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An Application on Fuzzy Relation Using in Difficult Problem Solved

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Abstract: As a basic concept in fuzzy theory, fuzzy relations are used in a variety of fields, such as fuzzy clustering, uncertainty reasoning, and fuzzy control. When fuzzy relations are applied in practise, it might be difficult to estimate and compare them. This method was therefore applied in this study to tackle a challenging issue. It turns out that a combination of this strategy and the three prior expanded ones can have a beneficial effect on the actual issue. Finally, using fuzzy quantifiers, we give a theoretical investigation into the capacity to solve systems of fuzzy relation equations. As we shall see, solutions to such systems are given in building a fashion, and they may be solvable.

Keywords: Fuzzy union and intersection, projection of fuzzy relation, cylindrical of fuzzy relation, height of fuzzy relation, fuzzy quantifiers.

1. Introduction:

In mathematics, fuzzy sets (also known as uncertain sets) are sets whose members may or may not be present. In 1965, Dieter Klaua and Lotfi A. Zadeh independently introduced fuzzy sets as an extension of the traditional notion of set. Fuzzy relations are specific examples of L-relations, which are used more and more frequently in fuzzy mathematics and have uses in areas like linguistics, decision-making, and clustering (De Cock, Bodenhofer, & Kerre 2000). (Bezdek 1978).In classical set theory, the membership of items in a set is assessed in binary terms in accordance with a bivalent condition; an element belongs to the set or it does not.

Contrarily, fuzzy set theory permits a gradual evaluation of an element's membership in a set, which is defined by a membership function with a value in the real unit interval [0, 1].Due to the fact that the membership functions of fuzzy sets, which can only take values of 0 or 1, can only take values of 0 or 1, the indicator functions of classical sets (also known as characteristic functions) are particular cases of these membership functions. Fuzzy set theory frequently refers to the traditional bivalent sets as crisp sets. When data is lacking or erroneous, the fuzzy set theory can be used to a variety of domains, including bioinformatics.

A fuzzy relation is the Cartesian product of mathematical fuzzy sets. When two fuzzy sets are supplied, the fuzzy relation is equal to the cross product of the sets, which is created by vector multiplication. In order to give the fuzzy controller the ability to change its internal values, rule bases are often preserved as matrices. In contrast to Zadeh's idea of fuzzy logic in the wide sense, the phrase "mathematical fuzzy logic" is frequently referred to as "fuzzy logic in the limited sense" (FLn). It can be described as a generalization of classical logic with particular qualities designed to address the vagueness phenomena.

Numerous additions have been made to it, such as fuzzy natural logic (FNL), which tries to create a mathematical representation of how people naturally reason and emphasizes the importance of natural language. To evaluate a formula, such (propositional) logic requires an algebraic structure of truth degrees, just as classical logic. In mathematical fuzzy logic, the structure of truth degrees typically takes the form of a resituated lattice, $\langle L, \wedge, \vee, \otimes, \rightarrow, 0, 1\rangle$, where L is a collection of truth values (support), generalizing the Boolean algebra, $\langle \{0, 1\}, \wedge, \vee, \neg, 0, 1\rangle$, as the structure of truth degrees in classical binary propositional logic. For additional information on mathematical fuzzy logic

Practically speaking, a fuzzy relationship is described in a second table. A table with fuzzy values between 0 and 1 is first created. The if-then rules will then be applied to the values. An array of the generated numbers is kept in the table. Fuzzy relations can be employed in fuzzy databases. As a fundamental concept in fuzzy theory, fuzzy relations have found use in a number of fields, such as fuzzy clustering, uncertainty reasoning, and fuzzy control. When fuzzy relations are used in practice, it can be challenging to estimate and compare them.

2. PRELIMINARY

A. Union and intersection of fuzzy regions

Definition 1

Let $\tilde{A}, \tilde{B} \subseteq$ R be general set, then

$\tilde{R} = \{((\tilde{a}, \tilde{b}), \mu_{\tilde{R}}\ (\tilde{a}, \tilde{b}\ )) \,|\, (\tilde{a}, \tilde{b}\ ) \in \tilde{A} \times \tilde{B}\ \}$ is so-called a fuzzy relation from $\tilde{A}$ to $\tilde{B}$.

Definition 2

Let $A, B \subseteq \tilde{R},$ and

$$A = \{a, \mu_A(a)) : a \in A\}; \quad B = \{b, \mu_B(b)) : b \in B\},$$ be two fuzzy sets.

Then, fuzzy relation $\tilde{R}(A, B)$ is well-defined as follows

$\tilde{R} \equiv \tilde{R}(A, B) = \{((a,b), \mu_{\tilde{R}}(a,b) : (a,b) \in A \times B\},$ is a fuzzy family member on $\tilde{A}$ *and* $\tilde{B}$ if

$$\mu_{\tilde{R}}(a,b) \le \mu_A(a), \qquad \forall (a,b) \in A \times B$$
$$\mu_{\tilde{R}}(a,b) \le \mu_B(b), \qquad \forall (a,b) \in A \times B$$
$$or \ \ \mu_{\tilde{R}}(a,b) \le \min(\mu_A(a), \mu_B(b))$$

Definition 3

Let $\tilde{R}$ *and* $\tilde{Z}$ be two fuzzy family members in the same product spaces. Then the union and intersection of $\tilde{R}$ *and* $\tilde{Z}$ is well - defined as follows:

$\tilde{R} \cup \tilde{Z} = \{[(a,b)\mu_{\tilde{R}\cup\tilde{Z}}(a,b)] : (a,b) \in A \times Z\}$

$where,\ \mu_{\tilde{R}\cup\tilde{Z}}(a,b) = \max\{\mu_{\tilde{R}}(a,b), \mu_{\tilde{Z}}(a,b) : (a,b) \in A \times Z\}$

$\tilde{R} \cap \tilde{Z} = \{[(a,b)\mu_{\tilde{R}\cap\tilde{Z}}(a,b)] : (a,b) \in A \times Z\}$

$where,\ \mu_{\tilde{R}\cap\tilde{Z}}(a,b) = \min\{\mu_{\tilde{R}}(a,b), \mu_{\tilde{Z}}(a,b) : (a,b) \in A \times Z\}$

Definition 4

An algebra L = ⟨L, ∧, ∨, ⊗, →, 0, 1⟩ is a resituated lattice if

1. ⟨L, ∧, ∨, 0, 1⟩ is a lattice with the least and greatest element.

2. ⟨L, ⊗, 1⟩ is a commutative monoid such that is isotone in both arguments.

3. The operation is a residue with respect to, that is, it does not change the value of ⊗.

$$a \otimes b \leq c \text{ if and if only } a \to c \geq b .$$

The relocated lattices depicted in the following photos are among the most common:

Example 2.4.1 (G¨odel algebra)

$$L_G = \langle [0, 1], \wedge, \vee, \to_\wedge, 0, 1\rangle$$

where the development ⊗ = ∧

Example 2.4.2.

L- Ukasiewicz algebra = ⟨[0, 1], ∧, ∨, ⊗ →⊗, 0, 1⟩
where

$a \otimes b = 0 \vee (a + b - 1)$ (2.1)

$a \to_\otimes b = 1 \wedge (1 - a + b)$,

$a \otimes b \leq a,\ a \otimes b \leq b$ (2.3)

$a \to b = 1$ whenever $a \leq b$ (2.4)

$a \to c \geq b \to c$ whenever $a \leq b$ (2.5)

$a \to b \leq a \to c$ whenever $b \leq c$ (2.6)

$a \otimes (a \to b) \leq b$ (2.7)

$a \to (b \to c) = (a \otimes b) \to c = (b \otimes a) \to c$ (2.8)

$(a \wedge b) \otimes c \leq (a \otimes c) \wedge (b \otimes c)$ (2.9)

$(a \vee b) \otimes c = (a \otimes b) \vee (a \otimes c)$ (2.10)

$(a \vee b) \to c = (a \to c) \wedge (b \to c)$ (2.11)

$(a \to b) \to b \geq a \vee b$ (2.12)

Other operations can be specified for all a, b ∈ L, including negation, addition, and biresiduation (bi-implication, residual equivalence).

a ↔ b = (a → b) ∧ (b → a),

$$\neg a = a \to 0 \, ,$$

a ⊕ b = ¬(¬a ⊗ ¬b) .

B. Projection and cylindrical fuzzy relations

Let $\tilde{R} = \{((a,b), \mu_{\tilde{R}}(a,b)) : (a,b) \in A \times B\}$ be the fuzzy dual relation, then

The first prediction of $\tilde{R}$ is defined as

$$\tilde{R}^{(1)} = \{(a, \max_b \mu_{\tilde{R}}(a,b)) : (a,b) \in A \times B\}$$

And the second prediction of $\tilde{R}$ is defined as

$$\tilde{R}^{(2)} = \{(b, \max_a \mu_{\tilde{R}}(a,b)) : (a,b) \in A \times B\}$$

Also, the total prediction is defined as

$$\tilde{R}^{(T)} = \max_a \max_b \{\mu_{\tilde{R}}(a,b) : (a.b) \in A \times B\}$$

C. Extension of the fuzzy relation in a cylinder

A fuzzy relation named cyl X with a membership function of equal to is the cylindrical extension $A \times B$ of a fuzzy set called X of a given A.

$$cyl\ X(a,b) = X(a), \quad \forall a \in A,\ b \in B$$

The B-projection filling all the columns of the connected matrix is referred to as "cylindrical extension from A-projection." Similar to cylindrical extension from projection, the relational matrix's rows of the B-projection are completely filled. Let U, V, W, Q be finite cosmoses, and let $P, P_1, P_2 \in$ F($X \times Y$), $S, S_1, S_2 \in$F($V \times W$) and T∈F (W×Q).

Furthermore, let ∪, ∩ denote the G odel union and intersection, correspondingly.

Proposition 2.3.1

$$(P \circ S)^T = S^T \circ P^T \, , \qquad (P \quad S)^T = S^T\, D\, P^T \, ,$$

$$(P\, D\, S)^T = S^T \quad P^T \, , \quad (P\, Q\, S)^T = S^T\, Q\, P^T \, .$$

Proposition 2.3.2.

$$P_1 \subseteq P_2 \Rightarrow (P_1 \circ S) \subseteq (P_2 \circ S) \, ,$$

$$S_1 \subseteq S_2 \Rightarrow (P \circ S_1) \subseteq (P \circ S_2) \, ,$$

$$P_1 \subseteq P_2 \Rightarrow (P_1 \quad S) \supseteq (P_2 \quad S) \, ,$$

$$S_1 \subseteq S_2 \Rightarrow (P \quad S_1) \subseteq (P \quad S_2) \, ,$$

$$P_1 \subseteq P_2 \Rightarrow (P_1\, D\, S) \subseteq (P_2\, D\, S) \, ,$$

$$S_1 \subseteq S_2 \Rightarrow (P\, D\, S_1) \supseteq (P\, D\, S_2) \, .$$

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Proposition 2.3.3.

$$(P_1 \cup P_2) \circ S = (P_1 \circ S) \cup (P_2 \circ S),$$

$$P \circ (S_1 \cup S_2) = (P \circ S_1) \cup (P \circ S_2),$$

$$(P_1 \cup P_2) \quad S = (P_1 \quad S) \cap (P_2 \quad S),$$

$$P \quad (S_1 \cup S_2) \supseteq (P \quad S_1) \cup (P \quad S_2),$$

$$(P_1 \cup P_2)\ D\ S \supseteq (P_1\ D\ S) \cup (P_2\ D\ S),$$

$$P\ D\ (S_1 \cup S_2) = (P\ D\ S_1) \cap (P\ D\ S_2).$$

Proposition 2.3.4.

$$(P_1 \cap P_2) \circ S \subseteq (P_1 \circ S) \cap (P_2 \circ S),$$

$$P \circ (S_1 \cap S_2) \subseteq (P \circ S_1) \cap (P \circ S_2), (P_1 \cap P_2)$$

$$S \supseteq (P_1 \quad S) \cup (P_2 \quad S), P \quad (S_1 \cap S_2) = (P \quad S_1) \cap$$

$$(P \quad S_2), (P_1 \cap P_2)\ D\ S = (P_1\ D\ S) \cap (P_2\ D\ S), P\ D\ (S_1 \cap S_2) \supseteq (P\ D\ S_1) \cup (P\ D\ S_2)$$

Proposition 2.3.5.

$$P \circ (S\ D\ T) \subseteq (P \circ S)\ D\ T,$$

$$(P \quad S) \circ T \subseteq P \quad (S \circ T).$$

Proposition 2.3.6.

$$P \circ (S \circ T) = (P \circ S) \circ T,$$

$$P \quad (S \quad T) = (P \circ S) \quad T,$$

$$P \quad (S\ D\ T) = (P \quad S)\ D\ T,$$

Proposition 2.3.7.

$$P\ Q\ S = (P \quad S) \cap (P\ D\ S)$$

Proposition 2.3.8.

$$P \circ S'E \subseteq (P \circ S) \cap \neg(P \circ E), \quad (2.13)$$

$$P \circ S'E \subseteq (P \circ S) \cap (P \quad \neg E). \ (2.14)$$

Proof.

It follows from property (2.7)

$$(P \circ S'E)(u,w) = P \circ S)(u,w) \otimes \neg(P \circ E)(u,w)$$

$$\leq (P \circ S)(u,w) \wedge \neg(P \circ E)(u,w)$$

$$= ((P \circ S) \cap \neg(P \circ E))\ (u,w)$$

For completely (u,w) ∈ U × W, which proves (2.13).

The proof of (2.14) uses the circumstance that

$$(P \circ S'E)(u,w) = (P \circ S'E)^{Q}(u,w)$$
$$= (P \circ S)(u,w) \otimes (P\neg E)(u,w)$$

and analogously to the preceding one also the

circumstance that lowest is the biggest t- norm.

Proposition 2.3.9.

$$(P \circ S) \circ T\,'F = P \circ (S \circ T\,)'(S \circ F\,)\ (3.17)$$

Proof.

Based on the associative property of the basic compositions, $(P \circ S) \circ T = P \circ (S \circ T\,)$, we have

$$((P \circ S) \circ T\,'F\,)(\mathrm{u}, u) = ((P \circ S) \circ T) (\mathrm{u}, u) \otimes \neg\, ((P \circ S) \circ F\,) (\mathrm{u}, u) = (P \circ (S \circ T\,)) (u, u) \otimes \neg\, (P \circ (S \circ F\,)) (u, u) = (P \circ (S \circ T\,)'(S \circ F\,)) (u,u)$$

for all (u, u) ∈ U × U , which verifies (2.14)
Proposition 2.3.10.

$$S_1 \subseteq S_2 \Rightarrow (P \circ S_1'E) \subseteq (P \circ S_2'E), E_1 \subseteq E_2$$
$$\Rightarrow (P \circ S'E_1) \supseteq (P \circ S'E_2).$$

Proof. Using belongings of the first argument of the suggestion and (2.7), we obtain

$(P \circ S_1'E)(u,w) = (P \circ S_1)(u,w) \otimes \neg(P \circ E)(u,w)$

$\leq (P \circ S_2)(u,w) \otimes \neg(P \circ E)(u,w)$

$= (P \circ S_2'E)(u,w)$

for all (u,w) ∈ X × Z. Thus, $(P \circ S_1'E) \subseteq (P \circ S_2'E)$ Additionally,

$P \circ S'E_1)(u,w) = (P \circ S)(u,w) \otimes \neg(P \circ E_1)(u,w)$

$\geq (P \circ S)(u,w) \otimes \neg(P \circ E_2)(u,w)$

$= (P \circ S'E_2)(u,w)$

for all (u,w) ∈ X × Z. Thus, (P ∘ S›E1) ⊇ (P ∘ S›E2).

Theorem 2.3.11.

$$P \circ (S_1 \cup S_2)'E = (P \circ S_1'E) \cup (P \circ S'\,E), \qquad (3.18)$$
$$P \circ S'(E_1 \cup E_2) \subseteq (P \circ S'E_1) \cap (P \circ S'E_2). \qquad (3.19)$$

Proof. Using the properties (2.9), (2.10) and (2.11), we obtain

$(P \circ (S_1 \cup S_2)'E)(u,w) = (P \circ (S_1 \cup S_2))(u,w) \otimes \neg(P \circ E)(u,w)$

$= ((P \circ S_1) \cup (P \circ S_2))\,(u,w) \otimes \neg(P \circ E)(u,w)$

$= ((P \circ S_1)(u,w) \vee (P \circ S_2)(u,w)) \otimes \neg(P \circ E)(u,w)$

$= ((P \circ S_1)(u,w) \otimes \neg(P \circ E)(u,w))$

$\vee ((P \circ S_2)(u,w) \otimes \neg(P \circ E)(u,w))$

$= ((P \circ S' E) \cup (P \circ S' E)) (u,w)$

for all (u,w) ∈ U × W, which proves (2.15). Additionally,

$(P \circ S'(E_1 \cup E_2))(u,w) = (P \circ S)(u,w) \otimes \neg(P \circ (E_1 \cup E_2))(u,w)$

$= (P \circ S)(u,w) \otimes \neg((P \circ E_1) \cup (P \circ E_2))(u,w)$

$= (P \circ S)(u,w) \otimes \neg((P \circ E_1)(u,w) \vee (P \circ E_2)(u,w))$

$= (P \circ S)(u,w) \otimes (\neg(P \circ E_1)(u,w) \wedge \neg(P \circ E_2)(u,w))$

$\leq ((P \circ S)(u,w) \otimes \neg(P \circ E_1)(u,w))$

$\wedge ((P \circ S)(u,w) \otimes \neg(P \circ E_2)(u,w))$

$= ((P \circ S'E_1) \cap (P \circ S'E_2))(u,w)$

for all (u,w) ∈ X × Z, which verifies (2.16).
Theorem 3.2.6.

$P \circ (S_1 \cap S_2)'E \subseteq (P \circ S_1'E) \cap (P \circ S' E),$ (2.20)

$P \circ S'(E_1 \cap E_2) \supseteq (P \circ S'E_1) \cup (P \circ S'E_2).$ (2.21)

Proof. Using the property (2.9), we obtain

$(P \circ (S_1 \cap S_2)'E)(u,w) = (P \circ (S_1 \cap S_2))(u,w) \otimes \neg(P \circ E)(u,w)$

$\leq ((P \circ S_1) \cap (P \circ S_2)) (u,w) \otimes \neg(P \circ E)(u,w)$

$= ((P \circ S_1)(u,w) \wedge (P \circ S_2)(u,w)) \otimes \neg(P \circ E)(u,w)$

$\leq ((P \circ S_1)(u,w) \otimes \neg(P \circ E)(u,w))$

$\wedge ((P \circ S_2)(u,w) \otimes \neg(P \circ E)(u,w))$

$= ((P \circ S' E) \cap (P \circ S' E)) (u,w)$

for all (u,w) ∈ U × W, which verifies (2.20). Similarly, using the property (1.10) and (2.11), we obtain

$(P \circ S'(E_1 \cap E_2))(u,w) = (P \circ S)(u,w) \otimes \neg(P \circ (E_1 \cap E_2))(u,w)$

$\geq (P \circ S)(u,w) \otimes \neg((P \circ E_1) \cap (P \circ E_2))(u,w)$

$= (P \circ S)(u,w) \otimes \neg((P \circ E_1)(u,w) \wedge (P \circ E_2)(u,w))$

$= (P \circ S)(u,w) \otimes (\neg(P \circ E_1)(u,w) \vee \neg(P \circ E_2)(u,w))$

$= ((P \circ S)(u,w) \otimes \neg(P \circ E_1)(u,w))$

$\vee ((P \circ S)(u,w) \otimes \neg(P \circ E_2)(u,w))$

$= ((P \circ S'E_1) \cup (P \circ S'E_2))(u,w).$

for all $(V, W) \in U \times W$, which verifies (3.21).

3. Methodology

Allow the two fuzzy relations $\tilde{R}\, and\, \tilde{Z}$ to exist. The following two relations matrix then give the union of $\tilde{R}\, and\, \tilde{Z}$ which can be understood as "a significantly larger than b" and "a is very close to b," i.e., "a considerably larger or very close to b." "a is significantly bigger than b"

$\tilde{R} =$

	b_1	b_2	b_3	b_4
a_1	.3	0	.8	.6
a_2	.8	.4	.5	.7
a_3	.4	0	.8	.5

"b is rather near a"

	b_1	b_2	b_3	b_4
a_1	.8	.9	.8	.7
a_2	.8	1	.5	.7
a_3	.6	.8	.8	.7

$\tilde{Z} =$

$$\tilde{R} \cup \tilde{Z} = \{[(a,b)\mu_{\tilde{R}\cup\tilde{Z}}(a,b)] : (a,b) \in A \times Z\}$$

$$where,\ \mu_{\tilde{R}\cup\tilde{Z}}(a,b) = \max\{\mu_{\tilde{R}}(a,b), \mu_{\tilde{Z}}(a,b) : (a,b) \in A \times Z\}$$

at that time

	b_1	b_2	b_3	b_4
a_1	.8	.9	0	.7
a_2	0	1	.3	.1
a_3	.6	.8	.7	.7

$\tilde{R} \cup \tilde{Z} =$

$$\tilde{R} \cap \tilde{Z} = \{[(a,b)\mu_{\tilde{R}\cap\tilde{Z}}(a,b)] : (a,b) \in A \times Z\}$$

$$where,\ \mu_{\tilde{R}\cap\tilde{Z}}(a,b) = \min\{\mu_{\tilde{R}}(a,b), \mu_{\tilde{Z}}(a,b) : (a,b) \in A \times Z\}$$

	b_1	b_2	b_3	b_4
a_1	.3	0	0	.6
a_2	0	.4	.3	.1
a_3	.4	0	.7	.5

$\tilde{R} \cap \tilde{Z} =$

Let $\tilde{R}(A,B)$ be a fuzzy relation well defined by the resulting relation matrix. Then the first projection take $\tilde{R} \cup \tilde{Z}$ values in above result substantially, First prediction $\tilde{R}^{(1)} =$ (.8, 1, .8) = max=1

Second projection $\tilde{R}^{(2)} =$ (.8, 1, .7, .7) = max=1

Total prediction $\tilde{R}^{(T)} = 1$

Then the cylindrical extension of $\tilde{R}^{(1)}$ is

$$\tilde{R}^{(1)} = \begin{bmatrix} .9 & .9 & .9 & .9 \\ 1 & 1 & 1 & 1 \\ .8 & .8 & .8 & .8 \end{bmatrix}$$

The cylindrical extension of $\tilde{R}^{(2)}$ is

$$\tilde{R}^{(2)} = \begin{bmatrix} .8 & 1 & .7 & .7 \\ .8 & 1 & .7 & .7 \\ .8 & 1 & .7 & .7 \end{bmatrix}$$

Because a fuzzy relation's first projection and domain, second projection and range, and overall projection and height are all the same, so too are all of these.

4. Results

The union of fuzzy relations is likewise satisfied for projection and cylindrical extension criteria, and the total projection is identical to the first and second projections.

5. Conclusion

Numerous academic disciplines as well as personal applications use fuzzy sets and their applications. I made the decision to concentrate my research on the foundations of fuzzy relations, such as the definitions of union and intersection as well as the projection and cylindrical aspects of fuzzy relations, due to developments in technology. I then went on to my next research endeavour, a district-by-district computation of accident leading utilising fuzzy relations equations and fuzzy quantifiers.

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