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Extended Reverse R Degrees of Vertices and Extended Reverse R indices of Graphs

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Abstract

A topological representation of a molecule is called molecular graph. A molecular graph is a collection of points representing the atoms in the molecule and set of lines represent the covalent bonds. Topological indices gather data from the graph of molecule and help to foresee properties of the concealing molecule. All the degree based topological indices have been defined through classical degree concept. In this paper, we define a novel degree concept for a vertex of a simple connected graph: Extended Reverse R degree and also, we define Extended Reverse R indices of a simple connected graph by using the Extended Reverse R degree concept. We compute the Extended Reverse R indices using the above contemporary degree concept for well-known simple connected graphs such as complete bipartite graph, Wheel graph, Generalized Peterson graph, Crown graph, Double star graph, and Windmill graph.

Keywords: Reverse degree, Topological indices, extended reverse R degree, extended reverse R indices.

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1. Introduction

A topological index is a mathematical invariant that characterize the chemical properties of a molecule. These indices are used in quantitative structure property relations (QSPR) research. Topological indices are important tools for analyzing some physicochemical properties of molecules without performing any experiment. The Wiener index W(G) is a distance-based topological invariant much used in the study of the structure-property and the structure-activity relationships of various classes of biochemically interesting compounds, which is introduced in 1947 for prognosticating boiling points by Harold Wiener [16]. As of now, myriad “Molecular descriptors” are being put forwarded. Recently, degree based topological indices are also formed a good correlation with chemical properties of a molecule. Some well-known degree based topological indices are Randic index, First and Second Zagreb indices, reformulated first and second Zagreb indices, Atom-Bond Connectivity index, Augmented Zagreb index, Harmonic index, Geometric-arithmetic index, Sum connectivity index are studied in [1-9], [11], [12], [15], [18] and [19]. The comparative testing of these well-known degree based topological indices were given in [10]. The concept of R degree of a vertex and R index of a graph were introduced by Siileyman Ediz[17]. In this paper, the extended reverse $\mathcal{R}$ indices for well-known simple connected graphs such as Complete bipartite graph, Wheel graph, Generalized Peterson graph, Crown graph, Double star graph and Windmill graph are obtained. Throughout this paper only simple connected graphs were considered, that is connected graphs without self-loops and parallel edges.

2. EXTENDED REVERSE R INDICES

The graph *G = (V, E) = (V(G),E(G))* have the set of all vertices *V(G)* and the set of all edges *E(G)* respectively. The degree of the vertex *v* is defined as the number of edges incident with *v* and denoted by *d(v)*. The set of all vertices which are adjacent to *v* is called the neighborhood of *v* and it is denoted by *N(v)*. The reverse degree of a vertex *v* is $Rd_V = \Delta - d(v) + 1$ where Δ is maximum degree of *G*.

Definition:2.1

The reverse sum degree of *v* is defined as $RS_V = \sum_{u\in N(v)} Rd_u$ and the reverse multiplication degree of a vertex *v* is defined as $RM_v = \prod_{u\in N(v)} Rd_u$.

Definition:2.2

The Extended Reverse $\mathcal{R}$ degree of a vertex *v* of a simple connected graph G is defined as $ER_{r(v)} = RS_V + RM_v$.

Definition:2.3

Let $G = (V, E)$ be a graph.

(a) The Extended Reverse first index of a simple connected graph *G* defined as

$$ER_r^1(G) = \sum_{v\in V(G)}[ER_{r(v)}]^2.$$

(b)The Extended Reverse Second index of a simple connected graph *G* defined as $ER_r^2(G) = \sum_{<uv>\in E(G)}[ER_{r(u)}ER_{r(v)}]$.

(b) The Extended Reverse third index of a simple connected graph *G* defined as $ER_r^3(G) = \sum_{<uv>\in E(G)}[ER_{r(u)} + ER_{r(v)}]$.

Hence, the extended reverse $\mathcal{R}$ indices are topological indices.

3. EXTENDED REVERSE $\mathcal{R}$ INDICES OF SOME GRAPHS

In this section, the Complete bipartite graph, Wheel graph, Generalized Peterson graph, Crown graph, Double star graph and Windmill graph are characterized using the extended reverse $\mathcal{R}$ indices.

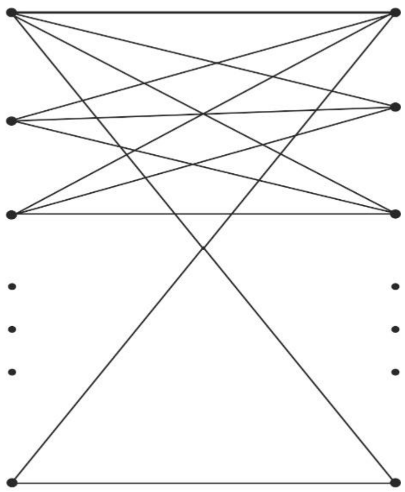
Theorem 3.1

If $K_{m,n}$ is the Complete bipartite graph with *n+m* vertices, *n > m* and *mn* edges then

$$\mathbb{E}\mathfrak{R}_{\mathcal{R}}^1(K_{m,n}) = n[m+1]^2 + m[(n-m+1)(n+(n-m+1)^{n-1})]^2,$$

$$\mathbb{E}\mathfrak{R}_{\mathcal{R}}^2(K_{m,n}) = mn\{[m+1][(n-m+1)(n+(n-m+1)^{n-1})]\},$$

$$\mathbb{E}\mathfrak{R}_{\mathcal{R}}^3(K_{m,n}) = mn\{[m+1] + [(n-m+1)(n+(n-m+1)^{n-1})]\}.$$



$K_{m,n}$

Proof:

Let $K_{m,n}$ be a Complete bipartite graph (V_i, V_j, E). The $\{u_1, u_2, \dots, u_m\} \subset V_i$ and $\{v_1, v_2, \dots, v_n\} \subset V_j$ are any two sets of vertices. Here, $(u_{i,} u_j), i = 1,2,3, \dots, m, j = 1,2,3, \dots, n$ are edges in E. A Complete bipartite graph with partitions of size $|V_i| = m, |V_j| = n, |V(K_{m,n})| = m + n$ and $|E(K_{m,n})| = mn$

Then the reverse vertex degree $\Re d_{u_i} = 1, \Re d_{v_j} = n - m + 1$, the reverse sum degree $\Re S_{u_i} = n(n - m + 1), \Re S_{v_j} = m$ and the reverse multiplication degree $\Re M_{u_i} = (n - m + 1)^n, \Re M_{v_j} = 1$. Then the extended reverse $\mathcal{R}$ degrees are $\mathbb{E}\Re^1_{\mathcal{R}u_i} = (n - m + 1)[(n + (n - m + 1)^{n-1})]^2$, $\mathbb{E}\Re^1_{\mathcal{R}v_j} = m + 1$.

Hence, the extended reverse $\mathcal{R}$ topological indices of $K_{m,n}$ are

$$\mathbb{E}\Re^1_{\mathcal{R}}(K_{m,n}) = n[m + 1]^2 + m[(n - m + 1)(n + (n - m + 1)^{n-1})]^2,$$

$$\mathbb{E}\Re^2_{\mathcal{R}}(K_{m,n}) = mn\{[m + 1][(n - m + 1)(n + (n - m + 1)^{n-1})]\},$$

$$\mathbb{E}\Re^3_{\mathcal{R}}(K_{m,n}) = mn\{[m + 1] + [(n - m + 1)(n + (n - m + 1)^{n-1})]\}.$$

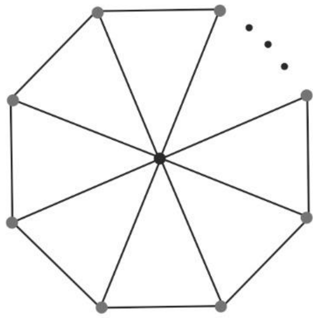
Theorem 3.2

If (W_n) is the Wheel graph with *n* vertices $n \geq 4$ then

$$\mathbb{E}\Re^1_{\mathcal{R}}(W_n) = [(n - 1)(n - 3) + (n - 3)^{n-1}]^2 + (n - 1)(n - 2)^4,$$

$$\mathbb{E}\Re^2_{\mathcal{R}}(W_n) = (n - 1)[\{(n - 1)(n - 3) + (n - 3)^{n-1}][n - 2]^2\} + (n - 2)^4,$$

$$\mathbb{E}\Re^3_{\mathcal{R}}(W_n) = (n - 1)(n - 3)^{n-1} + 4n^3 - 20n^2 + 31n - 15.$$



W_n

Proof:

A Wheel graph W_n with n vertices and *2n-2* edges is obtained by connecting a single vertex to a vertices of a cycle of length *n-1*. The set of vertices $\{v_1, v_2, v_3, \ldots, v_n\}$ can be classified into two sets of vertices such that v_1 and $\{v_j, j = 1,2, \ldots, n\}$. Then the reverse vertex degree $\Re d_{v_1} = 1, \Re d_{v_j} = n-3$. The reverse sum degree $\Re S_{v_1} = (n-1)(n-3), \Re S_{v_j} = 2n-5$ and the reverse multiplication degree $\Re M_{v_1} = (n-3)^{n-1}, \Re M_{v_j} = (n-3)^2$.

Then the extended reverse $\mathcal{R}$ degrees are $\mathbb{E}\Re_{\mathcal{R}v_1} = (n-1)(n-3) + (n-3)^{n-1}$, $\mathbb{E}\Re_{\mathcal{R}v_j} = (n-2)^2$.

$$\mathbb{E}\Re_{\mathcal{R}}^1(W_n) = [(n-1)(n-3) + (n-3)^{n-1}]^2 + (n-1)(n-2)^4,$$

$$\mathbb{E}\Re_{\mathcal{R}}^2(W_n) = (n-1)[\{(n-1)(n-3) + (n-3)^{n-1}][n-2]^2\} + (n-2)^4,$$

$$\mathbb{E}\Re_{\mathcal{R}}^3(W_n) = (n-1)(n-3)^{n-1} + 4n^3 - 20n^2 + 31n - 15.$$

Theorem 3.3

If $GP_{n,k}$ be the Generalized Peterson graph with n ≥ 3 and $1 \leq k \leq \left\lfloor \frac{(n-1)}{2} \right\rfloor$ then

$$\mathbb{E}\Re_{\mathcal{R}}^1(GP_{n,k}) = 2n(n+1)^2,$$

$$\mathbb{E}\Re_{\mathcal{R}}^2(GP_{n,k}) = 3n(n+1)^2,$$

$$\mathbb{E}\Re_{\mathcal{R}}^3(GP_{n,k}) = 6n(n+1)^2.$$

Proof:

The vertex and edge cardinality of Generalized Peterson graph is $|V(GP_{n,k})| = 2n, |E(GP_{n,k})| = 3n$ respectively. The reverse vertex degree

degree $\Re d_v = 1$. The reverse sum degree $\Re S_v = n$ and the reverse multiplication degree $\Re M_{v_1} = 1$. Then the extended reverse $\mathcal{R}$ degree $\mathbb{E}\Re_v = (n+1)$.

$$\mathbb{E}\Re_{\mathcal{R}}^1(GP_{n,k}) = 2n(n+1)^2,$$

$$\mathbb{E}\Re_{\mathcal{R}}^2(GP_{n,k}) = 3n(n+1)^2,$$

$$\mathbb{E}\Re_{\mathcal{R}}^3(GP_{n,k}) = 6n(n+1)^2.$$

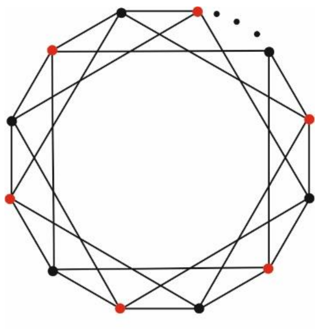
Theorem 3.4

If S_n is the Crown graph with 2n vertices and $n(n-1)$ edges then

$$\mathbb{E}\mathfrak{R}^1_{\mathcal{R}}(S_n) = 2n^3,$$

$$\mathbb{E}\mathfrak{R}^2_{\mathcal{R}}(S_n) = n^3(n-1),$$

$$\mathbb{E}\mathfrak{R}^3_{\mathcal{R}}(S_n) = 2n^2(n-1).$$



$S_{m,n}$

Proof:

The Crown graph S_n is a graph whose vertices can be subdivided into two sets of vertices $\{u_1, u_2, u_3, \ldots, u_n\}$ and $\{v_1, v_2, v_3, \ldots, v_n\}$ as v and with an edge from u_i to v_j whenever $i \neq j$.

A size of Crown graph is $|V(S_n)| = 2n, |E(S_n)| = n(n-1)$. Then the reverse vertex degree

degree $\mathfrak{R}d_v = 1$. The reverse sum degree $\mathfrak{R}S_v = n(n-1)$ and the reverse multiplication degree $\mathfrak{R}M_{v_1} = 1$. The extended reverse $\mathcal{R}$ degree $\mathbb{E}\mathfrak{R}_{R_v} = n$.

Hence, the extended reverse $\mathcal{R}$ topological indices of S_n are

$$\mathbb{E}\mathfrak{R}^1_{\mathcal{R}}(S_n) = 2n^3,$$

$$\mathbb{E}\mathfrak{R}^2_{\mathcal{R}}(S_n) = n^3(n-1),$$

$$\mathbb{E}\mathfrak{R}^3_{\mathcal{R}}(S_n) = 2n^2(n-1).$$

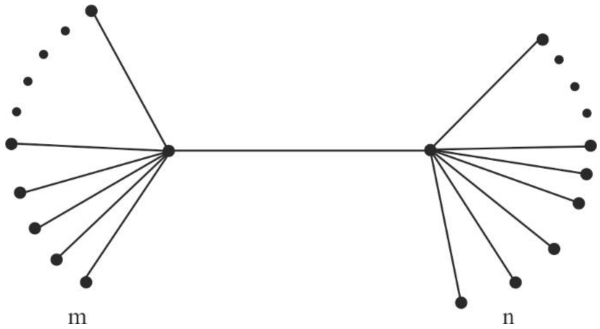
Theorem 3.5

If $S_{m,n}$ is the Double star graph with n+m+2 vertices (n > m) and n+m+1edges then

$$\mathbb{E}\mathfrak{R}^1_{\mathcal{R}}(S_{m,n}) = \{(n^2-m+1)n^{(m-1)} + n(m-1) + 1\}^2\{n^{n-1}(n-m+1) + n^2 - m + 1\}^2 + 4(m-1)\{n-m+1\}^2 + 4n - 4,$$

$$\mathbb{E}\mathfrak{R}^2_{\mathcal{R}}(S_{m,n}) = 2(m-1)\{(n^{(m-1)} + (n(m-1)+1))(n-m+1)\} + 2(n-1)\ (n^2 - m + 1 + n^{n-1}(n-m+1)) + (n^{(m-1)} + n(m-1) + 1)(n^{n-1}(n-m+1) + n^2 - m + 1),$$

$$\mathbb{E}\mathfrak{R}^3_{\mathcal{R}}(S_{m,n}) = (m-1)\{n^{(m-1)} + n(m-1) + 1 + 2(n-m+1)\} + (n-1)\{(n^2 - m + 3 + n^{n-1}(n-m+1)) + 2\} + (n^{(m-1)} + n(m-1) + 1) + (n^{n-1}(n-m+1) + n^2 - m + 1).$$



Proof:

The Double star graph $S_{m,n}$ n, m ≥ 2 and (n > m). Here, $\left|V(S_{m,n})\right| = n + m + 2$, $\left|E(S_{m,n})\right| = n - m + 1$. Then, reverse vertex degrees are

$\mathfrak{R}d_{V_I} = n - m + 1, \mathfrak{R}d_{V_{II}} = n$ and $\mathfrak{R}d_{V_{III}} = 1$ where V_I is the central vertex of *m* star, is the pendent vertex of *m* star, is the central vertex of *n* star and is the pendent vertex of *n* star graph.

The reverse sum degrees are $\mathfrak{R}S_{V_I} = mn - n + 1, \mathfrak{R}S_{V_{II}} = n - m + 1, \mathfrak{R}S_{V_{III}} = n^2 - m + 1, \mathfrak{R}S_{V_{IV}} = 1$ and the reverse multiplication degree $\mathfrak{R}M_{V_I} = (n^2 - m + 1)n^{m-1}, \mathfrak{R}M_{V_{II}} = n - m + 1, \mathfrak{R}M_{V_{III}} = (n - m + 1)n^{n-1}, \mathfrak{R}M_{V_{IV}} = 1.$

Then the extended reverse $\mathcal{R}$ degrees are $\mathbb{E}\mathfrak{R}_{\mathcal{R}v_I} = (n^2 - m + 1)n^{(m-1)} + n(m-1) + 1, \mathbb{E}\mathfrak{R}_{\mathcal{R}V_{II}} = 2(n - m + 1), \mathbb{E}\mathfrak{R}_{\mathcal{R}V_{III}} = n^{n-1}(n - m + 1) + n^2 - m + 1, \mathbb{E}\mathfrak{R}_{\mathcal{R}V_{IV}} = 2.$

Hence, the extended reverse $\mathcal{R}$ topological indices of $S_{m,n}$ are

$$\mathbb{E}\mathfrak{R}_{\mathcal{R}}^1(S_{m,n}) = \left\{(n^2 - m + 1)n^{(m-1)} + n(m-1) + 1\right\}^2 \{n^{n-1}(n - m + 1) + n^2 - m + 1\}^2 + 4(m-1)\{n - m + 1\}^2 + 4n - 4,$$

$$\mathbb{E}\mathfrak{R}_{\mathcal{R}}^2(S_{m,n}) = 2(m-1)\left\{(n^{(m-1)} + (n(m-1) + 1))(n - m + 1)\right\} + 2(n-1)(n^2 - m + 1 + n^{n-1}(n - m + 1)) + (n^{(m-1)} + n(m-1) + 1)(n^{n-1}(n - m + 1) + n^2 - m + 1),$$

$$\mathbb{E}\mathfrak{R}_{\mathcal{R}}^3(S_{m,n}) = (m-1)\left\{n^{(m-1)} + n(m-1) + 1 + 2(n - m + 1)\right\} + (n-1)\{(n^2 - m + 3 + n^{n-1}(n - m + 1)) + 2\} + (n^{(m-1)} + n(m-1) + 1) + (n^{n-1}(n - m + 1) + n^2 - m + 1).$$

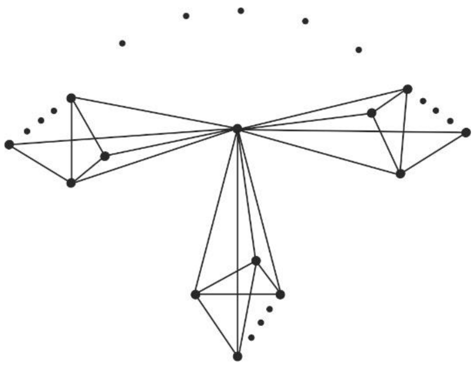
Theorem 3.6

If $W_m^{(n)}$ is the Windmill graph with m, n ≥ 2 then

$$\mathbb{E}\mathfrak{R}_{\mathcal{R}}^1(W_m^{(n)}) = \left\{n(m-1)[(m-1)(n-1) + 1] + [(m-1)(n-1) + 1]^{n(m-1)}\right\}^2 + n(m-1)[2(m-1)(n-1) + 3]^2,$$

$$\mathbb{E}\mathfrak{R}_{\mathcal{R}}^2(W_m^{(n)}) = n(m-1)\{(n(m-1)[(m-1)(n-1) + 1] + \left[(m-1)(n-1) + 1\right]^{n(m-1)}) (2(m-1)(n-1) + 3)\} + [2(m-1)(n-1) + 3]^2,$$

$$\mathbb{E}\mathfrak{R}_{\mathcal{R}}^3(W_m^{(n)}) = n(m-1)\{(n(m-1)[(m-1)(n-1) + 1] + \left[(m-1)(n-1) + 1\right]^{n(m-1)}) + (2(m-1)(n-1) + 3)\} + [2(m-1)(n-1) + 3]^2.$$



Proof:

The Windmill graph $W_m^{(n)}$ is an undirected graph constructed for $m, n \geq 2$ by joining n copies of the complete graph m at a shared universal vertex and $\left|V(W_m^{(n)})\right| = (m-1)n + 1, \left|E(W_m^{(n)})\right| = \frac{mn(m-1)}{2}$.

The reverse vertex degrees are $\Re d_{V_I} = 1, \Re d_{V_{II}} = (m-1)(n-1) + 1$ where V_I is the central vertex, V_{II} is the vertices which are all adjacent to the central vertex.

The reverse sum degrees are

$$\Re S_{V_I} = n(m-1)[(m-1)(n-1)+1], \Re S_{V_{II}} = (m-1)(n-1) + 2$$

and the reverse multiplicative degrees are

$$\Re M_{V_I} = [(m-1)(n-1)+1]^{n(m-1)}, \Re M_{V_{II}} = (m-1)(n-1) + 1.$$

Then the extended reverse $\mathcal{R}$ degrees are

$$\mathbb{E}\Re_{\mathcal{R}v_I} = n(m-1)[(m-1)(n-1)+1] + [(m-1)(n-1)+1]^{n(m-1)},$$

$$\mathbb{E}\Re_{\mathcal{R}V_{II}} = 2(m-1)(n-1) + 3.$$

Hence, the extended reverse $\mathcal{R}$ topological indices of $W_m^{(n)}$ are

$$\mathbb{E}\Re_{\mathcal{R}}^1\left(W_m^{(n)}\right) = \{n(m-1)[(m-1)(n-1)+1] + [(m-1)(n-1)+1]^{n(m-1)}\}^2 + n(m-1)[2(m-1)(n-1)+3]^2,$$

$$\mathbb{E}\Re_{\mathcal{R}}^2\left(W_m^{(n)}\right) = n(m-1)\{(n(m-1)[(m-1)(n-1)+1] + \left[(m-1)(n-1)+1\right]^{n(m-1)}\right) (2(m-1)(n-1)+3)\} + [2(m-1)(n-1)+3]^2,$$

$$\mathbb{E}\Re_{\mathcal{R}}^3\left(W_m^{(n)}\right) = n(m-1)\{(n(m-1)[(m-1)(n-1)+1] + \left[(m-1)(n-1)+1\right]^{n(m-1)}\right) + (2(m-1)(n-1)+3)\} + [2(m-1)(n-1)+3]^2.$$

4. Conclusion

In this paper, the brand new degree based topological indices such as extended reverse R indices are elucidated using the reverse sum degree, reverse multiplicative degree and extended reverse R degree. Make use of, extended reverse R indices for the Complete bipartite graph, Wheel graph, Generalized Peterson graph, Crown graph, Double star graph and Windmill graph are structurally characterized. This study may be regarded as an introduction to the topic and may seek to identify further recourse for simple connected graphs. Also one could concentrate on Chemical graphs.

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