

Exploration of $\mathcal{N}\alpha\mathcal{I}g$ -Closed and $\mathcal{N}\alpha\mathcal{I}g$ -Open Sets in Nano Ideal Topological Spaces with Practical Applications of Nano Ideal Topological spaces

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Abstract: In this study, we explore the concepts of $\mathcal{N}\alpha\mathcal{I}g$ -closed sets (nano $\alpha\mathcal{I}g$ -closed sets) and $\mathcal{N}\alpha\mathcal{I}g$ -open sets (nano $\alpha\mathcal{I}g$ -open sets) in nano ideal topological spaces. We discuss their relationships with other forms of nano ideal sets and illustrate the abstract concepts using appropriate examples. This research delves into the application of $\mathcal{N}\alpha\mathcal{I}g$ -closed and $\mathcal{N}\alpha\mathcal{I}g$ -open sets across diverse domains, ranging from network security to healthcare and environmental monitoring. By leveraging these mathematical concepts, we aim to enhance anomaly detection, optimize network operations, and improve decision-making processes across various sectors. This article outlines the methodologies and potential benefits of integrating $\mathcal{N}\alpha\mathcal{I}g$ -closed and $\mathcal{N}\alpha\mathcal{I}g$ -open sets in different application domains.

Keywords: Ideals, Nanotopology, $\mathcal{N}\alpha\mathcal{I}g$ -closed sets, $\mathcal{N}\alpha\mathcal{I}g$ -open sets, nano α -open.

1. Introduction

On topological spaces, Levine [1] introduced the concept of generalized closed sets during 1970. Numerous results in general topology have been developed using this concept. In 1991, Balachandran et. al [2] introduced and examined the notion of generalized continuous functions in topological spaces. The notion of α -open sets was introduced and investigated by Njastad [3]. By using α -open, Mashbour et al. [4] defined and studied the concept of α -closed sets, α -closure of a set, α -continuity. The concept of ideal topological space was introduced by kuratowski [5] in 1966. He also defined the local functions in ideal topological spaces. Furthermore, during the period 1990, Jankovic and Hamlett [6] investigated the properties of ideal topological spaces. In 2014, $\alpha\mathcal{I}g$ -closed is introduced in ideal topological spaces.

The notion of nano topology was introduced by Lellis Thivagar [7, 8, 9] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also established and analyzed the nano forms of weakly open sets such as nano α -open sets, nano

semi-open sets and nano pre-open sets. In 2016, Bhuvaneswari et. al [10], introduced and studied the characteristic of Nano generalized closed sets.

The structure of this manuscript is as follows. In section 2, we recall some fundamental definitions and results which are useful to prove our main results. In section 3, we define and study the notion of $\text{N}\alpha\text{Ig}$ - closed sets and $\text{N}\alpha\text{Ig}$ - open sets in nano ideal topological spaces. We also discuss the concept of $\text{N}\alpha\text{Ig}$ -closed sets and discussed the relationships between the other existing nano ideal sets. In section 4, The integration of $\text{N}\alpha\text{Ig}$ -closed and $\text{N}\alpha\text{Ig}$ -open sets in nano ideal topological spaces which presents a novel approach to address complex challenges across multiple domains is discussed. This research explores the diverse applications of these mathematical concepts, highlighting their potential to revolutionize various industries.

2. Preliminaries

Throughout this study $(U, rR(x))$ (or U) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space $(U, rR(x))$, $\text{Ncl}(A)$ and $\text{Nint}(A)$ denote the nano closure of A and the nano interior of A respectively. We recall the following definition which are useful in the sequel.

Definition 2.1. [11] Let U be a non-empty finite set of objects called the universe R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

- (1) The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $LR(X)$. That is , $LR(X) = \{Ux \in U \{R(x): R(x) \subseteq X\}\}$, where $R(x)$ denotes the equivalence class determined by x .
- (2) The Upper approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $UR(X)$. That is , $UR(X) = \{Ux \in U \{R(x): R(x) \cap X \neq \emptyset\}\}$
- (3) The Boundary region of X with respect to R is the set of all objects, which can be classified as neither as X nor as not X with respect to R and it is denoted by $BR(X)$. That is, $BR(X) = UR(X) - LR(X)$

Definition 2.2. Let U be the universe, R be an equivalence relation on U and $rR(x) = \{U, \emptyset, LR(X), UR(X), BR(X)\}$ where $X \subseteq U$. Then $rR(x)$ satisfies the following axioms:

- (1) U and $\emptyset \in rR(x)$
- (2) The union of elements of any subcollection of $rR(x)$ is in $rR(x)$.
- (3) The intersection of the elements of any finite subcollection of $rR(x)$ is in $rR(x)$.

That is , $rR(x)$ forms a topology on U called the nano topology on U with respect to X . We call $\{U, rR(x)\}$ is called the nano topological space. The elements of $rR(x)$ are called as nano-open sets. The complement of the nano-open sets are called nano-closed sets.

Definition 2.3. An ideal I on a topological space is a non-empty collection of subsets of X which satisfies

$$(1) A \in I \text{ and } B \subseteq A \Rightarrow B \in I.$$

$$(2) A \in I \text{ and } B \in I \Rightarrow A \cup B \in I.$$

Definition 2.4 [13] Let (X, τ) be a topological space and I be an ideal on X . A subset A of X is said to be α Ig-closed if $A^* \subseteq U$ whenever $A \subseteq U$ and U is α -open.

Definition 2.5. [12] A nano topological space $\{U, rR(x)\}$ with an ideal I on U is called a nano ideal topological space or nano ideal space and denoted as $(U, rR(x), I)$

Definition 2.6. Let $\{U, rR(x), I\}$ be a nano ideal topological space. A set operator $(A)^{*N}: P(U) \rightarrow P(U)$ is called the nano local function of I on U with respect to I on $rR(x)$ is defined as $(A)^{*N}$

$= \{x \in U: U \cap A \notin I; \text{ for every } U \in rR(x)\}$ and is denoted by $(A)^{*N}$, where nano closure operator is defined as $Ncl^*(A) = A \cup (A)^{*N}$.

Result 2.7. Let $(U, rR(x), I)$ be a nano ideal topological space and let A and B be subsets of U , then

$$(1) (\emptyset)^{*N} = \emptyset$$

$$(2) A \subset B \rightarrow (A)^{*N} \subset (B)^{*N}$$

$$(3) \text{ For another } J \supseteq I \text{ on } U, (A)^{*N}(J) \subset (A)^{*N}(I)$$

$$(4) (A)^{*N} \subset Ncl^*(A)$$

$$(5) (A)^{*N} \text{ is a nano closed set.}$$

$$(6) ((A)^{*N})^*N \subset (A)^{*N}$$

$$(7) (A)^{*N} \cup (B)^{*N} = (A \cup B)^{*N}$$

$$(8) (A \cap B)^{*N} = (A)^{*N} \cap (B)^{*N}$$

$$(9) \text{ For every nano open set } V, V \cap (V \cap A)^{*N} \subset (V \cap A)^{*N}$$

$$(10) \text{ For } I \in I, (A \cup I)^{*N} = (A)^{*N} = (A - I)^{*N}$$

Result 2.8 . Let $\{U, rR(x), I\}$ be a nano ideal topological space and A be a subset of U , If $A \subset (A)^{*N}$, then $(A)^{*N} = Ncl(A)^{*N} = Ncl^*(A)$.

Definition 2.9. Let $\{U, rR(x)\}$ be a nano topological space and $A \subseteq U$. Then A is said to be

$$(1) \text{ Nano semi- closed, if } Nint(Ncl(A)) \subseteq A.$$

$$(2) \text{ Ng-closed, } Ncl(A) \subseteq G \text{ whenever } A \subseteq G \text{ and } G \text{ is nano open.}$$

$$(3) \text{ Ng}\hat{\text{ }}\text{-closed, } Ncl(A) \subseteq G \text{ whenever } A \subseteq G \text{ and } G \text{ is nano semi-open.}$$

$$(4) \text{ Ng}^*\text{-closed, } Ncl(A) \subseteq G \text{ whenever } A \subseteq G \text{ and } G \text{ is nano g-open.}$$

$$(5) \text{ Nano pre closed if } Ncl(Nint(A)) \subseteq A.$$

$$(6) \text{ Nano } \alpha\text{-closed set if } Ncl(Nint(Ncl(A))) \subseteq A$$

The complements of the above mentioned sets are called their respective open sets.

Definition 2.10. A subset A of a nano ideal space. Let $(U, rR(x), I)$ is said to be

- (1) $*N$ - closed , if $(A)^{*N} \subseteq A$
- (2) $*N$ - dense, if $A \subseteq (A)^{*N}$
- (3) N Ig closed, if $(A)^{*N} \subseteq G$ whenever $A \subseteq G$ and G is nano open
- (4) N Ig* closed, if $(A)^{*N} \subseteq G$ whenever $A \subseteq G$ and G is nano g- open

3. $N \alpha$ Ig- CLOSED SETS

In this section we define and study the notion of $N \alpha$ Ig-closed sets and $N \alpha$ Ig-open sets in nano ideal topological spaces. Also we discuss their basic properties and study the relationship between other existing nano closed sets in nano ideal topological spaces.

Definition 3.1. A subset A of a nano ideal space $(U, rR(x), I)$ is said to be $N \alpha$ Ig-closed if $(A)^{*N} \subseteq G$ whenever $A \subseteq G$ and G is nano α open.

Example 3.2: Let $U = \{a, b, c, d\}$, $U/R = \{a\}, \{d\}, \{b, c\}$ and $X = \{a, d\}$. Let the nano ideal space $rR(x) = \{U, \emptyset, \{a, d\}\}$ with a nano ideal $I = \{\emptyset, \{a\}\}$. Then $N \alpha$ Ig-closed sets are $\{U, \emptyset, \{a\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}\}$.

Definition 3.3. Let $(U, rR(x), I)$ be a Nano ideal topological space. A subset A of X is said to be $N \alpha$ Ig-open if $X - A$ is $N \alpha$ Ig-closed.

Theorem 3.4. If $(U, rR(x), I)$ is any nano ideal space, then the following are equivalent

- (1) A is $N \alpha$ Ig-closed
- (2) $Ncl^*(A) \subseteq G$ whenever $A \subseteq G$ and G is nano α open in U
- (3) For all $x \in Ncl^*(A)$, $N \alpha cl(\{x\}) \cap A \neq \emptyset$
- (4) $Ncl^*(A) - A$ contains no non empty nano α closed set.
- (5) $(A)^{*N} - A$ contains no nonempty nano α closed set.

Proof: (1) $\Rightarrow$ (2) : If A is $N \alpha$ Ig-closed, then $(A)^{*N} \subseteq G$ whenever $A \subseteq G$ and G is nano α open in X and so $Ncl^*(A) = A \cup (A)^{*N} \subseteq G$ and G is nano α open in U . This proves (2).

(2) $\Rightarrow$ (3) : Suppose $x \in Ncl^*(A)$. If $N \alpha cl(\{x\}) \cap A = \emptyset$, then $A \subseteq X - N \alpha cl(\{x\})$. By (2), $Ncl^*(A) \subseteq X - N \alpha cl(\{x\})$, which is a contradiction to $x \in Ncl^*(A)$. This Proves (3)

(3) $\Rightarrow$ (4) : Suppose $F \subseteq Ncl^*(A) - A$, F is nano α -closed and $x \in F$. Since $F \subseteq X - A$ and F is nano α -closed, then $A \subseteq X - F$ and hence $N \alpha cl(\{x\}) \cap A = \emptyset$. Therefore, $Ncl^*(A) - A$ contains no non empty nano α -closed set.

(4) $\Rightarrow$ (5) : Since $Ncl^*(A) - A = (A \cup (A)^{*N}) - A = (A \cup (A)^{*N}) \cap A^c = (A \cap A^c) \cup ((A)^{*N} \cap A^c) = (A)^{*N} \cap A^c = (A)^{*N} - A$. Therefore, $(A)^{*N} - A$ contains no nonempty nano α closed set.

Theorem 3.5: Every $*^N$ closed set is $N \alpha$ Ig-closed but not conversely.

Proof: Let A be a $*^N$ closed, then $(A)^{*N} \subseteq A$. Let $A \subseteq G$ and G is Nano α open. This implies $(A)^{*N} \subseteq G$. Hence A is $N \alpha$ Ig-closed.

Example 3.6: Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Let the nano ideal space $rR(x) = \{U, \emptyset, \{a\}, \{a, b, d\}, \{b, d\}\}$ with a nano ideal $I = \{\emptyset, \{a\}, \{a, b, d\}\}$. Then $N \alpha$ Ig-closed sets are $\{U, \emptyset, \{a\}, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ and $*^N$ closed set are $\{U, \emptyset, \{a\}, \{c\}, \{a, c\}, \{b, c, d\}\}$. It is clear that $\{a, b, c\}$ is $N \alpha$ Ig-closed set but it is not $*^N$ closed.

Theorem 3.7: Every NI_g^* closed set is $N \alpha$ Ig-closed. But not conversely.

Proof: Let $A \subseteq G$ and G is Nano α open. Clearly every Nano α open set is Nano semi open. Since A is NI_g^* closed set, $(A^*)^N \subseteq G$, which implies that A is an $N \alpha$ Ig-closed set.

Example 3.8: Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{d\}, \{b, c\}\}$ and $X = \{a, d\}$. Let the nano ideal space $rR(x) = \{U, \emptyset, \{a, d\}\}$ with a nano ideal $I = \{\emptyset, \{a\}\}$. Then $N \alpha$ Ig-closed sets are $\{U, \emptyset, \{a\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}\}$ and NI_g^* closed sets are $\{U, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$. It is clear that $\{b, d\}$ is $N \alpha$ Ig-closed but it is not NI_g^* closed.

Theorem 3.9. Every $N \alpha$ Ig-closed set is NI_g closed. But Converse is not true.

Proof: Let $A \subseteq G$ and G is Nano α open. Clearly every nano open set is Nano α open. Since A is $N \alpha$ Ig-closed set, $(A^*)^N \subseteq G$, which implies that A is NI_g closed.

Example 3.10. Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{d\}, \{b, c\}\}$ and $X = \{a, d\}$. Let the nano ideal space $rR(x) = \{U, \emptyset, \{a, d\}\}$ with a nano ideal $I = \{\emptyset, \{a\}\}$. Then $N \alpha$ Ig-closed sets are $\{U, \emptyset, \{a\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}\}$ and NI_g closed sets are $\{U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. It is clear that $\{b\}$ is NI_g closed but it is not NI_g -closed.

Theorem 3.11. Let $(U, rR(x), I)$ be an nano ideal space. For every $A \in I$, A is $N \alpha$ Ig-closed set.

Proof: Let $A \subseteq G$ and G is Nano α open. Since $(A^*)^N = \emptyset$ for every $A \in I$, then $(A^*)^N \subseteq A$. This implies $(A^*)^N \subseteq G$. Hence for every $A \in I$, A is an $N \alpha$ Ig-closed set.

Theorem 3.12. If A and B are $N \alpha$ Ig-closed sets in $(U, rR(x), I)$, then $A \cup B$ is also an $N \alpha$ Ig-closed set.

Proof: Let $A \cup B \subseteq G$ where G is a Nano α -open set in X . Then, $A \subseteq G$ and $B \subseteq G$. By hypothesis, A and B are two $N \alpha$ Ig-closed set. This implies $(A^*)^N \subseteq G$ and $(B^*)^N \subseteq G$. Hence $(A^*)^N \cup (B^*)^N \subseteq G$. By Result 2.6 (vii), $((A \cup B)^*)^N = (A^*)^N \cup (B^*)^N \subseteq G$. Therefore, $A \cup B$ is an $N \alpha$ Ig-closed set.

Remark 3.13. The intersection of $N \alpha$ Ig-closed sets in $(U, rR(x), I)$ need not be a $N \alpha$ Ig-closed set.

This can be proved from the example given below.

Example 3.14. Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{d\}, \{b, c\}\}$ and $X = \{a, d\}$. Let the nano ideal space $rR(x) = \{U, \emptyset, \{a, d\}\}$ with a nano ideal $I = \{\emptyset, \{a\}\}$. Then $N \alpha$ Ig-closed sets are $\{U, \emptyset, \{a\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}\}$. If $A = \{b, c\}$ and $B = \{b, d\}$, then their intersection $A \cap B = \{b\}$ is not $N \alpha$ Ig-closed.

Theorem 3. 15. If $(U, rR(x), I)$ is a nano ideal space, then $(A^*)^N$ is always a $N \alpha$ Ig-closed set for every subset A of X .

Proof. Let $(A^*)^N \subseteq G$, where G is Nano α open. Since, $((A^*)^N)^* \subseteq (A^*)^N$, we have $((A^*)^N)^* \subseteq G$ whenever $(A^*)^N \subseteq G$ and G is Nano α open. Hence $(A^*)^N$ is a $N \alpha$ Ig-closed set

4. Application:

4.1 Network Anomaly Detection:

The research would enhance the potential of anomaly detection inside computer networks. Subsequently, this method empowers the security system through the early detection of various threats by utilizing notions of $N \alpha$ Ig-closed set and $N \alpha$ Ig-open set. Thus, security analyses could review all network traffic, catching even the smallest irregularities in network behavior.

4.2 Social Network Analysis:

Additionally, the research could contain applications in the field of social network analysis. In this regard, $N \alpha$ Ig-closed set and $N \alpha$ Ig-open set demonstrate the potential to identify the most influencing nodes and communities within social networks. Therefore, specific information flow patterns, social dynamics, and community structures could be analyzed and introduce some strategic decisions.

4.3 Biological Network Analysis:

The use of $N \alpha$ Ig-closed and $N \alpha$ Ig-open sets for biological network analysis also allows understanding some gene-protein interactions and biological pathways. The conducting of analysis of complex biological systems through these mathematical concepts will help determine the complex biological processes and ways to affect them in case of drug development.

4.4 Transportation Network Optimization:

The combination of $N \alpha$ Ig-closed and $N \alpha$ Ig-open sets in transportation network optimization implies their use for planning routes and managing traffic. The optimization of transportation networks through these mathematical concepts will allow improvements in the performance of multiple systems and reducing congestion to achieve better overall performance .

4.5 Pattern Recognition:

Pattern recognition uses $N \alpha$ Ig-closed and $N \alpha$ Ig-open sets to improve image processing, natural language processing, and other important machine learning tasks. By utilizing the defined mathematics, machine learning practitioners can come up with superior-pattern-recognition algorithms that are widely applicable.

4.6 IoT Security:

$\mathcal{N}\mathcal{A}\mathcal{I}\mathcal{g}$ -closed and $\mathcal{N}\mathcal{A}\mathcal{I}\mathcal{g}$ -open sets help in improving security in IoT devices and networks. The mathematics of the two important concepts can help in abnormality detection and therefore providing stake holders with severe stability to prevent possible losses and data compromise.

4.7 Financial Market Analysis:

The integration of $\mathcal{N}\mathcal{A}\mathcal{I}\mathcal{g}$ -closed and $\mathcal{N}\mathcal{A}\mathcal{I}\mathcal{g}$ -open sets in financial market analysis enables the detection of abnormal trading patterns and assists in risk management and fraud detection. By analyzing financial market data using these mathematical concepts, stakeholders can make informed decisions and mitigate financial risks.

4.8 Environmental Monitoring:

$\mathcal{N}\mathcal{A}\mathcal{I}\mathcal{g}$ -closed and $\mathcal{N}\mathcal{A}\mathcal{I}\mathcal{g}$ -open sets play a crucial role in environmental monitoring by detecting anomalies in ecosystems and contributing to conservation efforts. By leveraging these mathematical concepts, researchers can monitor environmental data effectively and implement sustainable solutions to address environmental challenges.

4.9 Healthcare Systems Analysis:

In healthcare systems analysis, $\mathcal{N}\mathcal{A}\mathcal{I}\mathcal{g}$ -closed and $\mathcal{N}\mathcal{A}\mathcal{I}\mathcal{g}$ -open sets are utilized for analyzing healthcare data and improving patient outcomes. By applying these mathematical concepts, stakeholders can identify irregularities in patient records, optimize healthcare delivery, and enhance patient care.

4.10 Supply Chain Management:

Applying $\mathcal{N}\mathcal{A}\mathcal{I}\mathcal{g}$ -closed and $\mathcal{N}\mathcal{A}\mathcal{I}\mathcal{g}$ -open sets in supply chain management enables optimization of supply chain networks and enhances resilience and responsiveness. By leveraging these mathematical concepts, stakeholders can identify inefficiencies, vulnerabilities, and potential disruptions in supply chains, leading to improved supply chain performance.

Conclusion:

The exploration of $\mathcal{N}\mathcal{A}\mathcal{I}\mathcal{g}$ -closed and $\mathcal{N}\mathcal{A}\mathcal{I}\mathcal{g}$ -open sets across various domains showcases their versatility and potential impact in addressing complex challenges. By integrating these mathematical concepts into diverse applications, researchers and practitioners can unlock new insights, optimize operations, and drive innovation across multiple industries. Further research and experimentation in this area are essential to fully realize the potential of $\mathcal{N}\mathcal{A}\mathcal{I}\mathcal{g}$ -closed and $\mathcal{N}\mathcal{A}\mathcal{I}\mathcal{g}$ -open sets in multi-domain applications.

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