bu_{2n+1}, ((1+p)q), M(Su_{2n}, Tu_{2n+1}, q)\}\}$

or, $M(v_{2n}, v_{2n+1}, \xi q) \geq$

$\varphi\{\min\{M(v_{2n-1}, v_{2n}, q), M(v_{2n}, v_{2n+1}, q), M(v_{2n}, v_{2n}, ((1-p))q), M(v_{2n-1}, v_{2n+1}, ((1+p)q), M(v_{2n-1}, v_{2n}, q)\}\}$

$\geq \varphi\{\min\{M(v_{2n-1}, v_{2n}, q), M(v_{2n}, v_{2n+1}, q), M(v_{2n-1}, v_{2n+1}, ((1+p)q), M(v_{2n-1}, v_{2n}, q)\}\}$

$\geq \varphi\{\min\{M(v_{2n-1}, v_{2n}, q), M(v_{2n}, v_{2n+1}, q), M(v_{2n-1}, v_{2n}, q), M(v_{2n}, v_{2n+1}, pq),$
$M(v_{2n-1}, v_{2n}, q)\}\}$ $\geq \varphi\{\min\{M(v_{2n-1}, v_{2n}, q), M(v_{2n}, v_{2n+1}, q), M(v_{2n}, v_{2n+1}, pq)\}\}$

As $p \to 1$, we obtain

$$M(v_{2n}, v_{2n+1}, \xi q) \geq \varphi\{\min\{M(v_{2n-1}, v_{2n}, q), M(v_{2n}, v_{2n+1}, q), M(v_{2n}, v_{2n+1}, q)\}\}$$
$$\geq \varphi\{\min\{M(v_{2n-1}, v_{2n}, q), M(v_{2n}, v_{2n+1}, q)\}$$

Or, $M(v_{2n}, v_{2n+1}, \xi q) \geq \varphi\{M(v_{2n-1}, v_{2n}, q)\} > M(v_{2n-1}, v_{2n}, q)$, by property of φ

Hence, we get $M(v_{2n}, v_{2n+1}, \xi q) \geq M(v_{2n-1}, v_{2n}, q)$

Similarly, we obtain

$$M(v_{2n+1}, v_{2n+2}, \xi q) \geq M(v_{2n}, v_{2n+1}, q)$$

Therefore, for every $n \in N$, $M(v_n, v_{n+1}, \xi q) \geq M(v_{n-1}, v_n, q)$

So, using Lemma (2.2), $\{v_n\}$ is a Cauchy sequence in K.

Since the Menger space (Y, M, t) is complete, so $\{v_n\}$ converges to a point z in Y and consequently the subsequences $\{A_{u_{2n}}\}$, $\{B_{u_{2n+1}}\}$, $\{S_{u_{2n}}\}$, $\{T_{u_{2n+1}}\}$of $\{v_n\}$ also converges to z.

Now, suppose that T is continuous. Then, since B & T are weakly compatible mappings of type (P) then by proposition 2.5, $BB_{u_{2n+1}}, TB_{u_{2n+1}} \to Tz\ as\ n \to \infty$. Putting $x = u_{2n}$ and $y = B_{u_{2n+1}}$ in relation (3.1.4) , we get

$M(Au_{2n}, BBu_{2n+1,}\ \ \xi q) \geq$
φ$\{\min\{M(Su_{2n}, Au_{2n,}\ \ q), M(TBu_{2n+1}, BBu_{2n+1,}\ \ q), M(TBu_{2n+1}, Au_{2n}\ , rq), M(Su_{2n}, BBu_{2n+1,}(2-r)q), M(Su_{2n}, TBu_{2n+1,}\ \ q)\}\}$

Taking $n \to \infty$, we have

$M(z, Tz, \xi\, q) \geq$ φ$\{\min\{M(z, z, q), M(Tz, Tz, q), M(Tz, z, rq), M(z, Tz(2-r)q), M(z, Tz, q)\}\}$

Letting $r = 1-p$ with $p \in (0,1)$ then

$M(z, Tz, \xi\, q) \geq$ φ$\{\min\{\ M(Tz, z, (1-p)q), M(z, Tz(2-(1-p)q), M(z, Tz, q)\}\}$

Or, $M(z, Tz, \xi\, q) \geq$ φ$\{\min\{\ M(Tz, z, (1-p)q), M(z, Tz(1+p)q), M(z, Tz, q)\}\}$

$\geq$ φ$\{\min\{\ M(Tz, z, (1-p+1+p)q), M(z, Tz, q)\}\}$
$\geq$ φ$\{\min\{\ M(Tz, z, 2q), M(z, Tz, q)\}\}$

$\geq$ φ$\{\min\{\ M(z, Tz, q)\}\}$

Therefore, $M(z, Tz, \xi\, q) \geq$ φ$\{M(z, Tz, q)\}$

Or, $M(z, Tz, \xi\, q) \geq M(z, Tz, q)$, by property of φ

which implies $z = Tz$ by Lemma 2.1.

Similarly, replacing x by u_{2n} and $y\ by\ z$ in relation **(3.1.4),** we have

$M(Au_{2n}, Bz, \xi\, q) \geq$ φ$\{\min\{M(Su_{2n}, Au_{2n,}\ \ q), M(Tz, Bz, q),\ M(Tz, Au_{2n,}\ rq),$
$M(Su_{2n}, Bz, (2-r)q), M(Su_{2n}, Tz, q)\}\}$

Taking $n \to \infty$, we get

$M(z, Bz, \xi\, q) \geq$ φ$\{\min\{M(z, z, q), M(z, Bz, q), M(z, z, rq), M(z, Bz(2-r)q), M(z, z, q)\}\}$

$\geq$ φ$\{\min\{\ M(z, Bz, q), M(z, Bz(2-(1-p))q)\}\}$

$\geq$ φ$\{\min\{\ M(z, Bz, q), M(z, Bz(1+p))q)\}\}$

$\geq$ φ$\{\min\{\ M(z, Bz, q),\ \ M(z, z, q), M(z, Bz, pq)\}\}$

$\geq$ φ$\{\min\{\ M(z, Bz, q),\ \ M(z, z, q), M(z, Bz, pq)\}\}$

$\geq$ φ$\{\min\{\ M(z, Bz, q), M(z, Bz, q)\}\}$, as $p \to 1$

So that $M(z, Bz, \xi\, q) \geq$ φ$\{M(z, Bz, q)\}$

Or, $M(z, Bz, \xi\, q) \geq M(z, Bz, q)$, by property of φ

which implies $z = Bz$ by Lemma 2.1.

Since, $B(Y) \subset S\ (Y)$, so there exists a point w in Y such that $Bz = Sw = z$.

By using relation (3.1.4) with $x = w, y = z$, we have

$M(Aw, z, \xi\, q) \geq \varphi\{\min\{M(Sw, Aw, q), M(Tz, Bz, q), M(Tz, Az, rq), M(Sw, Bz(2-r)q, M(Sw, Tz, q)\}\}$

$\geq \varphi\{\min\{M(z, Aw, q), M(Tz, z, q), M(z, Aw, (1-p)q), M(Sw, z(1+p)q, M(z, Tz, q)\}\}$

$\geq \varphi\{\min\{M(z, Aw, q), M(Tz, z, q), M(Aw, z, (1-p)q), M(Sw, z(1+p)q, M(z, Tz, q)\}\}$

$\geq \varphi\{\min\{M(z, Aw, q), M(z, z, q), M(Aw, Sw, (1-p+1+p)q)\}\}$

$\geq \varphi\{\min\{M(z, Aw, q), M(Aw, z, 2q)\}\}$

Therefore, $M(Aw, z, \xi\, q) \geq \varphi\{M(z, Aw, q)\}$

Or, $M(Aw, z, \xi\, q) \geq M(z, Aw, q)$, by property of φ

which implies $Aw = z$ by Lemma 2.1.

Again, since A and S are weakly compatible mappings of type (P) and $Aw = Sw = z$, by proposition 2.4, we have for every $\varepsilon > 0$

$1 = M(AAw, SSw, \epsilon) \geq M(Aw, Sw, \epsilon)$

Hence $Aw = AAw = SSw = Sw$

Finally, by relation (3.1.4) with $x = z, y = Bz = z$, we have

$M(Az, z, \xi\, q) = M(Az, Bz, \xi\, q) \geq \varphi\{\min\{M(Sz, Az, q), M(Tz, z, q), M(Tz, Az, rq), M(Sz, z(2-r)q, M(Sz, Tz, q)\}$

$\geq \varphi\{\min\{M(Az, Az, q), M(z, z, q), M(z, Az, rq), M(Az, z(2-r)q, M(Az, z, q)\}$

$\geq \varphi\{\min\{M(Az, z, rq), M(z, Az(2-r)q, M(Az, z, q)\}$

$\geq \varphi\{\min\{M(Az, Az, rq + (2-r)q, M(Az, z, q)\}$

$\geq \varphi\{\min\{M(Az, z, q)\}$

$\geq \varphi\{M(Az, z, q)\}$

Or, $M(Az, z, \xi\, q) \geq M(Az, z, q)$, by property of φ
$\therefore Az = z$, by Lemma 2.1.

Hence, $Az = Bz = Sz = Tz = z$.

That is, z is a common fixed point of given mappings A, B, S & T.

Uniqueness: Suppose z_1 is another point in Y such that

$$z_1 = Az_1 = Bz_1 = Sz_1 = Tz_1.$$

Then, putting $x = z$ and $y = z_1$, $r = 1$ in (3.1.4), we get

$M(Az, Bz_1, \xi\, q) = M(z, z_1, \xi\, q) \geq \varphi\{\min\{M(Sz, Az, q), M(Tz_1, Bz_1, q), M(Tz_1, Az, q), M(Sz, Tz_1, q)\}$

Or, $M(z, z_1, \xi q) \geq \varphi\{\min\{M(z, z_1, q), M(z, z, q)\}$

Or, $M(z, z_1, \xi q) \geq \varphi\{M(z, z_1, q)\}$

$M(z, z_1, \xi q) \geq M(z, z_1, q)$, by property of φ

$\therefore z = z_1$, by Lemma 2.1.

Hence, $z = Az = Bz = Sz = Tz$, and z is a unique common fixed point for A, B, S, and T in Y.

This completes the proof.

4. Conclusion: In conclusion, the result of Chaudhary et. al. [5] is a particular case of this theorem. Also, this theorem may apply to consequences results in metric space in four self-mappings and generalizes and improves other similar results in the literature.

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