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The Structure of Generalized Cayley Graph When $Cay(G,S) = P_2 \times P_2$ and $P_2 \times C_3$

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Abstract

This work aims to present the generalized Cayley graph and identify its structure in a few specific scenarios. Assume that Ψ is a finite-group and that S is a non-empty subset of Ψ.

$e \notin S$ and $S^{-1} \subseteq S$. As a result, the vertices of the Cayley graph Cay (Ψ,S) are all members of Ψ, and two nearby vertices, x and y, are only adjacent if $xy^{-1} \in S$. The given generalized Cayley graph is defined as $Cay_m(G,S)$This is a graph whose vertex set is made up of every column matrix X_m It has two vertices and all of its components in Ψ. X_m and Y_m are adjacent ↔ $X_m[(Y_m)^{-1}]^t \in M(S)$, where ${Y_m}^{-1}$ is a column matrix in which ∀ entry correlates to an associated element's inverse. Y_m and $M(S)$ is a m×m matrix where every entry is in S , $[Y^{-1}]^t$ is the opposite of Y^{-1} and $m \geq 1$. In this study, we assign the structure of the new graph and highlight some of its fundamental aspects $Cay_m(G,S)$ when $Cay(G,S)$ is the $P_2 \times P_2$ and $P_2 \times C_3$.

Keywords: Cayley Graph, Algebraic graph theory etc.

Introduction.

Algebraic graph theory has emerged as a prominent mathematical topic of interest to specialists in the domains of algebra and graph theory in recent years. Algebraic graph theory states that every graph may be associated with a group, ring, module, or any other algebraic structure. An algebraic graph that is particularly interesting is the Cayley graph for a group and related subset. In 1878, Arthur Cayley created the Cayley graph to provide clarification on the concept of abstract groups, which at the time were created by a group of generators. A graph with a group encoded is called a Cayley graph. Assuming Ψ is a group and S is its inverse closed subset, we may conclude that e∉S. As a result, the Cayley graph Cay(Ψ,S) is an undirected simpl-graph whose vertex set is made up of all of Ψ's members, and x is next to y only if $xy^{-1} \in S$. We note that $Cay(G,S)$ is a simple r −regular graph and it depends on to set S of the group. Also, $Cay(G,S)$is connected↔ S is a generating set of G. A new definition of the generalized Cayley graph, called Caym (Ψ,S), was recently provided by Erfanian in [4]. This new definition uses column m×1 matrices and is a novel extension of the standard Cay((Ψ,S)). The generalized Cayley graph, represented as Cay_m (G,S), is an undirected simple graph with two vertices and a vertex set made up of all m×1 matrices, where x_i∈G,1≤i≤m, for each positive integer m≥1 $X = [x_1, x_2, \dots, x_m]^t$ and $Y = [y_1, y_2, \dots, y_m]^t$ are contiguous only in the event that X(Y)

)t∈M(S). Since it is obvious that the standard Cayley graph Cay(Ψ,S) exists if m=1, we refer to this as the generalized Cayley graph. In this work, we consistently assume that S^(-1)⊆S, e∉S, and S is a entertain set of G. Cay(G,S) is therefore a connected graph in this case.
In this paper, we focus on the Cartesian product of two graphs in order to determine the generalized Cayley graph. $Cay(G,S) = P_2 \times P_2$ and $Cay(G,S) = P_2 \times C_3$.

Binary operations create a new graph from two initial graphs G, H, such as graph union, Cartesian graph product, Corona graph product, and generalized corona product. Here we define these graph operations.

Definition 1. Assuming Ψ and H represent two graphs. Following that, the graph represented by Ψ∪H, which is the union of Ψ and H $V(G \cup H) = V(G) \cup V(H)$ and $E(G \cup H) = E(G) \cup E(H)$.

Definition 2. The graph denoted by Ψ×H is the Cartesian product of Ψ and H, with V(G)×V(H) as its vertex set. There are two vertices $(g, h), (g', h')$ are next to each other if (gg'∈ G┤ and ├ h=h^'∈E(H)) or (g=g' ┤ and ├ hh'∈E(H)). Therefore, E(G×H)={(g,h)(g',h') │ g=g',hh'∈E(H) or gg'∈ E(G),├ h=h') } and V(G×H)={(g,h) ├|g∈┤ V(G),h∈V (H)}. Factors of G×H are represented by the graph G,H.

Definition 3. Assuming ψ and H are graphs, one may derive the Corona product of Ψ and H, represented as Ψ∘H, by linking each vertex of the i-th copy of H to the i-th vertex of G, where 1≤i≤|V(G)|, using one copy of Ψ and |V(G)| The functioning of copies of H. Corona product is non-commutative. $i.e.\ G \circ H \neq H \circ G.$.

Lemma 4. Let $X = [\varpi_1, \varpi_2, \dots, \varpi_m]^t$ and $Y = [y_1, y_2, \dots, y_m]^t$ be two arbitrary vertices of $Cay_m(G,S)$ where x_i and y_j are in G for all $i, j \in \{1,2,\dots,m\}$, then X and Y are adjacent $\leftrightarrow$ x_i is adjacent to y_j in $Cay(G,S)$ ∀l $i, j \in \{1,2,\dots,m\}$.

Here, we find the generalized Cayley graph when $Cay(G,S)$ is the Cartesian products $P_2 \times P_2$ and $P_2 \times C_3$.

Lemma 5. Let $Cay(G,S) = P_2 \times P_2$, then $Cay_2(G,S) = K_{4,4} \cup 8P_1$.

Proof: Suppose that $G_1 = P_2$ with vertex set $\{x_1, x_2\}$ and $G_2 = P_2$ with vertex set $\{x_3, x_4\}$ and $Cay(G,S) = P_2 \times P_2$. So, $Cay(G,S)$ is a cycle of length 4 and its vertex set is

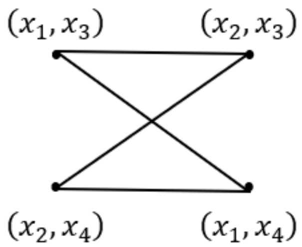
$$V(Cay(G,S)) = \{(x_1,x_3),(x_1,x_4),(x_2,x_3),(x_2,x_4)\}$$

and the set of four edges $\{(x_1,x_3)(x_1,x_4),(x_1,x_3)(x_2,x_3),(x_2,x_3)(x_2,x_4),(x_2,x_4)(x_1,x_4)\}$

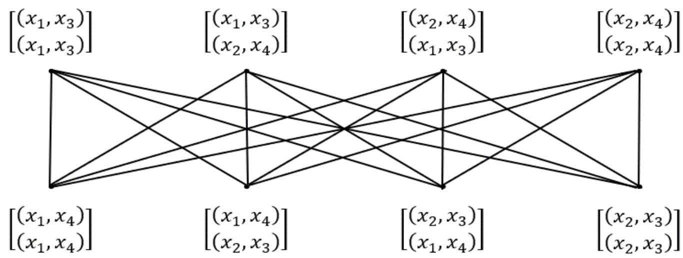
Since the Cayley graph is a cycle $(x_1,x_3) - (x_1,x_4) - (x_2,x_4) - (x_2,x_3) - (x_1,x_3)$. Then, we have $4^2 = 16$ vertices in $Cay_2(G,S)$ and $V(Cay_2(G,S)) = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} \middle| a, b \in V(Cay(G,S)) \right\}$

$$\left\{\begin{bmatrix}(x_1,x_3)\\(x_1,x_3)\end{bmatrix},\begin{bmatrix}(x_1,x_3)\\(x_1,x_4)\end{bmatrix},\begin{bmatrix}(x_1,x_3)\\(x_2,x_3)\end{bmatrix},\begin{bmatrix}(x_1,x_3)\\(x_2,x_4)\end{bmatrix},\right.$$
$$\begin{bmatrix}(x_1,x_4)\\(x_1,x_3)\end{bmatrix},\begin{bmatrix}(x_1,x_4)\\(x_1,x_4)\end{bmatrix},\begin{bmatrix}(x_1,x_4)\\(x_2,x_3)\end{bmatrix},\begin{bmatrix}(x_1,x_4)\\(x_2,x_4)\end{bmatrix},$$
$$\begin{bmatrix}(x_2,x_3)\\(x_1,x_3)\end{bmatrix},\begin{bmatrix}(x_2,x_3)\\(x_1,x_4)\end{bmatrix},\begin{bmatrix}(x_2,x_3)\\(x_2,x_3)\end{bmatrix},\begin{bmatrix}(x_2,x_3)\\(x_2,x_4)\end{bmatrix},$$
$$\left.\begin{bmatrix}(x_2,x_4)\\(x_1,x_3)\end{bmatrix},\begin{bmatrix}(x_2,x_4)\\(x_1,x_4)\end{bmatrix},\begin{bmatrix}(x_2,x_4)\\(x_2,x_3)\end{bmatrix},\begin{bmatrix}(x_2,x_4)\\(x_2,x_4)\end{bmatrix}\right\}$$

Consequently. Every vertex in set A is obviously adjacent to every vertex in set B, and vice versa. Thus, the bipartite graph is obtained $K_{4,4}$. We demonstrate that every other vertex is an independent vertex. Assume, without losing generality, that $\begin{bmatrix}(x_1,x_3)\\(x_1,x_4)\end{bmatrix}$ is not isolated. So, there is a vertex $\begin{bmatrix}(a,c)\\(b,d)\end{bmatrix} \in V(Cay_2(G,S))$ such that $(x_1,x_3)-(a,c)$, $(x_1,x_3)-(b,d)$, $(x_1,x_4)-(a,b)$ and $(x_1,x_4)-(b,d)$. So, $(a,c)=(x_1,x_4)$ or $(a,c)=(x_2,x_3)$. If $(a,c)=(x_1,x_4)$, then $(a,c)-(x_1,x_4)$ then it implies that $(x_1,x_4)-(x_1,x_4)$ which is a contradiction. Similarly, If $(a,c)=(x_2,x_3)$, then $(a,c)-(x_1,x_4)$ which implies that $(x_2,x_3)-(x_1,x_4)$ and gain it is a contradiction. Hence, $\begin{bmatrix}(x_1,x_3)\\(x_1,x_4)\end{bmatrix}$ is an isolated vertex. The following procedure may be used to other vertices as well. There are these solitary vertices in an amount of $4^2-8=8$, and hence $Cay_2(G,S)=K_{4,4}\cup 8P_1$. The graph of $Cay_2(G,S)$ in this case, is shown below. ■



The graph $P_2\times P_2$



A component of graph $Cay_2(G,S)$ of $P_2\times P_2$

In the next Theorem, we generalized the Cayley graph for each m=3 when the common Cayley graph is $P_2\times P_2$.

Lemma 6. Let $Cay(G,S)=P_2\times P_2$, then $Cay_3(G,S)=K_{8,8}\cup 48P_1$.

Proof: Suppose that $G_1=P_2$ with vertex set $\{x_1,x_2\}$ and $G_2=P_2$ with vertex set $\{x_3,x_4\}$ and $Cay(G,S)=P_2\times P_2$. So, $Cay(G,S)$ is a cycle of lenψth 4 and its vertex set is

$$V(Cay(G,S))=\{(x_1,x_3),(x_1,x_4),(x_2,x_3),(x_2,x_4)\}$$

and the set of four edψes $\{(x_1,x_3)(x_1,x_4),(x_1,x_3)(x_2,x_3),(x_2,x_3)(x_2,x_4),(x_2,x_4)(x_1,x_4)\}$

Since the Cayley graph is a cycle $(x_1,x_3)-(x_1,x_4)-(x_2,x_4)-(x_2,x_3)-(x_1,x_3)$. Then, we have

$4^3 = 64$ vertices in $Cay_3(G,S)$ and $V\big(Cay_3(G,S)\big) = \left\{\begin{bmatrix} a \\ b \\ c \end{bmatrix} \middle| a,b,c \in V\big(Cay(G,S)\big)\right\}$. So,

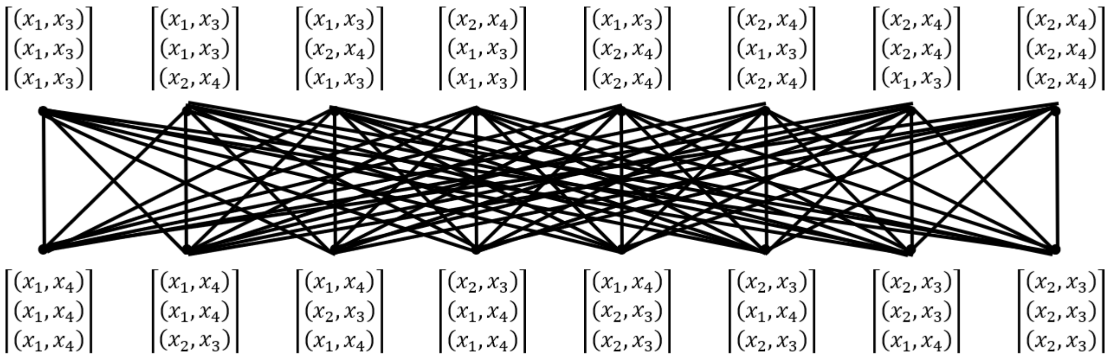
$V\big(Cay_3(G,S)\big) = \left\{\begin{bmatrix} (x_i,x_j) \\ (x_k,x_l) \\ (x_r,x_s) \end{bmatrix} \quad : \quad i,j,k,l,r,s = 1,2,3,4\right\}$. Therefore, we have two independent sets

$$A = \left\{\begin{bmatrix} (x_1,x_3) \\ (x_1,x_3) \\ (x_1,x_3) \end{bmatrix}, \begin{bmatrix} (x_1,x_3) \\ (x_1,x_3) \\ (x_2,x_4) \end{bmatrix}, \begin{bmatrix} (x_1,x_3) \\ (x_2,x_4) \\ (x_1,x_3) \end{bmatrix}, \begin{bmatrix} (x_2,x_4) \\ (x_1,x_3) \\ (x_1,x_3) \end{bmatrix}, \begin{bmatrix} (x_1,x_3) \\ (x_2,x_4) \\ (x_2,x_4) \end{bmatrix}, \begin{bmatrix} (x_2,x_4) \\ (x_1,x_3) \\ (x_2,x_4) \end{bmatrix}, \begin{bmatrix} (x_2,x_4) \\ (x_2,x_4) \\ (x_1,x_3) \end{bmatrix}, \begin{bmatrix} (x_2,x_4) \\ (x_2,x_4) \\ (x_2,x_4) \end{bmatrix}\right\}$$

and

$$B = \left\{\begin{bmatrix} (x_1,x_4) \\ (x_1,x_4) \\ (x_1,x_4) \end{bmatrix}, \begin{bmatrix} (x_1,x_4) \\ (x_1,x_4) \\ (x_2,x_3) \end{bmatrix}, \begin{bmatrix} (x_1,x_4) \\ (x_2,x_3) \\ (x_1,x_4) \end{bmatrix}, \begin{bmatrix} (x_2,x_3) \\ (x_1,x_4) \\ (x_1,x_4) \end{bmatrix}, \begin{bmatrix} (x_1,x_4) \\ (x_2,x_3) \\ (x_2,x_3) \end{bmatrix}, \begin{bmatrix} (x_2,x_3) \\ (x_1,x_4) \\ (x_2,x_3) \end{bmatrix}, \begin{bmatrix} (x_2,x_3) \\ (x_2,x_3) \\ (x_1,x_4) \end{bmatrix}, \begin{bmatrix} (x_2,x_3) \\ (x_2,x_3) \\ (x_2,x_3) \end{bmatrix}\right\}.$$

It is clear that every vertex in set A is adjacent to all vertices in set B and vice versa. Thus, we ψet the bipartite graph $K_{8,8}$. We demonstrate that every other vertex is an independent vertex. Absent loss of generality, suppose that $\begin{bmatrix} (x_i,x_j) \\ (x_k,x_l) \\ (x_r,x_s) \end{bmatrix}$ is not isolated where $i,k,r = 1,2$ and $j,l,s = 3,4$. So, there is a vertex $\begin{bmatrix} (a,b) \\ (c,d) \\ (e,f) \end{bmatrix} \in V(Cay_3(G,S))$ such that $(x_i,x_j) - (a,b)$, $(x_i,x_j) - (c,d)$, $(x_i,x_j) - (e,f)$ and $(x_k,x_l) - (a,b)$, $(x_k,x_l) - (c,d)$, $(x_k,x_l) - (e,f)$ and $(x_r,x_s) - (a,b)$, $(x_r,x_s) - (c,d)$, $(x_r,x_s) - (e,f)$. but (x_i,x_j) is of deψree 2. So, $(a,b) = (x_i,x_j)$ or $(a,b) = (x_k,x_l)$ or $(a,b) = (x_r,x_s)$. If $(a,b) = (x_i,x_j)$, then it implies that $(x_i,x_j) - (x_i,x_j)$ which is a contradiction. Likewise, If $(a,b) = (x_k,x_l)$ and $(a,b) = (x_r,x_s)$ we ψet the a contradiction. Hence, the rest vertices $\begin{bmatrix} (x_i,x_j) \\ (x_k,x_l) \\ (x_r,x_s) \end{bmatrix}$ are isolated vertices. We can prove by the same method as above for more vertices. There are these solitary vertices in an amount of $|V(Cay_3(G,S)| - (|A| + |B|) = 4^3 - (8+8) = 48$, and hence $Cay_3(G,S) = K_{8,8} \cup 48P_1$. The graph of $Cay_3(G,S)$ is shown below. ■



A component of graph $Cay_3(G,S)$ of $P_2 \times P_2$

In the next Theorem, we generalized the Cayley graph for each $m \geq 2$ when the common Cayley graph is $P_2 \times P_2$.

Theorem 7. Let $Cay(G,S) = P_2 \times P_2$, then the generalized Cayley graph $Cay_m(G,S)$ is the graph $K_{2^m,2^m} \cup \left(2^{m+1}(2^{m-1}-1)\right)P_1$ for all $m \geq 2$.

Proof: Suppose that $G_1 = P_2$ with vertex set $\{x_1,x_2\}$ and $G_2 = P_2$ with vertex set $\{x_3,x_4\}$ and $Cay(G,S) = P_2 \times P_2$. So, $Cay(G,S)$ is a cycle of lenψth 4 and its vertex set is

$$V(Cay(G,S)) = \{(x_1,x_3),(x_1,x_4),(x_2,x_3),(x_2,x_4)\}$$

and the set of edψes is $(x_1,x_3)-(x_1,x_4)-(x_2,x_4)-(x_2,x_3)-(x_1,x_3)$. So, $V = V\left(Cay_m(G,S)\right) = \left\{[a_1,a_2,\ldots,a_m]^t \;\middle|\; a_1,a_2,\cdots,a_m \in V\left(Cay(G,S)\right)\right\}$. Therefore, $\left|V\left(Cay_m(G,S)\right)\right| = 4^m$. Consider the subsets A and B of V as follows:

$A = \{\ [a_1,a_2,\ldots,a_m]^t : a_i \in \{x_1,x_3\}, i = 1,2,\ldots,m\}$ and $B = \{\ [a_1,a_2,\ldots,a_m]^t : a_i \in \{x_2,x_4\}, i = 1,2,\ldots,m\}$. We can see that A and B are independent sets and that every vertex from one is adjacent to another set using the same technique used in the demonstration of the preceding lemma. As a result, the entire bipartite network is induced by the union of disjoint sets A∪ Ɓ, and the remaining vertices are all isolated vertices. Consequently, $Cay_m(G,S) = K_{2^m,2^m} \cup$
$\left(2^{m+1}(2^{m-1}-1)\right)P_1$

for all $m \geq 2$. ∎

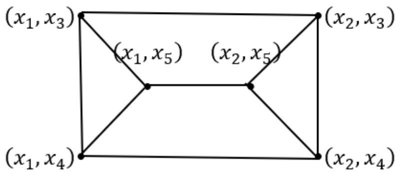
In the next lemma, we find the generalized Cayley graph for the special case $n = 2$ when $Cay(G,S) = P_2 \times C_3$.

Theorem 8. Let $Cay(G,S) = P_2 \times C_3$, then $Cay_2(G,S)$ has $\left((P_2 \times C_3) \circ 2P_1\right) \cup 18P_1$ as a subgraph.

Proof: Suppose that $G_1 = P_2$ with vertex set $\{x_1,x_2\}$ and $G_2 = C_3$ with vertex set $\{x_3,x_4,x_5\}$ and $Cay(G,S) = P_2 \times C_3$. So, $V\left(Cay(G,S)\right) = \{(x_1,x_3),(x_1,x_4),(x_1,x_5),(x_2,x_3)(x_2,x_4),(x_2,x_5)\}$

and $|V(Cay(G,S)| = 6$ and $E\left(Cay(G,S)\right) = \{(x_1,x_3)(x_1,x_4),(x_1,x_3)(x_2,x_3),(x_2,x_3)(x_2,x_4),$

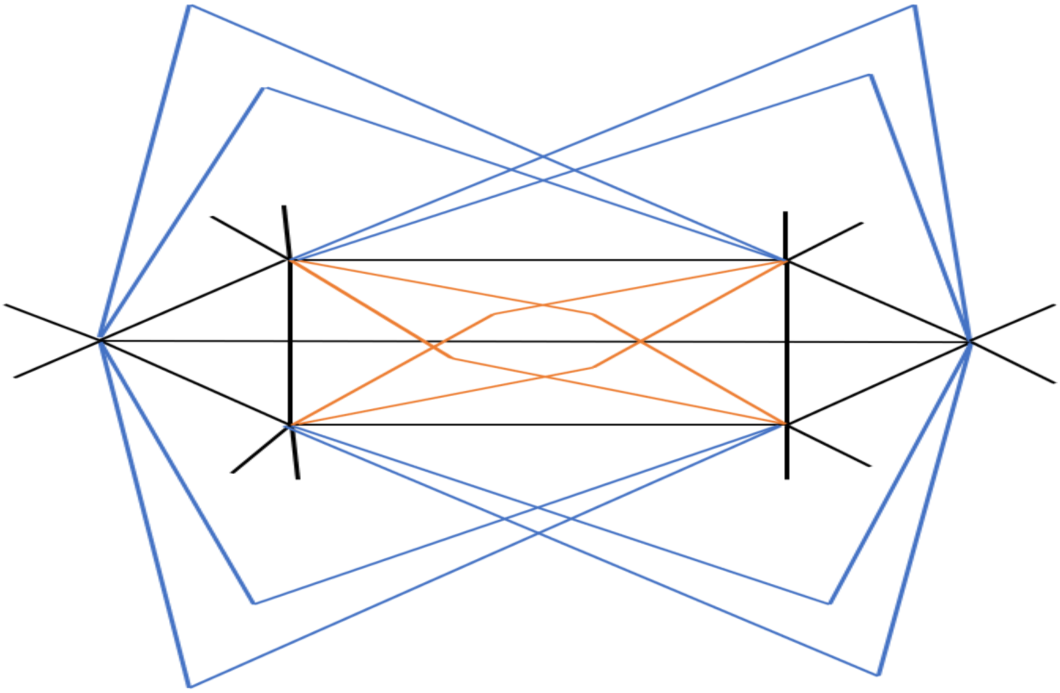
$(x_2,x_4)(x_1,x_4),(x_2,x_4)(x_2,x_5),(x_2,x_3)(x_2,x_5),(x_1,x_3)(x_1,x_5),,(x_1,x_4)(x_1,x_5),(x_2,x_5)(x_1,x_5)$T he graph $Cay(G,S) = P_2 \times C_3$ shown in below.



Then, we have $6^2 = 36$ vertices in $Cay_2(G, S)$ and $V\left(Cay_2(G,S)\right) = \left\{\begin{bmatrix} a \\ b \end{bmatrix} \middle| a, b \in V\left(Cay(G,S)\right)\right\} =$

$$\left\{\begin{bmatrix}(x_1,x_3)\\(x_1,x_3)\end{bmatrix}, \begin{bmatrix}(x_1,x_3)\\(x_1,x_4)\end{bmatrix}, \begin{bmatrix}(x_1,x_3)\\(x_1,x_5)\end{bmatrix}, \begin{bmatrix}(x_1,x_3)\\(x_2,x_3)\end{bmatrix}, \begin{bmatrix}(x_1,x_3)\\(x_2,x_4)\end{bmatrix}, \begin{bmatrix}(x_1,x_3)\\(x_2,x_5)\end{bmatrix},\right.$$
$$\begin{bmatrix}(x_1,x_4)\\(x_1,x_3)\end{bmatrix}, \begin{bmatrix}(x_1,x_4)\\(x_1,x_4)\end{bmatrix}, \begin{bmatrix}(x_1,x_4)\\(x_1,x_5)\end{bmatrix}, \begin{bmatrix}(x_1,x_4)\\(x_2,x_3)\end{bmatrix}, \begin{bmatrix}(x_1,x_4)\\(x_2,x_4)\end{bmatrix}, \begin{bmatrix}(x_1,x_4)\\(x_2,x_5)\end{bmatrix},$$
$$\begin{bmatrix}(x_1,x_5)\\(x_1,x_3)\end{bmatrix}, \begin{bmatrix}(x_1,x_5)\\(x_1,x_4)\end{bmatrix}, \begin{bmatrix}(x_1,x_5)\\(x_1,x_5)\end{bmatrix}, \begin{bmatrix}(x_1,x_5)\\(x_2,x_3)\end{bmatrix}, \begin{bmatrix}(x_1,x_5)\\(x_2,x_4)\end{bmatrix}, \begin{bmatrix}(x_1,x_5)\\(x_2,x_5)\end{bmatrix},$$
$$\begin{bmatrix}(x_2,x_3)\\(x_1,x_3)\end{bmatrix}, \begin{bmatrix}(x_2,x_3)\\(x_1,x_4)\end{bmatrix}, \begin{bmatrix}(x_2,x_3)\\(x_1,x_5)\end{bmatrix}, \begin{bmatrix}(x_2,x_3)\\(x_2,x_3)\end{bmatrix}, \begin{bmatrix}(x_2,x_3)\\(x_2,x_4)\end{bmatrix}, \begin{bmatrix}(x_2,x_3)\\(x_2,x_5)\end{bmatrix},$$
$$\begin{bmatrix}(x_2,x_4)\\(x_1,x_3)\end{bmatrix}, \begin{bmatrix}(x_2,x_4)\\(x_1,x_4)\end{bmatrix}, \begin{bmatrix}(x_2,x_4)\\(x_1,x_5)\end{bmatrix}, \begin{bmatrix}(x_2,x_4)\\(x_2,x_3)\end{bmatrix}, \begin{bmatrix}(x_2,x_4)\\(x_2,x_4)\end{bmatrix}, \begin{bmatrix}(x_2,x_4)\\(x_2,x_5)\end{bmatrix},$$
$$\left.\begin{bmatrix}(x_2,x_5)\\(x_1,x_3)\end{bmatrix}, \begin{bmatrix}(x_2,x_5)\\(x_1,x_4)\end{bmatrix}, \begin{bmatrix}(x_2,x_5)\\(x_1,x_5)\end{bmatrix}, \begin{bmatrix}(x_2,x_5)\\(x_2,x_3)\end{bmatrix}, \begin{bmatrix}(x_2,x_5)\\(x_2,x_4)\end{bmatrix}, \begin{bmatrix}(x_2,x_5)\\(x_2,x_5)\end{bmatrix}\right\}$$

Therefore, each vertex $\begin{bmatrix}(x_i, x_j)\\(x_i, x_j)\end{bmatrix}$ has deψree 4 and it is adjacent to the vertices $\begin{bmatrix}(x_i, x_{j+1})\\(x_{i+1}, x_j)\end{bmatrix}$ and $\begin{bmatrix}(x_{i+1}, x_j)\\(x_i, x_{j+1})\end{bmatrix}$ and $\begin{bmatrix}(x_i, x_{j+1})\\(x_i, x_{j+1})\end{bmatrix}$ and $\begin{bmatrix}(x_i, x_{j+2})\\(x_i, x_{j+2})\end{bmatrix}$. The other vertices are isolated. The graph $Cay_2(G,S)$ is shown in below. ■



References

[1] Farrokhi DΨ, M., Rajabian, M., & Erfanian, A. (2019). Relative Cayley graphs of finite ψroups. Asian-European Journal of Mathematics, 12(07), 2050003.

[2] Frucht, R., Harary, F. On the Corona of two graphs. Aeq. Math. 4, 322–325 (1970).

[3] S. Mohammadi, Some ψeneralizations of Cayley graph and Intersection graph, Ph.D Thesis, Ferdowsi University of Mashhad, Mashhad, Iran, 2020.

[4] Neamah, A. A., Erfanian, A., & Majeed, A. H. (2022). On A Generalized Cayley Graph of Column Matrices Of Elements Of A Finite Ψroup. Mathematica (1222-9016), 64(2).

[5] Neamah, A. A., Majeed, A. H., & Erfanian, A. (2022). The generalized Cayley graph of complete graph K_nand complete multipartite graphs K_(n,n) and K_(n,n,n). Iraqi Journal of Science, 3103-3110.

[6] Neamah, A. A., Erfanian, A., & Majeed, A. H. (2023, December). The structure of generalized Cayley graph when Cay (G,S)= K_(n,n,n,n). In AIP Conference Proceedinψs (Vol. 2834, No. 1). AIP Publishinψ.

[7] Neamah, S. A., & Erfanian, A. (2023). Some Results on the Generalized Cayley Graph of Complete Graphs. Iraqi Journal of Science, 3424-3436.

[8] Kelarev, A. V. (2002). On undirected Cayley graphs. Australasian Journal of Combinatorics, 25, 73-78.

[9] Rajabian, M., & Erfanian, A. (2018). Relative Cayley graphs of finite ψroups. Asian-European Journal of Mathematics, 12.