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K- Hausdorff Separation Axiom in Second Order Interval Valued Fuzzy Topological Spaces

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Abstract: We studied and introduced a concept SIVFT then present the concept of SIVF subspace and SIVF product topology in SIVF topological spaces. K-Hausdorff Separation Axiom in SIVF topological spaces and its basics are studied.

Keywords: IVF set (IVFs).IVF toplogy (IVFT) ,second order fuzzy set (SFs),Second order fuzzy topology (SFT),Second order IVF set (SIVFs), Second order IVF topology (SIVFT), SIVF subspace, SIVFK-Hausdorff space.

1. Introduction:

Decision making is always plagued by an element of uncertainty and it is an unavoidable aspect in every walk of life. To overcome the problems of uncertainty Zadeh [28] framed the fuzzy set with broad application in abundant fields and is used to express imprecise and vague concepts in natural language. Zadeh [29] initiated IVFs in which membership are intervals of numbers. IVFs furnish a more sufficient description of uncertainty than conventional Fs's. Chang [6] introduced fuzzy topological spaces based on Fs's of Zadeh [28]. Lowen [18] modified the definition of Chang [6] in order to study the structures of fuzzy topological spaces in detail. Further the author introduced two functors ω and i to investigate the connections between fuzzy topological spaces and topological spaces. Mondal and Samanta [21] introduced the concept of IVF topology by following the definition of Chang's and Lowen's FT. The continuous function from an IVFs to an IVFs is also defined. Liang [17] proposed SIVFs which is isolated case of type 2 Fs's.Using SFs's, Kalaichelvi [11] extended FT of Chang and Lowen to SFT. Several versions of Hausdorff separation axiom have been defined ([2],[8],[12],[14],[20],[23],[27]). K- Hausdorff separation axioms in fuzzy topological spaces introduced by Katsaras [16] were extended to second order fuzzy topological spaces by Kalaichelvi ([12],) and were denoted as K-Hausdorff axiom respectively. Using SIVFs's, We extend IVF of Mondal and Samantha's to SIVFT Here, We focused our work to develop SIVFK- Hausdorff space as an extension of first order IVFK-Hausdorff and its properties discussed.

2.Preliminaries:

Definition: 2.1 [28]

A Fs in Ʊ which is nonnull set mapping from Ʊ into I where I = [0, 1].

Definition: 2.2 [10]

$\mathcal{A}_{IV}$: Ʊ → [I] said IVFs in Ʊ, [I] be closed subintervals of [0, 1]. All IVFs's on Ʊ is denoted by IVF (Ʊ).

For every $\mathcal{A}_{IV}$ ∈ IVF(Ʊ) and $\mathscr{v}$ ∈ Ʊ, $\mathcal{A}_{IV}$ $(\mathscr{v})$ = [$\mathcal{A}^{LF}$ $(\mathscr{v})$, $\mathcal{A}^{UF}$ $(\mathscr{v})$] called membership for element $\mathscr{v}$ to $\mathcal{A}_{IV}$, where $\mathcal{A}^{LF}$: Ʊ → I and $\mathcal{A}^{UF}$: Ʊ → I called lower Fs and upper Fs in Ʊ. $\mathcal{A}_{IV}$ can also be represented as $\mathcal{A}_{IV}$ = [$\mathcal{A}^{LF}$, $\mathcal{A}^{UF}$]

Definition: 2.3[21]

$\mathfrak{Y}_{IV}$ be family of IVFs on Ʊ. The family $\mathfrak{Y}_{IV}$ $\widehat{\subset}$ IVF (Ʊ) called IVFT iff $\mathfrak{Y}_{IV}$ satisfies

(i) $0_{IV}, 1_{IV} \in \mathfrak{Y}_{IV}$

(ii) $\mathcal{A}_{IV}, \mathcal{B}_{IV} \in \mathfrak{Y}_{IV}$ implies $\mathcal{A}_{IV} \widehat{\cap} \mathcal{B}_{IV} \in \mathfrak{Y}_{IV}$

(iii) ${\mathcal{A}_{IV}}_j \in \mathfrak{Y}_{IV}$ for each j ∈ J implies $(\bigcup_{j\in J} ({\mathcal{A}_{IV}}_j) \in \mathfrak{Y}_{IV}$

(Ʊ, $\mathfrak{Y}_{IV}$) is IVF topological space. .

Definition: 2.4 [17]

A SIVFs in Ʊ is a map IVFs: Ʊ→ $[I]^I$. The family of SIVFs's is denoted by SIVF(Ʊ).

For every $\mathcal{A}_{SIV}$∈ SIVF(Ʊ), $\mathscr{v}$ ∈ Ʊ and α ∈ I. $\mathcal{A}_{SIV}$ $(\mathscr{v})$ (α) = [$\mathcal{A}^{LSF}$ $(\mathscr{v})$(α), $\mathcal{A}^{USF}$ $(\mathscr{v})$(α)] called membership element $\mathscr{v}$ to $\mathcal{A}_{SIV}$, where $\mathcal{A}^{LSF}$: Ʊ → I^I and $\mathcal{A}^{USF}$: Ʊ → I^I called lower SFs and upper SFs in Ʊ. Here $\mathcal{A}_{SIV}$ written [$\mathcal{A}^{LSF}$, $\mathcal{A}^{LSF}$].

Two SIVFss $\mathcal{A}_{SIV}, \mathcal{B}_{SIV}$ in SIVF(Ʊ), ∀ $\mathscr{v}$ ∈ Ʊ and ∀ $\alpha \in$ I

(i) $\mathcal{A}_{SIV} \hat{\hat{\subseteq}} \mathcal{B}_{SIV}$ iff $\mathcal{A}^{LSF}$ $(\mathscr{v})$ ≤ $\mathcal{B}^{LSF}$ $(\mathscr{v})$ and $\mathcal{A}^{USF}$ $(\mathscr{v})$ ≤ $\mathcal{B}^{USF}$ $(\mathscr{v})$,

iff $\mathcal{A}^{LSF}$ $(\mathscr{v})$ (α) ≤ $\mathcal{B}^{LSF}$ $(\mathscr{v})$ (α) and $\mathcal{A}^{USF}$ $(\mathscr{v})$ (α) ≤ $\mathcal{B}^{USF}$ $(\mathscr{v})$ (α),

(ii) $\mathcal{A}_{SIV}$ = $\mathcal{B}_{SIV}$ iff $\mathcal{A}_{SIV} \hat{\hat{\subseteq}} \mathcal{B}_{SIV}$ and $\mathcal{B}_{SIV} \hat{\hat{\subseteq}} \mathcal{A}_{SIV}$

(iii) The union $\mathcal{A}_{SIV} \hat{\hat{\cup}} \mathcal{B}_{SIV}$ and intersection $\mathcal{A}_{SIV} \hat{\hat{\cap}} \mathcal{B}_{SIV}$ are defined respectively, by

$$\mathcal{A}_{SIV} \hat{\hat{\cup}} \mathcal{B}_{SIV} = [\mathcal{A}^{LSF} \vee \mathcal{B}^{LSF}, \mathcal{A}^{USF} \vee \mathcal{B}^{USF}]$$

$$\mathcal{A}_{SIV} \hat{\hat{\cap}} \mathcal{B}_{SIV} = [\mathcal{A}^{LSF} \wedge \mathcal{B}^{LSF}, \mathcal{A}^{USF} \wedge \mathcal{B}^{USF}]$$

(iv) The constant SIVFs's 0_{SIV} and 1_{SIV} defined 0_{SIV} = [0_{SF}, 0_{SF}], 1_{SIV} = [1_{SF}, 1_{SF}]

3.SECOND ORDER IVF TOPOLOGICAL SPACES

Definition: 3.1

$\mathfrak{Y}_{SIV}$ collection of SIVFs on $\mho$ called SIVFT (Chang) iff $\mathfrak{Y}_{SIV}$ satisfies

(i) 0_{SIV}, $1_{SIV} \in \mathfrak{Y}_{SIV}$

(ii) $\mathcal{A}_{SIV}, \mathcal{B}_{SIV} \in \mathfrak{Y}_{SIV}$ implies $\mathcal{A}_{SIV} \cap \mathcal{B}_{SIV} \in \mathfrak{Y}_{SIV}$

(iii) ${\mathcal{A}_{SIV}}_j \in \mathfrak{Y}_{SIV}$ for each j ∈ J implies $(\bigcup_j {\mathcal{A}_{SIV}}_j) \in \mathfrak{Y}_{SIV}$

The pair $(\mho, \mathfrak{Y}_{SIV})$ is SIVF topological space (Chang).

Example : 3.2

A nonnull set $\mho$ and +ve integer n. $\mathfrak{Y}_{SIV} = \{1_{SIV}\} \cup \{\mathcal{A}_{SIV} \in$ SIVF($\mho$) / $\mathcal{A}^{LSF}(\mathcal{v})(\alpha) = 0$, $\mathcal{A}^{USF}(\mathcal{v})(\alpha) = 0$, for every $\mathcal{v} \in \mho$ and for $\alpha \neq \frac{r}{n}$, r = 0, 1, . . . n$\}$. Then $\mathfrak{Y}_{SIV}$ is a SIVFT on $\mho$.

Proof :

(i) Since $0_{SF}(\mathcal{v})(\alpha) = 0$, $0_{SF}(\mathcal{v})(\alpha) = 0$, for every $\mathcal{v} \in \mho$ and for every $\alpha \in$ I

Therefore $0_{SIV} \in \mathfrak{Y}_{SIV}$

(ii) ${\mathcal{A}_{SIV}}_j \in \mathfrak{Y}_{SIV}$ for j ∈ J

implies ${\mathcal{A}^{LSF}}_j(\mathcal{v})(\alpha) = 0$, ${\mathcal{A}^{USF}}_j(\mathcal{v})(\alpha) = 0$, for $\alpha \neq \frac{r}{n}$, r = 0, 1, .. n, for every $\mathcal{v} \in \mho$ and for every j ∈ J.

Therefore $\bigvee_{j \in J}({\mathcal{A}^{LSF}}_j(\mathcal{v})(\alpha)) = 0$, $\bigvee_{j \in J}({\mathcal{A}^{USF}}_j(\mathcal{v})(\alpha)) = 0$, for $\alpha \neq \frac{r}{n}$, r = 0, 1, . . . n, for every $\mathcal{v} \in \mho$

implies $(\bigcup_j {\mathcal{A}_{SIV}}_j) \in \mathfrak{Y}_{SIV}$

(iii) ${\mathcal{A}_{SIV}}_\mu \in \mathfrak{Y}_{SIV}$ for μ = 1 to m

implies ${\mathcal{A}^{LSF}}_\mu(\mathcal{v})(\alpha) = 0$, ${\mathcal{A}^{USF}}_\mu(\mathcal{v})(\alpha) = 0$, for $\alpha \neq \frac{r}{n}$, r = 0, 1, . . n, for every $\mathcal{v} \in \mho$ and for μ = 1 to m

Therefore $\bigwedge_{\mu=1}^{m}({\mathcal{A}^{LSF}}_\mu(\mathcal{v})(\alpha)) = 0$, $\bigwedge_{\mu=1}^{m}({\mathcal{A}^{USF}}_\mu(\mathcal{v})(\alpha)) = 0$, $\alpha \neq \frac{r}{n}$, r = 0,…. . . n, for every $\mathcal{v} \in \mho$

implies $(\bigcap_{\mu=1}^{m} {\mathcal{A}_{SIV}}_i) \in \mathfrak{Y}_{SIV}$

Therefore $\mathfrak{Y}_{SIV}$ is a SIVFT on $\mho$.

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4. K-Hausdorff Separation Axiom In Second Order Interval Valued Fuzzy Topological Spaces

Definition: 4.1

SIVF topological space $(\mho, \mathfrak{Y}_{SIV})$ said (SIVFK-H)$_1$ if $\forall$ $\mathscr{v}$, y $\in \mho$, $\mathscr{v} \neq$ y, there exists two IVF open sets $\mathcal{A}_{SIV}$ = $[\mathcal{A}^{LSF}, \mathcal{A}^{USF}]$, $\mathcal{B}_{SIV}$ = $[\mathcal{B}^{LSF}, \mathcal{B}^{USF}] \in \mathfrak{Y}_{SIV}$, such that $\mathcal{A}^{LSF}(\mathscr{v}) > 0$, $\mathcal{A}^{USF}(\mathscr{v}) > 0$, $\mathcal{B}^{LSF}(y) > 0$, $\mathcal{B}^{USF}(y) > 0$ and $\mathcal{A}_{SIV} \cap_1 \mathcal{B}_{SIV} = 0_{SIV}$.

Definition: 4.2

SIVF topological space $(\mho, \mathfrak{Y}_{SIV})$ said to SIVFK-Hausdorff of type 2, denoted (SIVFK-H)$_2$, by replacing $\mathcal{A}_{SIV} \cap_1 \mathcal{B}_{SIV} = 0_{SIV}$ in above definition by $\mathcal{A}_{SIV} \cap_2 \mathcal{B}_{SIV} = 0_{SIV}$.

Note: 4.3

$(\mho, \mathfrak{Y}_{SIV})$ is (SIVFK-H)$_1$ implies $(\mho, \mathfrak{Y}_{SIV})$ is (SIVFK-H)$_2$ since $\mathcal{A}_{SIV} \cap_1 \mathcal{B}_{SIV} = 0_{SIV}$ implies $\mathcal{A}_{SIV} \cap_2 \mathcal{B}_{SIV} = 0_{SIV}$. The converse of the above implication is not true. Since $\mathcal{A}_{SIV} \cap_2 \mathcal{B}_{SIV} = 0_{SIV}$ need not imply that $\mathcal{A}_{SIV} \cap_1 \mathcal{B}_{SIV} = 0_{SIV}$.

Definition : 4.4

Let $(\mho, \mathfrak{Y}_{SIV})$ be SIVF topological space, a nonnull $\mathfrak{M}$ is subset of $\mho$. Let $\mathcal{A}_{SIV} \in \mathfrak{Y}_{SIV}$. Define the restriction function $\mathcal{A}_{SIV}/\mathfrak{M} : \mathfrak{M} \to [I]^I$ as follows

$(\mathcal{A}^{LSF}/\mathfrak{M})(\mathfrak{p}) = \mathcal{A}^{LSF}(\mathfrak{p})$, $(\mathcal{A}^{USF}/\mathfrak{M})(\mathfrak{p}) = \mathcal{A}^{USF}(\mathfrak{p})$, if $\mathfrak{p} \in \mathfrak{M}$.

Define $\mathfrak{Y}_{SIV}/\mathfrak{M} = \{\mathcal{A}_{SIV}/\mathfrak{M}$ such that $\mathcal{A}_{SIV} \in \mathfrak{Y}_{SIV}\}$. Then $\mathfrak{Y}_{SIV}/\mathfrak{M}$ called SIVF subspace topology of $\mathfrak{M}$ and $(\mathfrak{M}, \mathfrak{Y}_{SIV}/\mathfrak{M})$ called SIVF subspace of $(\mho, \mathfrak{Y}_{SIV})$.

Theorem: 4.5

1. "SIVF subspace of (SIVFK-H)$_1$ space is (SIVFK-H)$_1$
2. SIVF subspace of (SIVFK-H)$_2$ space is (SIVFK-H)$_2$"

Proof:

$(\mho, \mathfrak{Y}_{SIV})$ is (SIVFK-H)$_1$ space and $\mathfrak{M}$ be nonempty subset of $\mho$.

Consider y_1, $y_2 \in \mathfrak{M}$ where $y_1 \neq y_2$. Then y_1, $y_2 \in \mho$, there exists two SIVF open set $\mathcal{A}_{SIV}$= $[\mathcal{A}^{LSF}, \mathcal{A}^{USF}]$, $\mathcal{B}_{SIV}$ = $[\mathcal{B}^{LSF}, \mathcal{B}^{USF}] \in \mathfrak{Y}_{SIV}$, such that $\mathcal{A}^{LSF}(y_1) > 0$, $\mathcal{A}^{USF}(y_1) > 0$, $\mathcal{B}^{LSF}(y_2) > 0$, $\mathcal{B}^{USF}(y_2) > 0$ and $\mathcal{A}_{SIV} \cap_1 \mathcal{B}_{SIV} = 0_{SIV}$

Let $\mathcal{A}_{SIV}/\mathfrak{M}$, $\mathcal{B}_{SIV}/\mathfrak{M} \in \mathfrak{Y}_{SIV}/\mathfrak{M}$, where

$(\mathcal{A}_{SIV}/\mathfrak{M}) = [\mathcal{A}^{LSF}/\mathfrak{M}, \mathcal{A}^{USF}/\mathfrak{M}]$, $(\mathcal{B}_{SIV}/\mathfrak{M}) = [\mathcal{B}^{LSF}/\mathfrak{M}, \mathcal{B}^{USF}/\mathfrak{M}]$

Therefore

$(\mathcal{A}^{LSF}/\mathfrak{M})(y_1) = \mathcal{A}^{LSF}(y_1) > 0$

$(\mathcal{A}^{USF}/\mathfrak{M})(y_1) = \mathcal{A}^{USF}(y_1) > 0$

$(\mathcal{B}^{LSF}/\mathfrak{M})(y_2) = \mathcal{B}^{LSF}(y_2) > 0$

$(\mathcal{B}^{USF}/\mathfrak{M})(y_2) = \mathcal{B}^{USF}(y_2) > 0$

Also $\mathcal{A}_{SIV} \cap_1 \mathcal{B}_{SIV} = 0_{SIV}$ implies $\mathcal{A}^{LSF}(\mathscr{v}) = 0$, $\mathcal{A}^{USF}(\mathscr{v}) = 0$ or $\mathcal{B}^{LSF}(\mathscr{v}) = 0$ and $\mathcal{B}^{USF}(\mathscr{v}) = 0$, $\forall\, \mathscr{v} \in \mathfrak{M} \subseteq \mho$

implies $(\mathcal{A}^{LSF}/\mathfrak{M})(\mathscr{v}) = 0$, $(\mathcal{A}^{USF}/\mathfrak{M})(\mathscr{v}) = 0$ or

$(\mathcal{B}^{LSF}$ / $\mathfrak{M}$)($\mathcal{v}$) = 0, $(\mathcal{B}^{USF}$ / $\mathfrak{M}$)($\mathcal{v}$) = 0 $\quad \forall\, \mathcal{v} \in \mathfrak{M} \subseteq \mho$
implies $\mathcal{A}_{SIV}/\mathfrak{M} \cap_1 \mathcal{B}_{SIV}/\mathfrak{M} = 0_{SIV}$
Hence ($\mathfrak{M}$, $\mathfrak{Y}_{SIV}$ / $\mathfrak{M}$) is $(\text{SIVFK-H})_1$
($\mathfrak{M}$, $\mathfrak{Y}_{SIV}$ / $\mathfrak{M}$) is a $(\text{SIVFK-H})_2$ as $(\frac{\mathcal{A}_{SIV}}{\mathfrak{M}} \cap_1 \frac{\mathcal{B}_{SIV}}{\mathfrak{M}}) = 0_{SIV}$
implies $(\mathcal{A}_{SIV}/\mathfrak{M} \cap_2 \mathcal{B}_{SIV}/\mathfrak{M}) = 0_{SIV}$.

Definition: 4.6

Let $(\mho, \mathfrak{Y}_{SIV_1})$, $(\mathfrak{M}, \mathfrak{Y}_{SIV_2})$ be two SIVF topological spaces. If $\mathcal{A}_{SIV} \in \mathfrak{Y}_{SIV_1}$ and $\mathcal{B}_{SIV} \in \mathfrak{Y}_{SIV_2}$, then the product $(\mathcal{A}_{SIV} * \mathcal{B}_{SIV})$ in $\mho$ x $\mathfrak{M}$ is defined as follows :
$[\mathcal{A}_{SIV} * \mathcal{B}_{SIV}] = [\mathcal{A}^{LSF} * \mathcal{B}^{LSF}, \mathcal{A}^{USF} * \mathcal{B}^{USF}]$ where
$(\mathcal{A}^{LSF} * \mathcal{B}^{LSF})(\mathcal{v}, y)(\alpha) = \mathcal{A}^{LSF}(\mathcal{v})(\alpha) \wedge \mathcal{B}^{LSF}(y)(\alpha)$
$(\mathcal{A}^{USF} * \mathcal{B}^{USF})(\mathcal{v}, y)(\alpha) = \mathcal{A}^{USF}(\mathcal{v})(\alpha) \wedge \mathcal{B}^{USF}(y)(\alpha)$
$\forall\, (\mathcal{v}, y) \in \mho \times \mathfrak{M}$, $\alpha \in I$.
Product topology $\mathfrak{Y}_{SIV_1}$ x $\mathfrak{Y}_{SIV_2}$ on $\mho$ x $\mathfrak{M}$ is SIVFT having the collection $\{\mathcal{A}_{SIV} * \mathcal{B}_{SIV} / \mathcal{A}_{SIV} \in \mathfrak{Y}_{SIV_1}, \mathcal{B}_{SIV} \in \mathfrak{Y}_{SIV_2}\}$ as basis.

Theorem:4.7

1. "Product of two $(\text{SIVFK-H})_1$ spaces is $(\text{SIVFK-H})_1$
2. Product of two $(\text{SIVFK-H})_2$ spaces is $(\text{SIVFK-H})_2$ "

Proof:
$(\mho, \mathfrak{Y}_{SIV_1})$ and $(\mathfrak{M}, \mathfrak{Y}_{SIV_2})$ be $(\text{SIVFK-H})_1$ spaces.
Consider distinct points $(\mathcal{v}_1, y_1), (\mathcal{v}_2, y_2) \in \mho \times \mathfrak{M}$
Either $\mathcal{v}_1 \neq \mathcal{v}_2$ or $y_1 \neq y_2$
take $\mathcal{v}_1 \neq \mathcal{v}_2$
Therefore there exists two SIVF open sets $\mathcal{A}_{SIV} = [\mathcal{A}^{LSF}, \mathcal{A}^{USF}]$, $\mathcal{B}_{SIV} = [\mathcal{B}^{LSF}, \mathcal{B}^{USF}] \in \mathfrak{Y}_{SIV_1}$ such that $\mathcal{A}^{LSF}(\mathcal{v}_1) > 0$, $\mathcal{A}^{USF}(\mathcal{v}_1) > 0$, $\mathcal{B}^{LSF}(\mathcal{v}_2) > 0$, $\mathcal{B}^{USF}(\mathcal{v}_2) > 0$ and
$\mathcal{A}_{SIV} \cap_1 \mathcal{B}_{SIV} = 0_{SIV}$
$\mathcal{A}_{SIV}, \mathcal{B}_{SIV} \in \mathfrak{Y}_{SIV_1}$ implies $\mathcal{A}_{SIV} * 1_{SIV} \in \mathfrak{Y}_{SIV_1} \text{x}\, \mathfrak{Y}_{SIV_2}$, $\mathcal{B}_{SIV} * 1_{SIV} \in \mathfrak{Y}_{SIV_1} \text{x}\, \mathfrak{Y}_{SIV_2}$
where
$\mathcal{A}_{SIV} * 1_{SIV} = [\mathcal{A}^{LSF} * 1_{SF}, \mathcal{A}^{USF} * 1_{SF}]$ and $\mathcal{B}_{SIV} * 1_{SIV} = [\mathcal{B}^{LSF} * 1_{SF}, \mathcal{B}^{USF} * 1_{SF}]$
Consider

$(\mathcal{A}^{LSF} * 1_{SF})(\mathcal{v}_1, y_1) = \mathcal{A}^{LSF}(\mathcal{v}_1) \wedge 1_{SF}(y_1)$, for every $(\mathcal{v}_1, y_1) \in \mho \times \mathfrak{M}$
> 0
$(\mathcal{A}^{USF} * 1_{SF})(\mathcal{v}_1, y_1) = \mathcal{A}^{USF}(\mathcal{v}_1) \wedge 1_{SF}(y_1)$, for every $(\mathcal{v}_1, y_1) \in \mho \times \mathfrak{M}$
> 0
$(\mathcal{B}^{LSF} * 1_{SF})(\mathcal{v}_2, y_2) = \mathcal{B}^{LSF}(\mathcal{v}_2) \wedge 1_{SF}(y_2)$, for every $(\mathcal{v}_2, y_2) \in \mho \times \mathfrak{M}$
> 0
$(\mathcal{B}^{USF} * 1_{SF})(\mathcal{v}_2, y_2) = \mathcal{B}^{USF}(\mathcal{v}_2) \wedge 1_{SF}(y_2)$, for every $(\mathcal{v}_2, y_2) \in \mho \times \mathfrak{M}$
> 0

For $(\mathcal{v}, y) \in \mho$ x $\mathfrak{M}$
Consider
$(\mathcal{A}^{LSF} * 1_{SF})$ $(\mathcal{v}, y) \neq 0$, $(\mathcal{A}^{USF} * 1_{SF})$ $(\mathcal{v}$, y) $\neq 0$
implies $(\mathcal{A}^{LSF} * 1_{SF})$ $(\mathcal{v}$, y) (α) ≠ 0, $(\mathcal{A}^{USF} * 1_{SF})$ $(\mathcal{v}$, y) (α) ≠ 0, for some α ∈ I
implies $\mathcal{A}^{LSF}$ $(\mathcal{v})$ (α) ∧ 1_{SF} (y) (α) ≠ 0,
$\mathcal{A}^{USF}$ $(\mathcal{v})$ (α) ∧ 1_{SF} (y) (α) ≠ 0, for some α ∈ I
implies $\mathcal{A}^{LSF}$ $(\mathcal{v})$ (α) ≠ 0, $\mathcal{A}^{USF}$ $(\mathcal{v})$ (α) ≠ 0, ∀$\mathcal{v} \in \mho$, for some α ∈ I
implies $\mathcal{A}^{LSF}$ $(\mathcal{v}) \neq 0$, $\mathcal{A}^{USF}$ $(\mathcal{v}) \neq 0$, ∀$\mathcal{v} \in \mho$
implies $\mathcal{B}^{LSF}$ $(\mathcal{v}) = 0$, $\mathcal{B}^{USF}$ $(\mathcal{v}) = 0$, ∀$\mathcal{v} \in \mho$ (since $\mathcal{A}_{SIV}$, $\cap_1$ $\mathcal{B}_{SIV}$, $= 0_{SIV}$)
implies $\mathcal{B}^{LSF}$ $(\mathcal{v})$ (α) = 0, $\mathcal{B}^{USF}$ $(\mathcal{v})$ (α) = 0, ∀$\mathcal{v} \in \mho$, ∀α ∈ I
implies $\mathcal{B}^{LSF}$ $(\mathcal{v})$ (α) ∧ 1_{SF} (y) (α) = 0, $\mathcal{B}^{USF}$ $(\mathcal{v})$ (α) ∧ 1_{SF} (y) (α) = 0, ∀$\mathcal{v} \in \mho$,
$y \in \mathfrak{M}$, α ∈ I
implies $(\mathcal{B}^{LSF} * 1_{SF})$ $(\mathcal{v}, y)$ (α) = 0, $(\mathcal{B}^{USF} * 1_{SF})$ $(\mathcal{v}, y)$ (α) = 0,
∀ $(\mathcal{v}, y) \in \mho$ x $\mathfrak{M}$ and α ∈ I
implies $(\mathcal{B}^{LSF} * 1_{SF})$ $(\mathcal{v}, y) = 0$, $(\mathcal{B}^{USF} * 1_{SF})$ $(\mathcal{v}, y) = 0$
Therefore $(\mathcal{A}_{SIV} * 1_{SIV})$ $\cap_1$ $(\mathcal{B}_{SIV} * 1_{SIV}) = 0_{SIV}$
Similarly, $y_1 \neq y_2$
Therefore ($\mho$ x $\mathfrak{M}$, ${\mathfrak{Y}_{SIV}}_1$x ${\mathfrak{Y}_{SIV}}_2$) is $(\text{SIVFK-H})_1$
($\mho$ x $\mathfrak{M}$, ${\mathfrak{Y}_{SIV}}_1$x ${\mathfrak{Y}_{SIV}}_2$) is $(\text{SIVFK-H})_2$ as $\mathcal{A}_{SIV}$ $\cap_1$ $\mathcal{B}_{SIV} = 0_{SIV}$ implies $\mathcal{A}_{SIV}$ $\cap_2$ $\mathcal{B}_{SIV} = 0_{SIV}$

Definition: 4.8

{($\mho_j$, ${\mathfrak{Y}_{SIV}}_j$) / j ∈ $\mathcal{T}$} Collection of SIVF topological spaces, $\mho = \prod_{j\epsilon\mathcal{T}} \mho$j. SIVF product topology on $\mho$ is SIVF open sets of the form $\mathcal{A}_{SIV}$ = [$\prod_{j\epsilon\mathcal{T}} \mathcal{A}^{LSF}$j, $\prod_{j\epsilon\mathcal{T}} \mathcal{A}^{USF}$j], where ${\mathcal{A}_{SIV}}_j \in {\mathfrak{Y}_{SIV}}_j$ and ${\mathcal{A}_{SIV}}_j = 1_{SIV}$ except j's. Here
($\prod_{j\epsilon\mathcal{T}} \mathcal{A}^{LSF}$j) ($\mathcal{v}_j$) (α) = $\bigwedge_{j \in \mathcal{T}}$ ${\mathcal{A}^{LSF}}_j$, ($\mathcal{v}_j$)(α),
($\prod_{j\epsilon\mathcal{T}} \mathcal{A}^{USF}$j) ($\mathcal{v}_j$) (α) = $\bigwedge_{j \in \mathcal{T}}$ ${\mathcal{A}^{USF}}_j$ ($\mathcal{v}_j$) (α)
∀ ($\mathcal{v}_j$) ∈ $\prod_{j\epsilon\mathcal{T}} \mho$j and for some α ∈ I.

Theorem: 4.9

1. "Arbitrary product of $(\text{SIVFK-H})_1$ space is $(\text{SIVFK-H})_1$
2. Arbitrary product of $(\text{SIVFK-H})_2$ space is $(\text{SIVFK-H})_2$ "

Proof:
{($\mho_j$, ${\mathfrak{Y}_{SIV}}_j$) / j ∈ $\mathcal{T}$ } be $(\text{SIVFK-H})_1$ spaces.
$\mho = \prod_{j\epsilon\mathcal{T}} \mho$j and $\mathfrak{Y}_{SIV} = \prod_{j\epsilon\mathcal{T}} {\mathfrak{Y}_{SIV}}_j$
Take distinct IVF points ($\mathcal{v}_j$), (y_j) ∈$\prod_{j\epsilon J} \mho$j.
Therefore $\mathcal{v}_k \neq y_k$, for some k ∈ $\mathcal{T}$,there exists SIVF open sets $\mathcal{A}_{SIV}$= [${\mathcal{A}^{LSF}}_k$, ${\mathcal{A}^{USF}}_k$,],

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$\mathcal{B}_{SIV}$= [$\mathcal{B}^{LSF}{}_{k}$, $\mathcal{B}^{USF}{}_{k}$,] ∈ $\mathfrak{Y}_{SIV_k}$, such that $\mathcal{A}^{LSF}{}_{k}$, ($\mathscr{v}_k$) > 0, $\mathcal{A}^{USF}{}_{k}$ ($\mathscr{v}_k$) > 0, $\mathcal{B}^{LSF}{}_{k}$ (y_k) > 0, $\mathcal{B}^{USF}{}_{k}$ (y_k) > 0 and $\mathcal{A}_{SIV_k} \cap_1 \mathcal{B}_{SIV_k} = 0_{SIV}$

Let $\mathcal{A}_{SIV} = \prod_{j\in\mathcal{T}} \mathcal{A}_{SIV_j}$, where $\mathcal{A}_{SIV_j} = 1_{SIV}$, for j ≠ k

$\mathcal{B}_{SIV} = \prod_{j\in\mathcal{T}} \mathcal{B}_{SIV_j}$, where $\mathcal{B}_{SIV_j} = 1_{SIV}$, for j ≠ k

Then $\mathcal{A}_{SIV}, \mathcal{B}_{SIV} \in \prod_{j\epsilon\mathcal{T}} \mathfrak{Y}_{SIV_j}$

$\mathcal{A}_{SIV} = \prod_{j\in\mathcal{T}} \mathcal{A}_{SIV_j} = [\prod_{j\epsilon\mathcal{T}} \mathcal{A}^{LSF}\text{j}, \prod_{j\epsilon\mathcal{T}} \mathcal{A}^{USF}\text{j}]$

$(\prod_{j\epsilon\mathcal{T}} \mathcal{A}^{LSF}\text{j})(\mathscr{v}_j) = \wedge_{j \in \mathcal{T}} \mathcal{A}^{LSF}{}_{j} (\mathscr{v}_j)$

$= \mathcal{A}^{LSF}{}_{k}, (\mathscr{v}_k)$, for some $k \in \mathcal{T}$

> 0

$(\prod_{j\epsilon\mathcal{T}} \mathcal{A}^{USF}\text{j}) (\mathscr{v}_j) = \wedge_{j \in \mathcal{T}} \mathcal{A}^{USF}{}_{j} (\mathscr{v}_j)$

$= \mathcal{A}^{USF}{}_{k} (\mathscr{v}_k)$, for some $k \in \mathcal{T}$

> 0

$\mathcal{B}_{SIV} = \prod_{j\in\mathcal{T}} \mathcal{B}_{SIV_j} = [\prod_{j\epsilon\mathcal{T}} \mathcal{B}^{LSF}\text{j}, \prod_{j\epsilon\mathcal{T}} \mathcal{B}^{USF}\text{j}]$

$(\prod_{j\epsilon\mathcal{T}} \mathcal{B}^{LSF}\text{j}) (y_j) = \wedge_{j \in \mathcal{T}} \mathcal{B}^{LSF}{}_{j} (y_j)$

$= \mathcal{B}^{LSF}{}_{k} (y_k)$, for some $k \in \mathcal{T}$

> 0

$(\prod_{j\epsilon\mathcal{T}} \mathcal{B}^{USF}\text{j}) (y_j) = \wedge_{j \in \mathcal{T}} \mathcal{B}^{USF}{}_{j} (y_j)$

$= \mathcal{B}^{USF}{}_{k} (y_k)$, for some $k \in \mathcal{T}$

> 0

Consider $\forall\, \mathscr{v} \in \mho$

$\prod_{j\epsilon\mathcal{T}} \mathcal{A}^{LSF}\text{j} (\mathscr{v}) \neq 0$ and $\prod_{j\epsilon\mathcal{T}} \mathcal{A}^{USF}\text{j} (\mathscr{v}) \neq 0$

$(\prod_{j\epsilon\mathcal{T}} \mathcal{A}^{LSF}\text{j} (\mathscr{v})) (\alpha) \neq 0$ and $(\prod_{j\epsilon\mathcal{T}} \mathcal{A}^{USF}\text{j} (\mathscr{v})) (\alpha) \neq 0$, for some $\alpha \in I$

$\wedge_{j \in \mathcal{T}}(\mathcal{A}^{LSF}{}_{j} (\mathscr{v}) (\alpha)) \neq 0$ and $\wedge_{j \in \mathcal{T}}(\mathcal{A}^{USF}{}_{j} (\mathscr{v}) (\alpha)) \neq 0$

$\mathcal{A}^{LSF}{}_{k} (\mathscr{v}) (\alpha) \neq 0$ and $\mathcal{A}^{USF}{}_{k} (\mathscr{v}) (\alpha) \neq 0$, for $k \neq j$

$\mathcal{B}^{LSF}{}_{k} (\mathscr{v}) (\alpha) = 0$ and $\mathcal{B}^{USF}{}_{k} (\mathscr{v}) (\alpha) = 0$, $\forall\ \alpha \in I$

$\wedge_{j \in \mathcal{T}}(\mathcal{B}^{LSF}{}_{k} (\mathscr{v}) (\alpha) = 0$ and $\wedge_{j \in \mathcal{T}}(\mathcal{B}^{USF}{}_{k} (\mathscr{v}) (\alpha) =$

$(\prod_{j\epsilon\mathcal{T}} \mathcal{B}^{LSF}\text{j}) (\mathscr{v}) (\alpha) = 0$ and $(\prod_{j\epsilon\mathcal{T}} \mathcal{B}^{USF}\text{j}) (\mathscr{v}) (\alpha) = 0$

$(\prod_{j\epsilon\mathcal{T}} \mathcal{B}^{LSF}\text{j}) (\mathscr{v}) = 0$ and $(\prod_{j\epsilon\mathcal{T}} \mathcal{B}^{USF}\text{j}) (\mathscr{v}) = 0$

implies $\mathcal{A}_{SIV} \cap_1 \mathcal{B}_{SIV} = 0_{SIV}$

Hence $(\mho, \mathfrak{Y}_{SIV})$ is $(\text{SIVFK-H})_1$

$(\mho, \mathfrak{Y}_{SIV})$ is $(\text{SIVFK-H})_2$ as $\mathcal{A}_{SIV} \cap_1 \mathcal{B}_{SIV} = 0_{SIV}$ implies $\mathcal{A}_{SIV} \cap_2 \mathcal{B}_{SIV} = 0_{SIV}$

5.Conclusions

The level of this research was to evolve K-Hausdorff Separation axioms in sequence to assemble a framework that will furnish method for object ranking. Thus ,We initiated a new definition of Second order IVFT,second order IVF Subspace and second order IVF product topology. K-Hausdorff separation axiom in second order IVF topological spaces were presented, and furthermore, its basics are studied.

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