

# Optimality of Finite Capacity Markovian Queues with Discouraged Arrivals and Singlehiatus with Waiting Server

J. Vimal Andrew<sup>1</sup>, V. Vijayalakshmi<sup>\*2</sup>

<sup>1</sup>Department of Mathematics, Karpagam Academy of Higher Education, Coimbatore - 641 021, Tamil Nadu, India.

<sup>2</sup>Department of Science and Humanities (Mathematics), Faculty of Engineering, Karpagam Academy of Higher Education, Coimbatore-641 021, Tamil Nadu, India.

<sup>1</sup>vimalandmat@gmail.com <sup>\*2</sup>viji.sanmugam@gmail.com

---

## Article History:

**Received:** 15-05-2024

**Revised:** 25-06-2024

**Accepted:** 10-07-2024

## Abstract:

We consider finite-capacity Markovian queues with a single hiatus scheme and waiting server. Customers are arriving at a Poisson arrival  $\lambda$  and exponential service distribution, with a mean service rate  $\mu$ . In which customers join the queue according to the number of customers in the system while the hiatus is in the service-providing process. For the assumed queuing model, steady-state probabilities were derived, and some important performance measures, such as the mean number of customers in the system and mean response time in the system and queue are analysed. The expected expense function is developed and formulated as an optimization problem in order to find the minimum expense. Numerical illustrations are given to show the effect of parameters on the performance measures.

**Keywords:** PGF, Vacations, Performance measures, PSO.

**Mathematics Subject Classification:** 90B22 and 60K25.

---

## 1. Introduction

In this paper, we consider a single server queueing system where the service time of each customer depends on the number of customers served prior to him in the current busy period and with server vacation. Several researchers have studied queueing systems in which the service time of a customer depends on the number of customers served in the current busy period. Doshi [5], Takagi [16] and Tian and Zhang [17] are excellent survey works on the subject. Teghem [10] has made a comprehensive survey of queueing system with vacation. Mishra et al. [19] discussed and investigated transient behavior of a M/M/1 waiting line undergoing multiple differentiated vacations in conjunction with impatient customers- manifested in the form of balking and probabilistically modified reneging. Vijayalakshmi et al. [20] discussed about arriving customers to receive only one service and may want to choose some optional service from the services available in the system. Baburaj and Sahana [21] studied the discrete time single arrival and single-batch service queue under policy 'C' is described in this study together with reneging and vacation interruption. Yumei Hou et al. [22] discussed optimization of beds allocation based on queueing model and by Particle Swarm Optimization Algorithm.

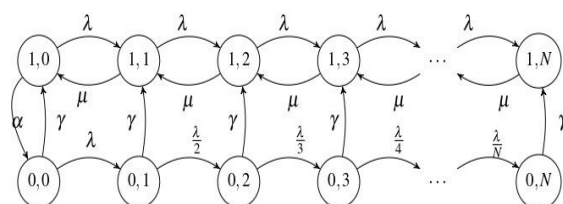
According to Ammar et al. [1], customers balk using a set probability and renege based on a negative exponential distribution. Bouchentouf et al. [3] examined a single-server M/M/1/N feedback queueing system that includes vacation, balking, reneging and customer retention. Abdul Rasheed et al. [2] investigated the discouraged arrival of markovian queueing systems, which regulate service speed according on customer count. Courtois and George [4] found that a customer's desire for service is

influenced by the duration of their wait time. Haddadi [6] proposed a method for studying busy period processes in queue models. Satish Kumar et al. [7] developed a single-server Markovian queueing system with limited capacity, encouraged or discouraged arrivals and a modified customer reneging policy. Reynolds [8] researched multi-server queueing models with diminishing arrival rates as queue length increases. Kumar [9] studied a reneged customer may be persuaded to stay in the wait for additional service with a probability of  $q$  or they may depart the queue without obtaining service with a probability of  $p = 1 - q$ . Parthasarathy et al. [11,12] conducted a transient analysis of a line with potential customers discouraged by its length and a fluid queue driven by discouraged arrivals. They obtained explicit formulas for the stationary distribution function. Sanga et al. [13] proposed a queueing model with a single server, finite capacity, discouraged customers, and distributed retry times. Sharma et al. [14] computed closed-form probability for transient states in a finite waiting space. Rao [15] mentioned, the M/G/1 queueing procedure involves units that balk and renege. Unit servicing may experience breakdowns due to disruptions, which must be addressed promptly. Van Doorn [18] investigated exact formulas for the birth-death process transition probability. The model treated in this paper is immediately applicable to many fields such as computer systems, telecommunication systems and production systems. The outlook of this paper is as follows. We describe the model and introduce notation in section 2. In section 3, we obtain the steady state probabilities and the moments of the queue length distribution. The performance measures are obtained in section 4. Optimization process is carried out in section 5 by Total cost method, Direct search method and particular swarm optimization (PSO) method. Numerical analysis is carried out in section 6.

## 2. Model Formulation

Consider a finite buffer Markovian queue with a single hiatus policy with the following assumptions.

- Beneficiary entries occur in a Poisson stream at a rate  $\lambda$
- The service times follow an exponential distribution with  $\mu$ .
- The beneficiaries are receiving service on a First Come First Serve (FCFS).
- A single hiatus( $\alpha$ ) scheme is followed. Once the system reaches the zero-beneficiary level, the service provider departs from the service station for a hiatus.
- If there are beneficiaries in the waiting line at the end of a hiatus, the server begins to provide service. Otherwise, the service provider remains waiting in the service station for the new beneficiaries. A hiatus period is exponentially distributed with  $\gamma$ .
- While the service provider is on hiatus, arriving beneficiaries are discouraged harmonically with respect to the number of beneficiaries waiting in system, that is,  $\lambda_n = \frac{\lambda}{n+1}$ ,  $n \geq 0$



**Figure 1:** State transition diagram

- The state transition diagram of the queueing model undertaken is displayed in figure 1.
- Let  $N(t)$  be the quantity of customers in the system at time  $t$ . At that point the bivariate process

$\{(C(t), N(t)), t \geq 0\}$  is a persistent time Markov chain. Let

$$P_{i,n}(t) = \text{Prob} \{C(t) = i, N(t) = n, i = 0, 1 \text{ and } n \geq 0\}$$

$$C(t) = \begin{cases} 0, & \text{if a vacation process takes place at time } t \\ 1, & \text{if the server is idle (or) a regular service takes place at time } t \end{cases}$$

### 3. Steady State Analysis

By Markov theory, the steady state balance flow equations of assumed model are as follows.

$$(\lambda + \gamma)p_{0,0} = \alpha p_{1,0} \quad (1)$$

$$\left(\frac{\lambda}{n+1} + \gamma\right)p_{0,n} = \frac{\lambda}{n}p_{1,n-1}, 1 \leq n \leq N-1 \quad (2)$$

$$\gamma p_{0,N} = \frac{\lambda}{N}p_{0,N-1} \quad (3)$$

$$(\lambda + \alpha)p_{1,0} = \gamma p_{0,0} + \mu p_{1,1} \quad (4)$$

$$(\lambda + \mu)p_{1,n} = \lambda p_{1,n-1} + \mu p_{1,n+1} + \gamma p_{0,n}, 1 \leq n \leq N-1 \quad (5)$$

$$(\lambda + \mu)p_{1,N} = \lambda p_{1,N-1} + \gamma p_{0,N} \quad (6)$$

Let

$$P_i(z) = \sum_{n=0}^N p_{i,n} z^n, \quad i = 0, 1 \quad (7)$$

be the partial probability generating functions.

From (3), 
$$p_{0,N} = \frac{\lambda}{N\gamma} p_{0,N-1}$$

The recursive application of (2) becomes

$$p_{0,N} = \frac{\lambda^N}{\gamma \prod_{i=2}^N (\lambda + i\gamma)} \quad (8)$$

Multiply (4) - (6) by appropriate  $z^n$  and adding, we have

$$P_1(Z) = \frac{\lambda z^N (1-z)p_{1,N} - \alpha p_{1,0} + \mu p_{1,0} + \gamma P_0(Z) - \frac{\mu p_{1,0}}{z}}{\lambda(1-z) - \frac{\mu}{z} + \mu} \quad (9)$$

Here the denominator of  $P_1(z)$  has two roots  $1, \frac{\mu}{\lambda}$

From this

$$P_0(Z) = \frac{\alpha}{\gamma} p_{1,0} \quad (10)$$

and  $z = \frac{\mu}{\lambda}$  is also the root of the numerator of  $P_1(z)$

$$0 = \lambda \left(\frac{\mu}{\lambda}\right)^N (1 - \frac{\mu}{\lambda}) p_{1,N} - \alpha p_{1,0} + \mu p_{1,0} + \gamma p_0 \left(\frac{\mu}{\lambda}\right) - \lambda p_{1,0} \quad (11)$$

From (1), (2) and (3)

$$P_0(Z) = p_{0,0} \left[ 1 + \sum_{n=1}^{N-1} \frac{(n+1)(\lambda z)^n}{\prod_{i=1}^n (\lambda + (i+1)\gamma)} + \frac{(\lambda z)^N}{\prod_{i=2}^N (\lambda + i\gamma)} \right] \quad (12)$$

Substitute  $z = \frac{\mu}{\lambda}$  in the above equation, we have

$$P_0\left(\frac{\mu}{\lambda}\right) = p_{0,0} \Theta \quad (13)$$

where

$$\Theta = 1 + \sum_{n=1}^{N-1} (n+1) \prod_{i=2}^{n+1} \frac{\lambda^i}{\lambda + i\gamma} \left(\frac{\mu}{\lambda}\right)^n + \frac{1}{\gamma} \prod_{i=2}^N \frac{\lambda^i}{\lambda + i\gamma} \left(\frac{\mu}{\lambda}\right)^N$$

From (11) and (13) we have

$$p_{1,N} = \frac{(\alpha - \mu + \lambda) \frac{\lambda + \mu}{\alpha} + \gamma \Theta}{(\lambda - \mu) \frac{\mu}{\lambda}} p_{0,0} \quad (14)$$

From the local balance equation,

$$\rho p_{1,0} + \frac{\rho}{1} p_{0,0} = p_{1,1}$$

$$\rho p_{1,1} + \frac{\rho}{1} p_{0,1} = p_{1,2}$$

.

.

.

$$\rho p_{1,N-1} + \frac{\rho}{N} p_{0,N-1} = p_{1,N}$$

Generally

$$\rho p_{1,n} + \frac{\rho}{n} p_{0,n-1} = p_{1,n+1}, \quad 0 \leq n \leq N-1 \quad (15)$$

After some mathematical manipulations equation (15), we obtain

$$\rho [P_1(z) - p_{1,N} z^N] + \rho \sum_{n=1}^N \frac{1}{n} p_{0,n-1} = \frac{1}{z} [P_1(z) - p_{1,0}]$$

put  $z = 1$

$$P_1(1) = \frac{\rho}{\rho-1} p_{1,N} - \frac{\rho}{\rho-1} \sum_{n=1}^N \frac{1}{n} p_{0,n-1} - \frac{1}{\rho-1} p_{1,0} \quad (16)$$

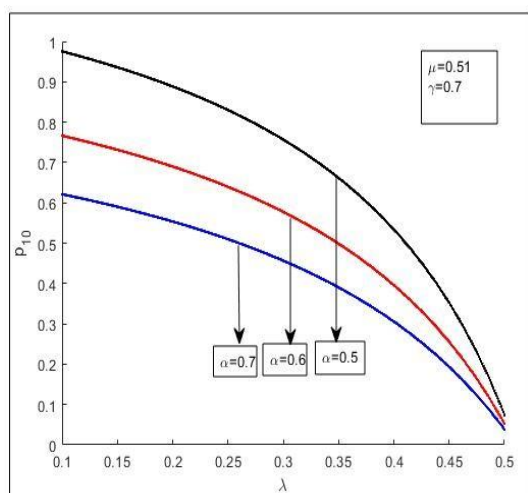
From the law of total probability,

$$p_{0,0} = \frac{1}{\frac{\lambda + \gamma}{\gamma} + \frac{\rho}{\rho-1} [\rho^N (\frac{\lambda + \gamma + \alpha}{\alpha}) + \frac{\lambda \rho}{\lambda + 2\gamma} (\rho^{N-2} + \sum_{i=3}^N \frac{\lambda^{i-2} \rho^{N-i}}{\prod_{j=3}^i (\lambda + j\gamma))}] - \frac{\lambda + \gamma}{(\rho-1)\alpha} - \frac{\rho}{\rho-1} \sum_{n=1}^N \frac{N \lambda^{N-1}}{n \prod_{i=2}^N (\lambda + i\gamma)}} \quad (17)$$

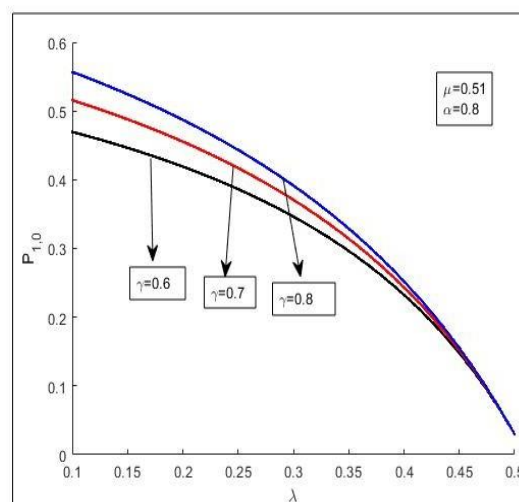
Substitute the value of  $p_{0,0}$  in (1),

$$p_{1,0} = \left( \frac{\lambda + \gamma}{\alpha} \right) p_{0,0}$$

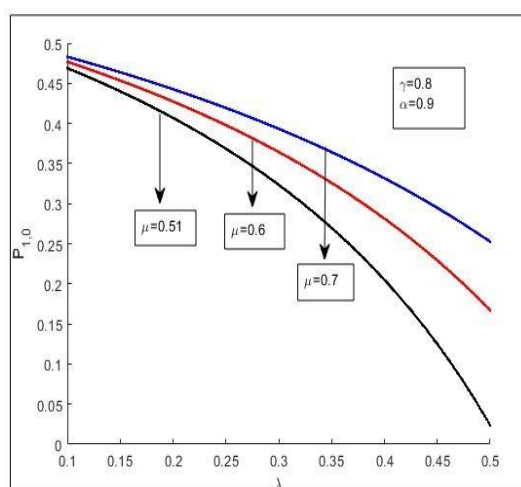
$$p_{1,0} = \left( \frac{\lambda + \gamma}{\alpha} \right) \frac{1}{\frac{\lambda + \gamma}{\gamma} + \frac{\rho}{\rho - 1} \left[ \rho^N \left( \frac{\lambda + \gamma + \alpha}{\alpha} \right) + \frac{\lambda \rho}{\lambda + 2\gamma} (\rho^{N-2} + \sum_{i=3}^N \frac{\lambda^{i-2} \rho^{N-i}}{\prod_{j=3}^i (\lambda + j\gamma)) \right] - \frac{\lambda + \gamma}{(\rho - 1)\alpha} - \frac{\rho}{\rho - 1} \sum_{n=1}^N \frac{N \lambda^{N-1}}{n \prod_{i=2}^N (\lambda + i\gamma)}} \quad (18)$$



**Figure 2:**  $P_{1,0}$  against  $\lambda$



**Figure 3:**  $P_{1,0}$  against  $\lambda$



**Figure 4:**  $P_{1,0}$  against  $\lambda$

As a regular service period ends, if there are customers in the system, the system will be resumed to the regular service period. Otherwise, the server will enter into a vacation, during which service is not rendered to any of new arrivals in the period completely. Ultimately as the arrival rate  $\lambda$  increases, the steady state probability which is in equation (18) decreased which is shown in Figure 2, for the fixed values of  $\mu = 0.51$  and  $\gamma = 0.7$ , by varying the values of  $\lambda$  from 0.1 to 0.5 with different values of  $\alpha = 0.5, 0.6, 0.7$ .

As aforesaid, in Figure 3, the nature of decreasing continues in the steady state probability for the fixed values of  $\mu = 0.51$  and  $\alpha = 0.8$ , by varying the values of  $\lambda$  from 0.1 to 0.5 with different values of  $\gamma = 0.6, 0.7, 0.8$ . This nature continues in Figure 4, for the fixed values of  $\gamma = 0.8$  and  $\alpha = 0.9$  by

varying the values of  $\lambda$  from 0.1 to 0.5 with different values of  $\mu = 0.51, 0.6, 0.7$ .

#### 4. Performance Measure

Expected number of customers in the system

$$E(L) = p_{0,0} \left\{ \sum_{i=1}^N \frac{i(i+1)\lambda^i}{\prod_{j=1}^i (\lambda + (j+1)\gamma)} + \frac{N\lambda^N}{\gamma \prod_{i=2}^N (\lambda + i\gamma)} + \rho \left( \frac{\lambda + \gamma + \alpha}{\alpha} \right) + \sum_{j=2}^N \left[ j\rho^j \left( \frac{\lambda + \gamma + \alpha}{\alpha} \right) + \frac{\lambda}{\lambda + 2\gamma} \left( \rho^{j-2} + \sum_{k=3}^j \frac{\lambda^{k-2} \rho^{j-k}}{\prod_{l=1}^k (\lambda + l\gamma)} \right) \right] \right\}$$

where  $p_{0,0}$  is already in (17) and the expected waiting time is

$$E(W) = \frac{E(L)}{\lambda}$$

The graph for the probability  $P_{1,0}$  against  $\lambda$ ,  $E(L)$  and  $E(W)$  for distinct parameters are as follows.

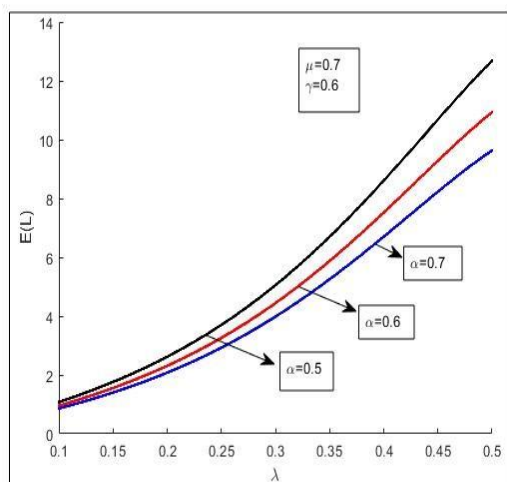


Figure 5:  $E(L)$  against  $\lambda$

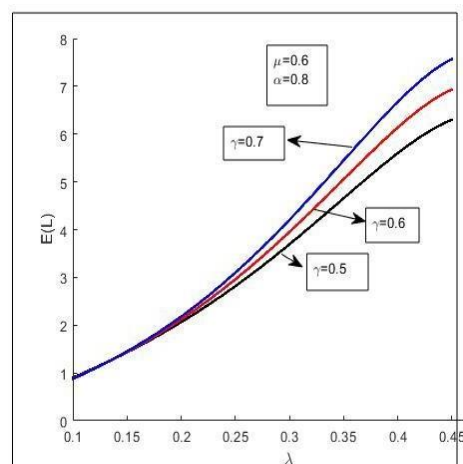


Figure 6:  $E(L)$  against  $\lambda$

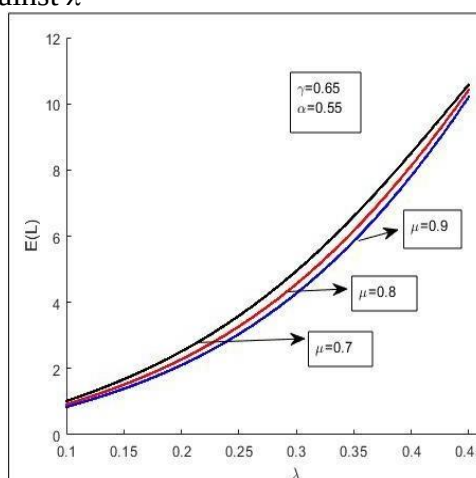
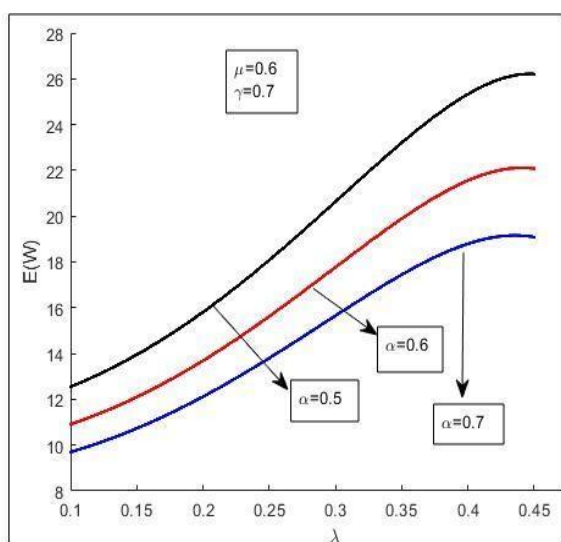
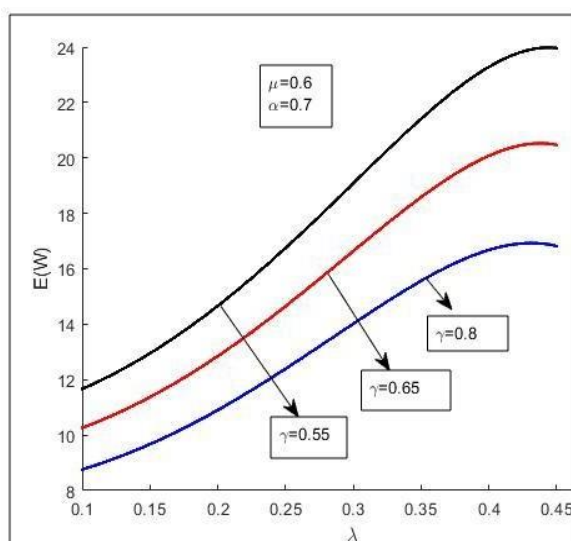


Figure 7:  $E(L)$  against  $\lambda$

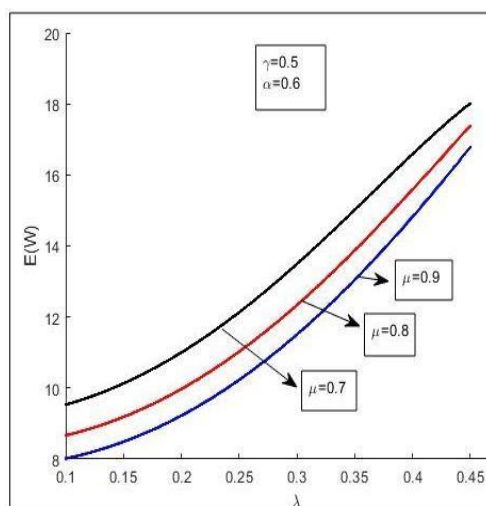
As the arrival rate  $\lambda$  increases then the number of customers in the line is also increased during the vacation period. In Figure 5, for the fixed values of  $\mu = 0.7$  and  $\gamma = 0.6$ , by varying the values of  $\lambda$  from 0.1 to 0.5 with different values of  $\alpha = 0.5, 0.6, 0.7$ . The value of  $E(L)$ , Figure 6 increases for the fixed values of  $\mu = 0.7$  and  $\alpha = 0.6$ , by varying the values of  $\lambda$  from 0.1 to 0.5 with different values of  $\gamma = 0.5, 0.6, 0.7$ . The expected number of customers in the line is also increased during the vacation period. In Figure 7, for the fixed values of  $\alpha = 0.55$  and  $\gamma = 0.65$ , by varying the values of  $\lambda$  from 0.1 to 0.5 with different values of  $\mu = 0.7, 0.8, 0.9$ .



**Figure 8:**  $E(W)$  against  $\lambda$



**Figure 9:**  $E(W)$  against  $\lambda$



**Figure 10:**  $E(W)$  against  $\lambda$

As the arrival rate  $\lambda$  increases then the number of waiting customers in the waiting line is also increased during the vacation period. In Figure 8, for the fixed values of  $\mu = 0.6$  and  $\gamma = 0.7$ , by varying the values of  $\lambda$  from 0.1 to 0.5 with different values of  $\alpha = 0.5, 0.6, 0.7$ . The value of  $E(W)$  in Figure 9 increases, for the fixed values of  $\mu = 0.6$  and  $\alpha = 0.7$ , by varying the values of  $\lambda$  from

0.1 to 0.5 with different values of  $\gamma = 0.55, 0.65, 0.8$ . The expected number of waiting customers in the waiting queue is also increased during the vacation period. In Figure 10, for the fixed values of  $\alpha = 0.6$  and  $\gamma = 0.5$ , by varying the values of  $\lambda$  from 0.1 to 0.5 with different values of  $\mu = 0.7, 0.8, 0.9$ .

## 5. Reliability Measures

Because of the importance of dependability measures under varied queueing conditions, a few investigations are dedicated to queueing hypothesis writing. Accessibility is defined as the likelihood that the system will function properly when it is mentioned for use. However, disappointed recurrence of the server is a possibility when the system is on vacation period. In this manner, we obtain the two key dependability measures

- The availability of the server

$$P_1(1) = \frac{\rho p_{0,0}}{\rho - 1} \left[ \rho^N \left( \frac{\lambda + \gamma + \alpha}{\alpha} \right) + \frac{\lambda \rho}{\lambda + 2\gamma} (\rho^{N-2} + \sum_{i=3}^N \frac{\lambda^{i-2} \rho^{N-i}}{\prod_{j=3}^i (\lambda + j\gamma)}) \right] - \frac{\lambda + \gamma}{\rho \alpha} - \sum_{n=1}^N \frac{N \lambda^{N-1}}{n \prod_{i=2}^N (\lambda + i\gamma)}$$

- The failure frequency of the server

$$P_0(1) = \left( \frac{\lambda + \gamma}{\gamma} \right) p_{0,0}$$

## 6. Optimization Analysis

### 6.1 Total Cost method

We develop the cost model to enable the minimum cost over the system. The various cost parameters are defined as

$CO_1 \equiv$  holding cost for every customer present in the system

$CO_2 \equiv$  waiting cost for every customer waits in the system

$CO_3 \equiv$  cost for the server in the busy period

$CO_4 \equiv$  cost for the server in the vacation period

$CO_5 \equiv$  cost for service

Using the definition of these cost elements listed above, the expected cost function per unit time is given by

$$TC \equiv CO_1 E(L) + CO_2 E(W) + CO_3 P_0(Z) + CO_4 P_1(Z) + CO_5 \mu$$

The cost minimization problem can be formulated as

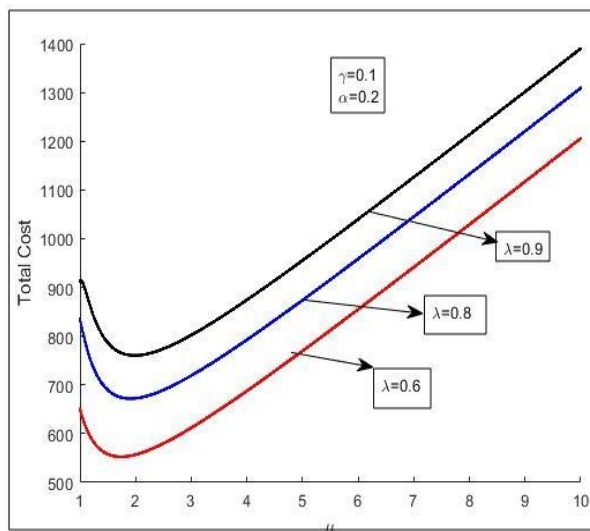
$$TC(\mu, \gamma) = \text{Minimize}(\mu, \gamma)$$

subject to  $\mu > \gamma$  and  $\rho < 1$



We consider the following cost parameters as

$$CO_1 = 20, CO_2 = 45, CO_3 = 60, CO_4 = 80, CO_5 = 90$$



**Figure 11:** Total Cost against  $\mu$

**Table 1:** Cost function versus  $\mu$

| $\mu$           | 1     | 2     | 3     | 4     | 5     | 6      | 7      | 8      | 9      | 10     |
|-----------------|-------|-------|-------|-------|-------|--------|--------|--------|--------|--------|
| $\lambda = 0.6$ | 650.3 | 557.1 | 612.1 | 687.6 | 769.8 | 854.9  | 941.5  | 1029.1 | 1117.2 | 1205.7 |
| $\lambda = 0.8$ | 835.1 | 672.5 | 719.1 | 792.5 | 873.9 | 958.6  | 1045.1 | 1132.5 | 1220.6 | 1309.1 |
| $\lambda = 0.9$ | 912.2 | 760.4 | 802.1 | 874.5 | 955.3 | 1039.9 | 1126.2 | 1213.6 | 1301.6 | 1390.1 |

In Figure 11, for the different arrival rate  $\lambda = 0.6, 0.8, 0.9$  and for the fixed values of  $\gamma = 0.1$  and  $\alpha = 0.2$ , we got the values while varying  $\mu$  from 1 to 10 as follows the table. The minimum values corresponding to the given values of  $\lambda$  are 557.1, 672.5 and 760.4 respectively. It is seen that initially the total cost diminishes and begins expanding with the ward of  $\mu$  for fixed estimations of  $\gamma$  and  $\alpha$ . The raised nature of the cost work concerning show the pattern for the ideal expense by expanding the typical assistance sace of the clients. This shows the convex nature.

Indeed, it is not possible to derive the analytic solutions for the optimal service rates at the minimum expected cost. Thus, we progress the approximations to achieve the optimal service rates by direct search method

## 6.2 Direct Search Method

We assumed the following cost parameters as

$$CO_6 = 30, CO_7 = 45, CO_8 = 90$$

The cost element is defined as

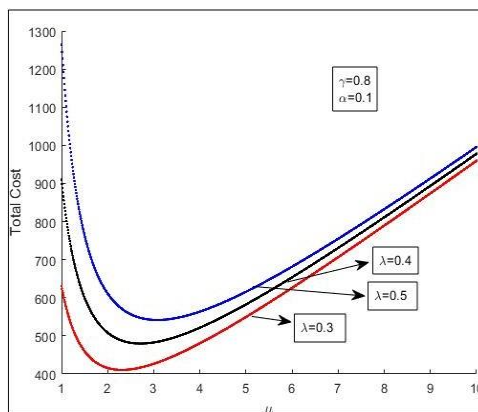
$CO_6 \equiv$  cost for expected number of customers in busy period

$CO_7 \equiv$  cost for expected number of customers in vacation period

$CO_8 \equiv$  cost for the server in the busy period

The cost function TC is defined as

$$TC \equiv CO_6 P_0^1(Z) + CO_7 P_1^1(Z) + CO_8 \mu$$

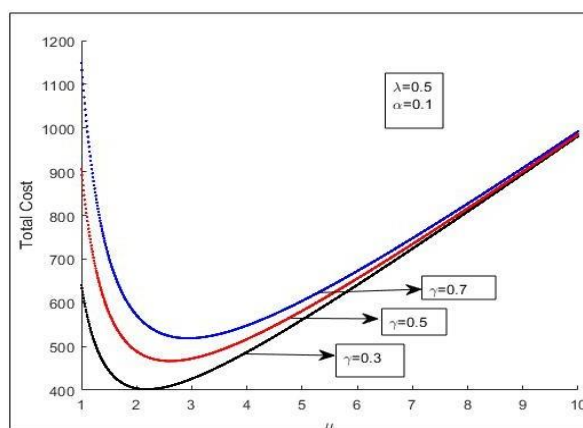


**Figure 12:** Total Cost against  $\mu$

**Table 2:** Cost function versus  $\mu$

| $\mu$           | 1      | 2     | 3            | 4            | 5     | 6     | 7     | 8     | 9     | 10     |
|-----------------|--------|-------|--------------|--------------|-------|-------|-------|-------|-------|--------|
| $\lambda = 0.3$ | 833.1  | 497.4 | <u>479.5</u> | 520.2        | 582.1 | 653.7 | 730.9 | 811.5 | 894.2 | 978.4  |
| $\lambda = 0.4$ | 1344.6 | 668.4 | <u>584.6</u> | 598.6        | 645.3 | 707.3 | 788.6 | 854.4 | 933.6 | 1015.1 |
| $\lambda = 0.5$ | 2088.4 | 885.9 | 713.8        | <u>693.1</u> | 722.1 | 773.3 | 836.6 | 907.1 | 982.1 | 1060.3 |

In Figure 12, for the different hiatus rate  $\lambda = 0.3, 0.4, 0.5$  and for the fixed values of  $\alpha = 0.1$  and  $\gamma = 0.7$  we got the values while varying  $\mu$  from 1 to 10 as follows the table. The minimum values corresponding to the given values for  $\lambda$  are 479.5, 584.6 and 693.1 respectively. It is evident that total cost function decreases first and then increases. Consequently, the convex nature arises in the total cost function. This confirms the possibility of obtaining the optimum service rates.

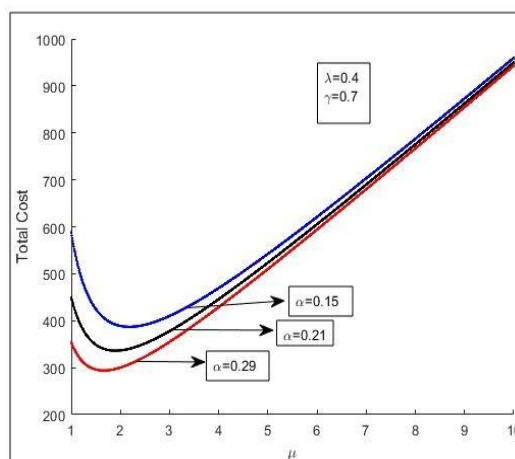


**Figure 13:** Total Cost against  $\mu$

**Table 3:** Cost function versus  $\mu$

| $\mu$          | 1      | 2     | 3            | 4            | 5     | 6     | 7     | 8     | 9     | 10     |
|----------------|--------|-------|--------------|--------------|-------|-------|-------|-------|-------|--------|
| $\gamma = 0.3$ | 1429.4 | 677.2 | <u>596.1</u> | 613.1        | 663.6 | 728.3 | 800.8 | 878.1 | 958.2 | 1040.5 |
| $\gamma = 0.5$ | 1683.5 | 752.7 | <u>635.5</u> | 637.9        | 679.6 | 739.1 | 807.9 | 882.4 | 960.6 | 1041.3 |
| $\gamma = 0.7$ | 1951.5 | 893.8 | 686.1        | <u>673.1</u> | 706.3 | 760.3 | 825.4 | 897.2 | 973.3 | 1052.4 |

In Figure 13, for the values of  $\gamma = 0.3, 0.5, 0.7$  and for the fixed values of  $\lambda = 0.5$  and  $\alpha = 0.1$  we got the values while varying  $\mu$  from 1 to 10 as follows the table. The minimum values corresponding to the given values for  $\gamma$  are 596.1, 635.5 and 673.1 respectively. The figure shows the convexity in the total cost function. The convex nature of the cost function with respect to  $\mu$  shows the trend for the optimum cost by increasing the normal service domain of the customers.



**Figure 14:** Total Cost against  $\mu$

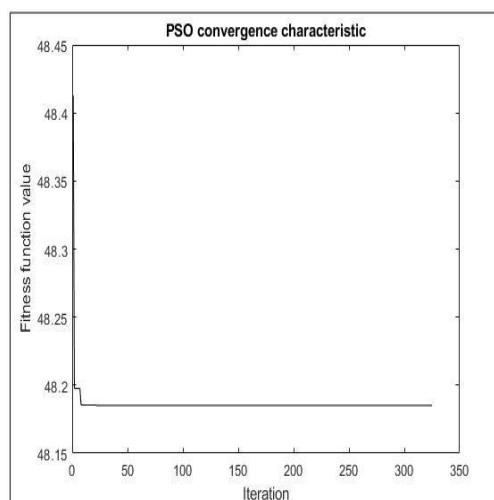
**Table 4:** Cost function versus  $\mu$

| $\mu$           | 1     | 2            | 3            | 4     | 5     | 6     | 7     | 8     | 9     | 10    |
|-----------------|-------|--------------|--------------|-------|-------|-------|-------|-------|-------|-------|
| $\alpha = 0.15$ | 912.5 | 509.8        | <u>487.9</u> | 528.8 | 591.2 | 663.4 | 741.1 | 822.1 | 905.1 | 989.5 |
| $\alpha = 0.21$ | 717.1 | <u>437.4</u> | 443.5        | 496.6 | 565.9 | 642.6 | 723.3 | 806.5 | 891.3 | 977.2 |
| $\alpha = 0.29$ | 581.9 | <u>387.2</u> | 412.6        | 474.1 | 548.2 | 627.9 | 710.8 | 795.5 | 881.5 | 968.3 |

In Figure 14, for the different hiatus rate  $\alpha = 0.15, 0.21, 0.29$  and for the fixed values of  $\lambda = 0.4$  and  $\gamma = 0.7$  we got the values while varying  $\mu$  from 1 to 10 as follows the table. The minimum values corresponding to the given values of  $\alpha$  are 487.9, 437.4 and 387.2 respectively. The figure shows the convexity in the total cost function. This confirms the possibility of obtaining the optimum service rate.

### 6.3 Particular Swarm Optimization

Particle swarm optimization is one of the most popular nature-inspired meta-heuristic optimization algorithms developed by James Kennedy and Russell Eberhart in 1995. Particle swarm optimization (PSO) is inspired by social and cooperative behavior displayed by various species to fill their needs in the search space. Recently, PSO has emerged as a promising algorithm in solving various optimization problems in the field of science and engineering



**Figure 15:** PSO Total Cost against Iteration

**Table 5:** PSO-Effect of  $\mu^*$  on  $TC^*$ ,  $E(L)$ ,  $E(W)$  for different value of  $\lambda$

| $\lambda$  | $\mu^*$       | $E(L)$ | $E(W)$ | $TC^*$         | Elapsed Time    |
|------------|---------------|--------|--------|----------------|-----------------|
| 0.3        | 1.3629        | 1.8445 | 6.1484 | 47.7613        | 291.0614        |
| <b>0.4</b> | <b>1.4473</b> | 2.3228 | 5.8070 | <b>48.1851</b> | <b>329.2360</b> |
| 0.5        | 1.5355        | 2.9194 | 5.8388 | 50.3765        | 321.3652        |
| 0.6        | 1.6241        | 3.6782 | 6.1304 | 53.9342        | 350.7149        |
| 0.8        | 1.7941        | 5.9485 | 7.4356 | 65.3713        | 359.1852        |

The snapshot of the optimized solution of the problem is shown in the above table. It is observed that the best value of the objective function obtained after 10 independent runs is 48.1851. This value is obtained at  $x(0.4) = 1.4473$ . Out of the 10 runs, 10th run gives this best results. The simulation total time taken is 329.2360 seconds. It is to be noted that the simulation time depends on computer configuration. Further, Figure 15 shows the convergence characteristic of PSO.

## 7. Conclusion

As the service rate  $\mu$  grows, the considered probabilities drop. This is apparent because growing signifies a higher chance of customers abandoning the system during the service receiving stage, resulting in a shorter system length. The purpose of this experiment is to investigate the effect of increasing the arrival rate  $\lambda$  on the average response time  $\mu$  of a client. Figure 4 shows that as  $\lambda$  grows, the average waiting time for clients in the system increases, as expected.

The purpose of this experiment is to evaluate the behavior of the mean system length versus the vacation rate for the three possibilities of the relationship between  $\lambda$  and  $\mu$ . Figure 5 shows that the following are true:

- With the exception of the case  $\lambda < \mu$ , the mean system size increases slowly with increasing.
- As the for the example increases, the mean system size falls steadily. This is due to the fact that as the vacation rate  $\gamma$  increases, the server returns to the system sooner. As a result, the predicted number of consumers increases for cases  $\lambda = \mu$  and  $\lambda < \mu$  and falls for the remaining case.

## References

- [1] S.I. Ammar, A.A. El-Sherbiny, S.A. El-Shehawy and R.O. Al-Seedy, *A matrix approach for the transient solution of an M/M/1/N queue with discouraged arrivals and reneging*, International Journal of Computer Mathematics, Vol 89(4), (2012), pp 482-491.
- [2] K.V. Abdul Rasheed and M. Manoharan, *Markovian queueing system with discouraged arrivals and self-regulatory servers*, Advances in Operations Research, Vol 2016, (2016), pp 2-12.
- [3] A.A. Bouchentouf, M. Cherfaoui and M. Bolaem, *Performance and economic analysis of a single server feedback queueing model with vacation and impatient customers*, OPSEARCH, Vol 56, (2019), pp 300-323.
- [4] P.J. Courtois and J. Georges, *On a single server finite queueing model with state dependent arrival and service processes*, Operations Research, Vol 19(2), (1971), pp 424-435.
- [5] B.T. Doshi, *Queueing Systems with Vacations-A Survey*, Queueing Systems, Vol 1, (1986).
- [6] N. Haddadi, *Busy period of queues with state dependent arrival and service rates*, Journal of Applied Probability, Vol 11(4), (1974), pp 842-848.
- [7] S. Hanumanth Rao, V. Vasanta Kumar and K. Satish Kumar, *Encouraged or discouraged arrivals of an M/M/1/N queueing system with modified reneging*, Advances in Mathematics: Science Journal, Vol 9(9), (2020), pp 6641-6647.
- [8] John F. Reynolds, *The Stationary Solution of a multiserver queueing model with discouragement*, Operations Research, Vol 16(1), (1968), pp 64-71.
- [9] R. Kumar and S. Kumar Sharma, *A single server markovian queueing system with discouraged arrivals and retention of reneged customer*, Yugoslav Journal of Operations Research, Vol 24(1), (2014), pp 119-126.
- [10] J. Loris- Teghem, *Analysis of single server queueing systems with vacation periods*, Belgium Journal of Operations Research, Statistics, Computer Science, Vol 25, (1985), pp 47-54.
- [11] P.R. Parthasarathy and N. Selvaraju, *Transient analysis of a queue where the potential customers discouraged by queue length*, Mathematical Problems in Engineering, Vol 7, (2001), pp 433-454.
- [12] P.R. Parthasarathy and K.V. Vijayashree, *Fluid queues driven by a discouraged arrivals queue*, International Journal of Mathematics and Mathematical sciences, Vol 24, (2003), pp 1509-1528.
- [13] S.S. Sanga and M. Jain, *Cost optimization and ANFIS computing for admission control of M/M/1/K queue with general retrial times and discouragement*, Applied Mathematics and Computation, Vol 363, (2019), pp 1-22.
- [14] O.P. Sharma and M.V.R. Maheswar, *Transient behaviour of a simple queue with discouraged arrivals*, Optimization, Vol 27(3), (1993), pp 283-291.
- [15] S. Subba Rao, *Queueing Models with Balking, Reneging, and Interruptions*, Operations Research, Vol 13(4), (1965) pp 596-608.
- [16] S. Takagi, *Queueing analysis: A foundation of performance evaluation (Volume I) vacation and priority systems*, ACM SIGMETRICS Performance Evaluation Review, Vol 19(2), (1991), pp 487.
- [17] N. Tian and Z.G. Zhang, *Vacation Queueing Models*, Springer, (2006).
- [18] Van Doorn, *the transient state probabilities for a queueing model where potential customers are discouraged by queue length*, Journal of Applied Probability, Vol 18(2), (1981), pp 499-506.
- [19] Vinoth Kumar Srivastava, *Analysis of Balking and Probabilistically Modified Reneging in a M/M/1 Waiting Line under Multiple Differentiated Vacations*, Journal of Algebraic Statistics, Vol 14(1), (2023), pp 87-106.
- [20] V. Vijayalakshmi, K. Kalidass and B. Deepa, *Cost Analysis of M/M/1/N queue with working breakdowns and a two-phase service*, Journal of Physics, Conference Series 1850, (2021).
- [21] C. Baburaj and P. Sahana, *A C Policy Batch Service Queueing System under Bernoulli Scheduled Vacation and Manual Service*, Journal of the Kerala Statistical Association, Vol 33, (2022), pp 71-87.
- [22] Yumei Hou<sup>1</sup>, Hui Zeng, Yimin Wang, Xueqin Wang and Qiuye Gao, *Optimization of Beds Allocation Based on Queueing Model and the Particle Swarm Optimization Algorithm*, International Journal of Circuits, Systems and Signal Processing, Vol 13, (2019).