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A Novel Application of Temporal Intuitionistic Fuzzy Hypergraphs in Mobile Networks

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Abstract

The relationships among the objects in actual scenario are more complex than pairwise. Information loss will unavoidably result from naively condensing complicated relationships into pairwise ones. As a result, hypergraphs are necessary to depict the intricate interactions between the items of our interest, which leads to the issue of learning with hypergraphs. Intuitionistic Fuzzy Hypergraph defined in a time domain, termed as Temporal Intuitionistic Fuzzy Hypergraph (TIFHG) is introduced. Dual Temporal Intuitionistic Fuzzy Hypergraph has also been offered as a concept. An application of TIFHG in mobile communication is discussed.

Keywords: Intuitionistic fuzzy hypergraph, Temporal intuitionistic fuzzy hypergraph, dual intuitionistic fuzzy hypergraph.

1 Introduction

The principle of graph theory was first introduced by Euler in 1736. Combinatorial problems can be solved with great benefit from the study of graph theory in a variety of disciplines, including computer science, topology, geometry, algebra, number theory, and optimization. The term “graph” was changed to “hypergraph” to broaden the scope of applications. Berge first presented the ideas behind hypergraph in 1976. Moderson introduced the phrases fuzzy graph and fuzzy hypergraph. A generalization of fuzzy sets, intuitionistic fuzzy sets were introduced by Atanassov in 1986. In later years, intuitionistic fuzzy graph (IFHG) and intuitionistic fuzzy hypergraph (IFHG) were made official.

Hypergraph reflects better features of relationship between individual entities than graphical structures. They are better at visualizing the complex situations that frequently occur in real-world settings. The heterogeneous characteristics in social networks can be mapped with help of hyperlinks than graphical notions. A detailed hypergraph representation was used to model the multiplex structure. It takes time to formulate real world problems, and they are not always paired. For instance, conversations via email and interactions in online communities, to mention a few. Combining a collection of nodes and a collection of hyperlinks, they create a hypergraph. Hypergraphs are made more powerfully expressive by hyperlinks, as each one represents a group interaction between multiple people or things and is a subset of any number of nodes.

In this study, we introduce the temporal domain into the intuitionistic fuzzy hypergraph. System analysis, circuit clustering, and pattern recognition have all been the subject of extensive research.

Furthermore, a large variety of significant real-world applications in computer science and mathematics, including image and video processing, inherently involve hypergraphs.

Why TIFS?

Recently, several studies have provided the properties of temporal hypergraph. They have their origin from different domains including email, interaction among people, threads, coauthorship for the publication. Vagueness related to time associated with every day problem leads to the introduction of intuitionistic fuzzy hypergraph in time domain. Thus, the authors are motivated to define temporal intuitionistic fuzzy hypergraph.

Definitions of hypergraph and Temporal Intuitionistic Fuzzy Hypergraph (TIFH) are covered in Section 2. The dual of TIFH is derived in Section 3. Moreover, a few applications are covered. The paper's conclusion is given in Section 4.

2 Preliminaries

This section includes relevant examples and definitions of the terms hypergraph, fuzzy hypergraph and temporal fuzzy hypergraph.

Definition 2.1. [3] A hypergraph H is the set of ordered pair H = (V,E) where

1. $V = \{n_1, n_2, ..., n_r\}$ a set of finite nodes
2. $E = \{E_1, E_2, ..., E_s\}$ a collection of subsets of V
3. $E_q \neq \phi$, q = 1, 2, ..., s and
4. $\cup_q E_q = V$

The collection of nodes is represented by the V, and the collection of hyperlinks is represented by the E.

Note

1. If $\|E_q\|=1$ by a cycle on the element, A solid line surrounding the link between the E_q's nodes is used to symbolize it.
2. An undirected, regular graph replaces the hypergraph if $\|E_q\|= 2$ for all q.
3. Another way to depict the hypergraph (V, E) is as $(V; E_1, E_2,..., E_s)$.

Definition 2.2. [6]

An Intuitionistic Fuzzy Hypergraph (IFHG) is the set of ordered pair $H = \langle V, E\rangle$ where

1. $V = \{n_1, n_2, ..., n_r\}$ is a set of finite intuitionistic fuzzy nodes
2. $E = \{E_1, E_2, ..., E_s\}$ is a collection of crisp subsets of V
3. $E_q = \{(v_p, \mu_q(v_p), \nu_q(v_p)): \mu_q(v_p), \nu_q(v_p) \geq 0 \text{ and } \mu_q(v_p) + \nu_q(v_p) \leq 1\}$, q = 1,2, ...,s,
4. $E_q \neq \phi$, q = 1,2, ..., s,
5. $\cup_q supp(E_q)=V$, q = 1, 2,..., s.

Here, the hyperlinks E_q are crisp sets of intuitionistic fuzzy nodes, $\nu_q(n_p)$ and $\mu_q(n_p)$ which respectively indicates the degrees of non-membership and membership of node n_p into hyperlink E_q. In light of this, the IFHG incidence matrix's elements have the form $(n_{pq}, \mu_q(n_p), \nu_q(n_p))$. The sets of nodes and hyperlinks V and E respectively are crisp sets.

Notations

1. Throughout this paper, $\langle\mu(n_p), \nu(n_p)\rangle$ or simply $\langle\mu_p, \nu_q\rangle$ indicates the degrees of the node $n_p \in$ V's membership and non-membership, with $0 \leq \mu_p + \nu_q \leq 1$.
2. $\langle\mu_q(n_p), \nu_q(n_p)\rangle$ or simply $\langle\mu_{pq}, \nu_{pq}\rangle$ represent the degrees of the hyperlink E_q's membership and non- membership, with $0 \leq \mu_{pq} + \nu_{pq} \leq 1$. Also, μ_{pq} is the membership value of p^{th} node in q^{th} hyperlink and ν_{pq}, be the non-membership value of p^{th} node in q^{th} hyperlink.
3. If $\mu_{pq} = 0$ and $\nu_{pq} = 1$ for some p and q, then $n_p \notin E_q$ and it is indexed by $\langle 0, 1\rangle$.

Note

1. The *support* of a hyperlink E_q in E, as shown by *supp*(E_q), is described as $supp(E_q) = \{n_p/\mu_p(n_p) > 0 \text{ and } \nu_q(n_p) > 0\}$.

Example 2.1.1

Consider an IFHG, H, such that V = $\{n_1, n_2, n_3, n_4\}$, E = $\{E_1, E_2, E_3, E_4\}$, given by the adjacency matrix

$$\begin{array}{c} \\ n_1 \\ n_2 \\ n_3 \\ n_4 \end{array} \begin{array}{c} \begin{array}{cccc} E_1 & E_2 & E_3 & E_4 \end{array} \\ \begin{pmatrix} \langle 0,1\rangle & \langle 0.5,0.4\rangle & \langle 0.2,0.6\rangle & \langle 0,1\rangle \\ \langle 0.4,0.2\rangle & \langle 0,1\rangle & \langle 0.4,0.3\rangle & \langle 0.2,0.4\rangle \\ \langle 0,1\rangle & \langle 0.4,0.5\rangle & \langle 0,1\rangle & \langle 0.3,0.4\rangle \\ \langle 0.2,0.7\rangle & \langle 0,1\rangle & \langle 0.3,0.4\rangle & \langle 0,1\rangle \end{pmatrix} \end{array}$$

Definition 2.3.

A triplet H (V, E, T) is a Temporal Intuitionistic Fuzzy Hypergraph (TIFHG), with

1. V = $\{n_1, n_2, ..., n_r\}$ is a group of fuzzy, finite intuitionistic nodes
2. E = $\{E_1, E_2, \dots, E_s\}$ is a set of fuzzy, intuitionistic subsets of V
3. T = $\{t_1, t_2, ..., t_j\}$, $T \neq \emptyset$ is the time domain
4. $E_q = \{\langle n_p, \mu_q(n_p,t), \nu_q(n_p,t)\rangle : \mu_q(n_p,t), \nu_q(n_p,t) \geq 0 \text{ and } \mu_q(n_p,t) + \nu_q(n_p,t) \leq 1\}$, q = 1, 2, ..., s, $t \in$ T
5. $E_q \neq \emptyset$, q = 1, 2, ..., s,
6. $\cup_q supp(E_q) = V$, q = 1, 2, ..., s.

In this instance, the hyperlinks E_q are crisp sets of IF fuzzy nodes, $\nu_q(n_p, t_k)$ and $\mu_q(n_p, t_k)$ which respectively represents the degrees of non-membership and membership of node n_p into hyperlink E_q at time t_k.

Note

1. The TIFHG incidence matrix's components have the following form (n_{pq}, $\mu_q(n_p,t_k)$, $\nu_q(n_p,t_k)$).
2. The node set V and link E are crisp sets.

Example.

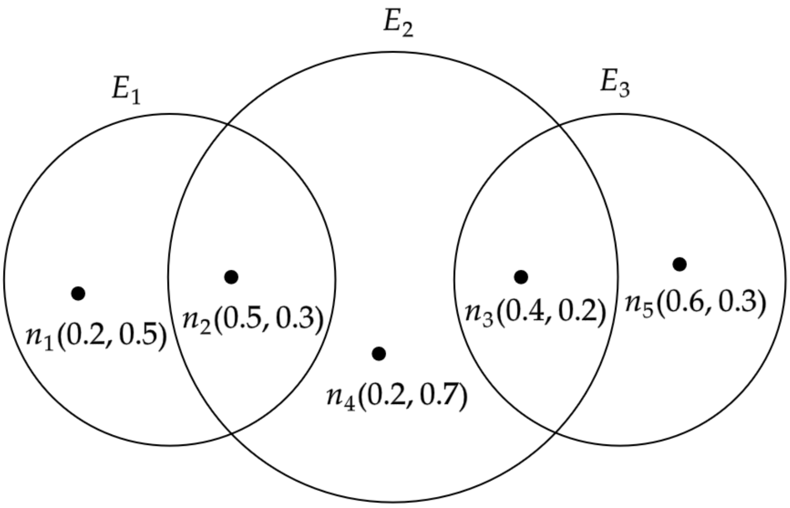

Figure 2.1: Temporal Intuitionistic Fuzzy Hypergraphs at time t_1

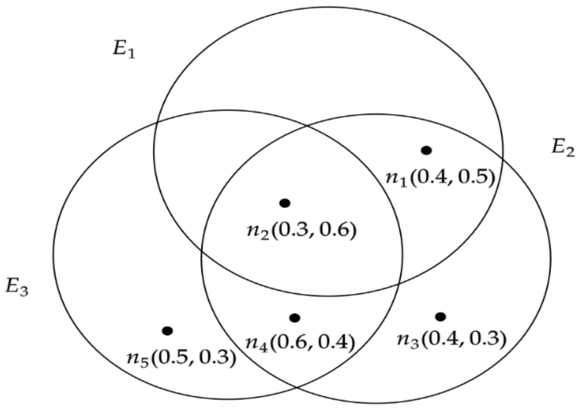

Figure 2.2: Temporal Intuitionistic Fuzzy Hypergraphs at time t_2

The index matrix representation of the Figure 2.1 and Figure 2.2 are represented as given. At time t_1 the Figure 2.1 is exhibited as:

$$\begin{array}{c c} & \begin{array}{ccc} E_1 & E_2 & E_3 \end{array} \\ \begin{array}{c} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \end{array} & \begin{pmatrix} \langle 0.2, 0.5\rangle & \langle 0,1\rangle & \langle 0,1\rangle \\ \langle 0.5, 0.3\rangle & \langle 0.5, 0.3\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0.4, 0.2\rangle & \langle 0.4, 0.2\rangle \\ \langle 0,1\rangle & \langle 0.2, 0.7\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0,1\rangle & \langle 0.6, 0.3\rangle \end{pmatrix} \end{array}$$

At time t_2 the Figure 2.2 is takes the form:

$$\begin{array}{c c} & \begin{array}{ccc} E_1 & E_2 & E_3 \end{array} \\ \begin{array}{c} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \end{array} & \begin{pmatrix} \langle 0.4, 0.5\rangle & \langle 0.4, 0.5\rangle & \langle 0,1\rangle \\ \langle 0.3, 0.6\rangle & \langle 0.3, 0.6\rangle & \langle 0.3, 0.6\rangle \\ \langle 0,1\rangle & \langle 0,1\rangle & \langle 0.4, 0.3\rangle \\ \langle 0,1\rangle & \langle 0.6, 0.4\rangle & \langle 0.6, 0.4\rangle \\ \langle 0,1\rangle & \langle 0,1\rangle & \langle 0.5, 0.3\rangle \end{pmatrix} \end{array}$$

3 Dual Temporal Intuitionistic Fuzzy Hypergraph (D-TIFHG)

Definition 3.1.

If an TIFHG H = (V, E, T), V = (n_1, n_2, ..., n_r), E = (E_1, E_2, ..., E_s) is given, its *Dual Temporal Intuitionistic Fuzzy Hypergraph* (D-TIFHG), H* = ($\underline{E}, \underline{V}, T$) is stated as follows:

H* = ($\underline{E}, \underline{V}, T$), with

1. $\underline{E}$ = (e_1, e_2, ..., e_s), set of nodes corresponding to $\underline{E_1}, \underline{E_2}, ..., \underline{E_s}$ respectively
2. $\underline{V}$ = {$\underline{n}_1, \underline{n}_2, ..., \underline{n}_r$}; set of hyperlinks corresponding to n_1, n_2, ..., n_r respectively,

   where $\underline{V}$ = {(e_q, $\mu_q(e_q, t)$, $\nu_q(e_q, t)$) : $\mu_q(e_q, t) = \mu_q(n_p, t)$, $\nu_q(e_q, t) = \nu_q(n_p, t)$}

Example 3.1.1

The DIFHG H* = {(E, $\underline{n}_1, \underline{n}_2, \underline{n}_3, \underline{n}_4, \underline{n}_5$)} of IFHG in figure 2.1 is given below. Here

H = {e_1, e_2, e_3},

$\underline{n}_1$ = {(e_1, 0.5, 0.4)}

$\underline{n}_2$ = {(e_1, 0.3, 0.6), (e_2, 0.3, 0.6)}

$\underline{n}_3$ = {(e_2, 0.5, 0.3), (e_3, 0.5, 0.3)}

$\underline{n}_4$ = {(e_2, 0.5, 0.3)}

$\underline{n}_5$ = {(e_3, 0.7, 0.3)}

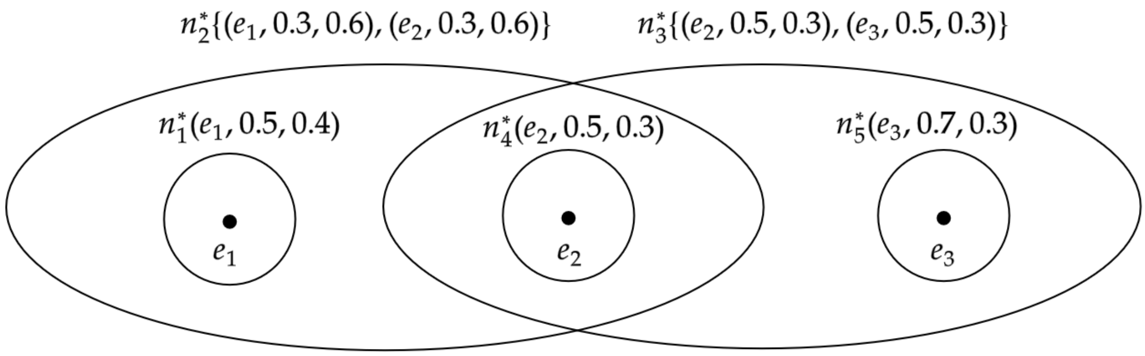

Figure 3.1: Dual Temporal Intuitionistic Fuzzy Hypergraph

The following is the appropriate incidence matrix:

$$\begin{array}{cc} & \begin{array}{ccccc} X_1 & X_2 & X_3 & X_4 & X_5 \end{array} \\ \begin{array}{c} e_1 \\ e_2 \\ e_3 \end{array} & \begin{pmatrix} \langle 0.5, 0.3\rangle & \langle 0.3, 0.6\rangle & \langle 0,1\rangle & \langle 0,1\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0.3, 0.6\rangle & \langle 0.5, 0.3\rangle & \langle 0.5, 0.3\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0,1\rangle & \langle 0.5, 0.3\rangle & \langle 0,1\rangle & \langle 0.7, 0.3\rangle \end{pmatrix} \end{array}$$

4 Numerical Example:

Hypergraphs generalize graphs by allowing multiple relationships between the nodes. Here, an application of Temporal Intuitionistic Fuzzy Hypergraph theory is considered in the area of mobile networking.

Let us consider m users as hyper nodes and n towers as hyperlinks. The finite set of users is represented by V. let it be m users and E be the hyperlink set of n towers receiving signals from mobile towers by different m users. Let the m users receive signals from different towers. This situation is expressed as a Temporal Intuitionistic Fuzzy Hypergraph H(V, E, T).

Interactions within objects at different time domains are often complex and they cannot be considered pairwise. These group together that can be represented as a Temporal Intuitionistic Fuzzy Hypergraph. Temporal Intuitionistic Fuzzy Hypergraph can be represented as a triplet set with set of nodes and set of hyperlinks at different time domains. In temporal perspective, the hyperlinks $e_1, e_2, ..., e_s$ are taken in account with hypernodes $v_1, v_2, ..., v_r$.

TIFGH can be static or dynamic depending upon the problem chosen. Images taken at different time are example for static case, while the problem of using mobile phone is a dynamic situation. At the outset, temporal properties are analysed in different time evolving hypergraphs and TIFHG is coined to represent as a model in vague situation at different time domains.

Let there be 10 users receiving signals from 5 towers. Let H(V,E,T) be the TIFHG. The users may receive their phone signals from different towers at different times. The figure 4.1 represents the model of TIFHG with 10 users at 5 towers.

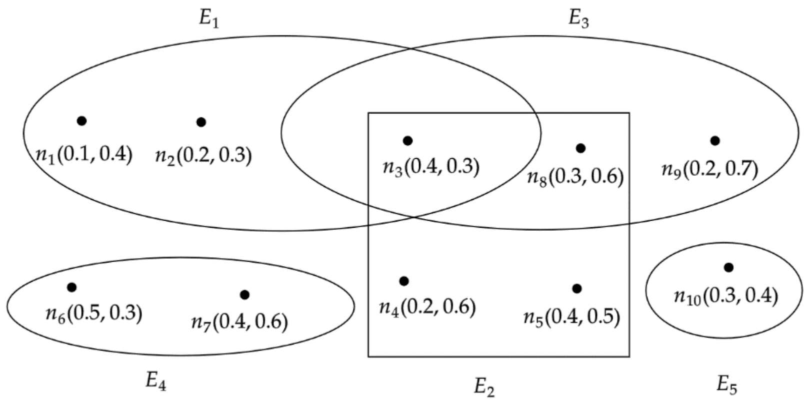

Figure 4.1: TIFHG in Mobile Network

The algorithm for Temp Hyp is exhibited in Algorithm 1:

Algorithm 1: Temporal Hypergraph H(V,E,T) (Temp Hyp)

Input: m users, p towers and n be the number of users connected with the p towers.

Output: Hypergraph H(V, E,T)

Set T = 0, where $T \neq \phi$, $V = \phi$

for temporal hypergraph e_j: $(V, t) \in E$, $t = t + \delta t$, $V \neq \phi$

insert (e_j)

if ($e_j \leq m$) && ($e_j \neq \phi$) for $t = t + \delta t$

then

$e_j + = V$

if ($e_j > m$) && ($V_i \neq \phi$)

then end

}

end

In the Algorithm 2 the procedure for generation of Temporal hypergraph is displayed. The algorithm for Temp IFHG is exhibited in Algorithm 2:

Algorithm 2: Temporal Intuitionistic Fuzzy Hypergraph (Temp IFHG)

Input: m users, p towers, n be the number of users connected with p towers.

Output: Temporal Intuitionistic Fuzzy Hypergraph H*(V, E, T)

Set T = 0, where $T \neq \phi$, $V \neq \phi$

for temporal IFHG $e_j = (V_i, t) \in E$

where

$e_j = \langle \mu_j(v_i, t_k), \nu_j(v_i, t_k) \rangle$

$e_j \neq \phi$

$V_i = \{v_1, v_2, ..., vr\}$ be the finite set of IF nodes.

insert (ej)

if

{

$(e_j \leq m)$ && $(ej \neq 0)$

then $e_j = \langle \mu_j(v_i, t), \nu_j(v_i, t) \rangle$

then $0 < \mu_j(v_i, t) \leq 1, \quad 0 < \nu_j(v_i, t) \leq 1, \; 0 \leq \mu_j + \nu_j \leq 1$

else

$\mu_j < 0$

No nodes

}

end

In the algorithm 2 the procedure for generation of TIFHG at different time domain is sketched.

Procedure con_edj_ver()

{

$T = \{t_1, t_2, .., t_p\}$

for(i = 0; i < n; i + +)

{

if ($e[i] \neq \phi$ && $V[i] >= 1$)

{

$V = \{v_1, v_2, ..., vr\}$ is a finite set of nodes

$E = \{E_1, E_2, ..., Es\}$ is a finite set of IF subsets of V

$Eq \neq \phi = \{V_i, \mu_j(V_i, t), V_j t) : \mu_j(V_i, t), \nu_j(V_i, t) \geq 0 \;\&\; \mu_j(V_i, t) + \nu_j(V_i, t) \leq 1\}, j = 1, 2, ..., m$

$T = \{t_1, t_2, ..., t_p\}$, $T \neq 0$ is the time domain

}

}

Temp IFHG generates a Temporal Intuitionistic Fuzzy Hypergraph $H^* = (V,E,T)$ consisting of a collection of IF nodes V and set of links E_q and time domain T. Initial assumptions from time t = 0 are considered where node count is taken from zero with link set $e(k) \neq \phi$, $E_q = \langle \mu_q(V_p, t_k), \nu_q(V_p, t_k)\rangle$ a set of temporal IF hyperlinks. Initiation starts from t = 0 and $V \neq \phi$ and $e(1) \neq \phi$, a temporal IFHG is formed with new node inserted and removed. A new TIFHG is generated and can generalised for 'm' users with p towers.

5 Conclusion

An attempt to define TIFHG and a dual TIFHG has been undertaken in this paper. Additionally, as a starting point, the authors suggested developing its applications and talked about one for mobile networks. Authors have gone through a simple illustration, that different users have different preference change to towers while using a mobile phone. In future, the authors planned to include different parameters for the users and look forward to test our model for other data sets with different kinds of social interactions available.

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