

Data Encryption and Decryption using Some Integral Transforms

¹Gobburi Rekha, ²V. Srinivas

¹ Department of Humanities and Sciences, Malla Reddy College of Engineering and Technology

² Department of Mathematics, Osmania university

Article History:

Received: 21-01-2024

Revised: 21-03-2024

Accepted: 18-04-2024

Abstract:

Ever since the beginning of time, people have valued privacy and information security. Information security is becoming increasingly vital in our digital society. The authentication system's protection of hidden information is crucial. One of the most significant methods used to protect data and secure message transmission is cryptography. There are the two components for the encoding and decoding process which includes the algorithm and key. Cryptography is made more secured by utilising a key for both encryption and decryption.

Introduction: In many circumstances, the sender wants to keep the message private from the public or from unauthorised users. Information is protected using encryption, and the original message is unlocked using decryption. Various Techniques are found in literature and further we extend the findings by combining the Elzaki and Aboodh transforms to encrypt the message and apply their inverse transforms to decrypt it with the ASCII values under modulo 255.

Objectives: In this paper we focused on two stages of encryption and two stages of decryption under modulo 255.

Methods: Using different transformation techniques, we have encrypted and decrypted the data. One is Laplace transform technique while the other is Aboodh-Elzaki transform technique.

Results: The text message is converted to coded message by using both Laplace transform technique and Aboodh-Elzaki transforms technique.

Conclusions: In the current paper, we present a novel method that encrypts data using Laplace transform and decrypts it using the private key, which is under modulo 255. The two integral transforms are used to increase the message's level of security during decryption by selecting a new function and altering the other elements.

Keywords: Cryptography, Plain text, Cipher text, Data encryption, Data decryption, Laplace, Elzaki - Aboodh transforms, ASCII code.

1.Introduction

The study of encrypted messages is known as cryptography. In many circumstances, the sender wants to keep the message private from the public or from unauthorised users. Information is protected using encryption, and the original message is unlocked using decryption. Integral

transformations have a wide range of uses, especially in the realm of cryptography. The plain text is encrypted using Laplace transformation, while the cipher text is decrypted using the inverse transformation. The following references are to literature utilising Laplace and other transformations.[2],[3],[4],[5],[6],[7],[8],[9],[10]and [11].

Various Techniques are found in literature and further we extend the findings of [1] by combining the Elzaki and Aboodh transforms to encrypt the message and apply their inverse transforms to decrypt it with the ASCII values under modulo 255.

2. Definitions:

2.1 Laplace Transformation:

If $f(t)$ is a function defined for all positive values of t , then the Laplace Transform of $f(t)$ is defined as:

$$L \{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t) dt ,$$

provided the integral exists. Here the parameter S is a real or complex number and the inverse Laplace transform is $L^{-1}\{F(s)\} = f(t)$.

2.2 Some standard results of Laplace transform:

$L \{1\} = \frac{1}{s}$	$L^{-1} \{ \frac{1}{s} \} = 1$
$L \{t^n\} = \frac{n!}{s^{n+1}}$	$L^{-1} \{ \frac{1}{s^{n+1}} \} = \frac{t^n}{n!}$
$L \{e^{at}\} = \frac{1}{s-a}$	$L^{-1} \{ \frac{1}{s-a} \} = e^{at}$
$L \{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$	$L^{-1} \{ \frac{d^n}{ds^n} F(s) \} = t^n f(t)$

2.3 Elzaki Transformation:

An integral transform defined for the function of exponential order; we consider function in the set E

$$\text{defined by } E = \left\{ f(t): \exists m, k_1, k_2 > 0 > , |f(t)| < m e^{\frac{|t|}{v}} \right\}$$

$$E [f(t)] = T(v) = v \int_0^\infty f(t) e^{-\frac{t}{v}} dt.$$

2.4 Some standard results of Elzaki transform:

$E[1] = v^2$	$E^{-1}[v^2] = 1$
$E[t^n] = n! v^{n+2}$	$E^{-1}[v^{n+2}] = \frac{t^n}{n!}$
$E[e^{at}] = \frac{v^2}{1-av}$	$E^{-1} \left[\frac{v^2}{1-av} \right] = e^{at}$

2.4 Aboodh Transformation:

A function in the set E defined by $E = \{f(t): \exists m, k_1, k_2 > 0 > , |f(t)| < m e^{-ut}\}$

$$A [f(t)] = T(u) = \frac{1}{u} \int_0^\infty f(t) e^{-ut} dt.$$

2.5 Some standard results of Aboodh transform:

$A[1] = \frac{1}{v^2}$	$A^{-1} \left[\frac{1}{v^2} \right] = 1$
$A[t^n] = \frac{n!}{v^{n+2}}$	$A^{-1} \left[\frac{1}{v^{n+2}} \right] = \frac{t^n}{n!}$

3. Methods

Method of encryption

- Select the message, M, to be transmitted, then convert it to ASCII format. The message should remain at n length.
- The message in plain text is structured as a limited series of numbers.
- Write the numbers as the coefficients in f(t).
- For the above f(t) apply laplace transform say, $G(s) = L\{f(t)\}$.
- Let $r_i \equiv f_i \text{ mod } 255$.
- Let $k_i = \frac{f_i - r_i}{255}$ where k_i is the key.

Method of decryption

- Compose the sender’s ciphertext and key.
- Create a finite numerical sequence from the encrypted text say G'_i .
- Let $f_i = 255 k_i + r_i$.
- Apply inverse laplace transform for $G(s)$ which is obtained in encryption say $L^{-1}\{G(s)\} = f(t)$.
- Write the coefficients of f(t).
- Convert the coefficients of f(t) into symbols we get the plain text which is sent.

Let us explain the above methodology using some illustrations.

4. Results

Illustration 1: Let the plain text be **Mathematician**.

Here $n = 13$. As per the steps mentioned above ASCII values of the plain text are $M = 77$, $a = 97$, $t = 116$, $h = 104$, $e = 101$, $m = 109$, $a = 97$, $t = 116$, $i = 105$, $c = 99$, $i = 105$, $a = 97$, $n = 110$.

Therefore, plain text finite sequence coefficients are given as $G_0 = 77, G_1 = 97, G_2 = 116, G_3 = 104, G_4 = 101, G_5 = 109, G_6 = 97, G_7 = 116, G_8 = 105, G_9 = 99, G_{10} = 105, G_{11} = 97$ and $G_{11} = 110$.

With r being a constant, write these values as the coefficients in $t^2 \cosh rt$.

The $\cosh rt$ expansion is

$$\cosh rt = 1 + \frac{(rt)^2}{2!} + \frac{(rt)^4}{4!} + \frac{(rt)^6}{6!} + \frac{(rt)^8}{8!} + \frac{(rt)^{10}}{10!} + \dots \quad \text{and}$$

$$t^2 \cosh rt = t^2 + \frac{r^2 t^4}{2!} + \frac{r^4 t^6}{4!} + \frac{r^6 t^8}{6!} + \frac{r^8 t^{10}}{8!} + \frac{r^{10} t^{12}}{10!} + \dots \quad \text{where r is a constant.}$$

Let us consider

$$f(t) = \sum_{i=0}^n G_i t^2 \cosh rt, \text{ where } r = 1.$$

$$= t^2 \left[G_0 + G_1 \frac{t^2}{2!} + G_2 \frac{t^4}{4!} + G_3 \frac{t^6}{6!} + G_4 \frac{t^8}{8!} + G_5 \frac{t^{10}}{10!} + G_6 \frac{t^{12}}{12!} + G_7 \frac{t^{14}}{14!} + G_8 \frac{t^{16}}{16!} + G_9 \frac{t^{18}}{18!} + G_{10} \frac{t^{20}}{20!} + G_{11} \frac{t^{22}}{22!} + G_{12} \frac{t^{24}}{24!} \right].$$

$$= \left[77t^2 + 97 \frac{t^4}{2!} + 116 \frac{t^6}{4!} + 104 \frac{t^8}{6!} + 101 \frac{t^{10}}{8!} + 109 \frac{t^{12}}{10!} + 97 \frac{t^{14}}{12!} + 116 \frac{t^{16}}{14!} + 105 \frac{t^{18}}{16!} + 99 \frac{t^{20}}{18!} + 105 \frac{t^{22}}{20!} + 97 \frac{t^{24}}{22!} + 110 \frac{t^{26}}{24!} \right].$$

Apply Laplace transform for $f(t)$

$$\begin{aligned} L\{f(t)\} &= L\{t^2 \cos ht\} = L\left\{77t^2 + 97 \frac{t^4}{2!} + 116 \frac{t^6}{4!} + 104 \frac{t^8}{6!} + 101 \frac{t^{10}}{8!} + 109 \frac{t^{12}}{10!} + 97 \frac{t^{14}}{12!} + 116 \frac{t^{16}}{14!} + 105 \frac{t^{18}}{16!} + 99 \frac{t^{20}}{18!} + 105 \frac{t^{22}}{20!} + 97 \frac{t^{24}}{22!} + 110 \frac{t^{26}}{24!}\right\}. \\ &= 77 \frac{2!}{s^3} + 97 \frac{4!}{2!s^5} + 116 \frac{6!}{4!s^7} + 104 \frac{8!}{6!s^9} + 101 \frac{10!}{8!s^{11}} + 109 \frac{12!}{10!s^{13}} + 97 \frac{14!}{12!s^{15}} + 116 \frac{16!}{14!s^{17}} + 105 \frac{18!}{16!s^{19}} + 99 \frac{20!}{18!s^{21}} + 105 \frac{22!}{20!s^{23}} + 97 \frac{24!}{22!s^{25}} + 110 \frac{26!}{24!s^{27}}. \\ &= \left(\frac{154}{s^3} + \frac{1164}{s^5} + \frac{3480}{s^7} + \frac{5824}{s^9} + \frac{9090}{s^{11}} + \frac{14388}{s^{13}} + \frac{17654}{s^{15}} + \frac{27840}{s^{17}} + \frac{32130}{s^{19}} + \frac{37620}{s^{21}} + \frac{48510}{s^{23}} + \frac{53544}{s^{25}} + \frac{71500}{s^{27}} \right). \end{aligned}$$

Now, find r_i such that $r_i \equiv f_i \pmod{255}$

Here $r_0 = 154, r_1 = 144, r_2 = 165, r_3 = 214, r_4 = 165, r_5 = 108, r_6 = 59, r_7 = 45,$
 $r_8 = 0, r_9 = 135, r_{10} = 60, r_{11} = 249, r_{12} = 100.$

$$= \frac{154}{s^3} + \frac{144}{s^5} + \frac{165}{s^7} + \frac{214}{s^9} + \frac{165}{s^{11}} + \frac{108}{s^{13}} + \frac{59}{s^{15}} + \frac{45}{s^{17}} + \frac{0}{s^{19}} + \frac{135}{s^{21}} + \frac{60}{s^{23}} + \frac{249}{s^{25}} + \frac{100}{s^{27}}.$$

The ASCII codes of the above remainders are the encoded message.

Now we find the key k_i such that $k_i = \frac{f_i - r_i}{255}$.

Therefore, the key is k_i : 0, 4, 13, 22, 35, 56, 69, 109, 126, 147, 190, 209, 280.

Hence, the sender sends the cipher text which is **ÛÉ ÑÖ¥!; -NUL‡<ùd** and the key.

Examine the sent cipher text and key.

Translate the cipher text to finite sequence of numbers.

Let $G'_0 = 154, G'_1 = 144, G'_2 = 165, G'_3 = 214, G'_4 = 165, G'_5 = 108, G'_6 = 59,$
 $G'_7 = 45, G'_8 = 0, G'_9 = 135, G'_{10} = 60, G'_{11} = 249, G'_{12} = 100.$

Using the given key k_i and assuming $f_i = 255k_i + G'_i$

Where $f_0 = 255 * 0 + 154 = 154, f_1 = 255 * 4 + 144 = 1164,$

$f_2 = 255 * 13 + 165 = 3480, f_3 = 255 * 22 + 214 = 5824,$

$f_4 = 255 * 35 + 165 = 9090, f_5 = 255 * 56 + 108 = 14388,$

$f_6 = 255 * 69 + 59 = 17654, f_7 = 255 * 109 + 45 = 27840,$

$f_8 = 255 * 126 + 0 = 32130, f_9 = 255 * 147 + 135 = 37620,$

$f_{10} = 255 * 190 + 60 = 48510, f_{11} = 255 * 209 + 249 = 53544,$

$$f_{12} = 255 * 280 + 100 = 71500.$$

Now consider $G(s) = \sum_{n=0}^{12} \frac{f_n}{s^{2n+3}}$

$$G(s) = \frac{154}{s^3} + \frac{1164}{s^5} + \frac{3480}{s^7} + \frac{5824}{s^9} + \frac{9090}{s^{11}} + \frac{14388}{s^{13}} + \frac{17654}{s^{15}} + \frac{27840}{s^{17}} + \frac{32130}{s^{19}} + \frac{37620}{s^{21}} + \frac{48510}{s^{23}} + \frac{53544}{s^{25}} + \frac{71500}{s^{27}}.$$

Apply inverse Laplace transform to $G(s)$

$$\begin{aligned} L^{-1}\{G(s)\} &= L^{-1}\left\{\frac{154}{s^3} + \frac{1164}{s^5} + \frac{3480}{s^7} + \frac{5824}{s^9} + \frac{9090}{s^{11}} + \frac{14388}{s^{13}} + \frac{17654}{s^{15}} + \frac{27840}{s^{17}} + \frac{32130}{s^{19}} + \frac{37620}{s^{21}} + \frac{48510}{s^{23}} + \frac{53544}{s^{25}} + \frac{71500}{s^{27}}\right\} \\ &= 154 \frac{t^2}{2!} + 1164 \frac{t^4}{4!} + 3480 \frac{t^6}{6!} + 5824 \frac{t^8}{8!} + 9090 \frac{t^{10}}{10!} + 14388 \frac{t^{12}}{12!} + 17654 \frac{t^{14}}{14!} + \\ &27840 \frac{t^{16}}{16!} + 32130 \frac{t^{18}}{18!} + 37620 \frac{t^{20}}{20!} + 48510 \frac{t^{22}}{22!} + 53544 \frac{t^{24}}{24!} + 71500 \frac{t^{26}}{26!} \\ &= 77t^2 + 97 \frac{t^4}{2!} + 116 \frac{t^6}{4!} + 104 \frac{t^8}{6!} + 101 \frac{t^{10}}{8!} + 109 \frac{t^{12}}{10!} + 97 \frac{t^{14}}{12!} + 116 \frac{t^{16}}{14!} + 105 \frac{t^{18}}{16!} + 99 \frac{t^{20}}{18!} + \\ &105 \frac{t^{22}}{20!} + 97 \frac{t^{24}}{22!} + 110 \frac{t^{26}}{24!}. \\ &= t^2 \left[77 + 97 \frac{t^2}{2!} + 116 \frac{t^4}{4!} + 104 \frac{t^6}{6!} + 101 \frac{t^8}{8!} + 109 \frac{t^{10}}{10!} + 97 \frac{t^{12}}{12!} + 116 \frac{t^{14}}{14!} + 105 \frac{t^{16}}{16!} + 99 \frac{t^{18}}{18!} + \right. \\ &\left. 105 \frac{t^{20}}{20!} + 97 \frac{t^{22}}{22!} + 110 \frac{t^{24}}{24!} \right]. \\ &= t^2 G \cosht \\ &= f(t). \end{aligned}$$

Consider the polynomial $f(t)$ coefficients as a finite sequence now.

$$G_0 = 77, G_1 = 97, G_2 = 116, G_3 = 104, G_4 = 101, G_5 = 109, G_6 = 97, G_7 = 116, G_8 = 105, G_9 = 99, G_{10} = 105, G_{11} = 97 \text{ and } G_{11} = 110.$$

Finally, finding the original plain text by converting the numbers in the finite sequence above to alphabets as **Mathematician**.

Illustration 2: Further we extend our findings by double encryption and decryption.

Method of encryption

Let the message, M , to be sent be convert it to ASCII value. Let the length of the message be n .

The plain text message is arranged as a finite sequence of numbers.

Let the message be BELIEVE. Here, $n = 7$. Based on the above steps ASCII values of the message are $B = 66, E = 69, L = 76, I = 73, E = 69, V = 86, E = 69$.

Therefore, plain text finite sequence coefficients are given as $G_0 = 66, G_1 = 69, G_2 = 76, G_3 = 73, G_4 = 69, G_5 = 86, G_6 = 69$.

Write these numbers as the coefficients in $t e^t$

Let $f(t) = \sum_{i=0}^6 G_i t^i e^t$

$$f(t) = G_0 t + G_1 t^2 + G_2 \frac{t^3}{2!} + G_3 \frac{t^4}{3!} + G_4 \frac{t^5}{4!} + G_5 \frac{t^6}{5!} + G_6 \frac{t^7}{6!}$$

$$f(t) = 66 t + 69 t^2 + 76 \frac{t^3}{2!} + 73 \frac{t^4}{3!} + 69 \frac{t^5}{4!} + 86 \frac{t^6}{5!} + 69 \frac{t^7}{6!}$$

Apply Elzaki transform

$$\begin{aligned} E\{f(t)\} &= E\left\{66 t + 69 t^2 + 76 \frac{t^3}{2!} + 73 \frac{t^4}{3!} + 69 \frac{t^5}{4!} + 86 \frac{t^6}{5!} + 69 \frac{t^7}{6!}\right\}. \\ &= 66 u^3 + 69 (2!) \frac{u^4}{1!} + 76 (3!) \frac{u^5}{2!} + 73 (4!) \frac{u^6}{3!} + 69 (5!) \frac{u^7}{4!} + 86 (6!) \frac{u^8}{5!} + 69 (7!) \frac{u^9}{6!}. \\ &= 66 u^3 + 138 u^4 + 228 u^5 + 292 u^6 + 345 u^7 + 516 u^8 + 483 u^9. \end{aligned}$$

Now, find r_i such that $r_i \equiv f_i \pmod{255}$

Here, $r_0 = 66 ; r_1 = 138 ; r_2 = 228 ; r_3 = 3; r_4 = 90 ; r_5 = 6 ; r_6 = 228$.

Treating 'u' as a variable, apply Aboodh transform

$$\begin{aligned} A[E\{f(t)\}] &= A[66 u^3 + 138 u^4 + 228 u^5 + 292 u^6 + 345 u^7 + 516 u^8 + 483 u^9]. \\ &= 66 \frac{3!}{v^5} + 138 \frac{4!}{v^6} + 228 \frac{5!}{v^7} + 292 \frac{6!}{v^8} + 345 \frac{7!}{v^9} + 516 \frac{8!}{v^{10}} + 483 \frac{9!}{v^{11}}. \\ &= \frac{396}{v^5} + \frac{3312}{v^6} + \frac{27360}{v^7} + \frac{210240}{v^8} + \frac{1738800}{v^9} + \frac{20805120}{v^{10}} + \frac{175271040}{v^{11}}. \end{aligned}$$

Now, find r'_i such that $r'_i \equiv f'_i \pmod{255}$

Here, $r'_i : 141 , 252 , 75 , 120 , 210 , 180 , 105$

The ASCII values for the above remainders is the encrypted message.

Hence, the sender sends the cipher text which is $\square \ddot{u} K x \ddot{O} \acute{i}$ and the key which are 1, 12, 107, 824, 6818, 81588, 687337.

Method of decryption:

Consider the sender's cipher text and key.

Translate the cipher text to finite sequence of numbers.

$$\begin{aligned} \text{Let } G'_0 &= 396 , G'_1 = 3312 , G'_2 = 27360 , G'_3 = 210240 , G'_4 = 1738800 , \\ G'_5 &= 20805120 , G'_6 = 175271040 \end{aligned}$$

Using the given key k_i and assuming $f'_i = 255 r'_i + G'_i$

$$\text{Now consider } G(s) = \sum_{n=0}^6 \frac{f'_n}{s^{2n+3}}$$

$$G(s) = A[E\{f(t)\}] = \frac{396}{v^5} + \frac{3312}{v^6} + \frac{27360}{v^7} + \frac{210240}{v^8} + \frac{1738800}{v^9} + \frac{20805120}{v^{10}} + \frac{175271040}{v^{11}}$$

1. For the above polynomial sequence apply inverse Aboodh transform

$$\begin{aligned}
& A^{-1} \left[\frac{396}{v^5} + \frac{3312}{v^6} + \frac{27360}{v^7} + \frac{210240}{v^8} + \frac{1738800}{v^9} + \frac{20805120}{v^{10}} + \frac{175271040}{v^{11}} \right] \\
&= 396 \frac{u^3}{3!} + 3312 \frac{u^4}{4!} + 27360 \frac{u^5}{5!} + 210240 \frac{u^6}{6!} + 1738800 \frac{u^7}{7!} + 20805120 \frac{u^8}{8!} + \\
& 175271040 \frac{u^9}{9!} \\
&= 66 u^3 + 138 u^4 + 228 u^5 + 292 u^6 + 345 u^7 + 516 u^8 + 483 u^9.
\end{aligned}$$

Now apply inverse Elzaki transform

$$\begin{aligned}
& E^{-1}[66 u^3 + 138 u^4 + 228 u^5 + 292 u^6 + 345 u^7 + 516 u^8 + 483 u^9] . \\
&= 66 \frac{t}{1!} + 138 \frac{t^2}{2!} + 228 \frac{t^3}{3!} + 292 \frac{t^4}{4!} + 345 \frac{t^5}{5!} + 516 \frac{t^6}{6!} + 483 \frac{t^7}{7!} \\
&= 66 t + 69 \frac{t^2}{1!} + 76 \frac{t^3}{2!} + 73 \frac{t^4}{3!} + 69 \frac{t^5}{4!} + 86 \frac{t^6}{5!} + 69 \frac{t^7}{6!} \\
&= f(t)
\end{aligned}$$

Hence, we have the coefficients of the above as $G_0 = 66, G_1 = 69, G_2 = 76, G_3 = 73, G_4 = 69, G_5 = 86, G_6 = 69$.

Finally, on converting the coefficients to ASCII values we get encrypted text as **BELIEVE**.

5. Discussion

In the current paper, we present a novel method that encrypts data using Laplace transform and decrypts it using the private key, which is under modulo255. Further more, to safeguard the information in the second illustration we used two stages of encryption and two stages of decryption. The two integral transforms are used to increase the message's level of security during decryption by selecting a new function and altering the other elements.

References

- [1] D.M.K. Kiran, M.V. R. Kameshwari, K. Sujatha, K.R.K. Sastry, B. Ramu Naidu, Data Encryption to Decryption by using Laplace Transform, International Journal of Innovative Technology an Exploring Engineering (IJITEE)ISSN:2278-3075, Volume-9 Issue -6, April 2020.
- [2] Jayanthi CH and Srinivas V, Mathematical Modelling for Cryptography using Laplace Transform, International Journal of Mathematics Trends and Technology, Vol.65, Issue.2, (2019).
- [3] Swati Dhingra, Archana A. Savalgi, Swati Jain, Laplace Transformation base Cryptographic Technique in Network Security, International Journal of Computer Applications (0975-8887), Vol 136-No.7, Feb 2016.
- [4] A.P. Hiwarekar, Applications of Laplace transforms for cryptographic scheme, proceeding of the world congress on engineering 2013 Vol I, WCE 2013, July3-5, 2013, London, U.K.
- [5] G. Naga Lakshmi, B. Ravi Kumar and A. Chandra Sekhar, A cryptographic scheme of Laplace transforms, International Journal of Mathematics Archive- 2, 2515-2519, (2011).
- [6] Jadhav Shaila Shivaji. and Hiwarekar A.P., Cryptographic Method Based on Laplace -Elzaki Transform, Journal of the Maharaja Sayajirao University of Baroda, ISSN:0025-0422, Vol-55, No. I (VIII), pp187-191, (2021).
- [7] Bhuvanewari K. and Bhuvanewari R., Application of Tarig transform in Cryptography, International Journal of creative research thoughts, Vol. 8, Issue 6 pp.1878-1880, (June2020).

- [8] Hiwarekar A.P., A new method of cryptography using Laplace transform, International Journal of Mathematical Archive, 3(3), pp 1193-1197, (2012).
- [9] Hiwarekar A.P., Application of Laplace transform for Cryptography, International Journal of Engineering & Science Research, Vol-5, Issue-4, pp 129-135, (April 2015).
- [10] Tarig. M. Elzaki., 2011, "The New Integral Transform Elzaki Transform ", Global Journal of Pure and Applied Mathematics ,7(1), pp57-64.
- [11] Tarig M Elzaki, Salil M Elzaki, Elnour EA. On the new integral transform Elzaki transform fundamental properties investigations and applications, Global Journal of Mathematical Sciences: Theory and Practical. 2012;4(1):1-13.