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Decision–Making Problem for Triangular Hesitant Fuzzy Set

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Abstract:

In this paper, we proposed two new algorithms to address the task of multicriteria decision-making problems in which the criteria weights which have been unidentified, and alternatives are provided by triangular fuzzy numbers' linguistic values. A decision-making problem with assessments has been proposed on the basis of expected values. The expected values for two algorithms are calculated with two different 'Ranking Functions'. The weights are obtained by calculating the variance and mean of expected values with the aid of the triangular hesitant fuzzy decision matrix. The all alternatives ranking order is determined, and the maximum one is the best that can be identified easily. Lastly, a representative example is provided regarding the health issues of a community.

Keywords: Expected Value, Triangular Hesitant Fuzzy Set, Variance, Mean, Multicriteria Decision Making.

1. Introduction

The idea of a fuzzy set is unable to accurately model unpredictability, inaccurate, and vague information when numerous sources of vagueness occur at the same time. To overcome this limitation, many fuzzy set extensions had been proposed in literature. A demonstration of interval-valued dual hesitant fuzzy information aggregation to multiple attribute decision-making has been presented by Peng X, Liu L, and Dai J [2]. A few noteworthy extensions are the interval-valued fuzzy set, which assigns a closed subinterval of [0, 1] as the membership degree for each component; the IFS (Intuitionistic Fuzzy Set), which takes in to the considerations both the degree of membership and non-membership degree of each element; the type-2 fuzzy set, which uses a fuzzy set over [0, 1] to incorporate uncertainty into the membership function definition; and the fuzzy multiset, on the basis of a multiset with possibly repeated elements. Nevertheless, hesitant fuzzy sets are a more popular and often utilized concept of fuzzy extensions. They are used to model a decision-making scenario where an expert may weigh the relative merits of each element in a set. In [1], they studied the speed forecasting system with novel defuzzification using the hesitant number. Humble fuzzy sets are thought to be the most comprehensive set because they support a flexible approach while decision-makers make their own decisions. This is because hesitant fuzzy sets have been formed by top researchers who has been given a broad conceptualization along with the application of the concept, which was later adopted by other researchers in the field. That's why hesitant fuzzy sets are more extensively used in many areas. In addition to researching the similarity metric with sign distance, Stephen Dinagar and Fany Helena [3,4] also presented a novel approach for computing the Centroids of both Vertical and Horizontal Axes as well as the Value and Ambiguity Indices using trapezoidal intuitionistic fuzzy numbers. Similarity Measures with Vector-Length under Fuzzy Environment were proposed in [5]. In this paper, we proposed a new expected value using different ranking procedures and the ranking functions in [3,4,5]. The order in which all options are ranked is determined, and the maximum one is the best, which can be identified easily through these ranking procedures.

2. Objectives

Definition:2.1

Let X be a fixed reference set. A HFS on X is defined in terms of a function from X to a subset of [0,1], that has been considered by

$$A = \{\langle x, \tilde{h}_A(x)\rangle / x \in X\},$$

in which $\tilde{h}_A(x)$ as hesitant fuzzy element (H.F.E.), and it represents a set of some values in [0,1]. Furthermore, the H.F.E. $\tilde{h}_A(x)$ stands for the possible membership degrees of the element x ∈ X to the set A.

Definition:2.2

For a hesitant, fuzzy element, h

$$S'(\tilde{h}) = \frac{1}{*\tilde{h}} \sum_{\Delta \in *\tilde{h}} \Delta$$

is called the score function of $\tilde{h}$, where $*\tilde{h}$ represents the number of elements.

For any 2 H.F.E

(i) If $S'(\tilde{h}_1) > S'(\tilde{h}_2)$ then $\tilde{h}_1 > \tilde{h}_2$

(ii) If $S'(\tilde{h}_1) = S'(\tilde{h}_2)$ then $\tilde{h}_1 = \tilde{h}_2$

Definition:2.3

Let $\tilde{h}_{A_1}$and $\tilde{h}_{A_1}$ Be two H.F.E.s, their union & intersection are, respectively, explained below:

$$\tilde{h}_{A_1} \cup \tilde{h}_{A_2} = \cup_{\Delta_{A_1} \in \tilde{h}_{A_1}, \Delta_{A_2} \in \tilde{h}_{A_2}} \max\{\Delta_{A_1}, \Delta_{A_2}\}$$

$$\tilde{h}_{A_1} \cap \tilde{h}_{A_2} = \cup_{\Delta_{A_1} \in \tilde{h}_{A_1}, \Delta_{A_2} \in \tilde{h}_{A_2}} \min\{\Delta_{A_1}, \Delta_{A_2}\}$$

From a mathematical perspective, for any x ∈ X, an H.F.E. can be thought of as the other well-known extensions of fuzzy sets.

Definition:2.4

Let x be a finite set. A triangular hesitant fuzzy set on X has been explained as

$$\tilde{T} = \{\{x, \tilde{h}_\eta(x) / x \in X\}$$

where $\tilde{h}_\eta(x)$ represents as a set of some triangular fuzzy numbers in the set of real numbers R, representing the possible membership function of the element $x \in X, \tilde{h}_\eta(x)$. It is called a T.H.F.E .

The TFN in $\tilde{h}_{\eta}(x)$ has been represented as Ҍ by and given by

$$\chi = (a^i, b^j, c^k), \text{ where } \chi \in Ҍ$$

2.5. Some Basic Operations of Triangular Hesitant Fuzzy Numbers (T.H.F.N.s)

Suppose Ҍ, $Ҍ_1$, $Ҍ_2$ be three T.H.F.E and $\lambda > 0$, then

1. λ Ҍ $= \bigcup_{\chi \in Ҍ} \{(\lambda a^i + \lambda\, b^j + \lambda c^k)\}$
2. $Ҍ_1 \oplus Ҍ_2 = \bigcup_{\chi_1 \in Ҍ_1, \chi_2 \in Ҍ_2} \{\, a_1^i + a_2^i, b_1^j + b_2^j, c_1^k + c_2^k\}$
3. $Ҍ_1 \otimes Ҍ_2 = \bigcup_{\chi_1 \in Ҍ_1, \chi_2 \in Ҍ_2} \{\, a_1^i a_2^i, b_1^j b_2^j, c_1^k c_2^k\}$
4. $Ҍ^c = \bigcup_{\chi \in Ҍ} \{(1 - a^i, 1 - \, b^j, 1 - \lambda c^k)\}$
5. $Ҍ_1 \cup Ҍ_2 = \bigcup_{\chi_1 \in Ҍ_1, \chi_2 \in Ҍ_2} \{max\,\{a_1^i, a_2^i\}, max\,\{b_1^j, b_2^j\}, max\,\{c_1^k, c_2^k\}\}$
6. $Ҍ_1 \cap Ҍ_2 = \bigcup_{\chi_1 \in Ҍ_1, \chi_2 \in Ҍ_2} \{min\,\{a_1^i, a_2^i\}, min\,\{b_1^j, b_2^j\}, min\,\{c_1^k, c_2^k\}\}$

3. Methods

Ranking Approach of Triangular Hesitant Fuzzy Set

Definition:3.1 (Existing Approach)

The T.H.F.E is denoted as Ҍ, and the expected value is defined as E(Ҍ) $= \frac{1}{3*Ҍ} \sum_{\chi * Ҍ} (\, a^i + b^j + c^k)$, where Ҍ as number of TFN in Ҍ.

Definition:3.2 (Mid Value Ranking)

The T.H.F.E is denoted as Ҍ, and the expected value is defined as E(Ҍ) $= \frac{1}{3*Ҍ} \sum_{\chi * Ҍ} \left(\frac{a^i + b^j + c^k}{3}\right)$, where Ҍ as number of TFN in Ҍ. This ranking is called mid-value ranking.

Theorem:3.3

Let Ҍ be any hesitant triangular fuzzy element, and their alpha cut be denoted as $Ҍ_{1l} = a_1 + \alpha(a_2 - a_1)$ and $Ҍ_{1r} = a_3 - \alpha(a_3 - a_2)$ then E(Ҍ) $= \frac{1}{3*Ҍ} \sum_{\chi * Ҍ} \left(\frac{a^i + b^j + c^k}{3}\right)$.

Proof:

Let us consider

$$E(Ҍ) = \frac{1}{3*Ҍ} \sum_{\chi * Ҍ} M_{\mu}(\hbar) \qquad - (1)$$

Where Ҍ = Number of Triangular Hesitant Fuzzy Set

i=1,2,….,n; j = 1,2,…..,n; k = 1,2,….,n

Where $M_{\mu}(\hbar) = \frac{1}{2} \int_0^1 \{[\, a^i + \left(\alpha\left(b^j - a^i\right)\right)\alpha] + [c^k - (\alpha\left(c^k - b^j\right))\alpha\} \, d\alpha$

After simplification, we get

$$M_{\mu}(\hbar) = \frac{a^i + b^j + c^k}{3} \qquad - (2)$$

Now substitute equation (2) in (1) we get

$$\mathrm{E}(\text{Ѣ}) = \frac{1}{3*\text{Ѣ}}\sum_{\chi*\text{Ѣ}}\left(\frac{a^i+b^j+c^k}{3}\right)$$

Definition:3.4 (Ambiguity Ranking)

The T.H.F.E is denoted as Ѣ, and the expected value explained as $\mathrm{E}(\text{Ѣ}) = \frac{1}{3*\text{Ѣ}}\sum_{\chi*\text{Ѣ}}\left(\frac{a^i+4b^j+c^k}{6}\right)$, where Ѣ is the number of TFN in Ѣ. The above ranking is called ambiguity ranking.

Theorem:3.5

Let Ѣ be any hesitant triangular fuzzy element, and their alpha cut be denoted as $\text{Ѣ}_{1l} = a_1 + \alpha(a_2 - a_1)$ and $\text{Ѣ}_{1r} = a_3 - \alpha(a_3 - a_2)$ then $\mathrm{E}(\text{Ѣ}) = \frac{1}{3*\text{Ѣ}}\sum_{\chi*\text{Ѣ}}\left(\frac{a^i+4b^j+c^k}{6}\right)$.

Proof:

Let us consider

$$\mathrm{E}(\text{Ѣ}) = \frac{1}{3*\text{Ѣ}}\sum_{\chi*\text{Ѣ}} A_\mu(\hbar) \qquad - (1)$$

Where Ѣ = Number of Triangular Hesitant Fuzzy Set

i=1,2,….,n; j = 1,2,…..,n; k = 1,2,….,n

Where $M_\mu(\hbar) = \int_0^1\{[\text{Ѣ}_{1l} - \text{Ѣ}_{1r}]\ f(\alpha)d\alpha$

$$= \int_0^1\{[\,a^i + \alpha\left(b^j - a^i\right)] + [c^k - \alpha\left(c^k - b^j\right)\}\ \alpha d\alpha$$

After simplification, we get

$$M_\mu(\hbar) = \frac{a^i+4b^j+c^k}{6} \qquad - (2)$$

Now substitute equation (2) in (1) we get

$$\mathrm{E}(\text{Ѣ}) = \frac{1}{3*\text{Ѣ}}\sum_{\chi*\text{Ѣ}}\left(\frac{a^i+b^j+c^k}{6}\right)$$

Algorithms of Triangular Hesitant Fuzzy Set for Multicriteria Decision-Making Problems

Algorithm: 3.6 (Existing Method)

Step 1: Construct the T.H.F. decision matrix. Suppose we have 'm' alternatives and 'n' criteria. The triangular hesitant matrix is $\mathrm{H} = \{\mathrm{h}_{ij}\}$ is an $m \times n$ matrix. Let two or more decisions are the same. Then the number should be considered as once in h_{ij}.

Step 2: Expected value is calculated.

Step 3: Obtain the criteria weight by using the standard deviation.

Step 4: Acquire the anticipated value that is weighted.

Step 5: Rank the alternatives.

Algorithm: 3.7 (New Method using Mid Value Ranking and Variance)

Let $\tilde{A}$ = { $\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_u$} be an alternative to a set. Let $\tilde{C}$ = { $\tilde{C}_1, \tilde{C}_2, \dots, \tilde{C}_v$} be a criterion of a set. The decision-makers' suggestions are obtained as

Step 1: A triangular, hesitant, fuzzy decision matrix is constructed with 'u' alternatives and 'v' criteria. The matrix is denoted as $\widetilde{H} = \{Ƕ_{ij}\}$ is an $u \times v$ matrix. Let 2 or more decision-makers' suggestions are the same. Then, the number should be considered as once. $Ƕ_{ij}$.

$$\widetilde{H} = \begin{pmatrix} Ƕ_{11} & Ƕ_{12} & \dots & Ƕ_{1v} \\ Ƕ_{21} & Ƕ_{22} & \dots & Ƕ_{2v} \\ \vdots & \vdots & \vdots & \vdots \\ Ƕ_{u1} & Ƕ_{u2} & \dots & Ƕ_{uv} \end{pmatrix}$$

Step 2: Use the following formula to determine the expected value.

$E_{ij} = \frac{1}{3*Ƕ_{ij}} \sum_{\chi_{ij}*Ƕ_{ij}} \left(\frac{a_{ij}+b_{ij}+c_{ij}}{3}\right)$, where the middle value is only taken from the T.H.F.s. Because with defuzzification, we will get the middle value.

In $Ƕ_{ij}$, where i = 1,2,…,u and j = 1,2,…..,v

$$\tilde{E} = \begin{pmatrix} E_{11} & E_{12} & \dots & E_{1v} \\ E_{21} & E_{22} & \dots & E_{2v} \\ \vdots & \vdots & \vdots & \vdots \\ E_{u1} & E_{u2} & \dots & E_{uv} \end{pmatrix}$$

Step 3: Obtain the criteria weight vector $\widetilde{W} = \{w_1, w_2, \dots, w_v\}$ by using the below formula

$$\widetilde{W}_j = \frac{g_j(c_j)}{\sum_{j=n}^{v} g_j(c_j)}$$

Where $\widetilde{W}_j \geq 0$ and $\sum_{j=n}^{v} \widetilde{W}_j = 1$. Then $g_j(c_j)$ is the variance of the expected values of various alternatives w.r.t criterion.

$$g_j(c_j) = \frac{1}{u} \sum_{j=1}^{u} (E_{ij} - \frac{1}{u} \sum_{i=1}^{v} E_{ij})^2$$

Step 4: Obtain the weighted expected value for each alternative $\tilde{A}_i$ where i = 1,2,…,u

$$\tilde{E}(\tilde{A}_i) = \sum_{j=1}^{v} \widetilde{W}_j E_{ij}$$

Step 5: Rank the alternatives as per the values.

Algorithm: 3.8 (New Method using Ambiguity Ranking and Mean)

Let $\tilde{A}$ = { $\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_u$} be an alternative to a set. Let $\tilde{C}$ = { $\tilde{C}_1, \tilde{C}_2, \dots, \tilde{C}_v$} be a criterion of a set. The decision-makers' suggestions are obtained as

Step 1: A triangular hesitant fuzzy decision matrix is constructed with 'u' alternatives and 'v' criteria. The matrix is denoted as $\widetilde{H} = \{Ƕ_{ij}\}$ is an $u \times v$ matrix. Suppose 2 or more decision-makers' suggestions are the same. Then the number should be considered as once in $Ƕ_{ij}$.

$$\widetilde{H} = \begin{pmatrix} Ƕ_{11} & Ƕ_{12} & \dots & Ƕ_{1v} \\ Ƕ_{21} & Ƕ_{22} & \dots & Ƕ_{2v} \\ \vdots & \vdots & \vdots & \vdots \\ Ƕ_{u1} & Ƕ_{u2} & \dots & Ƕ_{uv} \end{pmatrix}$$

Step 2: compute the expected value by utilizing the mentioned formula

$$E_{ij}{}' = \frac{1}{3*Ƕ_{ij}} \sum_{\chi_{ij}*Ƕ_{ij}} \left(\frac{a_{ij}+4b_{ij}+c_{ij}}{6} \right)$$

In $Ƕ_{ij}$, where i = 1,2,…,u and j = 1,2,…..,v

$$\tilde{E}' = \begin{pmatrix} E_{11} & E_{12} & \dots & E_{1v} \\ E_{21} & E_{22} & \dots & E_{2v} \\ \vdots & \vdots & \vdots & \vdots \\ E_{u1} & E_{u2} & \dots & E_{uv} \end{pmatrix}$$

Step 3: Obtain the criteria weight vector $\widetilde{W} = \{w_1, w_2, \dots, w_v\}$ by using the below formula

$$\widetilde{W}_j = \frac{g_j{}'(c_j)}{\sum_{j=n}^{v} g_j{}'(c_j)}$$

Where $\widetilde{W}_j \geq 0$ and $\sum_{j=n}^{v} \widetilde{W}_j = 1$. Then $g_j(c_j)$ is the mean of the expected values of various alternatives w.r.t criterion.

$$g_j{}'(c_j) = \frac{1}{u} \sum_{j=1}^{u} \left(\left| E_{ij}{}' - \frac{1}{u} \sum_{i=1}^{v} E_{ij}{}' \right| \right)$$

Step 4: Obtain the weighted expected value for each alternative $\tilde{A}_i$ where i = 1,2,…,u

$$\tilde{E}(\tilde{A}_i) = \sum_{j=1}^{v} \widetilde{W}_j \, E_{ij}{}'$$

Step 5: Rank the alternatives according to the values.

4. Results

The World Health Organization (WHO) was created in 1948. At the time of Creation, health was outlined as being "a state of complete physical, mental and social wellbeing and not merely the absence of disease or infirmity" Comprehensive preventive health sources that enable a significant portion of curative and rehabilitative services to be included in occupational health services. Subsequent research concentrated on the physiological and psychological characteristics of workers and their management.

Here, we considered Police officers to be the high-risk category for mental health development disturbances due to numerous serious incidents as well as potentially traumatic incidents that occurred during their careers. These include things like seeing children die, interacting with sexual harassment victims, experiencing major traffic mishaps, suicide, and violent incidents. These are referred to as operational aggravation. The likelihood of experiencing symptoms of hostility, anxiety, as well as fatigue may arise due to these operational stressors. Disorders like depression and post-traumatic stress disorder may strike some people.

As a result of psychological issues, the majority of the police officers experience the 10 diseases as mentioned below: D1- Irritable bowel syndrome / D2- Constipation / D3- High blood pressure / D4 Headaches/ D5- cardiovascular diseases/ D6- Insomnia/ D7Nausea/ D8- Diabetic/ D9- Arthritis/ D10- Mental Fatigue. The alternatives with respect to the criterion are A_1- Tightness; A_2- Tensity; A_3- Worry; A_4- Strain; A_5- Anger

Let's look at the 7 linguistic variable scale Chen (2000)

Table:1

"**Linguistic Variable**	**Linguistic Value**
Very Low (k_1)	(0,0,0.1)
Low (k_2)	(0,0.1,0.3)
Medium Low (k_3)	(0.1,0.3,0.5)
Medium (k_4)	(0.3,0.5,0.7)
Medium-High (k_5)	(0.5,0.7,0.9)
High (k_6)	(0.7,0.9,1)
Very High (k_7)	(0.9,1,1)"

These substitutes are given by 5 experts as a linguistic variable Chen (2000), as shown in the table below

Table:2

	$\tilde{D}_1$	$\tilde{D}_2$	$\tilde{D}_3$	$\tilde{D}_4$	$\tilde{D}_5$	$\tilde{D}_6$	$\tilde{D}_7$	$\tilde{D}_8$	$\tilde{D}_9$	$\tilde{D}_{10}$
$\tilde{A}_1$	k_6 k_4 k_5 k_3	k_6 k_4 k_5 k_3	k_6 k_4 k_6 k_6	k_7 k_6 k_6 k_5	k_6 k_6 k_5 k_6	k_6 k_7 k_5 k_6	k_6 k_5 k_3 k_6	k_5 k_4 k_4 k_5	k_2 k_1 k_1 k_2	k_7 k_6 k_6 k_5
$\tilde{A}_2$	k_6 k_5 k_4 k_6	k_6 k_5 k_4 k_5	k_6 k_7 k_6 k_6	k_7 k_5 k_6 k_4	k_6 k_4 k_5 k_4	k_4 k_4 k_3 k_4	k_3 k_2 k_3 k_2	k_4 k_3 k_2 k_3	k_2 k_1 k_1 k_1	k_6 k_6 k_5 k_6
$\tilde{A}_3$	k_6 k_6 k_6 k_6	k_6 k_7 k_6 k_5	k_7 k_6 k_7 k_6	k_7 k_6 k_7 k_7	k_6 k_5 k_6 k_6	k_7 k_6 k_7 k_7	k_5 k_6 k_3 k_6	k_3 k_4 k_2 k_5	k_3 k_2 k_2 k_4	k_7 k_7 k_6 k_7
$\tilde{A}_4$	k_3 k_6 k_6 k_5	k_2 k_1 k_2 k_3	k_6 k_5 k_3 k_4	k_1 k_4 k_3 k_4	k_2 k_1 k_1 k_2	k_6 k_7 k_6 k_6	k_5 k_6 k_3 k_4	k_1 k_2 k_1 k_2	k_2 k_1 k_2 k_1	k_6 k_5 k_7 k_6
$\tilde{A}_5$	k_3 k_5 k_4 k_2	k_2 k_3 k_1 k_2	k_6 k_5 k_7 k_6	k_6 k_7 k_6 k_5	k_6 k_5 k_3 k_4	k_2 k_1 k_3 k_4	k_1 k_3 k_4 k_2	k_2 k_1 k_4 k_3	k_2 k_1 k_2 k_2	k_4 k_3 k_4 k_5

Table:3

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	$\widetilde{D}_1$	$\widetilde{D}_2$	$\widetilde{D}_3$	$\widetilde{D}_4$	$\widetilde{D}_5$	$\widetilde{D}_6$	$\widetilde{D}_7$	$\widetilde{D}_8$	$\widetilde{D}_9$	$\widetilde{D}_{10}$
$\tilde{A}_1$	(0.7,0.9,1) (0.3,0.5,0.7) (0.5,0.7,0.9) (0.1,0.3,0.5)	(0.7,0.9,1) (0.3,0.5,0.7) (0.5,0.7,0.9) (0.1,0.3,0.5)	(0.7,0.9,1) (0.5,0.7,0.9) (0.7,0.9,1)	(0.9,1,1) (0.7,0.9,1) (0.7,0.9,1) (0.5,0.7,0.9)	(0.7,0.9,1) (0.7,0.9,1) (0.5,0.7,0.9) (0.7,0.9,1)	(0.7,0.9,1) (0.9,1,1) (0.5,0.7,0.9) (0.7,0.9,1)	(0.7,0.9,1) (0.5,0.7,0.9) (0.1,0.3,0.5) (0.7,0.9,1)	(0.5,0.7,0.9) (0.3,0.5,0.7) (0.3,0.5,0.7) (0.5,0.7,0.9)	(0,0.1,0.3) (0,0,0.1) (0,0,0.1) (0,0.1,0.3)	(0.9,1,1) (0.7,0.9,1) (0.7,0.9,1) (0.5,0.7,0.9)
$\tilde{A}_2$	(0.7,0.9,1) (0.5,0.7,0.9) (0.3,0.5,0.7) (0.7,0.9,1)	(0.7,0.9,1) (0.5,0.7,0.9) (0.3,0.5,0.7) (0.5,0.7,0.9)	(0.7,0.9,1) (0.9,1,1) (0.7,0.9,1)	(0.9,1,1) (0.5,0.7,0.9) (0.7,0.9,1) (0.3,0.5,0.7)	(0.7,0.9,1) (0.3,0.5,0.7) (0.5,0.7,0.9) (0.3,0.5,0.7)	(0.3,0.5,0.7) (0.3,0.5,0.7) (0.1,0.3,0.5) (0.3,0.5,0.7)	(0.1,0.3,0.5) (0,0.1,0.3) (0.1,0.3,0.5) (0,0.1,0.3)	(0.3,0.5,0.7) (0.1,0.3,0.5) (0,0.1,0.3) (0.1,0.3,0.5)	(0,0.1,0.3) (0,0,0.1) (0,0,0.1) (0,0,0.1)	(0.7,0.9,1) (0.7,0.9,1) (0.5,0.7,0.9) (0.7,0.9,1)
$\tilde{A}_3$	(0.7,0.9,1) (0.7,0.9,1) (0.7,0.9,1) (0.7,0.9,1)	(0.7,0.9,1) (0.9,1,1) (0.7,0.9,1) (0.5,0.7,0.9)	(0.9,1,1) (0.7,0.9,1) (0.9,1,1)	(0.9,1,1) (0.7,0.9,1) (0.9,1,1) (0.9,1,1)	(0.7,0.9,1) (0.5,0.7,0.9) (0.7,0.9,1) (0.7,0.9,1)	(0.9,1,1) (0.7,0.9,1) (0.9,1,1) (0.9,1,1)	(0.5,0.7,0.9) (0.1,0.3,0.5) (0.1,0.3,0.5) (0.9,1,1)	(0.7.0.9.1) (0.3,0.5,0.7) (0,0.1.0.3) (0.9,1,1)	(0.1.0.3,0.5) (0,0.1,0.3) (0,0.1,0.3) (0.7,0.9,1)	(0.7,0.9,1) (0.5,0.7,0.9) (0.3,0.5,0.7) (0.9,1,1)
$\tilde{A}_4$	(0.1,0.3,0.5) (0.7,0.9,1) (0.7,0.9,1) (0.5,0.7,0.9)	(0,0.1.0.3) (0,0,0.1) (0,0.1,0.3) (0.1,0.3,0.5)	(0.7.0.9,1) (0.5,0.7,0.9) (0.1,0.3,0.5)	(0.0,0.1) (0.3,0.5,0.7) (0.1,0.3,0.5) (0.3,0.5,0.7)	(0,0.1,0.3) (0,0,0.1) (0,0,0.1) (0,0.1,0.3)	(0.7,0.9,1) (0.9,1,1) (0.7,0.9,1) (0.7,0.9,1)	(0.5,0.7,0.9) (0.7,0.9,1) (0.1,0.3,0.5) (0.3,0.5,0.7)	(0,0,0.1) (0,0.1,0.3) (0,0,0.1) (0,0.1,0.3)	(0,0.1,0.3) (0,0,0.1) (0,0.1,0.3) (0,0,0.1)	(0.7,0.9,1) (0.5,0.7,0.9) (0.9,1,1) (0.7,0.9,1)
$\tilde{A}_5$	(-0.1,0.3,0.5) (0.5,0.7,0.9) (0.3,0.5,0.7) (0,0.1,0.3)	(0,0.1,0.3) (0.1,0.3,0.5) (0,0,0.1) (0,0.1,0.3)	(0.7,0.9,1) (0.5,0.7,0.9) (0.9,1,1	(0.7,0.9,1) (0.9,1,1) (0.7,0.9,1) (0.5,0.7,0.9)	(0.7,0.9,1) (0.5,0.7,0.9) (0.1,0.3,0.5) (0.3,0.5,0.7)	(0,0.1,0.3) (0,0,0.1) (0.1,0.3,0.2) (0.3,0.5,0.7)	(0,0,0.1) (0.1,0.3,0.5) (0.3,0.5,0.7) (0,0.1,0.3)	(0,0.1,0.3) (0,0,0.1) (0.3,0.5,0.7) (0.1,0.3,0.5)	(0,0.1,0.3) (0,0,0.1) (0,0.1,0.3) (0,0.1,0.3)	(0.3,0.5,0.7) (0.1,0.3,0.5) (0.3,0.5,0.7) (0.5,0.7,0.9)

Triangular hesitant fuzzy decision matrix

Calculate the anticipated value for every T.H.F.E in the decision matrix $\widetilde{H}$ Utilizing the algorithms listed above:

Algorithm: 4.1 (Existing Method)

By using all the steps in the existing method algorithm, we get the ranking as

$$\tilde{A}_3 > \tilde{A}_1 > \tilde{A}_2 > \tilde{A}_4 > \tilde{A}_5$$

This leads us to the conclusion that worry is the primary cause of illness.

Algorithm: 4.2 (New Method using Mid Value Ranking and Variance)

The expected value is calculated by using step 2 in the algorithm (2)

We get

$$\tilde{E} = \begin{pmatrix} 0.0667 & 0.0667 & 0.0889 & 0.0963 & 0.0889 & 0.0963 & 0.0704 & 0.0667 & 0.0056 & 0.0963 \\ 0.0778 & 0.0778 & 0.1056 & 0.0861 & 0.0778 & 0.0444 & 0.0222 & 0.0333 & 0.0056 & 0.0889 \\ 0.1 & 0.0963 & 0.1056 & 0.1056 & 0.0889 & 0.1056 & 0.0704 & 0.0444 & 0.0333 & 0.1056 \\ 0.0704 & 0.0148 & 0.0667 & 0.0296 & 0.0056 & 0.1056 & 0.0667 & 0.0056 & 0.0056 & 0.0963 \\ 0.0444 & 0.0148 & 0.0963 & 0.0963 & 0.0667 & 0.0250 & 0.0250 & 0.0250 & 0.0056 & 0.0556 \end{pmatrix}$$

Next, the weight of each criterion is obtained from $\tilde{E}$ By using step 3 in an algorithm (2), We have

$\widetilde{W}_1$ =0.2602; $\widetilde{W}_2$ = 0.08943; $\widetilde{W}_3$ =0.0163; $\widetilde{W}_4$ =0.0569; $\widetilde{W}_5$ =0.0732

$\widetilde{W}_6$ =0.0894; $\widetilde{W}_7$ =0.0407; $\widetilde{W}_8$ =0.3415; $\widetilde{W}_9$ =0.0081;$\widetilde{W}_{10}$ =0.0244

Also, the weighted expected value for each alternative $\tilde{A}_i$

$\tilde{E}_W(\tilde{A}_1)$ = 0.0734

$\tilde{E}_W(\tilde{A}_2)$ = 0.0510

$\tilde{E}_W(\tilde{A}_3)$ = 0.0792

$\tilde{E}_W(\tilde{A}_4)$ = 0.0392

$\tilde{E}_W(\tilde{A}_5)$ = 0.0380

Now rank the alternatives as per the expected value

$$\tilde{A}_3 > \tilde{A}_1 > \tilde{A}_2 > \tilde{A}_4 > \tilde{A}_5$$

This leads us to the conclusion that worry is the primary cause of the group of illnesses.

Algorithm:4. 3 (New Method using Ambiguity Ranking and Mean)

The expected value is calculated by using step 2 in an algorithm (3)

We get

$$\tilde{E}' = \begin{pmatrix} 0.1986 & 0.1986 & 0.2639 & 0.2852 & 0.2639 & 0.2852 & 0.2093 & 0.2 & 0.0222 & 0.2852 \\ 0.2315 & 0.2315 & 0.3111 & 0.2556 & 0.2315 & 0.1333 & 0.0695 & 0.1019 & 0.0222 & 0.2639 \\ 0.2944 & 0.2852 & 0.3111 & 0.3111 & 0.2639 & 0.3111 & 0.2093 & 0.1347 & 0.1019 & 0.3111 \\ 0.2093 & 0.0482 & 0.1986 & 0.0907 & 0.0222 & 0.3111 & 0.1986 & 0.0222 & 0.0222 & 0.2852 \\ 0.1347 & 0.0482 & 0.2852 & 0.2852 & 0.1986 & 0.0778 & 0.0778 & 0.0778 & 0.0222 & 0.1667 \end{pmatrix}$$

Next, the weight of each criterion is obtained from $\tilde{E}'$ By using step 3 in an algorithm (3), We have

$\widetilde{W}_1 = 0.0746$; $\widetilde{W}_2$ = 0.1729; $\widetilde{W}_3 = 0.0648$; $\widetilde{W}_4 = 0.1172$; $\widetilde{W}_5 = 0.1316$

$\widetilde{W}_6 = 0.1790$; $\widetilde{W}_7 = 0.1201$; $\widetilde{W}_8 = 0.0909$; $\widetilde{W}_9 = 0.0483$; $\widetilde{W}_{10} = 0.0006$

Also, the weighted expected value for each alternative $\tilde{A}_i$

$\tilde{E}'_W(\tilde{A}_1)$ = 0.2289

$\tilde{E}'_W(\tilde{A}_2)$ = 0.1370

$\tilde{E}'_W(\tilde{A}_3)$ = 0.2601

$\tilde{E}'_W(\tilde{A}_4)$ = 0.1311

$\tilde{E}'_W(\tilde{A}_5)$ = 0.1246

Now rank the alternatives according to the expected value

$$\tilde{A}_3 > \tilde{A}_1 > \tilde{A}_2 > \tilde{A}_4 > \tilde{A}_5$$

This leads us to the conclusion that worry is the primary cause of illness.

5. Discussion and Conclusion

Comparison Table of the Existing and Proposed Methods

Table:4

Existing Method	**Proposed Method – 1** **Mid value Ranking and variance**	**Proposed Method – II** **Ambiguity Ranking and Mean**
The rank according to the expected value is $\tilde{A}_3 > \tilde{A}_1 > \tilde{A}_2 > \tilde{A}_4 > \tilde{A}_5$	The rank according to the expected value is $\tilde{A}_3 > \tilde{A}_1 > \tilde{A}_2 > \tilde{A}_4 > \tilde{A}_5$	The rank according to the expected value is $\tilde{A}_3 > \tilde{A}_1 > \tilde{A}_2 > \tilde{A}_4 > \tilde{A}_5$

In this article, two algorithms for calculating the expected value are proposed to overcome the limitations and shortcomings of the existing method. This paper mainly concentrates on focusing on the newly emerging fuzzy set theory "Hesitant Fuzzy Set Theory". Here, the Triangular Hesitant Fuzzy Sets, which are made up of Triangular Hesitant Fuzzy Elements, were utilized. Under the triangular hesitant fuzzy environment, we provided the decision-making issue with linguistic evaluations and fully unknown criteria weights. Comparison is done between the algorithms. The "mid value ranking variance" of these two proposed methods is a good ranking principle that is also simple to apply to

MCDM problems with uncertain, imprecise, or vague circumstances. This approach in particular takes a lot of time.

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