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On Solving Cubic Equation $x^3 + y^3 = 7(z-w)^2(z+w)$

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Abstract: The cubic Quadratic Equation$x^3 + y^3 = 7(z-w)^2(z+w)$ is analyzed for its non-zero distinct integer solutions. Five different patterns of non-zero distinct integer solutions to the equation under consideration are obtained. A few applications of Diophantine equations are also presented.

Keywords: Integral solutions, cubic Diophantine

1. Introduction

The cubic Diophantine (homogeneous or non-homogeneous) equation offer an unlimited field for research due to their variety. Interesting methods like brute force methods and substitution strategies are used by few authors to solve cubic equations [1]. Some interesting results like obtaining Pythagorean triples using continued fractions through which one can solve Diophantine equation [2]. In particular,one may refer [3-18] for non-homogeneous cubic equations, with three and four unknowns. This communication concerns with yet another interesting homogenous cubic equation with four unknowns given. A few applications are also presented.

Lang was a prolific mathematician known for his work in algebra, number theory, and analysis. His book "Algebra" is a classic text that covers a wide range of topics in abstract algebra, including polynomial equations. While "Algebra" may not delve deeply into homogeneous cubic equations with three unknowns specifically, it provides a solid foundation in algebraic structures and methods that are relevant to understanding and solving such equations. Artin is another influential mathematician whose contributions span algebraic geometry, number theory, and group theory. His textbook "Algebra" is widely used in undergraduate and graduate courses. While it may not focus extensively on homogeneous cubic equations with three unknowns, it covers important algebraic concepts and techniques that are applicable to solving polynomial equations of various degrees. Dummit and Foote co-authored the textbook "Abstract Algebra," which is widely used in undergraduate algebra courses. This comprehensive text covers a broad range of topics in abstract algebra, including polynomial equations, group theory, ring theory, and field theory. While it may not specifically address homogeneous cubic equations with three unknowns, it provides a thorough introduction to the algebraic structures and techniques needed to understand and potentially solve such equations.

2. Objectives

The Objective of the article is to solve the cubic equation in five different patterns And observe the result for comparison.

3. Methods

The homogeneous cubic diophantine equation with four unknowns is

$\boldsymbol{x^3 + \ y^3 \ = 7(z-w)^2(z+w)}$-------(1)

Let $x \ = \ u+v \ ; \ y = \ u - v : z = u+p : w = u-p$----- (2) `in equation (1).

Then (1) becomes $u^2 + 3v^2 = \ 7p^2$--------(3)

A. *Pattern I*

Let $p = a^2 + 3b^2$-------------------------------(4)

Substituting (4) in (3) and using the method of factorization, define

$u^2 + 3v^2 = (u + i\sqrt{3}\ v)(2 - i\sqrt{3})$

$u + i\sqrt{3}\,v = (k+3i)(a+ib)^2$---------------(5)

Comparing real and imaginary parts in (5),

$u = 2a - 3b$

$v = 2b \ + a$ ------------------(6)

On substituting (4) and (6) in (2),

$x = x(k,a,b) = 3a - b$

$y = y(k,a,b) = a - 5b$

$z = z(k,a,b) = 2a - 3b + a^2 \ + 3b^2$

$w = w(k,a,b) = 2a - 3b - a^2 - 3b^2$

B. *Pattern II*

Equation (i) can be written in the form $u^2 + \ 3v^2 = \ 7p^2$

$7p^2 \ = 3p^2 + 4p^2$

This implies

$u^2 - 4p^2 = \ 3(p^2 - v^2)$

$\Rightarrow (u+2p)(u-2p) = 3(p+v)(p-v)$----------------(7)

$\frac{\Rightarrow(u+2p)}{((p+v))} = \frac{3(p-v)}{(u-2p)} = \frac{A}{B}, B \neq 0$ ---------------------------(8)

Using cross multiplication method .one can arrive the following patterns of solutions:

$x = -A^2 + 3B^2 - 10AB$

$y = -3A^2 + 9B^2 - 2AB$

$z = -3A^2 + 3B^2 - 6AB$

$w = -A^2 + 9B^2 - 6AB$

C. Pattern :III

$x = -3A^2 + 9B^2 - 2AB$

$y = -A^2 + 3B^2 - 10AB$

$z = -3A^2 + 3B^2 - 6AB$

$w = -A^2 + 3B^2 - 6AB$

D. Pattern :IV

$x = -4A^2 + 6B^2 + 6AB$

$y = 12AB$

$z = -A^2 + 6B^2 + 9AB$

$w = -3A^2 + 9AB$

E. Pattern : V

$x = 3A^2 - 6B^2 - 8AB - 6A + 9B$

$y = -5A^2 - 6B^2 - 8AB + 6A - 9B$

$z = 3A^2 - 6B^2 - 3AB + 9B$

$w = 3A^2 - 6B^2 - 6AB - 9B$

4. Applications and Discussions

Diophantine equations are used in many real time situations. It is widely applied in many fields like Public key cryptosystems, Data dependency in Super Computers, Integer factorization, Balancing chemical equations, Algebraic curves, Projective curves, Elliptic curves, Computable economics and Theoretical Computer Science.

Cryptography is one such kind, which helps in writing or solving the codes. The cryptographers in 1980 published that they found the art of writing codes which can be shared in public, yet cannot decrypt the messages. These are written by extending the large prime numbers as a powered value which is complex calculation to determine the prime factor. For instance, though 358^{143} could be possible with tools, to determine prime factors of a number such as 982,028,384,119, 448 however is challenging if one does not know to start with the right prime number. The Cryptography is executed in such a way that to use the similar kind of above mentioned numbers as a key to the enclosed messages which are not to be disclosed to the public. Hence the sender and receiver should be aware of the prime factor numbers to decrypt the messages. Having that said, in theory, any other person who knows this key would also be able to decrypt the messages. However practically it takes year's time and complex calculation with advanced computers to determine the key solution to decrypt the messages.

In Diophantine equations which are applied in real life, the most popular one is Cryptography with Public Key using elliptic curves which are projected in cubic form and it was directed by RogierBrussee. It is a magical abelian group structure. With 2 given positions A,B the simple geometric progression relates to the 3rd position called C where C = A + B. Position A, the addition of d copies of A is represented as dA. It is known as discrete log on elliptical curves which has a method to determine d if the position A and dA is known. It could be easy or difficult based on the structure of the elliptical curves. In real life, it is used in biometric for passport in Europe. Also in USA it is equipped with a chip which describes with the owner's identity which is secured by the elliptical curve. It is also used in solving data involving social economic problems.[19]

5. Conclusion

An interesting way of solving homogenous cubic equation with four unknowns is found. Five different patterns of solving methods are applied and solutions are obtained. Interesting Applications of Diophantine equations are also listed.

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